



HAL
open science

Internal ocean dynamics contribution to North Atlantic interdecadal variability strengthened by ocean-atmosphere thermal coupling

Olivier Arzel, Thierry Huck, Antoine Hochet, Alexandre Mussa

► To cite this version:

Olivier Arzel, Thierry Huck, Antoine Hochet, Alexandre Mussa. Internal ocean dynamics contribution to North Atlantic interdecadal variability strengthened by ocean-atmosphere thermal coupling. *Journal of Climate*, 2022, pp.1-43. 10.1175/JCLI-D-22-0191.1 . hal-03860612

HAL Id: hal-03860612

<https://hal.science/hal-03860612v1>

Submitted on 21 Nov 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

1 **Internal ocean dynamics contribution to North Atlantic interdecadal**
2 **variability strengthened by ocean-atmosphere thermal coupling**

3 Olivier Arzel,^a Thierry Huck,^a Antoine Hochet,^a and Alexandre Mussa,^b

4 ^a*Laboratoire d'Océanographie Physique et Spatiale, Univ. Brest, CNRS, IRD, IFREMER, Brest,*
5 *France*

6 ^b*Laboratoire Littoral, Environnement et Sociétés, La Rochelle Université*

7 *Corresponding author: Olivier Arzel, oarzel@univ-brest.fr*

8 ABSTRACT: Identifying the primary drivers of North Atlantic interdecadal climate variability is
9 crucial for improving climatic prediction over the coming decades. Here the effect of thermal cou-
10 pling on the leading energy sources of the interdecadal variability of the ocean-atmosphere system
11 is examined by means of a stochastically-forced idealized coupled model. The effect of coupling
12 is quantified from a comparison of the buoyancy variance budget of coupled and uncoupled model
13 configurations. The simplicity of the model allows us to contrast the effect of coupling between a
14 super-critical regime where the deterministic ocean dynamics drive the variability and a damped
15 regime where noise forcing is central to its existence. The results show that changes in surface
16 buoyancy fluxes act as a sink of temperature variance in the super-critical regime, and only be-
17 come a source in the strongly damped regime. By contrast, internal ocean dynamics associated
18 with the interaction of transient buoyancy fluxes with mean buoyancy gradients always act as a
19 source of interdecadal variability. In addition to the reduced thermal damping effect in coupled
20 integrations, thermal coupling with the atmosphere is shown to significantly increase the role of
21 internal ocean dynamics in the variability, in particular in the regime where interdecadal modes
22 are damped. Only for oceanic background states in the strongly damped regime do changes in
23 surface buoyancy fluxes play a leading role in the upper ocean variability. A stochastically-forced
24 coupled box model is proposed that captures the basic effect of thermal coupling on atmospheric
25 and oceanic energy sources of variability.

26 SIGNIFICANCE STATEMENT: The purpose of this study is to better understand the impact of
27 ocean-atmosphere thermal coupling on the leading energy sources of Atlantic interdecadal vari-
28 ability. Increasing our understanding of the physical mechanisms driving climate variability at
29 interdecadal timescales is important to improve climate prediction. We show that the effect of
30 ocean-atmosphere thermal coupling is to substantially increase the role of internal ocean dynam-
31 ics in the low-frequency variability of the upper ocean heat content and sea surface temperature.
32 Atmospheric stochastic forcing only becomes the primary driver of the oceanic temperature vari-
33 ability in the large dissipative limit, when internal ocean modes are strongly damped.

34 1. Introduction

35 Multidecadal variability of North Atlantic sea surface temperatures has been observed in both
36 the instrumental record (Delworth et al. 2007; Deser et al. 2010; Tung and Zhou 2013) and a num-
37 ber of climatic proxy reconstructions (Mann et al. 1998; Gray et al. 2004; Chylek et al. 2011).
38 Atlantic Multidecadal Variability (AMV, Sutton et al. 2018) is characterized by alternating bas-
39 inwide decadal-scale warming and cooling periods with maximum variance at subpolar latitudes.
40 Clement et al. (2015) suggested that the AMV could result from the thermodynamic response to
41 atmospheric stochastic forcing and that ocean circulation variations are unimportant. This mecha-
42 nism seems however incompatible with the paleorecord that exhibits statistically significant multi-
43 decadal periods above a red-noise background (e.g. Gray et al. 2004; Wang et al. 2017). O'Reilly
44 et al. (2016) showed instead that that ocean circulation variations are key to reproduce the sign
45 of the observed correlation between sea surface temperatures (SST) and the AMV. Zhang et al.
46 (2019) reviewed the observational and modelling evidence for the origins of the AMV and con-
47 cluded that internal ocean dynamics is a key driver. The existence of such internal variability was
48 recently questioned by Mann et al. (2021) who found instead a prominent role of volcanic forcing
49 and anthropogenic aerosols. Which of internal ocean dynamics, atmospheric stochastic forcing
50 or external radiative forcing lies at the very origin of the observed AMV remains therefore much
51 debated. The instrumental record is unfortunately too short compared to the timescale of the vari-
52 ability and too sparse in terms of spatial coverage to obtain a definitive answer to this question and
53 climate models have yet to reach a consensus (Zhang et al. 2019). The present study will solely
54 focus on the variability generated internally in the ocean-atmosphere system. Even in this con-

55 text numerical models are inconclusive, in part because they show that the relative contributions
56 of the ocean and atmosphere to the variability are a strong function of poorly constrained critical
57 parameters, such as turbulent eddy diffusivities associated with unresolved scales (Arzel and Huck
58 2020). In addition ocean-atmosphere interactions certainly have an impact on the amplitude of
59 the variability, at least at low-frequencies (Barsugli and Battisti 1998), but the net effect of those
60 interactions on the leading energy sources of the variability have yet to be found. This study will
61 thus focus on the effect of thermal coupling on the primary drivers of the interdecadal variability
62 of the extratropical ocean-atmosphere system.

63 The effect of ocean-atmosphere coupling on the decadal or longer variability is traditionnaly
64 evaluated by comparing fully coupled ocean-atmosphere simulations to ocean-only experiments
65 where the surface heat, momentum and freshwater fluxes are diagnosed from the coupled run. Un-
66 der such flux boundary conditions atmospheric damping on sea surface temperature (SST) anoma-
67 lies is reduced to zero and the amplitude of oscillations is maximized. This approach has been used
68 in many modelling studies and across a large diversity of model configurations and complexities.
69 Delworth and Greatbatch (2000) and Gastineau et al. (2018) performed such experiments with
70 comprehensive coupled general circulation models (GCMs) to show that coupling with the atmo-
71 sphere was not essential to the variability, and that atmospheric stochastic forcing associated with
72 mid-latitudes disturbances was necessary to sustain interdecadal oscillations against dissipation.
73 One drawback of such an approach is that the surface fluxes used to drive ocean-only integrations
74 contain information about the response of the atmosphere to SST changes. The comparison of
75 those ocean-only runs with coupled integrations does not therefore allow to isolate the effect of
76 the feedback of the atmosphere on the SST anomalies, but instead to assess the entire effect of
77 surface heat flux damping on SST anomalies. Another approach consists in forcing the ocean with
78 the time history of surface air temperatures (SAT) and winds diagnosed from an atmospheric-only
79 integration itself forced by climatological SSTs from the coupled simulation (Barsugli and Battisti
80 1998). These “uncoupled” ocean-only experiments therefore lack the feedback of the atmosphere
81 onto the SST anomalies. A direct comparison with the coupled runs thus allows to unambiguously
82 isolate the basic impact of this feedback (i.e. thermal coupling) on the variability of the coupled
83 system. The absence of atmospheric feedback in uncoupled experiments makes the phasing of SST
84 and SAT anomalies less coherent, leading to a smaller persistence of SST anomalies in uncoupled

85 than in coupled integrations. The basic effect of thermal coupling is therefore to enhance the
86 temperature variance in both the ocean and atmosphere, and more specifically at low-frequencies
87 (Barsugli and Battisti 1998). Farneti and Vallis (2009) performed uncoupled atmosphere-only ex-
88 periments in the context of interdecadal climate variability and reached the same conclusions. The
89 spectral peak at interdecadal periods in the SAT was shown to be only present when prescribed,
90 time-varying, SST from the coupled run were used, indicating an influence of either an oceanic
91 interdecadal mode or some form of coupled interaction. Wu and Liu (2005) run an uncoupled
92 experiment with their realistic coupled model to show the critical role of coupling in sustaining
93 North Atlantic Ocean decadal variability.

94 Arzel and Huck (2020) used a realistic ocean general circulation model forced by combination of
95 steady surface buoyancy and momentum fluxes and North Atlantic Oscillation-related stochastic
96 forcing to show that internal ocean dynamics plays a leading role in the growth of SST variance in
97 the super-critical regime (i.e. the regime where the multidecadal variability develops without any
98 noise forcing, at relatively low diffusivities), but that the atmospheric stochastic forcing represents
99 the major energy source for the variability in the damped regime (i.e. the regime where the vari-
100 ability requires some atmospheric noise to emerge, at relatively high diffusivities). Therefore the
101 leading energy source of the variability is a strong function of critical parameters (such as eddy dif-
102 fusivities) and background climate conditions, at least in the forced case. How ocean-atmosphere
103 thermal coupling modifies this behaviour remains to be determined. The present study will assess
104 the impact of thermal coupling on the leading energy sources of the variability and is therefore a
105 natural extension of Arzel and Huck (2020) who used an ocean-only configuration. The aim of the
106 present study is thus to progress in the understanding of the physical processes driving Atlantic
107 climate variability at interdecadal timescales. The main purpose is to quantify the impact of ocean-
108 atmosphere thermal coupling on the *drivers* of temperature variance at interdecadal timescales. As
109 such the present study is also a natural extension of Barsugli and Battisti (1998) who focused their
110 analysis on the impact of coupling on the temperature variance of the ocean-atmosphere system.
111 A joint objective is to determine how thermal coupling impact the energy sources of the variability
112 in both the super-critical and damped regimes. Different variability regimes will be obtained by
113 systematically varying the magnitude of the turbulent eddy diffusivity K which has been shown
114 to be one of the most critical parameter to the interdecadal variability of the overturning circula-

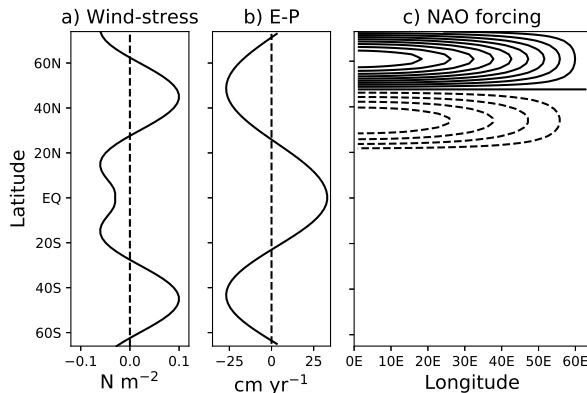
tion (Huck et al. 1999a). For each value of K the effect of thermal coupling is inferred from a comparison of coupled, uncoupled and forced integrations. Given the relatively large number of millennial-scale experiments required to achieve this task a computationally efficient coupled model is required. Following the views Held (2005) we believe that the use of a model hierarchy such as the one proposed here is necessary to help gain understanding of the mechanisms driving climate variability at interdecadal timescales. The model comprises a planetary geostrophic oceanic component coupled to a stochastically-forced energy balance model. The geometry is idealized with a two-hemisphere sector of sphere and a southern periodic channel. The model captures the large-scale features of the Atlantic circulation and for some parameter range exhibit interdecadal oscillations under deterministic forcing conditions. The cause of the variability was shown to be a large-scale baroclinic instability of the mean flow driving interdecadal oscillations with the oscillation period set by the zonal transit time of long baroclinic planetary waves (Colin de Verdière and Huck 1999; te Raa and Dijkstra 2002). This mode of variability has been shown to persist in realistic configurations of ocean-only (Sévellec and Fedorov 2013; Arzel et al. 2018) and coupled models (Ortega et al. 2015; Muir and Fedorov 2016; Gastineau et al. 2018).

The outline of this paper is as follows. Section 2 provides a description of the idealized coupled ocean-atmosphere model as well as the experimental procedure used to quantify the effect of ocean-atmosphere thermal coupling. The main characteristics of the variability are presented in section 3, and its sensitivity to horizontal turbulent diffusivity is presented in section 4 for the coupled, uncoupled and forced cases. Section 5 quantifies the leading energy sources of the interdecadal variability and their sensitivity to K using an approach based on the buoyancy variance budget. The model hierarchy is finally extended to a coupled stochastically-forced ocean-atmosphere dynamical system in section 6 that reproduces with very high fidelity the results obtained with the 3D model. The results are then summarized and discussed in section 7.

2. Model and experiments

a. The coupled model

The 3D ocean model is based on the planetary geostrophic equations, valid for time scales much larger than the inertial period and spatial scales much larger than the internal Rossby radius of deformation (Salmon 1986; Colin de Verdière 1988). The domain is a flat-bottom sector of a



157 FIG. 1. (a) Zonal (solid) and meridional (dashed) surface wind-stress. (b) Imposed freshwater forcing (evap-
 158 oration minus precipitation), (c) spatial pattern of the stochastic forcing (c.i. $10 W m^{-2}$) entering the atmospheric
 159 thermodynamic balance. The amplitude in the subpolar gyre ($100 W m^{-2}$) is twice that of the subtropical gyre
 160 ($50 W m^{-2}$) similar to Herbaut et al. (2002). The sign change in the stochastic forcing roughly occurs at the
 161 intergyre boundary at $47^{\circ}N$.

144 sphere, with dimensions appropriate for the Atlantic: it extends from $66^{\circ}S$ to $74^{\circ}N$ and is 64°
 145 wide in zonal extent. A zonal subpolar channel analogous to the Drake Passage in the Southern
 146 Ocean is represented by applying cyclic boundary conditions between $66^{\circ}S$ and $52^{\circ}S$ throughout
 147 the entire water column. Lateral boundaries are solid vertical walls where no-slip and no-flux
 148 boundary conditions are applied. The horizontal resolution is 2° and there are 15 layers whose
 149 vertical thickness increases unevenly from 50 m at the surface to 550 m at the bottom (4500
 150 m depth). Static instability is removed by instantaneously restoring vertical density profiles to
 151 neutrality. The vertical mixing is fixed at $10^{-4} m^2 s^{-1}$, in agreement with values inferred from the
 152 large-scale abyssal stratification (Munk and Wunsch 1998). The horizontal Laplacian viscosity is
 153 fixed at $10^5 m^2 s^{-1}$. A linear equation of state is used with constant thermal expansion and haline
 154 contraction coefficients. The surface wind-stress forcing is distributed over the surface layer (50 m
 155 depth), it is purely zonal and follows the analytical profile function of latitude proposed by Weaver
 156 and Sarachik (1990) (Fig. 1a).

