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1 **Frequency dependence and the predictability of** 2 **evolution in a changing environment**

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8 **Abstract**

9 Frequency-dependent (FD) selection, whereby fitness and selection depend on the genetic or phenotypic
10 composition of the population, arises in numerous ecological contexts (competition, mate choice,
11 crypsis, mimicry, etc.), and can strongly impact evolutionary dynamics. In particular, negative
12 frequency-dependent selection (NFDS) is well known for its ability to potentially maintain stable
13 polymorphisms, but it has also been invoked as a source of persistent, predictable frequency fluctuations.
14 However, the conditions under which such fluctuations persist are not entirely clear. In particular,
15 previous work rarely considered that FD is unlikely to be the sole driver of evolutionary dynamics when
16 it occurs, because most environments are not static but instead change dynamically over time. Here we
17 investigate how FD interacts with a temporally fluctuating environment to shape the dynamics of
18 population genetic change. We show that a simple metric introduced by Lewontin (1958), the slope of
19 frequency change against frequency near equilibrium, works as a key criterion for distinguishing
20 microevolutionary outcomes, even in a changing environment. When this slope D is between 0 and -2
21 (consistent with the empirical examples we review), substantial fluctuations would not persist on their
22 own in a large population occupying a constant environment, but they can still be maintained indefinitely
23 as quasi-cycles fueled by environmental noise or genetic drift. However, such moderate NFDS buffers
24 and temporally shifts evolutionary responses to periodic environments (e.g. seasonality). Stronger FD,
25 with slope $D < -2$, can produce self-sustained cycles that may overwhelm responses to a changing
26 environment, or even chaos that fundamentally limits predictability. This diversity of expected
27 outcomes, together with the empirical evidence for both FD and environment-dependent selection,
28 suggest that the interplay of internal dynamics with external forcing should be investigated more
29 systematically to reach a better understanding and prediction of evolution.

30 **Keywords**

31 Frequency-dependence – Changing environment – Fluctuating selection – Chaos - Predictability.

32 **Introduction**

33 What causes variation in evolutionary trajectories, and to what extent can we predict these trajectories
34 over meaningful timescales? Beyond randomness (drift and contingency of mutation) and uncertainty
35 (measurement error) reducing the predictability of evolution (Crow and Kimura 1970; Gould 1989;
36 Lenormand et al. 2009; Blount et al. 2018; Nosil et al. 2020), an important question in many long-term
37 studies of natural populations is: What causes temporal variation in natural selection? And can we
38 predict how these causes vary over time, to predict in turn variation in selection and evolutionary
39 change? Numerous investigations of natural selection over repeated years in the wild have shown that
40 the direction and/or strength of selection may vary over time (Reimchen 1995; Grant and Grant 2002;
41 Reimchen and Nosil 2002; Siepielski et al. 2009; Bell 2010; Morrissey and Hadfield 2012; Rouzic et al.
42 2015; Nosil et al. 2018; de Villemereuil et al. 2020). However the reason for this variation is less often
43 demonstrated, not to mention directly quantified, for instance by regressing selection gradients, optimum
44 phenotypes, or selection coefficients, against putatively causal environmental variables (Wade and
45 Kalisz 1990; MacColl 2011; Chevin et al. 2015; Siepielski et al. 2017; Gompert 2021). Yet the search
46 for these causes is a necessary step towards understanding and projecting evolutionary change.

47 In particular, a critical question that has yet received little attention is: When natural selection varies
48 over time, is it because a variable external biotic or abiotic environment acts as a forcing factor on the
49 population, as suspected for instance for seasonal cycles in allelic frequency in fruit flies (Bergland et
50 al. 2014), or adaptation to climate change across a range of organisms (Hoffmann and Sgro 2011)? Or
51 is it instead because ecological feedbacks cause natural selection to depend on the current state of the
52 population, leading to internally driven dynamics, as also clearly established *in natura* (Sinervo and
53 Lively 1996; Olendorf et al. 2006; Rouzic et al. 2015; Chouteau et al. 2016; Bolnick and Stutz 2017;
54 Nosil et al. 2018; Goldberg et al. 2020)? Different traditions in evolutionary biology (both theoretical
55 and empirical) tend to favor one or the other explanation, sometimes based on prior knowledge and
56 experience of a study system, but often also on the preference and scientific background of the authors.

57 On the one hand, a large body of literature focuses on adaptation to changing environments and its
58 interplay with extinction risk, in particular with respect to global climate change and environmental
59 degradation (Lynch and Lande 1993; Bürger and Lynch 1995; Chevin et al. 2010; Hoffmann and Sgro
60 2011; Kopp and Matuszewski 2014). In this context, natural selection and its variation over time are
61 generally assumed to result from change in the external environment. This is envisioned to cause the
62 displacement of an optimum phenotype, which the population then has to track by evolution, phenotypic
63 plasticity, or their combination, as demonstrated empirically in some case studies (Vedder et al. 2013;
64 Chevin et al. 2015; Gamelon et al. 2018; de Villemereuil et al. 2020; Gauzere et al. 2020), and invoked
65 verbally in many others. Empirical work in this field often aims at testing or applying predictions from
66 an abundant theoretical literature on adaptation to a moving optimum (Lynch and Lande 1993; Bürger
67 and Lynch 1995; Chevin et al. 2010; reviewed by Kopp and Matuszewski 2014).

68 On the other hand, studies that focus on eco-evolutionary feedbacks (Hendry 2016; Lion 2018;
69 Govaert et al. 2019) or genetic conflicts (Hurst et al. 1996; Chapman et al. 2003) tend to emphasize
70 situations where the evolutionary dynamics of a population are mostly driven by its own evolution. This
71 includes a large body of empirical work on the maintenance of visible polymorphisms (Sinervo and
72 Lively 1996; Halkka et al. 2001; Oxford 2005; Nosil et al. 2018; Goldberg et al. 2020; reviewed by
73 Svensson 2017), and abundant theory on evolution driven by within-species interactions - resource
74 competition (Ackermann and Doebeli 2004), cooperation (Axelrod and Hamilton 1981), or mate choice
75 (Lande 1980) - and interactions with other species (e.g. predation, parasitism, etc.) (Abrams 2001;
76 Senthilnathan and Gavrillets 2021). Such scenarios are prone to evolutionary feedbacks, because they
77 cause natural selection to depend on the current genetic and phenotypic composition of the population;
78 in other words, to be frequency-dependent (Wright 1969), hereafter FD. In particular, negative
79 frequency-dependent selection (NFDS), where less common variants are favored (Wright and
80 Dobzhansky 1946; Wright 1969), arises in ecological scenarios such as crypsis, where search images by
81 predators penalize common prey types (Nosil et al. 2018); sexual conflict - where a similar process
82 penalizes common female types via male harassment (Svensson et al. 2005; Rouzic et al. 2015); or self-
83 incompatibility in plants, where common pollen types have fewer pistils to fertilize (Wright 1939;
84 Castric and Vekemans 2004). NFDS causes negative feedbacks, which often stabilize dynamical
85 systems, but it may also yield cycling, or even complex dynamics (Lewontin 1958; Altenberg 1991;
86 Gavrillets and Hastings 1995; Sinervo and Lively 1996).

87 While most studies of adaptation tend to favor one or the other explanation (external forcing by the
88 environment, vs internal feedbacks) for variation in natural selection, many real-world situations likely
89 include both. For instance, rising temperature may affect the way individuals within a species interact,
90 through e.g. competition (Mitchell and Angilletta 2009; Germain et al. 2018) or mating (as recently
91 shown for sex-specific ornaments in dragonflies, Moore *et al.* 2021). More specifically, Svensson et al.
92 (2020) recently showed that a female polymorphism maintained by negative frequency dependence was
93 also under temperature-dependent, frequency-independent selection at an earlier life stage. Reciprocally,
94 ecological interactions can modify the impacts of environmental change on organisms, such as
95 ecological facilitation alleviating the detrimental effects of drought (Bruno et al. 2003). Therefore, a
96 question that is likely highly relevant to many real-life situations is: When a population is subject to
97 both a changing external environment and internally driven dynamics caused by ecological interactions,
98 which of these factors is likely to dominate the evolutionary dynamics? And how does the answer to
99 this question influence the repeatability and predictability of selection and evolution?

100 These questions have received surprisingly little attention from evolutionary biologists. Svensson
101 et al. (2005) simulated a combination of NFDS with environmental noise, and Svensson and Connallon
102 (2019) recently investigated how FD affects adaptation and evolutionary rescue in a directionally
103 changing environment. Rego-Costa *et al.* (2018) showed that a cycling environment can modify the
104 predictability of evolution for quantitative traits undergoing complex forms of FD that can lead to

105 chaotic dynamics. Here, we ask more generally how FD affects the temporal variability and
106 predictability of selection and evolution in a temporally fluctuating environment. Such a coupling
107 between external forcing and internal feedbacks is an important element of realism for many populations
108 in the wild, so our aim here is to provide a simple formalism to guide our understanding and prediction
109 of their dynamics.

110 **When does frequency dependence alone cause predictable** 111 **fluctuations?**

112 Before proceeding further, it is worth clarifying when NFDS alone is likely to cause persistent
113 fluctuations in selection. Throughout this work, we focus for simplicity on discrete polymorphisms
114 determined by a single locus, as described in many empirical examples (Svensson 2017).

115 *Insights from a local stability analysis*

116 We first go back to a simple framework introduced by Lewontin (1958) to broadly characterize
117 evolutionary dynamics generated by FD in discrete generations. Lewontin (1958) showed that
118 alternative dynamic outcomes (stable equilibria, unstable equilibria, cycling) can be distinguished based
119 on a simple metric, which we here denoted as D , defined as

$$120 \quad D = \left. \frac{\partial \Delta p}{\partial p} \right|_{p=\hat{p}} \quad (1)$$

121 In Figure 1, D is the slope of the green line plotting frequency change per generation Δp against
122 frequency p , evaluated where it intersects the x-axis (equilibrium frequency \hat{p} , black dot). Negative FD
123 maintaining polymorphism is characterized by a negative slope near an internal equilibrium (with \hat{p}
124 different from 0 or 1). Note that such relationship could also be explained by other forms of balancing
125 selection (such as overdominance; see also Discussion), but we here assume it is caused by frequency-
126 dependent selection.

127 Indeed, starting at a small deviation from equilibrium $\delta = p - \hat{p}$, frequency change under selection
128 can be approximated as linear in p , that is, $\Delta p = D\delta$. Iterating over multiple generations yields

$$129 \quad \delta_t = (1 + D)^t \delta_0. \quad (2)$$

130 Equation (2) makes it clear that D determines the system behavior near an equilibrium \hat{p} . When $D > 0$
131 (positive FD), small initial deviations from equilibrium get amplified exponentially over time, and the
132 equilibrium is unstable. In contrast, negative D leads to a diversity of outcomes. If $-1 < D < 0$, then
133 δ_t decays exponentially over time, causing a gradual approach to the stable equilibrium \hat{p} (Figure 1,
134 top), with timescale $-1/\ln(1 + D)$ (faster with stronger NFDS, with D closer to -1). If $-2 < D < -1$,
135 the frequency overshoots its equilibrium in each generation, causing oscillations around \hat{p} with period
136 2, alternating ups and downs. However these oscillations are damped (Figure 1, middle), and eventually
137 vanish (persisting over a timescale of $-1/\ln[-(1 + D)]$, longer when NFDS is stronger), and a stable
138 equilibrium is again reached. Finally if $D < -2$, the frequency oscillates around the equilibrium but

139 with exponentially increasing magnitude (Figure 3, bottom).

140 This simple stability analysis (which applies more broadly to any discrete-time dynamical system,
141 e.g. Otto and Day 2007, pp163-170) shows that, when Δp is well approximated as linear in p , the system
142 moves from gradual approach to equilibrium, to damped oscillations, to unstable expanding oscillations,
143 as the strength of NFDS increases (Fig 1). Stable fluctuations, with fully predictable alternations of ups
144 and downs of fixed magnitude around the equilibrium, only occur when $D = -2$ under this linear
145 approximation, but transient fluctuations may still persist for some time if D is very close to -2 (for
146 instance, the magnitude of fluctuations is halved in about 3, 7, and 13 generations if $D = -1.8, -1.9,$
147 or -1.95 , respectively).

148 ***Strong frequency dependence can produce predictable fluctuations, but also unpredictable***
149 ***chaos***

150 We relied above on a local approximation near equilibrium, but the dependency of Δp on p cannot
151 remain linear - or even just monotonic - over the full range of relative frequencies (from 0 to 1) if FD
152 causes an internal equilibrium, because this would lead to unrealistically large frequency change near
153 fixation. Indeed, frequency change under selection can generally be written as (Wright 1969; Crow and
154 Kimura 1970)

$$\Delta p = p(1 - p) s(p), \quad (3)$$

156 where $s(p)$ is a frequency-dependent selection coefficient, and $p(1 - p)$ quantifies genetic diversity at
157 the locus. As $p(1 - p) = 0$ when $p = 0$ or 1 , frequency change Δp also must tend to 0 as alleles approach
158 fixation. This implies that the simplest way for Δp to have a negative slope with respect to p near an
159 internal equilibrium \hat{p} is by having *positive* slopes at $p = 0$ and $p = 1$, as illustrated by orange portions
160 of the curves in Figure 2. Hence, negative FD near an internal equilibrium may often imply positive FD
161 near fixation (unless the fitness function is more complex), and the same holds for other forms of
162 balancing selection such as overdominance (in fact, this was identified as a sufficient condition for
163 protected polymorphism by Prout 1968). An important question therefore is: how likely is it that
164 frequencies mostly remains within a region with negative slope, and near-linear relationship, between
165 Δp and p ? As we show below, the answer largely depends on the slope D near equilibrium.

166 Previous work has shown that generalized diploid FD, where the fitnesses of all three genotypes at
167 a bi-allelic locus (heterozygote and both homozygotes) depend linearly on all of their frequencies
168 (Altenberg 1991; Gavrillets and Hastings 1995; Rice 2004; Cockerham et al. 2015), can lead to complex
169 evolutionary dynamics, notably when heterozygotes exert strong detrimental effects on all genotypes,
170 including themselves (as shown by Altenberg 1991; Gavrillets and Hastings 1995). We here rely on
171 Rice's (2004) model (Appendix), focusing for simplicity on symmetric FD with an equilibrium
172 frequency at $\hat{p} = 1/2$ (as done by previous authors). The relationship between Δp and p can be highly
173 nonlinear in this model (Figure 2), but Lewontin (1958)'s simple criterion above still provides a useful
174 guideline. When $-2 < D < 0$, the system behaves as predicted by its linear approximation near

175 equilibrium: a stable equilibrium \hat{p} is reached regardless of initial frequency (Figure 2, top left),
 176 preceded by damped oscillations if $-2 < D < -1$ (Figure 2 top middle-left). In contrast for steeper
 177 slopes $D < -2$, the behavior is influenced by the non-linearity of Δp with respect to p . In this symmetric
 178 model, the behavior of the system is then determined by where the line with slope -2 going through \hat{p}
 179 (that is, the $1 - 2p$ line, in dashed black in Figure 2, top) intersects with the frequency change curve. If
 180 intersections occur in the part of the curve with negative FD (in green in Figure 2), then a limit cycle is
 181 reached (independent of initial conditions), where frequencies oscillate between these intersections (red
 182 dots in Figure 2). The magnitude of these fluctuations increases as the steepness of NFDS increases,
 183 causing the red dots in Figure 2 to move farther apart. Under very strong NFDS ($D \ll -2$), FD is positive
 184 at the intersection (red dots in orange part of the curve, right panel of Figure 2). The frequency p thus
 185 regularly explores regions with both positive and negative FD. Interestingly, this causes the dynamics
 186 to become chaotic, such that frequency change displays no obvious pattern, and slight differences in
 187 initial conditions can lead to very different evolutionary trajectories (Altenberg 1991; Gavrilets and
 188 Hastings 1995). When this occurs, even though the dynamics are completely deterministic, they cannot
 189 be predicted even over short timescales, because the strong dependency on initial conditions means that
 190 any measurement error is going to be amplified considerably.

191 In summary, NFDS by itself can only produce persistent frequency fluctuations if the relationship
 192 between Δp and p is very steep ($D < -2$). However, if this relationship is too steep, the system will
 193 regularly explore regions with positive and negative FD and become chaotic, so the fluctuations will not
 194 be predictable.