162 The atmospheric model is a standard, two-dimensional, dry energy balance model. It closely
 163 follows the scheme developed by Fanning and Weaver (1996). The freshwater forcing is imposed
 164 and follows the latitudinal profile shown in Fig. 1b. The atmospheric eddy temperature diffusivity
 165 K_a is uniform and fixed to $1.3 \times 10^6 m^2 s^{-1}$. The atmosphere-ocean heat exchange coefficient λ falls

166 within the range of observed values and is fixed to $30 \text{ W m}^{-2} \text{ K}^{-1}$ in all experiments. Both the
 167 incoming solar radiation S at the top of the atmosphere and the planetary albedo α_p follow annual
 168 mean latitudinal profiles given by North et al. (1981) and Graves et al. (1993), respectively. In the
 169 absence of atmospheric dynamics, stochastic forcing is introduced in the atmospheric temperature
 170 equation as

$$Q_N(x, y, t) = \sigma_N Q_{NAO}(x, y) \zeta(t), \quad (1)$$

171 This term parameterizes the stochastic forcing associated with the divergence of eddy temperature
 172 fluxes which are typically enhanced at mid-latitudes. There the observed transient sensible and
 173 latent heat fluxes amount to about $P = 3.5 \text{ PW}$ (Peixoto and Oort 1992). Using an eddy length
 174 scale L_{eddy} of 1000 km and a zonal length scale L_x of 25,000 km for the length of a latitude
 175 circle at mid-latitudes, we obtain $\sigma_N = P/L_x L_{eddy} = O(100) \text{ W m}^{-2}$ for the amplitude of the eddy
 176 forcing. We then specify $\sigma_N = 100 \text{ W m}^{-2}$ in all stochastically-forced experiments. The pattern
 177 Q_{NAO} is similar to that used by Herbaut et al. (2002). It mimicks the observed North Atlantic
 178 Oscillation pattern with a dipolar structure centered at mid-latitudes and with maximum values
 179 in the west of the basin. The random discrete timeseries $\zeta(t)$ has been built from a first order
 180 auto-regressive process with a decorrelation timescale of 10 days. This timescale corresponds to
 181 estimates inferred by Feldstein (2000) using daily means from the NCEP-NCAR reanalysis. To
 182 assess the effect of noise coherence additional experiments using a white noise forcing and a longer
 183 temporal coherence (30 days) have been conducted. The noise forcing has a sampling frequency
 184 (0.05 days) corresponding to the oceanic timestep of the model. The variance of $\zeta(t)$ is 1. Note
 185 finally that the stochastic timeseries are strictly identical in all stochastically-forced integrations.

186 *b. Experimental design*

187 Three different model configurations (coupled, uncoupled, forced) are used to assess the impact
 188 of ocean-atmosphere thermal coupling on the variability. For each of those three configurations
 189 the model is run with and without atmospheric stochastic forcing. The latter case is said to be
 190 deterministic. This procedure is repeated for a wide range of oceanic horizontal eddy diffusivities
 191 K between 500 and 2600 m^2s^{-1} . Those values of K span the observed range of eddy diffusivities
 192 but do not attempt to capture the strong spatial variations (Abernathey and Marshall 2013). This
 193 approach allows us to explore the effect of ocean-atmosphere coupling in two different variability

194 regimes, namely a super-critical regime where the variability spontaneously develops without
 195 atmospheric stochastic forcing and a damped regime where the variability requires some noise to
 196 emerge. Deterministic runs are integrated for 5000 years and are initialized from a resting ocean
 197 with uniform temperature (4°C) and salinity (35 psu). Stochastic runs are integrated for 1000
 198 years and are initialized from the end state of deterministic runs.

199

200 (1) **Coupled.** The model is integrated in its standard coupled mode (denoted by superscript C)
 201 where the atmospheric temperature T_a and the surface buoyancy (heat and freshwater) flux B_O
 202 toward the ocean are given by

$$C_a dT_a^C / dt = F(T_a^C, T_o^C) + Q_N, \quad (2)$$

$$B_O^C = G(T_a^C, T_o^C), \quad (3)$$

203 where C_a is the atmospheric heat capacity and T_o the sea surface temperature. The operators F
 204 and G include both turbulent and radiative fluxes,

$$F(T_a, T_o) = \lambda(T_o - T_a) + \epsilon_o \sigma T_o^4 - \epsilon_a \sigma T_a^4 - \epsilon_p \sigma T_a^4 + S a_s (1 - \alpha_p) + K_a \nabla^2 T_a, \quad (4)$$

$$B_O(T_a, T_o) = \lambda(T_a - T_o) - \epsilon_o \sigma T_o^4 + \epsilon_a \sigma T_a^4 + S(1 - a_s)(1 - \alpha_p) \quad (5)$$

205 where σ is the Stefan-Boltzmann constant, $\epsilon_o = 0.96$ is the longwave emissivity of the surface,
 206 $\epsilon_a = 0.85$ is the longwave emissivity of the atmosphere, $\epsilon_p = 0.63$ is the planetary emissivity to
 207 space, $a_s = 0.3$ is the shortwave absorptivity. The terms on the right hand side of (4) are, from
 208 first to last, the sensible heat exchange with the surface, the upward longwave emission from the
 209 surface, the downward longwave emission from the atmosphere, the outgoing longwave radiation
 210 to space, the absorbed solar radiation within the atmosphere and the diffusive heat transport. The
 211 last term on the right hand side of (5) is the absorbed solar flux at the surface.

212

213 (2) **Uncoupled.** (a) The atmosphere is first run with the fixed 100 years averaged SST distribution
 214 diagnosed from deterministic coupled integrations. This SST field is defined here as the climato-
 215 logical SST field T_o^{clim} . (b) The ocean model is then forced by the surface fluxes computed using
 216 the time history of atmospheric temperatures from phase (a) and the actual SST field from the

217 uncoupled integration. Those two phases are run synchronously with the same coupling frequency
 218 as the coupled model (0.05 days). Hence

$$C_a dT_a^U / dt = F(T_a^U, T_o^{clim}) + Q_N, \quad (6)$$

$$B_O^U = G(T_a^U, T_o^U), \quad (7)$$

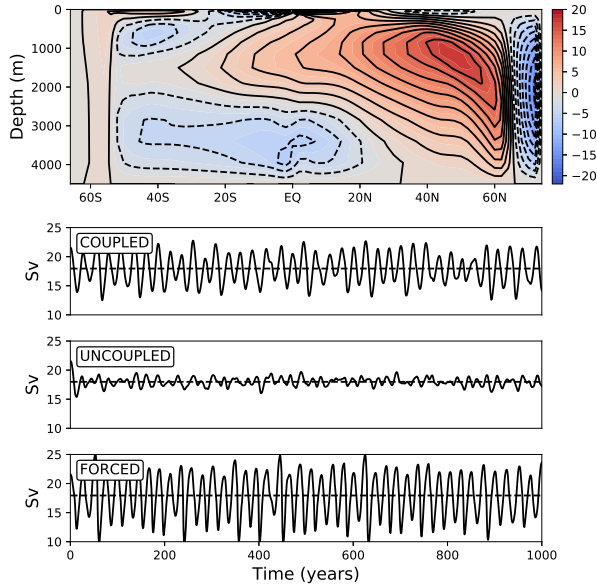
219 where the superscript U stands for uncoupled. In those uncoupled runs the atmospheric temper-
 220 ature can only change in response to the stochastic forcing, all other forcings including the SST
 221 field being kept constant in time. The atmosphere therefore does not see the SST variations that
 222 can occur in response to either atmospheric stochastic forcing (if any, embedded in T_a^U) or changes
 223 due to the existence of an intrinsic interdecadal ocean mode. In the coupled integrations by
 224 contrast the atmosphere can vary in response to both atmospheric stochastic forcing and changing
 225 SSTs. The comparison of the coupled and uncoupled runs therefore allows to unambiguously
 226 isolate the impact of the atmospheric feedback on SST anomalies.

227

228 (3) **Forced.** The main motivation behind these ocean-only forced experiments is to determine
 229 whether the variability obtained in the coupled model can be interpreted as a manifestation of a
 230 self-sustained ocean mode or as an excitation of a damped ocean mode. The procedure here is
 231 very similar to that used for building uncoupled runs except for the previous phase (b) where the
 232 surface heat flux to the ocean B_O^F is now computed using the climatological SST T_o^{clim} rather than
 233 the actual SST obtained in those ocean-only integrations,

$$B_O^F = G(T_a^U, T_o^{clim}), \quad (8)$$

234 where the superscript F stands for forced. Because surface heat fluxes are completely decorrelated
 235 from SST anomalies in those forced experiments, surface heat flux damping on SST anomalies is
 236 reduced to zero and the amplitude of the variability is maximized. Forced experiments therefore
 237 lack thermal damping on SST anomalies. Uncoupled experiments by contrast lack thermal cou-
 238 pling and have increased surface heat flux damping on SST anomalies and subsequently a much
 239 weaker variability compared to either the forced or coupled runs. As such the forced runs have a



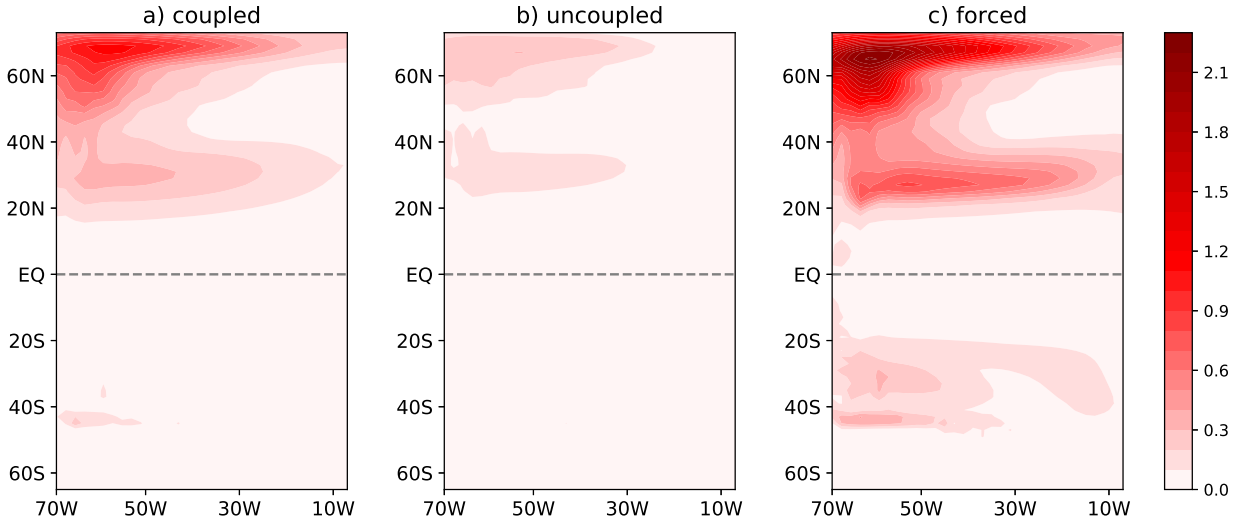
246 FIG. 2. (top) Time-mean meridional overturning circulation (MOC) in the stochastic coupled integration for
 247 $K = 1000 \text{ m}^2\text{s}^{-1}$. (bottom) Time-series of the MOC index in the stochastic experiments. The MOC index is
 248 defined as the maximum of the meridional overturning streamfunction north of 30°N and below 850m. The
 249 stochastic experiments are run for 1000 years and start from the end state of the corresponding deterministic
 250 solutions. These later solutions are obtained from a 5000 years long integration which was necessary to reach
 251 statistical equilibrium.

240 thermal damping on SST anomalies that is at the exact opposite of what is expected when thermal
 241 coupling with the atmosphere is suppressed.

242 3. Interdecadal variability

243 In what follows we focus on the stochastic integrations and contrast the variability patterns
 244 obtained for the coupled, uncoupled and forced systems. The comparison is made for the canonical
 245 horizontal eddy diffusivity value $K = 1000 \text{ m}^2\text{s}^{-1}$.

256 Figure 2 shows the time-mean meridional overturning streamfunction in the stochastic coupled
 257 run, as well as the timeseries of the MOC index over a 1000 years period for the coupled, un-
 258 coupled and forced systems. The MOC index is defined as the maximum value of the meridional
 259 overturning streamfunction north of 30°N and below 850m. Despite the relative simplicity of the
 model, in terms of both geometry and physics, the observed North Atlantic meridional overturning



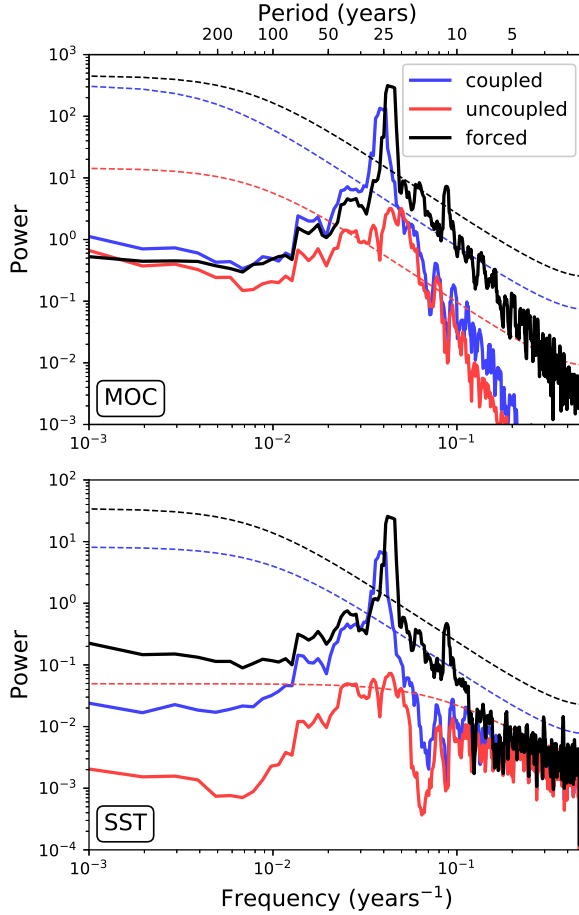
252 FIG. 3. SST standard deviations diagnosed from the annual mean output of SST anomalies over a 100 years
 253 period for the stochastic case with $K = 1000 \text{ m}^2\text{s}^{-1}$ for the coupled (left), uncoupled (middle) and forced (right)
 254 configurations.