195 **How does frequency dependence affect the predictability of** 196 **evolution in a fluctuating environment?**

197 We have just seen that moderately strong NFDS (with $-2 < D < 0$) cannot maintain persistent
 198 fluctuations on its own. But what if an external perturbation, such as a temporally varying environment,
 199 interacts with the internal dynamics caused by FD?

200 *Moderate NFDS can increase the predictability of evolution in unpredictable environments*

201 We start by considering perturbations that are themselves random, and thus unpredictable, such as
 202 environmental noise or genetic drift. For simplicity we rely on Lewontin's (1958) linearized model of
 203 NFDS (Figure 1), and add a noise component to it. When the equilibrium is attracting ($-2 < D < 0$),
 204 the recursion for $\delta = p - \hat{p}$ becomes

$$205 \quad \delta_{t+1} = (1 + D)\delta_t + \sigma W_t, \quad (4)$$

206 where W_t is drawn from a standard normal (white noise), and σ is the standard deviation of the noise
 207 process. Beyond the assumption of linear FD, equation (4) further assumes that noise variance of
 208 frequency change is independent of frequency, which is generally not true under fluctuating selection
 209 (Wright 1948; Kimura 1954; Gillespie 1973; Chevin 2019), but may be a good approximations if

210 frequencies remain sufficiently close to 1/2.

211 Equation (4) implies that in a white noise (non-autocorrelated) environment, δ may be
212 approximated as a first-order autoregressive process (AR1). Such a process is stationary, such that the
213 variance of random fluctuations in frequency eventually reaches a constant value, $V(\delta) = V(p) =$
214 $-\frac{\sigma^2}{D(2+D)}$, which is highest towards $D = -2$ and $D = 0$, and minimum at $D = -1$, where it equals the
215 variance of the external perturbation, $V(p) = \sigma^2$. In finite populations, σ^2 also includes a component
216 caused by genetic drift, with variance $\frac{p(1-p)}{2N_e}$ (in diploids), but this component should cause moderate
217 fluctuations unless the variance effective population size N_e is very small.

218 In the presence of noise, although fluctuations in frequency are random, they still have some
219 predictable aspects. In particular frequency change, which is often the main focus in studies of
220 evolutionary dynamics (Nosil et al. 2018; Goldberg et al. 2020), has autocorrelation $\rho(\Delta p) = \frac{D}{2}$ over 1
221 generation. Hence, frequency changes is negatively autocorrelated under NFDS, all the more so as the
222 absolute strength of FD increases, but with no influence of the magnitude of noise, as long as noise
223 exists and can be accurately modeled by eq. (4). For large negative slopes ($D \rightarrow -2$), autocorrelation
224 tends towards $\rho(\Delta p) = -1$, such that increases in frequency are almost certainly followed by decreases
225 in frequency of similar magnitude (and vice versa). The short-term predictability of evolution can be
226 defined as the proportion of variance in frequency change that is explained by the previous frequency
227 change. From eq. (4), this is simply

$$228 \quad \rho^2(\Delta p) = \frac{D^2}{4}, \quad (5)$$

229 which saturates at its maximum of 1 for $D = -2$, and should remain high even under stronger FD ($D <$
230 -2), as long the dynamics are not chaotic (see below). This analysis can easily be extended to the case
231 where noise is itself autocorrelated. For instance if the noise process is autoregressive of order r , then
232 fluctuations in allelic frequency become autoregressive of order $r + 1$ (Karlín and Taylor 1981; Box et
233 al. 2008). These have more complex dynamics, with $r + 1$ embedded time scales providing more
234 “memory” to the process, but they should still be characterized by rapid fluctuations around the
235 equilibrium frequency \hat{p} as long as $-2 < D < -1$.

236 To investigate the robustness of these predictions to the approximations in eq. (4), we carried out
237 simulations with randomly fluctuating selection. Without FD, maintenance of polymorphism in such
238 temporally varying environments is possible when temporal variation in selection leads to associative
239 overdominance, whereby the long-term, geometric fitness of the heterozygote is larger than those of
240 both homozygotes, even when there is no overdominance in any specific generation (Haldane and
241 Jayakar 1963). This may occur under beneficial reversal of dominance, such that that the heterozygote’s
242 fitness is always closer to that of the most-fit homozygote (Posavi et al. 2014; Wittmann et al. 2017), or
243 even without dominance in any generation (Haldane and Jayakar 1963; Lande 2008). Here, we model
244 the latter for simplicity, by assuming that selection on codominant alleles is reversed symmetrically

245 across environments (following Lande 2008), and combine this with the diploid FD model in Figure 2
246 (more details in the Appendix).

247 Figure 3 shows that without FD, the frequency fluctuates erratically under the influence of the
248 random environment, with a temporal mean of $p = 1/2$ set by associative overdominance. Under
249 intermediate NFDS ($D = -1.8$), frequency fluctuates less erratically than without FD, instead
250 displaying alternations of ups and downs around the equilibrium frequency $\hat{p} = \frac{1}{2}$. Remarkably, this FD
251 strength would lead to damped oscillations in a constant environment (Figure 2), but when combined
252 with a random environment these oscillations are maintained indefinitely as quasi-cycles, through a
253 mechanism called stochastic resonance (Nisbet and Gurney 1976; Boettiger 2018). Under stronger FD
254 ($D = -3$), fluctuations display internally driven 2-generation cycles, with a magnitude influenced by
255 the random perturbations, but not much their general pattern. Lastly under very strong FD, fluctuations
256 become erratic again, and with much larger magnitude than those caused by the randomly fluctuating
257 environment, as a result of chaos driven by FD.

258 Figure 4 shows how the temporal predictability of frequency change over 1 generation depends on
259 the strength of frequency dependence in these simulations. Predictability is 0 without FD, as expected
260 since environmental forcing is white noise. Our simple approximation that assumes linear frequency
261 dependence (eq. 5, dashed red line in Figure 4) works remarkably well over the entire range over which
262 it is defined ($0 \leq -D \leq 2$), and even though the true FD of Δp is clearly not linear (Figure 4). The
263 autocorrelation of these fluctuations is negative (inset in Fig. 4), since NFDS causes alternations of ups
264 and downs. The predictability of fluctuations remains close to its maximum of 1 under stronger
265 frequency dependence $2 \leq -D \leq 3$ (horizontal red dashed line in Fig. 4), because FD then causes
266 predictable internal fluctuations, which are only marginally perturbed by the random noise (as seen in
267 Figure 3). Beyond this point, FD starts to decrease the predictability of evolution, because the dynamics
268 become chaotic. Interestingly, the transition in predictability is not abrupt as chaos arises. This is perhaps
269 because the contribution of chaos to predictability depends on how the magnitude of chaotic fluctuations
270 relates to that of random noise in selection.

271 ***Strong NFDS can decrease the predictability of evolution in a predictable environment***

272 Let us now turn to the opposite situation, where the changing environment is highly predictable, but FD
273 perturbs evolutionary dynamics in a way that reduces their predictability. We illustrate this scenario by
274 considering a highly predictable aspect of seasonality (eg photoperiod), causing yearly cycles in
275 selection with a period of 20 generations, consistent with observed seasonal fluctuations of allelic
276 frequencies across the genome of *Drosophila* in northern America (Bergland et al. 2014; Wittmann et
277 al. 2017). We use the same model as above for the influence of the environment on selection (Haldane
278 and Jayakar 1963; Lande 2008), but now assume that this environment is periodic, such that the selection
279 coefficient has mean 0, and cycles from positive to negative once every year, which lasts 20 generations
280 (Appendix).

281 The strength of FD also has large impacts on evolutionary dynamics in this context. Without FD,
 282 allelic frequencies settle into periodic fluctuations, well approximated by a sine wave with the same
 283 period T as the selection coefficient. The amplitude of these cycles is approximately multiplied by $\frac{T}{8\pi}$
 284 relative to cycles in selection (Appendix). This amplitude increases with increasing period of
 285 fluctuations, because more generations per year allow more accumulation of frequency change in each
 286 cycle. In addition, cycles in frequency lag behind cycles in selection by a quarter period (as shown in
 287 the Appendix), such that increases in frequency coincide with positive selection coefficients (gray
 288 shading in Figure 5). These analytical predictions appear as red dashed line in Figure 5, left.

289 NFDS modifies these patterns in a number of ways. When the strength of FD is appreciable, but
 290 not sufficient to cause fluctuations by itself, the cycles retain the same period as the environment, but
 291 with a smaller amplitude, and a shifted phase. Approximating FD as linear as previously (Appendix),
 292 the amplitude of fluctuations is multiplied by

$$293 \quad R_A = \frac{2\pi}{\sqrt{4\pi^2 + D^2 T^2}} \quad (6)$$

294 relative to the case without FD (Appendix). R_A is at most 1 under weak FD and rapid environmental
 295 fluctuations ($D^2 T^2 \ll 4\pi^2$), and decreases with increasing absolute strength of FD and period of
 296 environmental fluctuations, tending towards $\frac{2\pi}{DT}$ when both are large. The periodic lag, or phase shift,
 297 between the dynamics of allelic frequencies and fluctuating selection, is approximately

$$298 \quad L = \frac{\text{ArcTan}[-\frac{2\pi}{DT}]}{2\pi}, \quad (7)$$

299 which tends to 1/4 (as without FD) under weak FD ($-DT \ll 2\pi$), but decreases with increasing strength
 300 of FD, tending to 0 under strong FD ($-DT \gg 2\pi$). Hence, as the strength of FD increases, the frequency
 301 cycles are increasingly buffered, and synchronized with cycles in selection, such that the highest
 302 frequency coincides with the largest selection coefficient (Figure 5, second panel; gray shadings are
 303 periods with positive selection coefficients).

304 Under stronger NFDS ($D = -2.3$ in Figure 5), FD by itself generates cycles with period 2,
 305 superimposed on the buffered cycles of period 20 caused by the fluctuating environment (which are still
 306 well described by the analytical prediction, dashed red line in Figure 5). Although the pattern of
 307 fluctuations are more complex, evolutionary trajectories remain fully repeatable (all replicates are
 308 confounded with their average, black line in Figure 5). This is in sharp contrast with what happens under
 309 very strong FD ($D = -4.33$ in Figure 5). In this chaotic regime, even minute differences in initial
 310 conditions lead to completely different and erratic evolutionary trajectories (colored lines for different
 311 replicates), such that the average trajectory (in black) displays no clear pattern over time, and tends
 312 towards the equilibrium frequency $\hat{p} = \frac{1}{2}$.

313 Figure 6 shows how the variability and repeatability of evolutionary trajectories changes with the
 314 strength of FD in such a periodic environment. A sharp threshold can be seen towards $D = -3.5$. Below
 315 this threshold, there is essentially no variance across replicates starting from very similar initial

316 conditions (Figure 6A), as they all converge to limit cycles determined by the environment (and possibly
317 also by FD). Above the threshold, the variance among replicates first increases abruptly, then keeps
318 increasing more smoothly with the strength of FD. The temporal variance of evolutionary trajectories
319 (Fig. 6B), which corresponds to the variance over time of the black line in Figure 5, first decreases with
320 increasing strength of FD, consistent with the buffering effect of FD (eq. 6). This decline over $0 \leq$
321 $-D \leq 2$ is well captured by our analytical approximation (red dashed line in Figure 6B). However when
322 FD becomes strong enough to cause fluctuations by itself ($2 \leq -D \leq 3.5$), the temporal variance
323 increases with increasing FD, because steeper FD should lead to fluctuations of higher magnitude
324 (Figure 2). Finally under chaotic fluctuations ($-D \geq 3.5$), the absence of consistent pattern across
325 replicates translates into a mean trajectory that does not vary much over time.

326 These effects can be summarized by computing the repeatability of evolutionary trajectories,
327 defined as the proportion of their total variance that is explained by the variance in the mean trajectory
328 over time (as used by Rego-Costa *et al.* 2018 in a similar context). This measures the extent to which
329 evolution in one replicate can be predicted from the average of other replicates. Strikingly, the
330 repeatability of evolution in a periodic environment remains close to 1 for all FD strengths that do not
331 produce chaos ($-D \leq 3.5$), but then suddenly shifts to almost 0 past this threshold. Hence, evolution
332 in response to a predictable environment can switch abruptly from highly predictable to highly
333 unpredictable as the strength of FD increases, causing the dynamics to become chaotic.

334 Discussion

335 The combination of internal feedbacks caused by ecological interactions with external forcing caused
336 by a changing environment is likely to be common and widespread in nature (Germain *et al.* 2018;
337 Svensson and Connallon 2019; Svensson *et al.* 2020; Grainger *et al.* 2021; Moore *et al.* 2021). We thus
338 wished to understand (1) how frequency dependence interacts with a changing environment (or
339 equivalently, random perturbations caused by genetic drift) in driving evolutionary dynamics; and (2)
340 how this impacts the pattern and predictability of evolution. Our analysis reveals that whether, how, and
341 how strongly FD influences evolutionary dynamics and their predictability, crucially depends on the
342 strength of FD, and on how FD interacts with a changing environment. In addition, we show that a
343 simple criterion proposed over 60 years ago by Lewontin (1958) serves as a very useful yardstick for
344 understanding these dynamics, even in regimes it was not originally designed for.

345 In the absence of any external perturbation, FD of moderate strength is unlikely to maintain
346 predictable patterns of frequency change for long in large populations. Fluctuations are instead likely to
347 be transient, leading to a stable equilibrium. Although a stable equilibrium is predictable in a sense, and
348 absence of evolution can inform about the existence of selective processes (Eldredge *et al.* 2005), a
349 constant frequency would generally not be analyzed in terms of the predictability of evolutionary
350 dynamics. Very strong FD can maintain long-term, predictable fluctuations in frequencies, but may also
351 lead to unpredictable chaotic dynamics (Altenberg 1991; Gavrillets and Hastings 1995).

352 Although this may suggest the FD should not influence patterns of fluctuating selection unless it is
353 very strong, this is not necessarily true. The reason is that (i) virtually any population is exposed to
354 temporal changes in its natural environment, causing natural selection to vary over time (Reimchen
355 1995; Grant and Grant 2002; Bell 2010; Chevin et al. 2015; de Villemereuil et al. 2020); and (ii) FD can
356 alter evolutionary responses to such temporally varying selection. Interestingly, we here show that even
357 FD that would be too weak to maintain long-term fluctuations by itself can still induce partly predictable
358 fluctuations, when noise also perturbs frequency change. Here, we assumed that this noise was caused
359 by a randomly fluctuating environment, but it may also be due to genetic drift, with the relative
360 importance of these two sources of randomness depending on the product of the variance in selection
361 by the effective population size (Chevin 2019). Drift may thus be likely to play a more prominent role
362 in vertebrates (e.g. side-blotched lizards, Sinervo and Lively 1996; Sinervo et al. 2000) than it does in
363 insects (Oxford 2005; Rouzic et al. 2015). Regardless of its origin, when noise is added to an
364 evolutionary system subject to FD, it can reveal its intrinsically cycling nature. This occurs because
365 noise causes the system to enter a regime known as stochastic resonance (Nisbet and Gurney 1976;
366 Boettiger 2018), where it undergoes quasi-cycles that are much more predictable than the noise itself
367 (Figs 3-4). An unexpected consequence of this phenomenon is that factors thought to decrease the
368 predictability of evolution (unpredictable environmental noise, or drift) can actually contribute to
369 establishing persistent, partly predictable fluctuations in frequency. Noise can therefore contribute to
370 improving information about evolutionary processes, as previously described for ecology (Boettiger
371 2018). Or to put it differently, NFDS can transform inherently unpredictable evolutionary responses to
372 stochastic noise into largely predictable ones. Previous work had suggested that random perturbations
373 may be necessary to reveal the fluctuations inherent to NFDS (Svensson et al. 2005; Rouzic et al. 2015),
374 but we here demonstrate this principle more formally, and quantify it. For instance, our results in Figure
375 4 confirm the intuition by Oxford (2005) that an almost flat relationship between Δp and p where it
376 crosses the x-axis (D close to 0) would lead to little contribution of NFDS to frequency change and weak
377 predictability of evolution over most observed range of frequencies.