260 circulation is well reproduced with a thermally direct cell intensified in the Northern Hemisphere
 261 with a peak value close to that observed (18 Sv, Colin de Verdière et al. 2019), and a weaker (5
 262 Sv) thermally-indirect cell at depth reminiscent of the Antarctic Bottom Cell. Quasi-regular oscil-
 263 lations with interdecadal periods (~ 25 years) are found in all cases, with peak-to-peak amplitudes
 264 reaching 38% (56%) of the mean transport in the coupled (forced) case, but only 12% in the un-
 265 coupled system. Thermal coupling with the atmosphere, as inferred from the comparison between
 266 the coupled and uncoupled systems, therefore acts as a strong amplifier of interdecadal oscilla-
 267 tions in agreement with Farneti and Vallis (2009) and Wu and Liu (2005). As originally stated
 268 by Barsugli and Battisti (1998), coupling with the atmosphere makes SST and SAT anomalies to
 269 vary more in phase in the coupled case, thereby reducing the damping on SST anomalies com-
 270 pared to the uncoupled case. In the forced case the damping of SST anomalies is at its minimum
 271 (but non-zero because of the presence of mixing processes) leading to much greater variability.
 272 This feature can be seen in the standard deviations of SST anomalies in Fig. 3. For all cases
 273 SST variance is maximum in the northwest corner of the basin and is about three times greater
 274 in the coupled compared to the uncoupled case. There is virtually no variability in the Southern
 275 Hemisphere: the presence of the southern channel induces a relatively weak upper circulation that

276 does not favor the baroclinic instability mechanism (Arzel et al. 2007). The spectral peaks (Fig.
277 4) in both the MOC index and SST anomalies (computed as the weighted average in the region
278 of maximum SST variance, 70-50°E, 46°N-74°N) greatly exceed the red-noise background (but
279 barely for the uncoupled case which just passes the 95% confidence level) indicating that inter-
280 decadal oscillations cannot be simply interpreted as an integration of the atmospheric noise by the
281 oceanic mixed layer. The first and second leading patterns of variability have been estimated in
282 terms of Empirical Orthogonal Functions (EOFs) and are presented in Fig. 5. The patterns in the
283 coupled and forced systems are very similar but with a greater amplitude for the latter. The first
284 EOF features a monopole intensified in the northwestern corner of the basin, whereas the second
285 EOF features a monopole of opposite sign centered in the subtropics and with a much weaker
286 amplitude. These two EOFs vary in quadrature (not shown) with the first EOF leading the second
287 one by about 6 years, thereby covering a full cycle in about 25 years. A lagged regression analysis
288 between SST anomalies and the MOC index highlights the emergence and growth of perturbations
289 in the eastward extension of the western boundary current in the northwestern corner of the basin
290 (not shown). The perturbations then propagate westward toward the western boundary where they
291 subduct and quickly dissipate (not shown). This spatio-temporal organization of the variability is
292 similar to several previous idealized ocean-only model studies using fixed flux boundary condi-
293 tions without noise forcing (Greatbatch and Zhang 1995; Colin de Verdière and Huck 1999; te Raa
294 and Dijkstra 2002). In the uncoupled case, the NAO signature along with its dipolar structure is
295 clearly evident in the first EOF of SST anomalies (Fig. 5). Therefore thermal coupling does not
296 only increase the amplitude of the mode but also significantly alter the leading pattern of the vari-
297 ability, at least for this specific value of $K = 1000 \text{ m}^2\text{s}^{-1}$. The second EOF of the uncoupled case
298 exhibits strong similarities with the leading EOF of the coupled and forced cases, suggesting an
299 implication of internal ocean dynamics in the uncoupled variability.

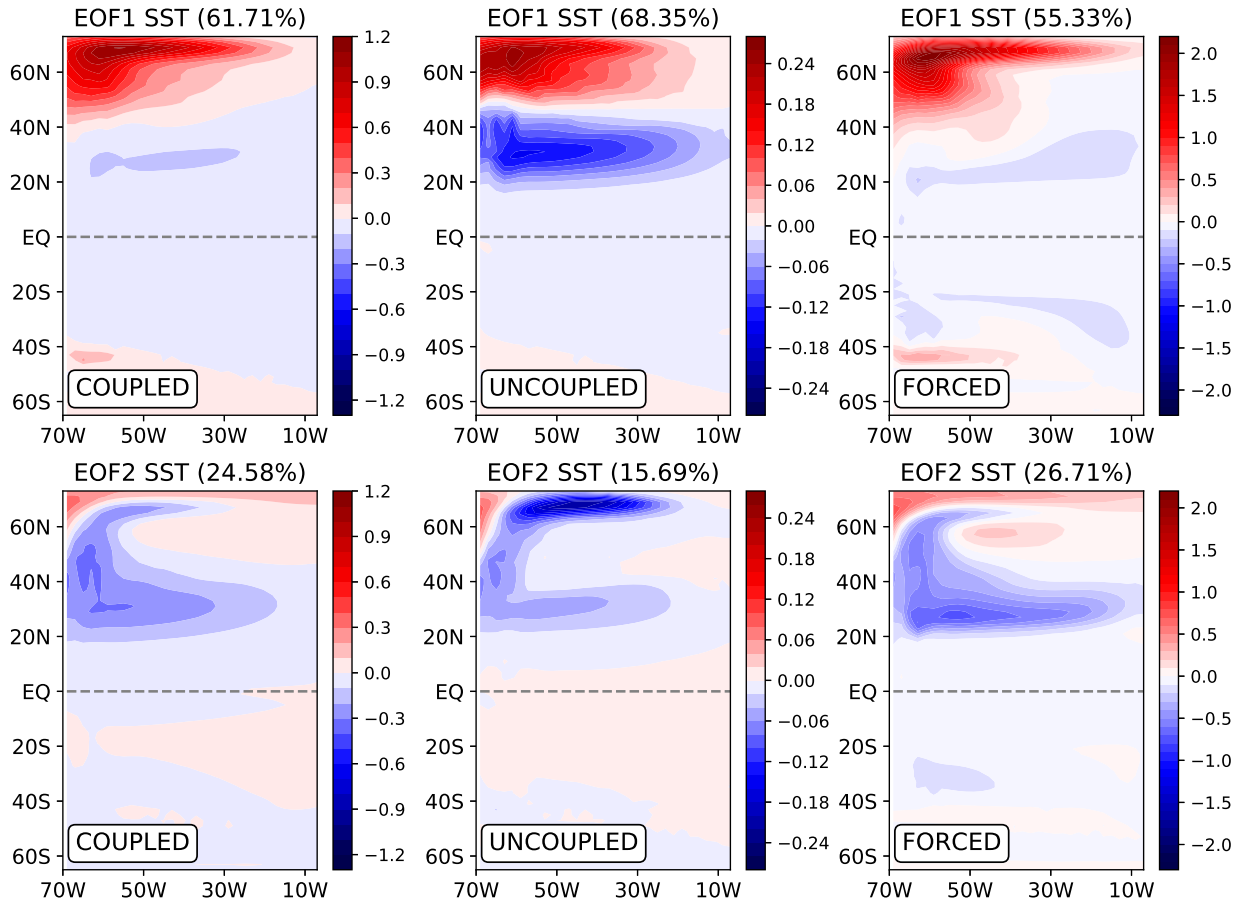
308 **4. Sensitivity to K**

309 Figure 6 presents the sensitivity of some key quantities to the horizontal diffusivity K in the
310 coupled, uncoupled and forced systems, and under both deterministic and stochastic forcing con-
311 ditions. We first see that the strength of the mean overturning increases with K in all cases. This
312 behaviour sharply contrasts with results obtained with models where the eddy-induced diffusivity



300 FIG. 4. Power spectra of (top) the MOC index ($\text{Sv}^2 \text{ years}$) and (bottom) SST anomalies ($^{\circ}\text{C}^2 \text{ years}$) timeseries
 301 averaged in the northwest corner ($70\text{-}50^{\circ}\text{W}$ and north of 46°N) in the stochastic experiments with $K = 1000$
 302 m^2s^{-1} and for the coupled (blue), uncoupled (red) and forced (black) systems. The calculation uses a multi-
 303 taper technique with 3 tapers and is based on 1000 years of annual mean model output. The smooth dashed lines
 304 indicate the red noise 95% confidence level.

313 rather the horizontal diffusivity is varied (Marshall et al. 2017; Arzel and Huck 2020). The reason
 314 for this discrepancy may be caused by the “Veronis effect” whereby horizontal diffusion induces
 315 large diapycnal fluxes once the isopycnals are tilted by coastal upwellings, in particular along the
 316 western boundary (Veronis 1975; Huck et al. 1999b). Quite remarkably the strength of the mean
 317 overturning for a given value of K keeps nearly the same value whatever the configuration used
 318 (coupled, forced, uncoupled) and irrespective of the stochastic forcing being present or not. Hence
 319 rectification of the mean flow by stochastic forcing does not occur in our simulations as opposed



305 FIG. 5. First (top) and second (bottom) EOFs of annual mean SST anomalies along with the explained variance
 306 (%) in the stochastic experiments with $K = 1000 \text{ m}^2\text{s}^{-1}$ and for the coupled (left), uncoupled (middle) and forced
 307 (right) cases.

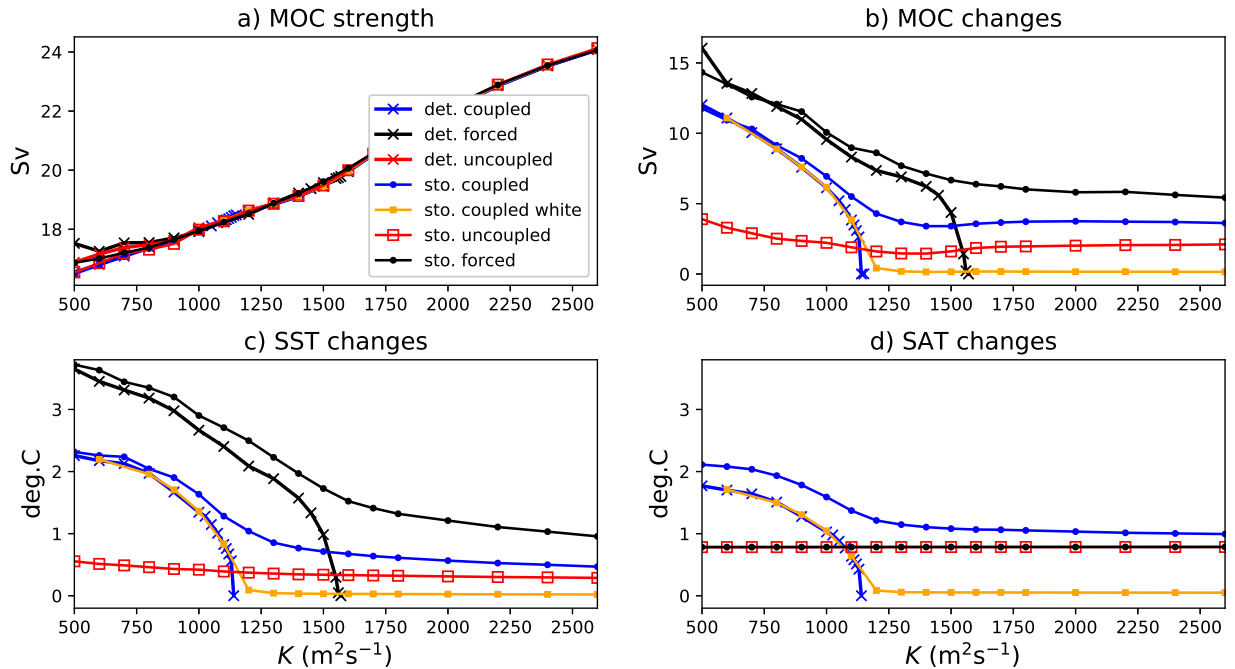
320 to Frankcombe and Dijkstra (2009). This implies that the different variability characteristics as-
 321 sociated with the coupled, uncoupled and forced systems, in particular when stochastic forcing is
 322 present, cannot be ascribed to modifications of the mean circulation.

323 The transition from a steady circulation to an interdecadal oscillatory behaviour was shown to
 324 occur through a Hopf bifurcation as the horizontal diffusivity (Colin de Verdière and Huck 1999)
 325 or ocean-atmosphere heat exchange coefficient (Chen and Ghil 1996) is decreased. Here Hopf
 326 bifurcations under deterministic conditions are successively found at $K = 210, 1140$ and 1560
 327 m^2s^{-1} for the uncoupled, coupled and forced cases respectively (note that only the last two bifur-
 328 cations are shown in Fig. 6). This sequence of bifurcations is consistent with the magnitude of

329 atmospheric damping acting on SST anomalies which is at its largest in the uncoupled case and at
330 its lowest in the forced case, as discussed in Section 2b. The suppression of variability for large
331 K is not caused by changes in the circulation, which favors increasingly baroclinically unstable
332 situations as K increases (Fig. 6a), but instead by the direct damping effect of dissipation on per-
333 turbations. When stochastic forcing is present interdecadal variability exists over the full range of
334 diffusivities explored here, but only as long as the noise forcing has some temporal coherence, in
335 agreement with Frankcombe and Dijkstra (2009) and Arzel and Huck (2020). Additional spectral
336 analyses of the MOC and SST (averaged in the region $70 - 50^\circ\text{W}$, $46 - 74^\circ\text{N}$) indices show that the
337 interdecadal variability is always statistically significant in both fields, even for the most diffusive
338 cases. The effect of stochastic forcing on the variability is large in the damped regime (for diffusiv-
339 ity values larger than the critical value at bifurcation) and much weaker in the super-critical regime
340 (for diffusivity values weaker than the critical value at bifurcation). The comparison of coupled
341 and uncoupled configurations of the model clearly shows that thermal coupling between the ocean
342 and atmosphere increases the temperature variance in both fluids, in agreement with Barsugli and
343 Battisti (1998), as well as the amplitude of ocean circulation changes. The novel aspect is that the
344 amplifying effect of thermal coupling on the variability is much stronger in the super-critical than
345 in the damped regime. For SST for instance, thermal coupling typically quadruples the amplitude
346 of the variability in the super-critical regime (from 0.55 to 2.31°C for $K = 500 \text{ m}^2\text{s}^{-1}$) but less than
347 doubles it in the damped regime (from 0.28 to 0.45°C for $K = 2600 \text{ m}^2\text{s}^{-1}$). Let us finally mention
348 that the amplitude of the changes in surface heat flux (diagnosed from a composite analysis identi-
349 cal to that used for SST and SAT in Fig. 6) in the region of maximum SST variance are relatively
350 independent of K and amount to about 25 W m^{-2} (not shown), an amplitude comparable to the
351 observed annual mean changes in turbulent fluxes associated with the NAO (e.g. Fig. 1 in Arzel
352 and Huck 2020).