378 At the other end of the spectrum, FD can interfere with highly predictable dynamics driven by a
379 periodic environment, such as seasonality. First, FD that is too weak to lead to fluctuations on its own
380 can still buffer evolutionary responses to periodic cycles in the environment, as illustrated in Figs 5 and
381 6. This buffering may make the influence of the periodic environment more difficult to detect
382 empirically. Stronger FD further causes cycles with their own periodicity, which may conceal the
383 influence of the periodic environment (in addition to also buffering it). Finally, extremely strong FD can
384 lead to chaotic dynamics, making evolution highly unpredictable because of a strong dependence on
385 initial conditions, thus overwhelming the responses to the predictable environment.

386 Strikingly, this diversity of outcomes is well predicted by a simple criterion proposed by Lewontin
387 (1958), based on the slope D of frequency change against frequency near an equilibrium frequency (eq.
388 1 and Figure 1). This criterion, typical of stability analysis (Otto and Day 2007), was designed for

389 constant environments, but also largely drives evolutionary outcomes when FD is combined with a
390 changing environment (Figures 3-6), so it should be a key ingredient for understanding and predicting
391 evolution in this context. Empirical estimates for D can be extracted from a few examples from the
392 literature. Goldberg *et al.* (2020) recently reported a slope of $D = -0.23$ for changes in the frequencies
393 of morphs of the plant *Datura wrightii* over 20 years. Nosil *et al.* (2018) studied changes in the frequency
394 of a striped morph among all green morphs of the walking stick *Timema cristinae* over 18 years; by
395 reanalyzing their dataset, we find that $D = -1.06$. Similarly reanalyzing the dataset of Rouzic *et al.*
396 (2015), which consists of multiple populations of the damselfly *Ischnura elegans*, we find $D = -0.95$
397 for the frequency of a male mimic morph in females. Wright and Dobzhansky (1946) analyzed changes
398 in the frequencies of inversions in experimental populations of the fruit fly *Drosophila pseudoobscura*,
399 over 3 to 4 generations in the laboratory. Transforming from their slightly different estimate of frequency
400 dependence (Appendix), we find $D = -0.27$. In all these examples, the strength of FD is thus moderate,
401 but not weak: it falls within the interesting range where NFDS would not cause persistent fluctuations
402 on its own, but can modify responses to a fluctuating environment (Figs 3-6). This is all the more striking
403 as the initial aim of Wright and Dobzhansky's (1946) experiment was to reproduce experimentally, and
404 thus better understand, seasonal cycles in frequency, as still currently observed in fruit flies using
405 genomic data (Bergland *et al.* 2014). Similarly, the demonstration by Svensson *et al.* (2020) that
406 temperature drives a frequency-independent component of viability selection on female color morphs in
407 *Ischnura elegans* damselflies suggests that seasonality could lead to periodic selection in this species
408 (although at a within-generation timescale).

409 That the parameter D captures important features of evolutionary dynamics with FD does not mean
410 that it is sufficient by itself to understand how selection operates in any particular system. Indeed, D is
411 a very summarized metric, and different selective scenarios may lead to undistinguishable slopes, or
412 even overall relationships, between Δp and p . This was already emphasized by Wright and Dobzhansky
413 (1946), who showed that the relationship between Δp and p that they observed was as consistent with
414 frequency dependence as it was with (possibly sex-specific) overdominance. A formal demonstration of
415 FD thus requires demonstrating that the individual (or marginal) fitness of each genotype/phenotype
416 depends on the genetic/phenotypic composition of the population, as done experimentally in e.g. guppies
417 (Olendorf *et al.* 2006), sticklebacks (Bolnick and Stutz 2017), stick insects (Nosil *et al.* 2018), or
418 *Heliconius* butterflies (Chouteau *et al.* 2016, involving positive rather than negative FD). On the other
419 hand, FD of *individual fitness* only leads to FD *selection* if genotypes/phenotypes differ in how their
420 fitness depends on frequency.

421 We have used one of the simplest population genetic models of FD at a single bi-allelic locus
422 (leading to e.g. discrete morphs), allowing the argument to be expressed in terms of empirically
423 accessible quantities. This is in line with most empirical investigations of FD in the wild, which have
424 typically focused for simplicity on discrete categories, such as color polymorphisms (Sinervo and Lively
425 1996; Halkka *et al.* 2001; Oxford 2005; reviewed by Svensson 2017). Nevertheless, the prevalence of

426 discrete traits in work on FD is only witness to their ease of study, and many ecologically relevant traits
427 instead exhibit polygenic, quantitative heritable variation (Walsh and Lynch 2018). There is no reason
428 why FD selection should be less prevalent for quantitative traits, although it is clearly less investigated.
429 FD selection can readily be inferred empirically on quantitative traits, by including phenotypes of
430 interactors when estimating fitness surfaces (Wolf et al. 1999; Santostefano et al. 2020). On the
431 theoretical side, FD selection on quantitative traits has long been modeled, by letting the individual
432 fitness function depend on the mean phenotype, or other aspects of the phenotype distribution (Slatkin
433 1979; Doebeli 1996; Burger and Gimelfarb 2004; Svensson and Connallon 2019). However,
434 understanding whether a simple metric (such as D) also delineates evolutionary outcomes in this context
435 – including in a changing environment – would require further work. For instance, evolutionary theory
436 has made it clear that typical measurements of selection on quantitative traits (selection gradients and
437 differentials) need to be handled with care in the presence of FD (Lande 1976; Abrams et al. 1993). In
438 addition, some evolutionary outcomes may differ qualitatively for quantitative traits. For instance, the
439 evolutionary dynamics of quantitative traits may remain partly predictable even when chaotic if
440 environmental fluctuations are larger than the chaotic attractor, such that the mean phenotype still
441 overall tracks a periodic optimum phenotype (Rego-Costa et al. 2018), whereas frequencies of discrete
442 morphs are necessarily bounded between 0 and 1.

443 Even with discrete types, evolutionary dynamics under NFDS could differ from our model in a
444 number of ways. First, we assumed discrete non-overlapping generations, which are generally more
445 prone to fluctuations in ecology and evolution (e.g. May 1976). Interestingly, most of the empirical
446 examples highlighted above (from univoltine insects to short-lived lizards) are in fact very close to
447 having discrete non-overlapping generations, which may explain why they also display fluctuations in
448 the field. Second, eco-evolutionary feedbacks may be more complex than can be summarized by a
449 simple dependence of selection on frequency. For instance, such feedbacks may materialize as a
450 combination of FD with density dependence, mediated by environmental factors such as resources or
451 interacting species (Heino et al. 1998; Lion 2018). Interestingly, such an interplay of FD selection with
452 density-dependent r/K -selection was shown to cause persistent fluctuations in the frequencies of female
453 color morphs in the side-blotched lizard (Sinervo et al. 2000), while intransitive rock-paper-scissor FD
454 interactions with no density dependence only led to transient fluctuations for male color morphs in the
455 same species (Sinervo and Lively 1996; Sinervo 2001). Likewise, an interplay between FD and density-
456 dependent selection (along with selection based on temperature) appears to act in *Timema* stick-insects
457 (Farkas and Montejo-Kovacevich 2014; Nosil et al. 2018). Third, when the changing environment is an
458 interacting species (predator, competitor, parasite, etc.), internal dynamics may influence the external
459 forcing, causing temporal changes in the FD function illustrated in Figs 1-2. All these scenarios would
460 be worth investigating thoroughly in future studies.

461 Despite these complexities and challenges, our simple theoretical results may help understand and
462 interpret temporal microevolutionary patterns, by providing clear predictions based on population

463 metrics that are relatively simple to obtain empirically (e.g., Wright and Dobzhansky 1946; Rouzic et
464 al. 2015; Nosil et al. 2018; Goldberg et al. 2020), ideally coupled with manipulative, individual-level
465 evidence (Olendorf et al. 2006; Chouteau et al. 2016; Bolnick and Stutz 2017). Our hope is that this
466 work will stimulate empirical approaches that account for what should be an important aspect of many
467 evolutionary systems: an interplay of internal dynamics caused by frequency-dependent interactions,
468 with external forcing caused by a changing environment.