365 5. Energy sources

366 We now turn to the analysis of the effect of thermal coupling on the leading energy sources
367 of the variability. These energy sources are identified as the major terms driving the growth of
368 buoyancy variance in the ocean against all sources of damping. As such the buoyancy variance
369 budget provides a quantitative estimate of the contribution of the ocean and atmosphere to the



353 FIG. 6. Statistics of key indices as a function of the horizontal eddy diffusivity K under deterministic and
 354 stochastic forcing conditions for the coupled, uncoupled and forced cases. (a) Time-mean MOC index. The
 355 index is defined as the maximum value of the meridional overturning streamfunction north of 30°N and below
 356 850m . (b) Amplitude of MOC variations (Sv). (c) Amplitude of SST changes ($^{\circ}\text{C}$) averaged in the north-
 357 western corner ($70 - 50^{\circ}\text{W}$, $46 - 74^{\circ}\text{N}$), which corresponds to the region of largest temperature changes in all
 358 experiments. (d) Same as (c) but for the SAT ($^{\circ}\text{C}$). The amplitude of the variability in (b-c-d) has been estimated
 359 from a composite analysis of the last 1000 years of each experiment where the max (min) of the timeseries is
 360 computed as the time average of the values greater (smaller) than the mean plus (minus) one standard deviation.
 361 Note that only the changes greater than zero are shown. The uncoupled deterministic system does not have
 362 variability in the range of diffusivity values shown here. When the stochastic forcing is white (orange dotted
 363 line) the amplitude of the variability in the damped regime is greatly reduced compared to a situation where a
 364 noise forcing has a finite temporal coherence.

370 growth of perturbations. Such an approach has been previously and successfully applied to the
 371 interdecadal climate variability problem in either stand-alone ocean models (Colin de Verdière
 372 and Huck 1999; Arzel et al. 2006, 2018) or coupled models (Arzel et al. 2007, 2012; Buckley
 373 et al. 2012; Jamet et al. 2016; Gastineau et al. 2018) with complexities ranging from idealized to
 374 fully coupled and realistic.

375 *a. Method*

376 The linearized buoyancy variance equation is given by

$$\frac{1}{2} \frac{\overline{\partial b'^2}}{\partial t} = -\overline{\mathbf{u}'_h b'} \cdot \nabla_h \bar{b} - \overline{w' b'} \partial_z \bar{b} - \frac{1}{2} \overline{\mathbf{u}} \cdot \nabla \overline{b'^2} + \overline{b' Q'_b} + \overline{b' D'_b}, \quad (9)$$

377 where the overbar denotes a time average and the prime the perturbation. Integrated over the do-
 378 main and weighted appropriately, it provides an equation for the available potential energy in the
 379 quasi-geostrophic approximation (Huang 1998). The third-order term associated with advection
 380 of buoyancy variance by the disturbed flow is between one and three orders of magnitude smaller
 381 than $-\overline{\mathbf{u}'_h b'} \cdot \nabla \bar{b}$ for all values of K (not shown) and has been dropped during the linearization pro-
 382 cedure. The first term on the right hand side is a source of buoyancy variance when transient
 383 buoyancy fluxes $\overline{\mathbf{u}'_h b'}$ are oriented down the mean buoyancy gradient, where \mathbf{u}_h is the horizontal
 384 Eulerian velocity. This configuration is typical of baroclinic instability for which potential energy
 385 is extracted from the mean stratification. This term has been pinpointed as the primary source of
 386 the variability in many ocean-only and coupled models (see references above). Baroclinic insta-
 387 bility is associated with a conversion of potential to kinetic energy of perturbations through the
 388 positive exchange term $\overline{w' b'}$. Under such unstable conditions, the second term in (9) is always
 389 negative (provided $\partial_z \bar{b} > 0$ in stably stratified waters) and is therefore a sink of buoyancy variance.
 390 The third term represents the spatial redistribution of buoyancy variance by the three dimensional
 391 background flow $\overline{\mathbf{u}}$. It plays an important role at the regional scale by decreasing or increasing the
 392 variance, but cannot be at the very origin of the variability at the basin scale since its global average
 393 is identically zero. The fourth term is a source of buoyancy variance when the surface buoyancy
 394 anomalies and the surface buoyancy flux anomalies $Q'_b = g_0 \alpha_T Q' / C_O$ (with g_0 the acceleration of
 395 gravity at the sea surface, C_O the specific heat capacity of the forcing layer and Q' the anomalous
 396 surface heat flux) are positively correlated. The dissipation term $\overline{b' D'_b}$, which contains contribu-
 397 tions from horizontal and vertical mixing processes (including convective mixing), is a sink of
 398 buoyancy variance at the basin scale and will not be considered. In what follows, we will restrict
 399 our attention to the two only terms in (9) that can take positive values at the basin scale, namely
 400 the atmospheric energy source associated with surface buoyancy fluxes anomalies and the oceanic

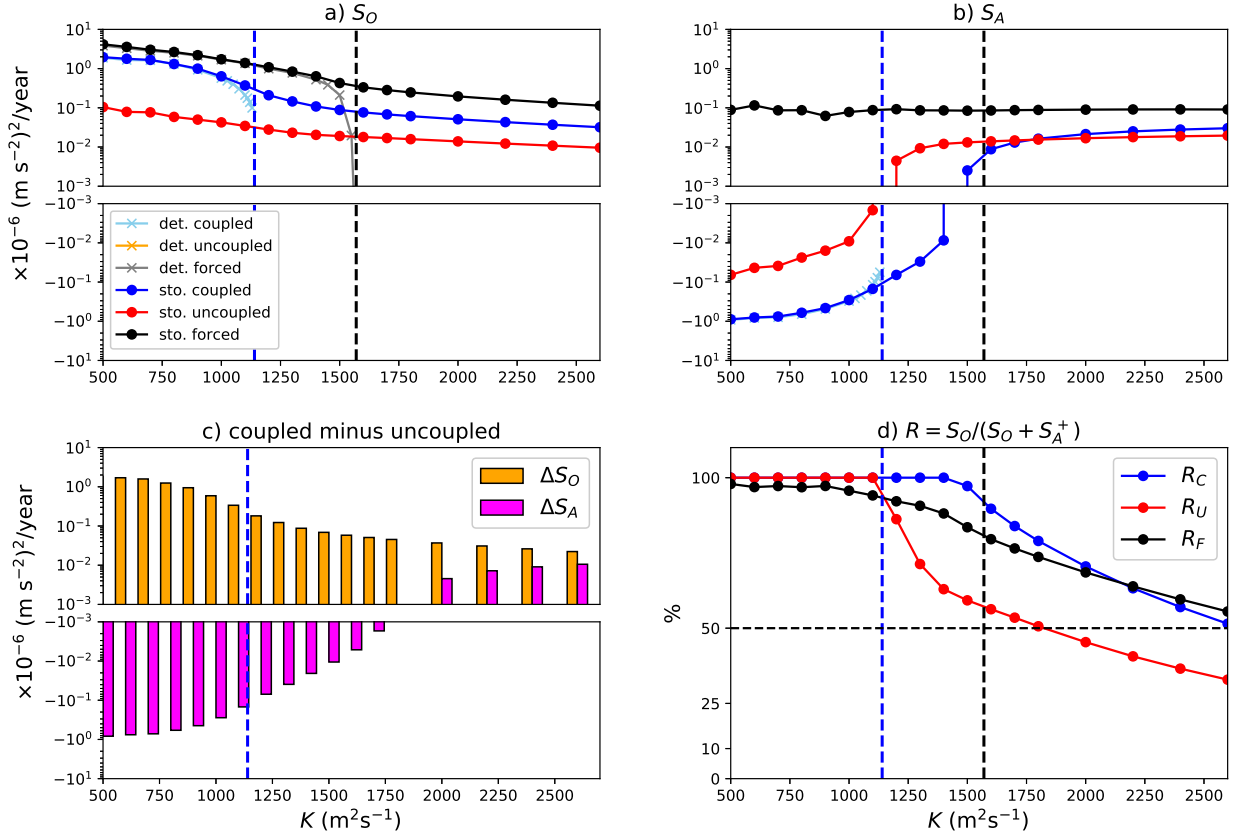
401 energy source associated with the interaction of transient buoyancy fluxes with mean horizontal
402 buoyancy gradients.

403 *b. Results*

404 The buoyancy variance budget is applied to the northwestern corner of the basin (70-50°W, 46-
405 74°N) which roughly coincides with the region of maximum temperature (buoyancy) changes in
406 all experiments (Fig. 3). The buoyancy changes are largely controlled by temperature variations so
407 that the buoyancy variance budget presented hereafter is to a large extent identical to the temper-
408 ature variance budget (not shown). We define the quantities $S_A = \langle \overline{b'Q'_b} \rangle$, $S_O = - \langle \overline{\mathbf{u}'_h b'} \cdot \nabla_h \bar{b} \rangle$
409 where the brackets denote volume averaging. The analysis entirely focuses on the budget within
410 the uppermost 250 m of the ocean model. Because our region of averaging encompasses the bulk
411 of the variability the advection of buoyancy variance into or out of the region by the mean flow is
412 always an order of magnitude smaller than S_O (not shown).

425 Figure 7ab presents the sensitivity of S_O and S_A to K for the coupled, uncoupled, and forced
426 systems, and under both deterministic and stochastic forcing conditions. We first see that S_O is
427 positive for all model configurations and for all values of K . Consistent with the amplitude of
428 the variability, the largest values of S_O are obtained for the forced system, and the lowest values
429 for the uncoupled one. The oceanic energy source term in the coupled case is about an order of
430 magnitude larger than that in the uncoupled one. The effect of thermal coupling is therefore to
431 significantly increase the creation of buoyancy variance by internal ocean dynamics for all values
432 of K , with the strongest impact in the super-critical regime (Fig. 7c).

433 Focusing now on S_A in the coupled case, we see that this term is strongly negative in the super-
434 critical regime and even beyond (up to $K = 1400 \text{ m}^2\text{s}^{-1}$, Fig. 7b). In this range of K values, surface
435 heat flux anomalies therefore act to damp the variability. For larger K values, S_A becomes positive,
436 which along with S_O contributes to the production of buoyancy variance in the northwestern corner
437 of the domain. The change in the sign of S_A with K can be rationalized as follows. Density
438 anomalies are largely controlled by temperature changes so that $b' \propto T'$. Using this result, and
439 assuming that the upwelling and downwelling longwave radiative fluxes nearly cancel out, the
440 covariance term between the surface buoyancy flux anomalies and the buoyancy anomalies can be
441 approximated as $S_A \propto \langle \lambda(\overline{T'T'_a} - \overline{T'^2}) \rangle$, where λ is the air-sea heat exchange coefficient (recall



413 FIG. 7. Buoyancy variance budget in the northwestern corner of the domain (70-50°W, 46-74°N) and in the
 414 uppermost 250m as a function of the horizontal diffusivity K . Top panels show the oceanic $S_O = -\langle \mathbf{u}'_h \bar{b}' \cdot \nabla_h \bar{b} \rangle$
 415 (left) and atmospheric $S_A = \langle \bar{b}' Q'_b \rangle$ (right) energy source terms under both deterministic and stochastic bound-
 416 ary conditions and for the coupled, uncoupled and forced systems. The uncoupled case under deterministic
 417 forcing conditions does not exhibit variability in the range of diffusivities considered here which explains why
 418 it is absent. The bottom left panel shows the changes (coupled minus uncoupled) in atmospheric and oceanic
 419 source terms caused by coupling with the atmosphere. Note that a log vertical scale is used for both posi-
 420 tive and negative values in the upper and bottom left panels. The bottom right panel shows the contribution
 421 $R = S_O / (S_O + S_A^+)$ (%) of internal ocean dynamics to the production of buoyancy variance in the northwestern
 422 corner of the domain for the coupled (R_C), uncoupled (R_U) and forced (R_F) cases, with $S_A^+ = S_A$ if $S_A > 0$ and 0
 423 otherwise (see text for details). In all panels the vertical dashed lines indicate the position of the Hopf bifurcation
 424 of the coupled (blue, $K = 1140 \text{ m}^2\text{s}^{-1}$) and forced (black, $K = 1560 \text{ m}^2\text{s}^{-1}$) cases.

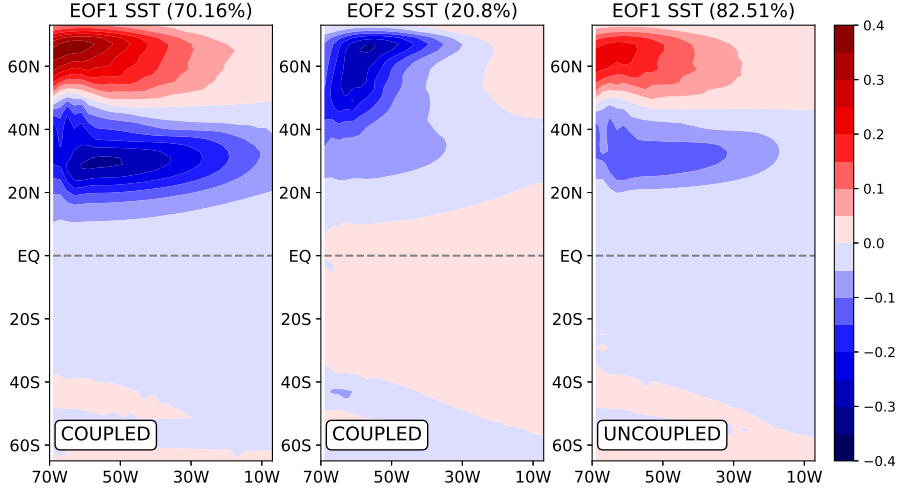
442 that both the solar and freshwater forcings are steady). Now in the super-critical regime, SST
 443 anomalies are relatively large implying that the covariance term S_A is dominated by $-\lambda < \overline{T'^2} >$

444 which is always negative. Larger negative values of S_A are obtained in the coupled case in the
 445 super-critical regime because of the much greater oceanic temperature variance $\overline{T'^2}$ compared to
 446 the uncoupled case. In the damped regime, SST anomalies have a much weaker magnitude and
 447 the leading term in S_A is $\lambda < \overline{T'T'_a} >$ which is positive since both SST and SAT anomalies tend to
 448 vary in phase. In the uncoupled system, the same behaviour occurs but the change in S_A occurs
 449 for lower values of K , consistent with the lower SST variance compared to the coupled case.

450 Interestingly the change in the sign of S_A in the uncoupled case occurs near the Hopf bifurcation
 451 of the coupled system. Whether this occurs by pure coincidence or not remains to be found.
 452 Consistent with the larger SST variance in the coupled system compared to the uncoupled one,
 453 thermal coupling acts to increase the sink of buoyancy variance associated with surface fluxes
 454 across a large range of K values, up to $1700 \text{ m}^2\text{s}^{-1}$ beyond the bifurcation at $1140 \text{ m}^2\text{s}^{-1}$, but
 455 increase the production of buoyancy variance for the most diffusive cases (Fig. 7c).

456 In the forced case surface flux anomalies are always a source of buoyancy variance (Fig. 7).
 457 We also note that S_A in this case is nearly independent on K despite the significant changes in the
 458 amplitude of the variability between the super-critical and damped regimes. An explanation for
 459 this behaviour was provided by Arzel and Huck (2020). In the damped regime, the kinetic energy
 460 variability is much weaker than in the super-critical regime. Low anomalous oceanic advection
 461 tends to keep the noise-forced SST anomalies in the forcing region, a process that favors relatively
 462 high correlations between the forcing and the SST field. The increase of the correlation with K
 463 compensates for the decrease in the SST variance leading to an almost unchanged covariance term
 464 S_A as K increases. Compared to the forced system, the coupled system decreases the production
 465 of buoyancy variance associated with both the surface fluxes and internal ocean dynamics.

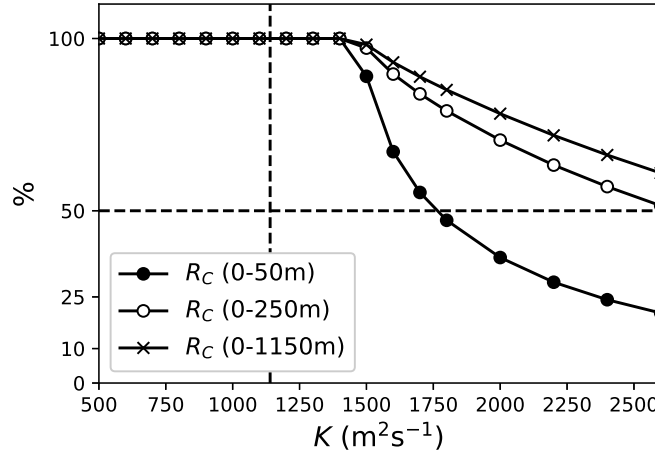
469 Which of the ocean or atmosphere represents the major energy source for the variability? How
 470 does the contribution of the atmosphere and ocean to the production of temperature variance de-
 471 pend on thermal coupling and oceanic eddy diffusivities? To answer these questions we now
 472 compute the ratio $R = S_O / (S_O + S_A^+)$ where $S_A^+ = S_A$ if $S_A > 0$ and equal to 0 otherwise. This
 473 ratio thus provides the fraction of the production of buoyancy variance associated with S_O and S_A
 474 explained by internal ocean dynamics. If $S_A \leq 0$ internal ocean dynamics (S_O) explains 100% of
 475 the production of the buoyancy variance associated with the ensemble S_O and S_A . The results are
 476 presented in Fig. 7d. Quite clearly the leading role of internal ocean dynamics in the variability ex-



466 FIG. 8. Leading patterns of SST variability ($^{\circ}\text{C}$) and explained variance (%) in the damped regime shown
 467 here for the specific diffusivity value $K = 1400 \text{ m}^2\text{s}^{-1}$. Shown are the first two EOFs and the first EOF in the
 468 stochastic coupled and uncoupled systems, respectively.

477 tends beyond the super-critical regime to a much larger range of K values when thermal coupling
 478 with the atmosphere is enabled. Of course stochastic forcing is required to generate variability
 479 in the damped regime, but the growth of upper ocean heat content (250 m) anomalies is however
 480 mostly explained by changing ocean currents rather than by the direct thermodynamical response
 481 of the mixed layer to the noise forcing. Importantly this behaviour occurs despite the fact that the
 482 leading pattern of the variability in the damped regime bears some resemblance with the imposed
 483 NAO forcing (Fig. 8). The imprint of the internal ocean mode in this damped regime is clearly
 484 seen in the second EOF of SST anomalies (Fig. 8) which appears to be similar to the leading
 485 pattern obtained in the super-critical regime (Fig. 5). We further note that the pattern correlation
 486 between the coupled and uncoupled systems in the damped regime is high (Fig. 8), a behaviour
 487 that sharply contrasts with that obtained in the super-critical regime (Fig. 5). In the forced config-
 488 uration, internal ocean dynamics is the primary driver of the upper ocean heat content (0-250 m)
 489 variability for all values of K (Fig. 7d).

490 The results obtained here have been obtained using a very specific timescale of noise coherence
 491 of 10 days. Additional experiments not presented here reveal that increasing this timescale to 30
 492 days has the effect of increasing the production of buoyancy variance by both surface fluxes (S_A)
 493 and internal ocean dynamics (S_O), the effect being much more pronounced in the damped regime.



499 FIG. 9. Contribution $R = S_O / (S_O + S_A^+)$ (%), see text for details) of internal ocean dynamics to the production of
500 buoyancy variance in the northwestern corner of the domain (70-50°W, 46-74°N) as a function of the horizontal
501 diffusivity K . The calculation is done for the stochastic coupled (R_C) case only and in the upper 50, 250 and
502 1150m. The vertical dashed (dotted) line indicates the corresponding position of the Hopf bifurcation at $K =$
503 $1140 \text{ m}^2\text{s}^{-1}$ whereas the horizontal dashed line corresponds to the pivotal value where $S_A = S_O$.

494 As a whole the increase in S_A is larger than the increase in S_O implying that a more coherent
495 noise tends to increase (decrease) the role of the surface fluxes (internal ocean dynamics) in the
496 variability. However the ocean is still the dominant contributor to the growth of buoyancy variance
497 in the northwestern corner of the basin and in the upper 250m for diffusivity values up to 2200
498 m^2s^{-1} (not shown).

504 Figure 9 illustrates finally how the contribution of the ocean and atmosphere to the tempera-
505 ture variability of the coupled system changes as the averaging depth increases from 50 m (the
506 depth of the forcing layer) to 1150 m (roughly the thermocline depth), the 250 m averaging depth
507 chosen previously being intermediate. Quite clearly the greater the averaging depth the stronger
508 the contribution of internal ocean dynamics to the variability, irrespective of the values of K . For
509 instance changes in surface heat fluxes represent the leading energy source of the SST variability
510 in the strongly damped regime, for $K > 1700 \text{ m}^2\text{s}^{-1}$. Upper ocean heat content variability (0-1150
511 m) in this range is however mostly constrained by the production of temperature variance asso-
512 ciated with internal ocean dynamics. The reason for this behaviour is that the S_A term is only
513 nonzero in the forcing layer (upper 50 m) whereas S_O monotonically decreases with depth, con-

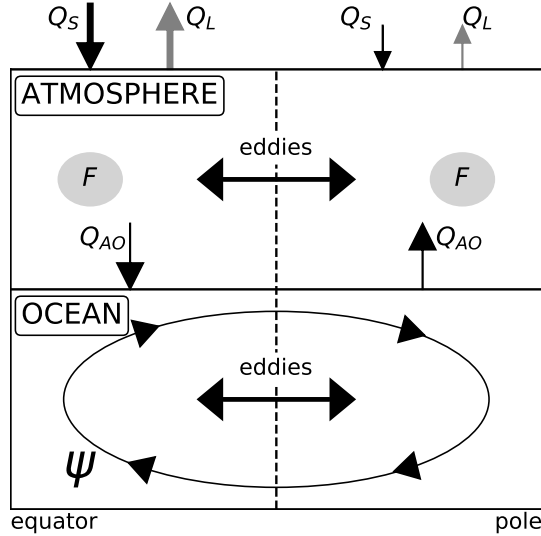
514 sistent with the result that the temperature anomalies mostly project onto the first baroclinic mode
515 whose vertical extent is $O(1000)$ m. Performing the variance budget over a greater depth has the
516 effect of decreasing S_O but at a much lower rate than S_A , implying a greater role of internal ocean
517 dynamics in the variability.

518 **6. A coupled ocean-atmosphere box model**

519 To better understand the way thermal coupling reinforces the internal oceanic contribution to
520 the variability we propose herein a conceptual two degree of freedom dynamical system first in-
521 troduced by Colin de Verdière and Huck (2000) to study interdecadal oscillations of the oceanic
522 overturning under deterministic conditions. The strength of this conceptual model resides in the
523 fact that it captures the behaviour obtained in GCMs, and in particular the Hopf bifurcation at the
524 transition between the steady and oscillatory regimes (Arzel et al. 2018). This model is modified
525 here to include atmospheric stochastic forcing (Fig. 10).

533 *a. Model description*

534 A detailed derivation of the model from first principles can be found in the Appendix. We sim-
535 ply provide here a brief description of the model and governing equations before going on to its
536 analysis in the following sections. The model represents advection, dissipation and exchange of
537 heat within and between the ocean and atmosphere in a single-hemisphere configuration (Fig. 10).
538 The meridional redistribution of heat by advective processes between the tropical and polar boxes
539 in the ocean is assumed to be entirely accomplished by the meridional overturning circulation. The
540 oscillations we are studying here arise under constant surface wind-stress forcing. As such, heat
541 transport by the gyre circulation does not play a fundamental role in the variability and will there-
542 fore be discarded, as opposed for instance to Marshall et al. (2001). The overturning circulation is
543 assumed to vary in quadrature with the anomalous temperature contrast, in agreement with results
544 from previous studies (Huck et al. 1999a; te Raa and Dijkstra 2002). This feature is also clearly
545 apparent here with the rate of change in the MOC being highly correlated ($r = 0.75$) with the
546 anomalous SST contrast between the subtropics (20°N - 48°N) and the subpolar area (48°N - 74°N)
547 in all model configurations, irrespective of the oceanic state being in the damped or super-critical
548 regime. The time lag between meridional circulation anomalies and changes in the meridional



526 FIG. 10. Geometry of the single-hemisphere coupled ocean-atmosphere box model. Two oceanic boxes are
 527 coupled to two atmospheric boxes in the meridional plane. Radiative fluxes at the top of the atmosphere are
 528 the solar forcing Q_S and outgoing longwave radiation Q_L . The ocean and atmosphere exchange heat through
 529 turbulent sensible fluxes Q_{AO} . The term F acting on the atmospheric layer represents the dynamical part of the
 530 forcing and is taken to be stochastic. Mixing by turbulent motions between the tropical and polar boxes within
 531 the atmosphere and ocean is taken into account. Advective heat exchange between the two oceanic boxes is
 532 entirely accomplished by the meridional overturning circulation Ψ .

549 temperature contrast represents the delay associated with westward propagating planetary waves.
 550 Turbulent mixing by mesoscale eddies in the ocean and by synoptic-scale disturbances in the at-
 551 mosphere between the tropical and polar boxes is included. Stochastic forcing is applied to the
 552 atmospheric layer with the same decorrelation time ($\tau_N = 10$ days) and amplitude ($\sigma_N = 100 \text{ W}$
 553 m^{-2}) as the 3D model. Oceanic baroclinic instability is parameterized through a linear growth rate
 554 μ for the strength of the meridional circulation. This parameter constitutes the most important pa-
 555 rameter of the box model because it tells us whether the oceanic state belongs to the super-critical
 556 or damped regime depending on the choice of oceanic and atmospheric turbulent diffusivities and
 557 other thermal damping coefficients. A dynamical ocean is thus added in the simpler way to the
 558 Hasselmann (1976)'s formulation. The nondimensional model equations (see Appendix, equations

559 A5 and A7) are given by

$$\dot{x}' = -2\psi' - \delta x' + \beta N_0, \quad (10a)$$

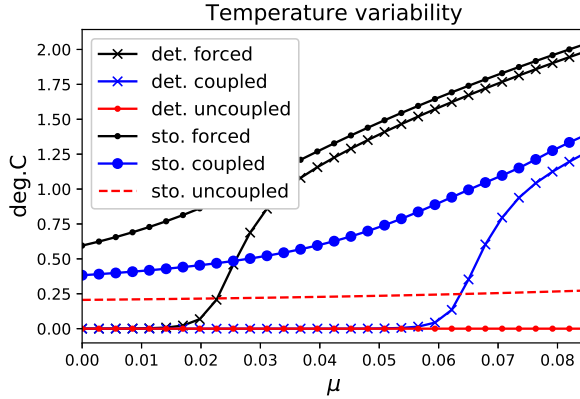
$$\dot{\psi}' = \kappa x' + \mu \psi' - \gamma \psi'^3, \quad (10b)$$

560 where x' is the perturbation oceanic temperature difference between the tropical and polar boxes
 561 and ψ' the perturbation oceanic meridional overturning circulation. There is no evolution equation
 562 of the atmospheric temperature since on the long, decadal, timescales of interest here, the atmo-
 563 sphere is in instantaneous equilibrium with its fluxes. Under this approximation, the atmospheric
 564 temperature becomes diagnostic and the atmosphere enters the problem only through the parame-
 565 ters δ , β and N_0 . More specifically, δ sums up the oceanic and atmospheric contributions to oceanic
 566 thermal damping (oceanic and atmospheric eddy diffusivity, turbulent air-sea heat exchange, long-
 567 wave radiative feedback) and β represents the amplitude of the atmospheric stochastic forcing N_0
 568 seen by the ocean. The last cubic term has been introduced to stabilize the system at large am-
 569 plitudes. We choose $\gamma = 300$ in all experiments. Following the previous approach, three different
 570 configurations of the box model impacting the coefficients δ and β are considered. The first one is
 571 the standard case where ocean-atmosphere thermal coupling is enabled, with coefficients

$$\delta = \alpha(2K_O + \lambda(1 - \sigma)) \quad \text{and} \quad \beta = \alpha\sigma\sigma_N/\bar{x} \quad (11)$$

572 with $\alpha = \tau/C_O$ and $\sigma = \lambda/(\lambda + B + 2K_A)$. Here $\tau = 1$ year is the scale for time, $C_O = 4 \times 10^9 \text{ J K}^{-1}$
 573 m^{-2} is the heat capacity of the thermocline whose depth is fixed to 1000 m, $\bar{x} = 20^\circ\text{C}$ is the mean
 574 meridional temperature contrast between the tropical and polar boxes, $\lambda = 30 \text{ W m}^{-2} \text{ K}^{-1}$ is the
 575 air-sea heat exchange coefficient, $K_O = 1 \text{ W m}^{-2} \text{ K}^{-1}$ (corresponding to $1000 \text{ m}^2\text{s}^{-1}$), $K_A = 2.275$
 576 $\text{W m}^{-2} \text{ K}^{-1}$ is the atmospheric eddy diffusivity (corresponding to $1.3 \times 10^6 \text{ m}^2\text{s}^{-1}$), and $B = 1.7$
 577 $\text{W m}^{-2} \text{ K}^{-1}$ is the longwave feedback at the top of the atmosphere. The second case is the forced
 578 ocean-only case where

$$\delta = 2\alpha K_O \quad \text{and} \quad \beta = \alpha\sigma\sigma_N/\bar{x} \quad (12)$$



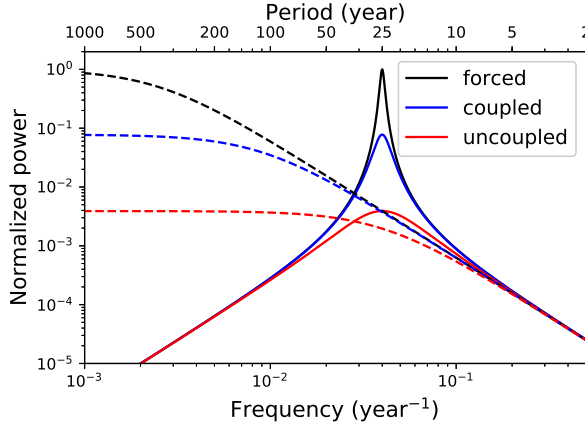
591 FIG. 11. Standard deviation of the anomalous meridional temperature contrast in the box model. For each
 592 value of the baroclinic growth rate μ the coupled system is integrated for 10,000 years using a fourth order
 593 Runge-Kutta scheme and the statistics are computed over the second half of the integration. The integration
 594 is done for the coupled, uncoupled and forced cases under both deterministic and stochastic conditions. Hopf
 595 bifurcations occur successively at $\mu = 0.016$, 0.057 and 0.25 for the forced, coupled and uncoupled cases. Over
 596 the range of values of μ considered here, the deterministic uncoupled system is always stable.