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476 **Supporting information**

477 Online Appendix

478 Mathematica notebook with simulations and derivations.

479 **References**

- 480 Abrams, P. A. 2001. Modelling the adaptive dynamics of traits involved in inter- and intraspecific
481 interactions: An assessment of three methods. *Ecol. Lett.* 4:166–175.
- 482 Abrams, P. A., Y. Harada, and H. Matsuda. 1993. On the relationship between quantitative genetic and
483 ESS models. *Evolution* (N. Y). 47:982–985.
- 484 Ackermann, M., and M. Doebeli. 2004. Evolution of niche width and adaptive diversification. *Evolution*
485 (N. Y)., doi: 10.1111/j.0014-3820.2004.tb01614.x.
- 486 Altenberg, L. 1991. Chaos from linear frequency-dependent selection. *Am. Nat.*, doi: 10.1086/285204.
- 487 Axelrod, R., and W. Hamilton. 1981. The evolution of cooperation. *Science* (80-.). 211:1390–1396.
488 American Association for the Advancement of Science.
- 489 Bell, G. 2010. Fluctuating selection: the perpetual renewal of adaptation in variable environments.
490 *Philos. Trans. R. Soc. B-Biological Sci.* 365:87–97.
- 491 Bergland, A. O., E. L. Behrman, K. R. O’Brien, P. S. Schmidt, and D. A. Petrov. 2014. Genomic
492 Evidence of Rapid and Stable Adaptive Oscillations over Seasonal Time Scales in *Drosophila*.
493 *PLoS Genet.* 10:e1004775.
- 494 Blount, Z. D., R. E. Lenski, and J. B. Losos. 2018. Contingency and determinism in evolution: Replaying
495 life’s tape. *Science* (80-.). 362:6415.

496 Boettiger, C. 2018. From noise to knowledge: how randomness generates novel phenomena and reveals
497 information. *Ecol. Lett.* 21:1255–1267.

498 Bolnick, D. I., and W. E. Stutz. 2017. Frequency dependence limits divergent evolution by favouring
499 rare immigrants over residents. *Nature*, doi: 10.1038/nature22351.

500 Box, G. E. P., G. M. Jenkins, and G. C. Reinsel. 2008. *Time series analysis*. Wiley, Hoboken, NJ.

501 Bruno, J. F., J. J. Stachowicz, and M. D. Bertness. 2003. Inclusion of facilitation into ecological theory.
502 *Trends Ecol. Evol.* 18:119–125.

503 Burger, R., and A. Gimelfarb. 2004. The effects of intraspecific competition and stabilizing selection on
504 a polygenic trait. *Genetics* 167:1425–1443.

505 Bürger, R., and M. Lynch. 1995. Evolution and extinction in a changing environment - a quantitative-
506 genetic analysis. *Evolution* (N. Y). 49:151–163.

507 Castric, V., and X. Vekemans. 2004. Plant self-incompatibility in natural populations: A critical
508 assessment of recent theoretical and empirical advances. *Mol. Ecol.* 13:2873–2889.

509 Chapman, T., G. Arnqvist, J. Bangham, and L. Rowe. 2003. Sexual conflict. *Trends Ecol. Evol.* 18:41–
510 47.

511 Chevin, L.-M., R. Lande, and G. M. Mace. 2010. Adaptation, plasticity, and extinction in a changing
512 environment: Towards a predictive theory. *PLoS Biol.* 8:e1000357.

513 Chevin, L.-M., M. E. Visser, and J. Tufto. 2015. Estimating the variation, autocorrelation, and
514 environmental sensitivity of phenotypic selection. *Evolution* (N. Y). 69:2319–2332.

515 Chevin, L. M. 2019. Selective sweep at a QTL in a randomly fluctuating environment. *Genetics*
516 213:987–1005.

517 Chouteau, M., M. Arias, and M. Joron. 2016. Warning signals are under positive frequency-dependent
518 selection in nature. *Proc. Natl. Acad. Sci. U. S. A.* 113:2164–2169.

519 Cockerham, C. C., P. M. Burrows, S. S. Young, and T. Prout. 2015. *Frequency-Dependent Selection in*
520 *Randomly Mating Populations*. <https://doi-org.inee.bib.cnrs.fr/10.1086/282790> 106:493–515.
521 University of Chicago Press .

522 Crow, J. F., and M. Kimura. 1970. *An Introduction to Population Genetics Theory*. Harper and Row,
523 New York.

524 de Villemereuil, P., A. Charmantier, D. Arlt, P. Bize, P. Brekke, L. Brouwer, A. Cockburn, S. D. Côté,
525 F. Stephen Dobson, S. R. Evans, M. Festa-Bianchet, M. Gamelon, S. Hamel, J. Hegelbach, K.
526 Jerstad, B. Kempnaers, L. E. B. Kruuk, J. Kumpula, T. Kvalnes, A. G. McAdam, S. Eryn
527 McFarlane, M. B. Morrissey, T. Pärt, J. M. Pemberton, A. Qvarnström, O. W. Røstad, J. Schroeder,
528 J. C. Senar, B. C. Sheldon, M. van de Pol, M. E. Visser, N. T. Wheelwright, J. Tufto, and L. M.
529 Chevin. 2020. Fluctuating optimum and temporally variable selection on breeding date in birds
530 and mammals. *Proc. Natl. Acad. Sci. U. S. A.* 117:31969–31978. National Academy of Sciences.

531 Doebeli, M. 1996. A quantitative genetic competition model for sympatric speciation. *J. Evol. Biol.*,
532 doi: 10.1046/j.1420-9101.1996.9060893.x.

533 Eldredge, N., J. N. Thompson, P. M. Brakefield, S. Gavrilets, D. Jablonski, J. B. C. Jackson, R. E.
534 Lenski, B. S. Lieberman, M. A. McPeck, and W. Miller. 2005. The dynamics of evolutionary
535 stasis. *Paleobiology* 31:133–145.

536 Farkas, T. E., and G. Montejó-Kovacevich. 2014. Density-dependent selection closes an eco-
537 evolutionary feedback loop in the stick insect *Timema cristinae*. *Biol. Lett.* 10. The Royal Society.

538 Gamelon, M., J. Tufto, A. L. K. Nilsson, K. Jerstad, O. W. Røstad, N. C. Stenseth, and B. E. Sæther.
539 2018. Environmental drivers of varying selective optima in a small passerine: A multivariate,
540 multiepisodic approach. *Evolution* (N. Y.), doi: 10.1111/evo.13610.

541 Gauzere, J., B. Teuf, H. Davi, L.-M. Chevin, T. Caignard, B. Leys, S. Delzon, O. Ronce, and I. Chuine.
542 2020. Where is the optimum? Predicting the variation of selection along climatic gradients and the
543 adaptive value of plasticity. A case study on tree phenology. *Evol. Lett.* 4:109–123. John Wiley &
544 Sons, Ltd.

545 Gavrilets, S., and A. Hastings. 1995. Intermittency and transient chaos from simple frequency-
546 dependent selection. *Proc. R. Soc. B Biol. Sci.* 261:233–238.

547 Germain, R. M., M. M. Mayfield, and B. Gilbert. 2018. The ‘filtering’ metaphor revisited: Competition
548 and environment jointly structure invasibility and coexistence. *Biol. Lett.*, doi:
549 10.1098/rsbl.2018.0460.

550 Gillespie, J. H. 1973. Natural selection with varying selection coefficients – a haploid model. *Genet.*
551 *Res.* 21:115. Cambridge University Press.

552 Goldberg, J. K., C. M. Lively, S. R. Sternlieb, G. Pintel, J. D. Hare, M. B. Morrissey, and L. F. Delph.
553 2020. Herbivore-mediated negative frequency-dependent selection underlies a trichome
554 dimorphism in nature. *Evol. Lett.* 4:83–90.

555 Gompert, Z. 2021. A population-genomic approach for estimating selection on polygenic traits in
556 heterogeneous environments. *Mol. Ecol. Resour.*, doi: 10.1111/1755-0998.13371.

557 Gould, S. J. 1989. *Wonderful life : the Burgess Shale and nature of history*. W.W. Norton, New York.

558 Govaert, L., E. A. Fronhofer, S. Lion, C. Eizaguirre, D. Bonte, M. Egas, A. P. Hendry, A. De Brito
559 Martins, C. J. Melián, J. A. M. Raeymaekers, I. I. Ratikainen, B. E. Saether, J. A. Schweitzer, and
560 B. Matthews. 2019. Eco-evolutionary feedbacks—Theoretical models and perspectives. *Funct.*
561 *Ecol.*, doi: 10.1111/1365-2435.13241.

562 Grainger, T. N., S. M. Rudman, P. Schmidt, and J. M. Levine. 2021. Competitive history shapes rapid
563 evolution in a seasonal climate.

564 Grant, P. R., and B. R. Grant. 2002. Unpredictable evolution in a 30-year study of Darwin’s finches.
565 *Science* (80-.). 296:707–711.

566 Haldane, J. B. S., and S. D. Jayakar. 1963. Polymorphism Due to Selection of Varying Direction. *J.*
567 *Genet.* 58:237–242.

568 Halkka, O., L. Halkka, and K. Roukka. 2001. Selection often overrides the effects of random processes
569 in island populations of *Philaenus spumarius* (Homoptera). *Biol. J. Linn. Soc.* 74:571–580. Oxford

570 Academic.

571 Heino, M., J. A. J. Metz, and V. Kaitala. 1998. The enigma of frequency-dependent selection. *Trends*
572 *Ecol. Evol.*, doi: 10.1016/S0169-5347(98)01380-9.

573 Hendry, A. P. 2016. *Eco-evolutionary dynamics*. Princeton University Press.

574 Hoffmann, A. A., and C. M. Sgro. 2011. Climate change and evolutionary adaptation. *Nature* 470:479–
575 485.

576 Hurst, L. D., A. Atlan, and B. O. Bengtsson. 1996. Genetic conflicts. *Q. Rev. Biol.*, doi: 10.1086/419442.

577 Karlin, S., and H. M. Taylor. 1981. *A second course in stochastic processes*. Academic Press, San Diego,
578 Ca.

579 Kimura, M. 1954. Process Leading to Quasi-Fixation of Genes in Natural Populations Due to Random
580 Fluctuation of Selection Intensities. *Genetics* 39:280–95. Genetics Society of America.

581 Kopp, M., and S. Matuszewski. 2014. Rapid evolution of quantitative traits: theoretical perspectives.
582 *Evol. Appl.* 7:169–191.

583 Lande, R. 2008. Adaptive topography of fluctuating selection in a Mendelian population. *J. Evol. Biol.*
584 21:1096–1105.

585 Lande, R. 1976. Natural selection and random genetic drift in phenotypic evolution. *Evolution* (N. Y).
586 30:314–334.

587 Lande, R. 1980. Sexual dimorphism, sexual selection, and adaptation in polygenic characters. *Evolution*
588 (N. Y). 292–305.

589 Lenormand, T., D. Roze, and F. Rousset. 2009. Stochasticity in evolution. *Trends Ecol. Evol.* 24:157–
590 165. Elsevier.

591 Lewontin, R. C. 1958. A General Method for Investigating the Equilibrium of Gene Frequency in a
592 Population. *Genetics* 43:419.

593 Lion, S. 2018. Theoretical approaches in evolutionary ecology: Environmental feedback as a unifying
594 perspective. *Am. Nat.*, doi: 10.1086/694865.

595 Lynch, M., and R. Lande. 1993. Evolution and extinction in response to environmental change. Pp. 234–
596 250 in P. Kareiva, J. Kingsolver, and R. Huey, eds. *Biotic Interactions and Global Change*. Sinauer,
597 Sunderland, Ma.

598 MacColl, A. D. 2011. The ecological causes of evolution. *Trends Ecol. Evol.* 26:514–522.

599 May, R. M. 1976. Simple mathematical models with very complicated dynamics. *Nature*, doi:
600 10.1038/261459a0.

601 Mitchell, W. A., and M. J. Angilletta. 2009. Thermal games: frequency-dependent models of thermal
602 adaptation. *Funct. Ecol.* 23:510–520. John Wiley & Sons, Ltd.

603 Moore, M. P., K. Hersch, C. Sricharoen, S. Lee, C. Reice, P. Rice, S. Kronick, K. A. Medley, and K. D.
604 Fowler-Finn. 2021. Sex-specific ornament evolution is a consistent feature of climatic adaptation
605 across space and time in dragonflies. *Proc. Natl. Acad. Sci.* 118:e2101458118. National Academy
606 of Sciences.

607 Morrissey, M. B., and J. D. Hadfield. 2012. Directional selection in temporally replicated studies is
608 remarkably consistent. *Evolution* (N. Y). 66:435–442. Wiley Online Library.

609 Nisbet, R. M., and W. S. C. Gurney. 1976. A simple mechanism for population cycles. *Nature*, doi:
610 10.1038/263319a0.

611 Nosil, P., S. M. Flaxman, J. L. Feder, and Z. Gompert. 2020. Increasing our ability to predict
612 contemporary evolution. *Nat. Commun.* 2020 111 11:1–6. Nature Publishing Group.

613 Nosil, P., R. Villoutreix, C. F. de Carvalho, T. E. Farkas, V. Soria-Carrasco, J. L. Feder, B. J. Crespi,
614 and Z. Gompert. 2018. Natural selection and the predictability of evolution in *Timema* stick
615 insects. *Science* (80-.). 359:765–770.

616 Olendorf, R., F. H. Rodd, D. Punzalan, A. E. Houde, C. Hurt, D. N. Reznick, and K. A. Hughes. 2006.
617 Frequency-dependent survival in natural guppy populations. *Nature*, doi: 10.1038/nature04646.

618 Otto, S. P., and T. Day. 2007. *A Biologist’s Guide to Mathematical Modeling in Ecology and Evolution*.
619 Princeton University Press, Princeton, NJ.

620 Oxford, G. S. 2005. GENETIC DRIFT WITHIN A PROTECTED POLYMORPHISM: ENIGMATIC
621 VARIATION IN COLOR-MORPH FREQUENCIES IN THE CANDY-STRIPE SPIDER,
622 ENOPLOGNATHA OVATA. *Evolution* (N. Y). 59:2170–2184. John Wiley & Sons, Ltd.

623 Posavi, M., G. W. Gelembiuk, B. Larget, and C. E. Lee. 2014. Testing for beneficial reversal of
624 dominance during salinity shifts in the invasive copepod *Eurytemora affinis*, and implications for
625 the maintenance of genetic variation. *Evolution* (N. Y). 68:3166–3183. John Wiley & Sons, Ltd.

626 Prout, T. 1968. Sufficient Conditions for Multiple Niche Polymorphism. *Am. Nat.* 102:493–496.

627 Rego-Costa, A., F. Débarre, and L.-M. Chevin. 2018. Chaos and the (un)predictability of evolution in a
628 changing environment. *Evolution* (N. Y). 72:375–385.

629 Reimchen, T. E. 1995. Predator-Induced Cyclical Changes in Lateral Plate Frequencies of *Gasterosteus*.
630 *Behaviour*, doi: 10.1163/156853995X00469.

631 Reimchen, T. E., and P. Nosil. 2002. Temporal variation in divergent selection on spine number in
632 threespine stickleback. *Evolution* (N. Y). 56:2472–2483.

633 Rice, S. H. 2004. *Evolutionary theory: mathematical and conceptual foundations*. Sinauer, Sunderland,
634 Massachussets.

635 Rouzic, A. Le, T. F. Hansen, T. P. Gosden, and E. I. Svensson. 2015. Evolutionary Time-Series Analysis
636 Reveals the Signature of Frequency-Dependent Selection on a Female Mating Polymorphism. *Am.*
637 *Nat.* 185:E182–E196. University of Chicago PressChicago, IL.

638 Santostefano, F., D. Garant, P. Bergeron, P. O. Montiglio, and D. Réale. 2020. Social selection acts on
639 behavior and body mass but does not contribute to the total selection differential in eastern
640 chipmunks. *Evolution* (N. Y)., doi: 10.1111/evo.13875.

641 Senthilnathan, A., and S. Gavrillets. 2021. Ecological consequences of intraspecific variation in
642 coevolutionary systems. *Am. Nat.* 197:1–17.

643 Siepielski, A. M., J. D. DiBattista, and S. M. Carlson. 2009. It’s about time: the temporal dynamics of

644 phenotypic selection in the wild. *Ecol Lett* 12:1261–1276.

645 Siepielski, A. M., M. B. Morrissey, M. Buoro, S. M. Carlson, C. M. Caruso, S. M. Clegg, T. Coulson,
646 J. DiBattista, K. M. Gotanda, C. D. Francis, J. Hereford, J. G. Kingsolver, K. E. Augustine, L. E.
647 B. Kruuk, R. A. Martin, B. C. Sheldon, N. Sletvold, E. I. Svensson, M. J. Wade, and A. D. C.
648 MacColl. 2017. Precipitation drives global variation in natural selection. *Science* (80-.). 355.