579 The third case is the uncoupled one where the feedback of the atmosphere on oceanic temperature
 580 anomalies is suppressed where

$$\delta = \alpha(2K_O + \lambda) \quad \text{and} \quad \beta = \alpha\sigma\sigma_N/\bar{x} \quad (13)$$

581 *b. Bifurcations, oscillation period and power spectra*

582 Using the values of δ for the coupled, uncoupled and forced cases, we see that as μ increases,
 583 oscillations first emerge in the forced case ($\mu_C = 0.016$), then in the coupled case ($\mu_c = 0.057$), and
 584 lastly in the uncoupled case ($\mu_C = 0.25$). The transition from the non-oscillatory to the oscillatory
 585 regime occurs through a genuine Hopf bifurcation (see Appendix for details). This sequence of
 586 bifurcations is presented in Fig. 11 for the amplitude of the changes in the meridional temperature
 587 contrast, as obtained from a 10,000 years numerical integration of the box model. This sequence of
 588 bifurcations is consistent with the results from the 3D model (Fig. 6), demonstrating the relevance
 589 of this box model to study the physics of interdecadal oscillations of the overturning circulation
 590 and the effect of ocean-atmosphere thermal coupling as we shall now see.



597 FIG. 12. Theoretical power spectra (A8) of temperature anomalies for $\mu = 0$ and for the coupled, uncoupled
 598 and forced cases (solid lines). When circulation changes are inhibited ($\psi' = 0$) the canonical red noise response
 599 is obtained (dashed lines) with a temperature spectra given by $|\hat{X}(\nu)|^2 = \beta^2 |\hat{N}(\nu)|^2 / (4\pi^2 \nu^2 + \mu_c^2)$. All the spectra
 600 are normalized by the peak value of the forced case.

601 Figure 12 shows the theoretical power spectra (A8) for the meridional temperature contrast at
 602 $\mu = 0$ for the coupled, uncoupled and forced cases and for cases where the circulation is free to
 603 interact with the temperature field (solid lines) and prescribed to its background value (dashed
 604 lines). If the circulation is not allowed to vary, technically by setting $\psi' = 0$ in (10a), the canonical
 605 response of Hasselmann (1976) is obtained: a red spectrum at high frequencies that levels out when
 606 $\omega < \mu_c$. Circulation changes are thus essential in producing a preferred timescale in the system,
 607 consistent with realistic ocean-only simulations of the MITgcm (Arzel and Huck 2020). The
 608 effect of thermal coupling is to reduce the damping of oceanic temperature anomalies compared
 609 to the uncoupled case. There is therefore more variability in the coupled case compared to the
 610 uncoupled case. This result is independent of the circulation being prescribed or not. The forced
 611 response exhibits the highest variance since the damping of temperature anomalies is even weaker
 612 in this case, with an effective damping timescale $\tau\delta^{-1}$ of 63 years (compared to 18 and 4 years
 613 in the coupled and uncoupled cases respectively). These results are in good agreement with those
 614 deduced from the 3D model (Fig. 4).

615 *c. Energy sources*

616 Following previous studies and the present approach, insight into the physical mechanisms driv-
617 ing the variability can be obtained by multiplying the temperature equation by x' and averaging
618 over a long time period (denoted by an overbar),

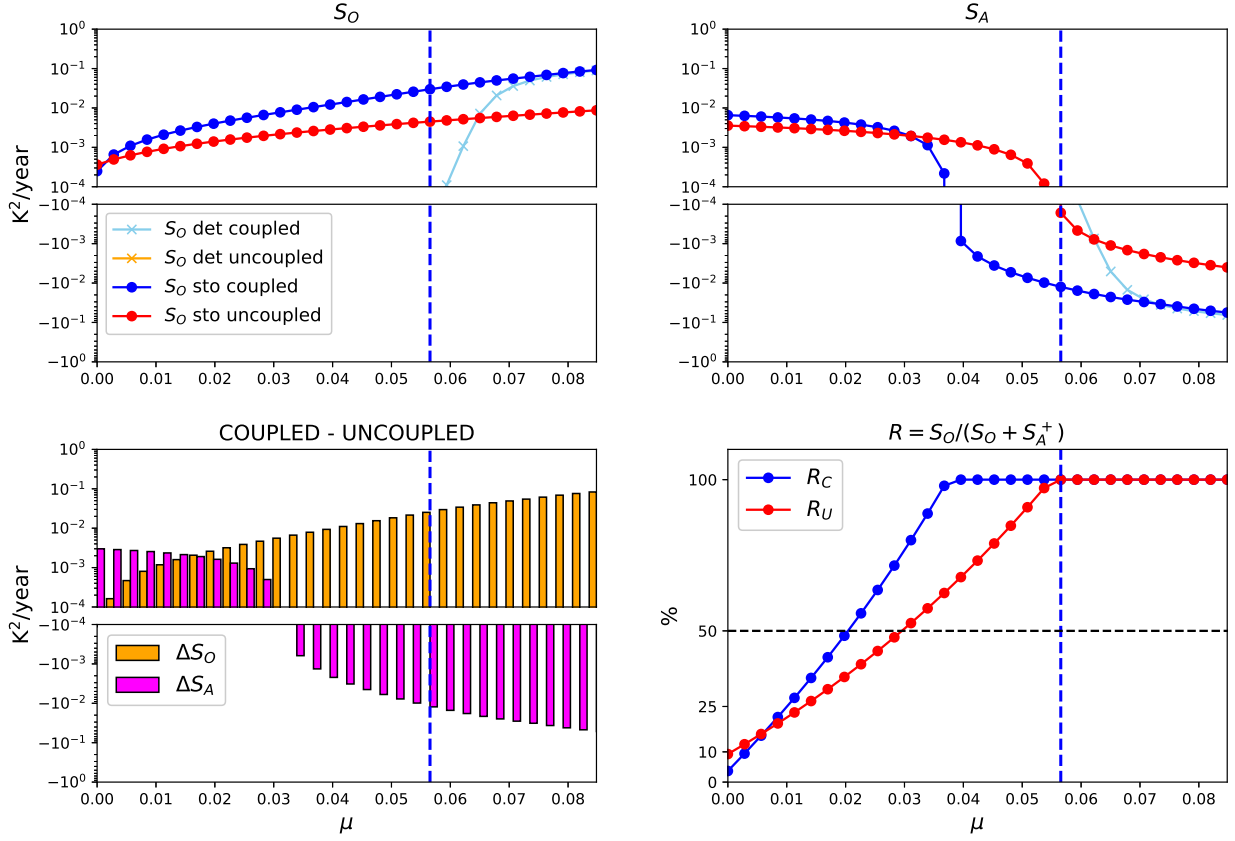
$$\frac{1}{2}\overline{\dot{x}'^2} = -2\overline{\psi'x'} + \overline{\beta x'N_0} - \overline{\delta x'^2} \quad (14)$$

619 Similar to (9), the growth of temperature variance can either originate from internal ocean dynam-
620 ics ($-2\overline{\psi'x'}$) or from the direct effect of stochastic forcing on the oceanic temperature ($\overline{\beta x'N_0}$).
621 The last term is always negative and represents a sink of temperature variance. The results for
622 the coupled and uncoupled cases are presented in Fig. 13. As can be seen the box model repro-
623 duces with very high fidelity the results obtained with the full 3D model (Fig. 7). The major
624 points are that thermal coupling (1) significantly increases the production of temperature variance
625 by internal oceanic processes, (2) significantly increases the destruction of temperature variance
626 by air-sea heat fluxes, but in the most weakly unstable cases where the effect of coupling is to
627 reinforce the production of temperature variance by air-sea heat fluxes, (3) and as a whole signifi-
628 cantly reinforces the role of internal ocean dynamics in the variability, in particular in the damped
629 regime.

637 **7. Summary and discussion**

638 The impact of ocean-atmosphere thermal coupling on the primary drivers of interdecadal vari-
639 ability has been investigated using an idealized coupled model. The main advantage of such a
640 model is its ability to explore a greater parameter regime than that would be possible with a com-
641 prehensive coupled GCM. The effect of thermal coupling on the primary drivers of the variability
642 was quantified across a wide range of oceanic eddy diffusivities from a comparison of the buoy-
643 ancy variance budget of coupled and uncoupled integrations.

644 The results first confirm that the primary effect of thermal coupling is to reduce the internal
645 damping of temperature anomalies due to surface heat fluxes (Barsugli and Battisti 1998). This
646 reduced damping leads to a greater thermal variance in both the ocean and atmosphere as well as
647 greater ocean circulation changes in coupled integrations compared to uncoupled ones, in agree-



630 FIG. 13. Temperature variance budget in the coupled-ocean-atmosphere box model, as obtained from a
 631 10,000 years long numerical integration. Shown are the oceanic ($S_O = -2\overline{x'\Psi'}$) and atmospheric energy source
 632 ($S_A = \beta\overline{x'N_0}$) terms for both the coupled and uncoupled systems under both deterministic and stochastic forcing
 633 conditions as a function of the growth rate μ , the differences in S_O and S_A between the coupled and uncoupled
 634 systems and the ratio $R = S_O / (S_O + S_A^+)$ measuring the fraction of the total production of temperature variance
 635 explained by internal ocean dynamics. In the calculation of R only positive values of S_A are retained so that
 636 $S_A^+ = 0$ wherever $S_A < 0$.

648 ment with Wu and Liu (2005). One new aspect is that the amplifying effect of thermal coupling on
 649 the variability is much stronger in the super-critical than in the damped regime. The presence of
 650 an internal ocean mode in the super-critical regime therefore provides a significant positive feed-
 651 back on the amplitude of the variability through reduced thermal damping by surface heat fluxes.
 652 Thermal coupling is also shown to significantly alter the spatial pattern of the variability, in partic-
 653 ular in the super-critical regime. When the interdecadal internal ocean mode is damped however,
 654 the pattern correlation between the coupled and uncoupled leading EOFs is large. The analysis

655 presented by Wu and Liu (2005) falls within this latter regime suggesting that the internal ocean
656 mode in their model is damped.

657 The buoyancy variance budget then reveals that internal ocean dynamics always acts as a source
658 of temperature variance, irrespective of the coupling with the atmosphere being present or not
659 or the oceanic state being in the super-critical or damped regimes. On the contrary changes in
660 surface buoyancy fluxes always act to damp the variability in the super-critical regime. Only
661 for the most diffusive, weakly unstable states, does the atmosphere act as a source of energy
662 for the variability through the stochastic forcing exceeding surface heat flux damping. These
663 behaviours are present in both the coupled and uncoupled configurations of the model, but with
664 different magnitudes. Overall thermal coupling with the atmosphere is shown to significantly
665 reinforces the role of internal ocean dynamics in the interdecadal variability of the coupled ocean-
666 atmosphere system, in particular in the damped regime. This result constitutes the major finding of
667 our study. Specifically it is shown that the leading role of internal ocean dynamics in the variability
668 extends beyond the super-critical regime to a much larger range of diffusivity values when thermal
669 coupling with the atmosphere is enabled. Put another way, thermal coupling with the atmosphere
670 significantly widens the range of diffusivity values over which internal ocean dynamics drives the
671 variability. Of course stochastic forcing is required to generate variability in the damped regime.
672 The growth of upper ocean heat content anomalies in this regime is however mostly constrained
673 by changing ocean currents associated with the noise excitation of the internal ocean mode rather
674 than by the direct thermodynamical response of the mixed layer to the noise forcing. Importantly
675 this behaviour occurs despite the fact that the leading pattern of the variability in the damped
676 regime bears some resemblance with the imposed NAO forcing. The same conclusion applies to
677 the underlying mechanism driving interdecadal sea surface temperature variability. These results
678 are in line with the more general statement that the ocean drives the mid-latitude North Atlantic
679 variability on decadal and longer timescales (Bjerknes 1964; Gulev et al. 2013; O'Reilly et al.
680 2016). Our results are also consistent with Garuba et al. (2018) who showed on the basis of
681 coupled and partially-coupled integrations of a realistic coupled climate model that the AMV is
682 mostly driven by ocean circulation variability.

683 A stochastic coupled ocean-atmosphere model is finally proposed that captures the basic effect
684 of ocean-atmosphere thermal coupling, as obtained in the 3D model. The box model builds upon

685 a stochastically-forced atmospheric component coupled to a dynamical ocean. The presence of
686 the latter significantly modifies the response of the coupled system to stochastic forcing, first by
687 introducing an additional and significant source of temperature variance in the system, and second
688 by selecting an interdecadal timescale as opposed to pure thermodynamical models (Hasselmann
689 1976; Barsugli and Battisti 1998; Clement et al. 2016). The fact that the effect of coupling is
690 captured by a simple two-degree of freedom dynamical system suggests that the proposed mech-
691 anism does not depend on the details of the 3D model and is therefore a robust feature of the
692 ocean-atmosphere system.

693 There are of course several fundamental aspects of the real climate system that are missing from
694 our idealized framework. Perhaps the most critical is the absence of a dynamical atmospheric
695 component. With a full dynamical atmosphere, NAO related stochastic forcing would adjust its
696 amplitude and spatial pattern to the changing ocean surface conditions and feed back onto the SST
697 field such that coupled air-sea modes or resonant behaviours could emerge (Weaver and Valcke
698 1998; Wu and Liu 2005; Ortega et al. 2015). The absence of a seasonal cycle and the limited
699 poleward extent of the ocean domain does not allow the emergence of sea ice although a sim-
700 ple one-layer thermodynamic sea ice component identical to that used by Colin de Verdière and
701 te Raa (2010) is present. The variability studied here is typically enhanced at mid-latitudes along
702 the North Atlantic Current (Ortega et al. 2015; Arzel et al. 2018; Gastineau et al. 2018; Arzel and
703 Huck 2020; Gastineau et al. 2018) so that direct sea ice effects are unimportant. It would however
704 be interesting to assess the effect of thermal coupling on the variability of coupled climate mod-
705 els presenting a source of variability related to Arctic-Atlantic interactions (Jungclauss et al. 2005;
706 Escudier et al. 2013). The lack of oceanic turbulence is another limitation of our model. With
707 resolved mesoscale eddies the explicit horizontal diffusivity used here becomes irrelevant and an-
708 other critical parameter, such as vertical diffusivity used by Farneti and Vallis (2009) for instance,
709 would be required to explore the physics of the variability in both the damped and super-critical
710 regimes. The presence of oceanic mesoscale eddies does not modify the generic mechanism of
711 baroclinic instability captured in our simple model (Huck et al. 2015), but simply acts as a sink of
712 temperature variance at interdecadal timescales (Hochet et al. 2020, 2022). As such the leading
713 role of internal ocean dynamics in the variability advocated in the present study is probably overes-
714 timated compared to a situation where mesoscale eddies are resolved. Additional studies based on

715 strongly eddying quasi-geostrophic coupled models point to a strong coupling between the ocean
716 and atmosphere on decadal and longer timescales (Kravtsov et al. 2007; Martin et al. 2021). These
717 quasi-geostrophic models cannot capture the large-scale baroclinic instability mechanism that is at
718 play in our model, because it develops in regions of steep isopycnal slopes. Future work based on
719 primitive ocean models at eddying resolutions will need to be pursued to assess which mechanism
720 among the few listed here is the most relevant for driving temperature variance from mid to subpo-
721 lar latitudes as well as the role of thermal coupling which is still debated (e.g. Weaver and Valcke
722 1998; Timmermann et al. 1998; Delworth and Greatbatch 2000; Dong and Sutton 2005; Gastineau
723 et al. 2018). The generic large-scale baroclinic instability mechanism driving internal oceanic vari-
724 ability in the planetary geostrophic ocean component of the present model (Colin de Verdière and
725 Huck 1999; Huck et al. 2001) has been shown to be robust in comprehensive ocean climate models
726 (Sévellec and Fedorov 2013; Ortega et al. 2015; Arzel et al. 2018; Gastineau et al. 2018). We thus
727 feel confident that the mechanisms presented here might play a role in more realistic contexts. As
728 a final note we wish to stress the extreme usefulness of the buoyancy variance budget combined
729 with the coupled/uncoupled modelling approach to identify the primary drivers of the variability
730 and unravel the role of thermal coupling. Applying such a budget in coupled and uncoupled con-
731 figurations of comprehensive GCMs and realistic climate models will undoubtedly improve our
732 understanding of the mechanisms driving climate variability on interdecadal timescales in those
733 models.

734 *Acknowledgments.* We thank the three anonymous reviewers for their helpful suggestions. We
735 are grateful to Alain Colin de Verdière who provided helpful criticism on an early version of
736 this manuscript. The authors acknowledge the Pôle de Calcul et de Données Marines (PCDM at
737 Ifremer, Brest) for providing DATARMOR computational resources.

738 *Data availability statement.* The data that support the findings of this study are stored on IFRE-
739 MER (Brest, France) servers and can be made available upon reasonable request to the corre-
740 sponding author.

741 APPENDIX

742 **The coupled ocean-atmosphere box model**

743 *a. Model equations*

744 Radiative imbalance at the top of the atmosphere (Fig. 10) arises due to differences between in-
 745 coming solar flux Q_S (assumed constant) and outgoing longwave radiation Q_L which is linearized
 746 around the atmospheric temperature T_a , i.e. $Q_L = A + BT_a$. The term F acting on the atmospheric
 747 layer represents the dynamical part of the forcing which is taken to be stochastic. The turbulent
 748 surface heat flux Q_{AO} (assumed positive downward) between the ocean and atmosphere is equal to
 749 $\lambda(T_a - T_o)$ where λ is the air-sea heat exchange coefficient and T_o the oceanic temperature. Using
 750 $\lambda = 30 \text{ W m}^{-2} \text{ K}^{-1}$, an atmospheric heat capacity $C_a = \rho_a C_{pa} h_a = 7 \times 10^6 \text{ J m}^{-2} \text{ K}^{-1}$ (assuming
 751 an atmospheric scale height $h_a = 7 \text{ km}$, and standard values for air density ρ_a and specific heat
 752 capacity of air C_{pa}), we obtain an atmospheric damping timescale C_a/λ of $\mathcal{O}(1)$ day. Since we
 753 are interested in timescales of the order of 10 years or longer, it is legitimate to assume that the
 754 atmosphere is in instantaneous equilibrium with the ocean and the stochastic forcing (Saravanan
 755 and McWilliams 1998). The heat budget of the atmospheric box i then reads

$$Q_S^i - Q_L^i - Q_{ao}^i + K_A(T_a^j - T_a^i) + F^i = 0 \quad (\text{A1})$$

756 where $j = 3 - i$ and K_A parameterizes the turbulent heat exchange by large-scale atmospheric eddies
 757 between the tropical ($i = 1$) and the polar ($i = 2$) boxes. The heat budget of the oceanic box i is

$$C_O \frac{\partial T^i}{\partial t} = Q_{ao}^i + \frac{C_O}{a_i} \psi (T^j - T^i) + K_O (T^j - T^i), \quad (\text{A2})$$

758 where ψ ($\text{m}^3 \text{ s}^{-1}$) is the overturning strength, $C_O = \rho_o C_{po} h$ is the heat capacity of the ocean where
 759 ρ_o , C_{po} and h are the density, specific heat of water and ocean depth respectively, and K_O parame-
 760 terizes the turbulent heat exchange by mesoscale eddies between the two oceanic boxes. Following
 761 Colin de Verdière and Huck (2000) we concentrate on the variability of the temperature differences
 762 between the two boxes. Introducing $x = T^1 - T^2$ for the ocean and $y = T_A^1 - T_A^2$ for the atmosphere
 763 and assuming equal areas a_i ($a_i = a = 4 \times 10^6 \text{ km}^2$) for the two boxes allows us to write

$$S - By - \lambda(y - x) - 2K_A y + N = 0 \quad (\text{A3})$$

764 and

$$\dot{x} = \alpha\lambda(y - x) - 2\psi x - 2K_O\alpha x, \quad (\text{A4})$$

765 where $\alpha = \tau/C_O$ and ψ is scaled by ha/τ where $\tau = 1$ year. The terms S and N are the differences of
766 solar flux and noise forcing between the tropical and polar boxes respectively. The noise forcing
767 is written as $N = \sigma_N N_0$ where σ_N is the amplitude of the forcing and N_0 obeys to a first order
768 autoregressive process with standard deviation equal to 1 and decorrelation time $\tau_N = 10$ days.
769 Linearizing now the thermodynamic equations (A3 and A4) around a time mean state (denoted by
770 an overbar), substituting (A3) into (A4), and nondimensionalizing the oceanic temperature anomaly
771 x' by \bar{x} the mean meridional temperature contrast, we obtain

$$\dot{x}' = -2\psi' - \delta x' + \beta N_0, \quad (\text{A5})$$

772 where ψ' is the anomalous overturning circulation. It has been shown from 3D numerical ex-
773 periments (Huck et al. 1999a) that temperature anomalies are mostly constrained by anomalous
774 advection rather than mean flow effects. Hence the term $-2\bar{\psi}x'$ is neglected. The coefficients δ
775 and β are given by

$$\delta = \alpha(2K_O + \lambda(1 - \sigma)) \quad \text{and} \quad \beta = \alpha\sigma\sigma_N/\bar{x} \quad (\text{A6})$$

776 with $\sigma = \lambda/(\lambda + B + 2K_A)$. Using $B = 1.7 \text{ W m}^{-2} \text{ K}^{-1}$, $K_A = 2.275 \text{ W m}^{-2} \text{ K}^{-1}$ (equivalent to an
777 atmospheric eddy diffusivity $K_A \times a/C_a = 1.3 \times 10^6 \text{ m}^2 \text{ s}^{-1}$), $K_O = 1 \text{ W m}^{-2} \text{ K}^{-1}$ (equivalent to an
778 oceanic eddy diffusivity of $K_O \times a/C_O = 1000 \text{ m}^2 \text{ s}^{-1}$, with $C_O = 4 \times 10^9 \text{ J m}^{-2} \text{ K}^{-1}$, for a 1000m
779 ocean depth), we obtain a damping timescale $\tau\delta^{-1}$ of oceanic temperature anomalies of about 18
780 years. This timescale is much longer than the typical e-folding times of $O(1)$ year of unstable
781 planetary waves computed from OGCM studies (Arzel et al. 2018) suggesting the potential for
782 spontaneous oscillations developping in ocean-only integrations to survive to coupling with the
783 atmosphere. Notice that the magnitude of the noise forcing seen by the ocean is proportional to
784 σ which decreases with decreasing λ values and increasing K_A values. To close the system, an
785 equation for the anomaly in the strength of the circulation is needed. With an attempt to encapsulate
786 the ideas developed in the previous sections, at least two features must be represented. The
787 first one is the baroclinic instability mechanism, which is parameterized here through the use of a
788 linear growth rate μ . The second one is the apparent phase lag that exists between the meridional

789 temperature gradient and the MOC. These aspects encourage us to follow Colin de Verdière and
 790 Huck (2000) and use the same dynamics, that is

$$\dot{\psi}' = \kappa x' + \mu \psi' - \gamma \psi'^3, \quad (\text{A7})$$

791 where the last cubic term has been introduced to stabilize the system at large amplitudes. The
 792 sensitivity of model solutions is studied in terms of the baroclinic growth rate μ .

793 *b. Properties of the solutions*

794 In the deterministic case ($\beta = 0$) and in the small amplitude limit, it can easily be shown (Colin de
 795 Verdière and Huck 2000) that the eigensolutions are unstable for $\mu > \mu_c = \delta$ provided that $\delta < \sqrt{2\kappa}$.
 796 At $\mu = \mu_c$ a super-critical Hopf bifurcation occurs. Physically this means that when the growth rate
 797 of perturbations becomes larger than all sources of thermal damping, the oscillations can grow and
 798 eventually settle into a finite amplitude limit cycle. In the stochastic regime, the statistics of the
 799 variability can be deduced from the temperature spectra in the small amplitude limit,

$$|\hat{X}(\omega)|^2 = \frac{\beta^2 |\hat{N}_0(\omega)|^2}{\omega^2 a(\omega)^2 + b(\omega)^2}, \quad (\text{A8})$$

800 where $\hat{N}_0(\omega)$ is the Fourier transform of the noise forcing and

$$a(\omega) = 1 - \frac{2\kappa}{\omega^2 + \mu^2} \quad (\text{A9a})$$

$$b(\omega) = \mu_c - \frac{2\kappa\mu}{\omega^2 + \mu^2} \quad (\text{A9b})$$

801 For timescales much longer than $\mathcal{O}(10)$ days, the stochastic forcing is essentially white and
 802 $\hat{N}_0(\omega) = 1$. In this case an analytical expression for the most energetic timescale $T_0 = 2\pi\omega_0^{-1}$
 803 of the variability can be deduced from (A8) and will be valid as long as it is much longer than the
 804 atmospheric spin down timescale,

$$T_0 = 2\pi[2\sqrt{\kappa}(\mu^2 - \mu\mu_c + \kappa)^{1/2} - \mu^2]^{-1/2}, \quad (\text{A10})$$

805 while the amplitude of temperature variations is given by $(\int |\hat{X}(\omega)|^2 d\omega)^{1/2}$. With $\kappa = 0.0315$, a
806 period of 25 years is obtained in the coupled, uncoupled and forced cases, similar to the 3D model.
807 It is readily seen that the period in the deterministic case $2\pi[2\kappa - (\mu + \mu_c)^2/4]^{-1/2}$ closely follows
808 that obtained under stochastic forcing as long as $\mu < \mu_c \ll 1$. At bifurcation $\mu = \mu_c$ the period of
809 the variability predicted by the deterministic solution is exactly recovered by (A10) but the linear
810 assumption breaks down since $|\hat{X}(\omega_0)|^2$ becomes infinite. The stochastic linear solutions become
811 invalid when approaching the Hopf bifurcation and are mostly useful in the most diffusive, weakly
812 unstable cases where $\mu \ll \mu_c$.

813 **References**

- 814 Abernathey, R. P., and J. Marshall, 2013: Global surface eddy diffusivities derived from satellite
815 altimetry. *J. Geophys. Res.*, **118**, 901–916.
- 816 Arzel, O., A. Colin de Verdière, and T. Huck, 2007: On the origin of interdecadal oscillations in a
817 coupled ocean-atmosphere model. *Tellus*, **59**, 367–383.
- 818 Arzel, O., M. H. England, A. Colin de Verdière, and T. Huck, 2012: Abrupt millennial vari-
819 ability and interdecadal-interstadial oscillations in a global coupled model: sensitivity to the
820 background climate state. *Clim. Dyn.*, **39**, 259–275.
- 821 Arzel, O., and T. Huck, 2020: Contributions of atmospheric stochastic forcing and intrinsic ocean
822 modes to North Atlantic ocean interdecadal variability. *J. Clim.*, **33**, 2351–2370.
- 823 Arzel, O., T. Huck, and A. Colin de Verdière, 2006: The different nature of interdecadal variability
824 of the thermohaline circulation under mixed and flux boundary conditions. *J. Phys. Oceanogr.*,
825 **36**, 1703–1718.
- 826 Arzel, O., T. Huck, and A. Colin de Verdière, 2018: The internal generation of the Atlantic Ocean
827 interdecadal variability. *J. Clim.*, **31**, 6411–6432.
- 828 Barsugli, J. J., and D. S. Battisti, 1998: The basic effects of atmosphere-ocean thermal coupling
829 on midlatitude variability. *J. Atmos. Sci.*, **55**, 477–493.
- 830 Bjerknes, J., 1964: Atlantic air-sea interaction. *Adv. Geophys.*, **10**, 1–82.

- 831 Buckley, M. W., D. Ferreira, J.-M. Campin, J. Marshall, and R. Tulloch, 2012: On the relationship
832 between decadal buoyancy anomalies and variability of the Atlantic meridional overturning
833 circulation. *J. Clim.*, **25**, 8009–8030.
- 834 Chen, F., and M. Ghil, 1996: Interdecadal variability in a hybrid coupled ocean-atmosphere model.
835 *J. Phys. Oceanogr.*, **26**, 1561–1578.
- 836 Chylek, P., C. K. Folland, H. A. Dijkstra, G. Lesins, and M. K. Dubey, 2011: Ice-core data evi-
837 dence for a prominent near 20 year time-scale of the Atlantic multidecadal oscillation. *Geophys.*
838 *Res. Lett.*, **38**, doi:10.1029/2011GL047501.
- 839 Clement, A., K. Bellomo, L. N. Murphy, M. A. Cane, T. Mauritsen, G. Radel, and B. Stevens,
840 2015: The Atlantic Multidecadal Oscillation without a role for ocean circulation. *Science*, **350**,
841 320–324.
- 842 Clement, A., M. A. Cane, L. N. Murphy, K. Bellomo, T. Mauritsen, and B. Stevens, 2016: Re-
843 sponse to Comment on "The Atlantic Multidecadal Oscillation without a role for ocean circula-
844 tion.". *Science*, **352**, 1527.
- 845 Colin de Verdière, A., 1988: Buoyancy driven planetary flows. *J. Mar. Res.*, **46**, 215–265.
- 846 Colin de Verdière, A., and T. Huck, 1999: Baroclinic instability: An oceanic wavemaker for
847 interdecadal variability. *J. Phys. Oceanogr.*, **29**, 893–910.
- 848 Colin de Verdière, A., and T. Huck, 2000: A 2 degree of freedom dynamical system for inter-
849 decadal oscillations of the ocean–atmosphere. *J. Clim.*, **13**, 2801–2817.
- 850 Colin de Verdière, A., T. Meunier, and M. Ollitrault, 2019: Meridional overturning and heat trans-
851 port from argo floats displacements and the planetary geostrophic method (pgm): application to
852 the subpolar north atlantic. *J. Geophys. Res.: Oceans*, **124**, 6270–6285.
- 853 Colin de Verdière, A., and L. te Raa, 2010: Weak oceanic heat transport as a cause of the instability
854 of glacial climates. *Clim. Dyn.*, **35**, 1237–1256.
- 855 Delworth, T. L., and R. J. Greatbatch, 2000: Multidecadal thermohaline circulation variability
856 excited by atmospheric surface flux forcing. *J. Clim.*, **13**, 1481–1495.

- 857 Delworth, T. L., R. Zhang, and M. E. Mann, 2007: Decadal to centennial variability of the Atlantic
858 from observations and models. *Geophysical Monograph*, **173**, 131–148, Ocean Circulation:
859 mechanisms and impacts.
- 860 Deser, C., M. A. Alexander, S.-P. Xie, and A. S. Phillips, 2010: Sea surface temperature variability:
861 patterns and mechanisms. *Annu. Rev. Mar. Sci.*, **2**, 115–143.
- 862 Dong, B., and R. T. Sutton, 2005: Mechanism of interdecadal thermohaline circulation variability
863 in a coupled ocean-atmosphere GCM. *J. Clim.*, **18**, 1117–1135.
- 864 Escudier, R., J. Mignot, and D. Swingedouw, 2013: A 20-year coupled ocean-sea ice-atmosphere
865 variability mode in the north atlantic in an aogcm. *Clim. Dyn.*, **40**, 619–636.
- 866 Fanning, A. F., and A. J. Weaver, 1996: An atmospheric energy-moisture balance model: clima-
867 tology, interpentadal climate change, and coupling to an ocean general circulation model. *J.*
868 *Geophys. Res.*, **101**, 111–128.
- 869 Farneti, R., and G. K. Vallis, 2009: Mechanisms of interdecadal climate variability and the role of
870 ocean-atmosphere coupling. *Clim. Dyn.*, **36**, 289–308.
- 871 Feldstein, S. B., 2000: The timescale, power spectra, and climate noise properties of teleconnec-
872 tion patterns. *J. Clim.*, **13**, 4430–4440.
- 873 Frankcombe, L. M., and H. A. Dijkstra, 2009: Coherent multidecadal variability in North Atlantic
874 sea level. *Geophys. Res. Lett.*, **36**, doi:10.029/2009GL039455.
- 875 Garuba, O. A., J. . Lu, H. A. Singh, F. Liu, and P. Rasch, 2018: On the relative roles of the
876 atmosphere and ocean in the Atlantic multidecadal variability. *Geophys. Res. Lett.*, **45**, 9186–
877 9196.
- 878 Gastineau, G., J. Mignot, O. Arzel, and T. Huck, 2018: North Atlantic Ocean internal decadal
879 variability: role of the mean state and ocean-atmosphere coupling. *J. Geophys. Res.*, **123**,
880 <https://doi.org/10.1029/2018JC014074>.
- 881 Graves, C. E., W. H. Lee, and G. R. North, 1993: New parametrizations and sensitivities for simple
882 climate models. *J. Geophys. Res.*, **98**, 5025–5036.

- 883 Gray, S., L. Graumlich, J. Betancourt, and G. Pederson, 2004: A tree-ring based reconstruction
884 of the Atlantic Multidecadal Oscillation since 1567 AD. *Geophys. Res. Lett.*, **31** (L12205),
885 <https://doi.org/10.1029/2004GL019932>.
- 886 Greatbatch, R. J., and S. Zhang, 1995: An interdecadal oscillation in an idealized ocean basin
887 forced by constant heat flux. *J. Clim.*, **8**, 81–91.
- 888 Gulev, S. K., M. Latif, N. Keenlyside, W. Park, and K. P. Koltermann, 2013: North Atlantic ocean
889 control on surface heat flux on multidecadal timescales. *Nature*, **499**, 464–467.
- 890 Hasselmann, K., 1976: Stochastic climate models. Part I: theory. *Tellus*, **28**, 473–484.
- 891 Held, I. M., 2005: The gap between simulating and understanding in climate modeling. *Bull. Am.*
892 *Meteorol. Soc.*, **86**, 1609–1614.
- 893 Herbaut, C., J. Sirven, and S. Fevrier, 2002: Response of a simplified oceanic general circulation
894 model to idealised NAO-like stochastic forcing. *J. Phys. Oceanogr.*, **32**, 3182–3192.
- 895 Hochet, A., T. Huck, O. Arzel, F. Sévellec, and A. C. de Verdière, 2022: Energy transfers between
896 multidecadal and turbulent variability. *J. Clim.*, **35**, 1157–1178.
- 897 Hochet, A., T. Huck, O. Arzel, F. Sévellec, A. C. de Verdière, M. Mazloff, and B. Cornuelle, 2020:
898 Direct temporal cascade of temperature variance in eddy-permitting simulations of multidecadal
899 variability. *J. Clim.*, **33**, 9409–9425.
- 900 Huang, R. X., 1998: Mixing and available potential energy in a Boussinesq ocean. *J. Phys.*
901 *Oceanogr.*, **28**, 669–678.
- 902 Huck, T., O. Arzel, and F. Sévellec, 2015: Multidecadal variability of the overturning circulation
903 in presence of eddy turbulence. *J. Phys. Oceanogr.*, **45**, 157–173.
- 904 Huck, T., A. Colin de Verdière, and A. Weaver, 1999a: Interdecadal variability of the thermohaline
905 circulation in box-ocean models forced by fixed surface fluxes. *J. Phys. Oceanogr.*, **29**, 865–892.
- 906 Huck, T., G. Vallis, and A. Colin de Verdière, 2001: On the robustness of the interdecadal modes
907 of the thermohaline circulation. *J. Clim.*, **14**, 940–963.

908 Huck, T., A. J. Weaver, and A. Colin de Verdière, 1999b: On the influence of the parametrization
909 of lateral boundary layers on the thermohaline circulation in coarse resolution ocean-models. *J.*
910 *Mar. Res.*, **57**, 387–426.

911 Jamet, Q., T. Huck, A. Colin de Verdière, O. Arzel, and J.-M. Campin, 2016: Oceanic control of
912 multidecadal variability in an idealized coupled GCM. *Clim. Dyn.*, **46**, 3079–3095.

913 Jungclauss, J. H., H. Haak, U. Mikolajewicz, and M. Latif, 2005: Arctic-North Atlantic interactions
914 and multidecadal variability of the meridional overturning circulation. *J. Clim.*, **18**, 4016–4034.

915 Kravtsov, S., W. K. Dewar, P. Berloff, J. McWilliams, and M. Ghil, 2007: A highly nonlinear cou-
916 pled mode of decadal variability in a mid-latitude ocean-atmosphere model. *Dyn. Atm. Oceans*,
917 **43**, 123–150.

918 Mann, M. E., R. S. Bradley, and M. K. Hughes, 1998: Global-scale temperature patterns and
919 climate forcing over the past six centuries. *Nature*, **392**, 779–787.

920 Mann, M. E., B. A. Steinman, D. J. Brouillette, and S. K. Miller, 2021: Multidecadal climate
921 oscillations during the past millennium driven by volcanic forcing. *science*, **371**, 1014–1019.

922 Marshall, J., J. R. Scott, A. Romanou, M. Kelley, and A. Leboissetier, 2017: The dependence of
923 the ocean’s MOC on mesoscale eddy diffusivities: A model study. *Ocean Modelling*, **111**, 1–8.

924 Marshall, J., and Coauthors, 2001: North atlantic climate variability: phenomena, impacts and
925 mechanisms. *Int. J. Climatol.*, **21**, 1863–1898.

926 Martin, P. E., B. K. Arbic, and A. Hogg, 2021: Drivers of atmospheric and oceanic surface tem-
927 perature variance: a frequency domain approach. *J. Clim.*, [https://doi.org/10.1175/JCLI-D-20-](https://doi.org/10.1175/JCLI-D-20-0557.1)
928 [0557.1](https://doi.org/10.1175/JCLI-D-20-0557.1).

929 Muir, M. C., and A. Fedorov, 2016: Evidence for the AMOC interdecadal mode re-
930 lated to westward propagation of temperature anomalies in CMIP5 models. *Clim. Dyn.*,
931 [doi:10.1007/s00382-016-3157-9](https://doi.org/10.1007/s00382-016-3157-9).

932 Munk, W., and C. Wunsch, 1998: Abyssal recipes ii: Energetics of tidal and wind mixing. *Deep-*
933 *Sea Res.*, **45A**, 1977–2010.

- 934 North, G. R., R. F. Cahalan, and J. A. Coakley, 1981: Energy balance climate models. *Rev. Geo-*
935 *phys. Space Phys.*, **19**, 91–121.
- 936 O'Reilly, C. H., M. Huber, T. Woollings, and L. Zanna, 2016: The signature of low-frequency
937 oceanic forcing in the Atlantic Multidecadal Oscillation. *Geophys. Res. Lett.*, **43**, 2810–2818.
- 938 Ortega, P., J. Mignot, D. Swingedouw, F. Sévellec, and E. Guilyardi, 2015: Reconciling two
939 alternative mechanisms behind bi-decadal variability in the North Atlantic. *Prog. Oceanogr.*,
940 **137**, 237–249.
- 941 Peixoto, J. P., and A. H. Oort, 1992: Physics of climate. *American Institute of Physics*, 520pp.
- 942 Salmon, R., 1986: A simplified linear ocean circulation theory. *J. Mar. Res.*, **44**, 695–711.
- 943 Saravanan, R., and J. C. McWilliams, 1998: Advective ocean-atmosphere interaction: An analyti-
944 cal stochastic model with implications for decadal variability. *J. Clim.*, **11**, 165–188.
- 945 Sévellec, F., and A. Fedorov, 2013: The leading, interdecadal eigenmode of the Atlantic merid-
946 ional overturning circulation in a realistic ocean model. *J. Clim.*, **26**, 2160–2183.
- 947 Sutton, R. T., G. D. M. Carthy, J. Robson, B. Sinha, A. T. Archibald, and L. J. Gray, 2018: Atlantic
948 Multidecadal Variability and the U.K. ACSIS program. *Bull. Am. Meteorol. Soc.*, **99**, 415–425.
- 949 te Raa, L. A., and H. A. Dijkstra, 2002: Instability of the thermohaline circulation on interdecadal
950 timescales. *J. Phys. Oceanogr.*, **32**, 138–160.
- 951 Timmermann, A., M. Latif, R. Voss, and A. Grötzner, 1998: Northern hemispheric interdecadal
952 variability: a coupled air-sea mode. *J. Clim.*, **11**, 1906–1931.
- 953 Tung, K.-K., and J. Zhou, 2013: Using data to attribute episodes of warming and cooling in
954 instrumental records. *Proc. Natl. Acad. Sci. USA*, **110** (6), 2058–2063.
- 955 Veronis, G., 1975: *The role of models in tracer studies*, chap. Numerical models of the ocean
956 circulation, 133–146. National Academy of Science.
- 957 Wang, J., B. Yang, F. C. Ljungqvist, J. Luterbacher, T. J. Osborn, K. R. Briffa, and E. Zorita, 2017:
958 Internal and external forcing of multidecadal Atlantic climate variability over the past 1,200
959 years. *Nature Geosc.*, **10**, 512–517.

- 960 Weaver, A. J., and E. S. Sarachik, 1990: On the importance of vertical resolution in certain ocean
961 general circulation models. *J. Phys. Oceanogr.*, **20**, 600–609.
- 962 Weaver, A. J., and S. Valcke, 1998: On the variability of the thermohaline circulation in the GFDL
963 coupled model. *J. Clim.*, **11**, 759–767.
- 964 Wu, L., and Z. Liu, 2005: North Atlantic decadal variability: air-sea coupling, oceanic memory,
965 and potential Northern Hemisphere resonance. *J. Clim.*, **18**, 331–349.
- 966 Zhang, R., R. Sutton, G. Danabasoglu, Y.-O. Kwon, R. Marsh, S. G. Yeager, D. E. Amrhein, and
967 C. M. Little, 2019: A review of the role of the Atlantic Meridional Overturning Circulation in
968 Atlantic Multidecadal Variability and associated climate impacts. *Rev. Geophys.*, **57**, 316–375.