649 Sinervo, B. 2001. Runaway social games, genetic cycles driven by alternative male and female
650 strategies, and the origin of morphs. *Genetica* 112–113:417–434. Springer, Dordrecht.

651 Sinervo, B., and C. M. Lively. 1996. The rock–paper–scissors game and the evolution of alternative
652 male strategies. *Nature* 380:240–243. Nature Publishing Group.

653 Sinervo, B., E. Svensson, and T. Comendant. 2000. Density cycles and an offspring quantity and quality
654 game driven by natural selection. *Nature* 406:985–988. Nature Publishing Group.

655 Slatkin, M. 1979. Frequency-dependent and density-dependent selection on a quantitative character.
656 *Genetics* 93:755–771.

657 Svensson, E. I. 2017. Back to basics: Using colour polymorphisms to study evolutionary processes. *Mol.*
658 *Ecol.* 26:2204–2211.

659 Svensson, E. I., J. Abbott, and R. Härdling. 2005. Female Polymorphism, Frequency Dependence, and
660 Rapid Evolutionary Dynamics in Natural Populations. *Am. Nat.* 165:567–576. The University of
661 Chicago Press.

662 Svensson, E. I., and T. Connallon. 2019. How frequency-dependent selection affects population fitness,
663 maladaptation and evolutionary rescue. *Evol. Appl.* 12:1243-1258.

664 Svensson, E. I., B. Willink, M. C. Duryea, and L. T. Lancaster. 2020. Temperature drives pre-
665 reproductive selection and shapes the biogeography of a female polymorphism. *Ecol. Lett.* 23:149–
666 159.

667 Vedder, O., S. Bouwhuis, and B. C. Sheldon. 2013. Quantitative assessment of the importance of
668 phenotypic plasticity in adaptation to climate change in wild bird populations. *PLoS Biol*
669 11:e1001605.

670 Wade, M. J., and S. Kalisz. 1990. The causes of natural selection. *Evolution* (N. Y). 44:1947–1955.

671 Walsh, B., and M. Lynch. 2018. *Evolution and selection of quantitative traits*. Oxford University Press.
672 Oxford University Press, Oxford, UK.

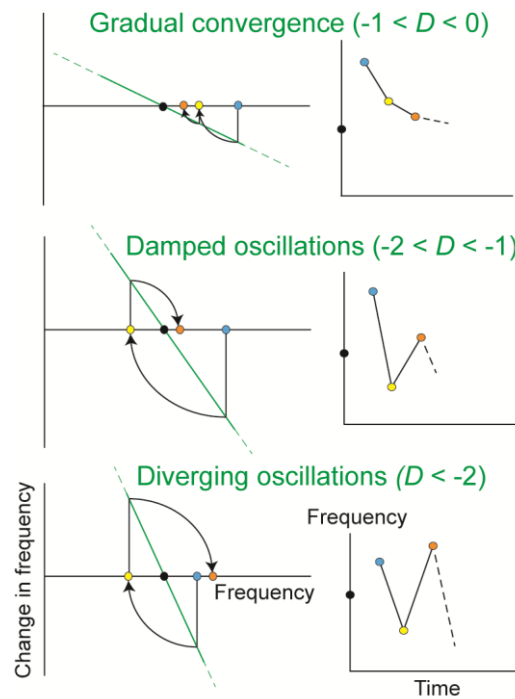
673 Wittmann, M. J., A. O. Bergland, M. W. Feldman, P. S. Schmidt, and D. A. Petrov. 2017. Seasonally
674 fluctuating selection can maintain polymorphism at many loci via segregation lift. *Proc. Natl.*
675 *Acad. Sci.* 114:E9932–E9941. National Academy of Sciences.

676 Wolf, J. B., E. D. Brodie, and A. J. Moore. 1999. Interacting phenotypes and the evolutionary process.
677 II. Selection resulting from social interactions. *Am. Nat.*, doi: 10.1086/303168.

678 Wright, S. 1969. *Evolution and the genetics of populations. Volume 2: The theory of gene frequencies*.
679 University of Chicago Press, Chicago.

680 Wright, S. 1948. On the roles of directed and random changes in gene frequency in the genetics of

681 populations. *Evolution* (N. Y). 279–294.
682 Wright, S. 1939. THE DISTRIBUTION OF SELF-STERILITY ALLELES IN POPULATIONS.
683 *Genetics* 24:538–552.
684 Wright, S., and T. Dobzhansky. 1946. *Genetics of Natural Populations*. Xii. Experimental Reproduction
685 of Some of the Changes Caused by Natural Selection in Certain Populations of *Drosophila*
686 *Pseudoobscura*. *Genetics* 31:125. Oxford University Press.
687

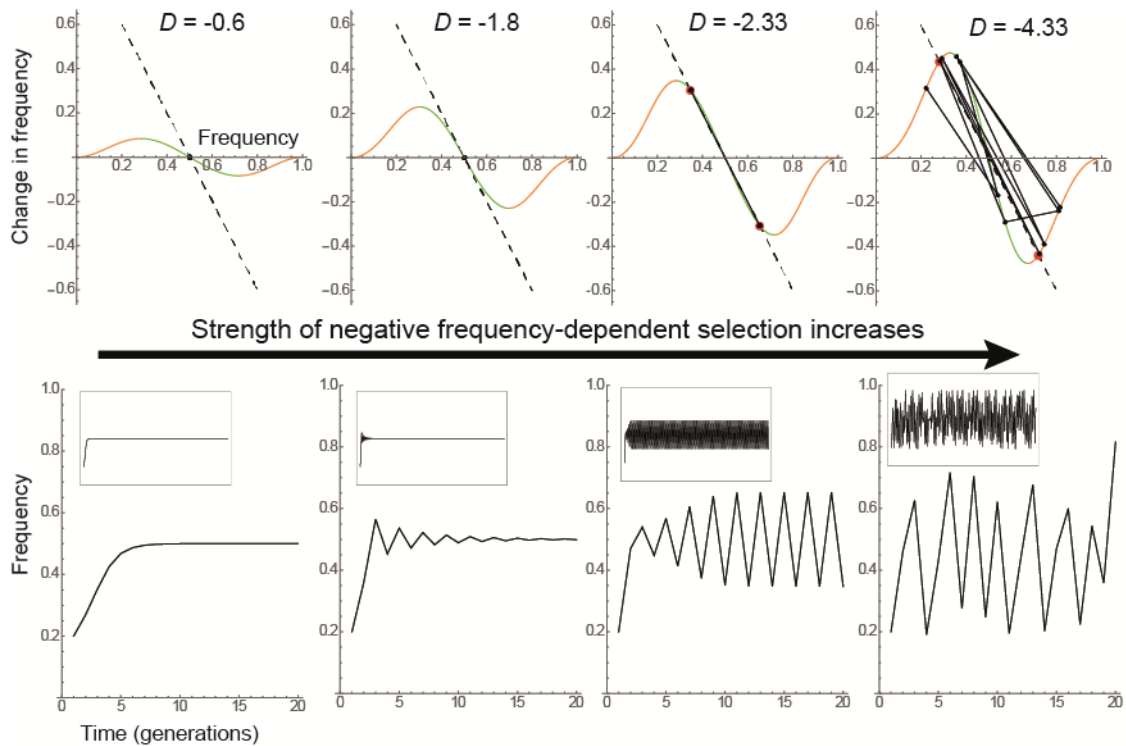


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691 **Figure 1: How the frequency dependence slope influences evolutionary dynamics.** On the left
 692 panels, the green lines plot frequency change Δp against current frequency p . This relationship is
 693 approximated as linear near an equilibrium frequency \hat{p} (black dot), and has negative slope under NFDS.
 694 Its steepness, measured by D , determines the frequency dynamics near \hat{p} . The cyan dot represents the
 695 initial frequency. Moving vertically towards the green line yields the corresponding frequency change,
 696 which increments the current frequency (via the circular arc arrow), yielding the frequency in the next
 697 generation (yellow dot). Iterating the process for one more generation yields orange dot. The system
 698 moves from gradual approach of equilibrium (top) to damped oscillations (middle) to diverging
 699 oscillations (bottom) as the steepness of the green line increases (larger negative D), as also illustrated
 700 by the frequency dynamics in the right panels.

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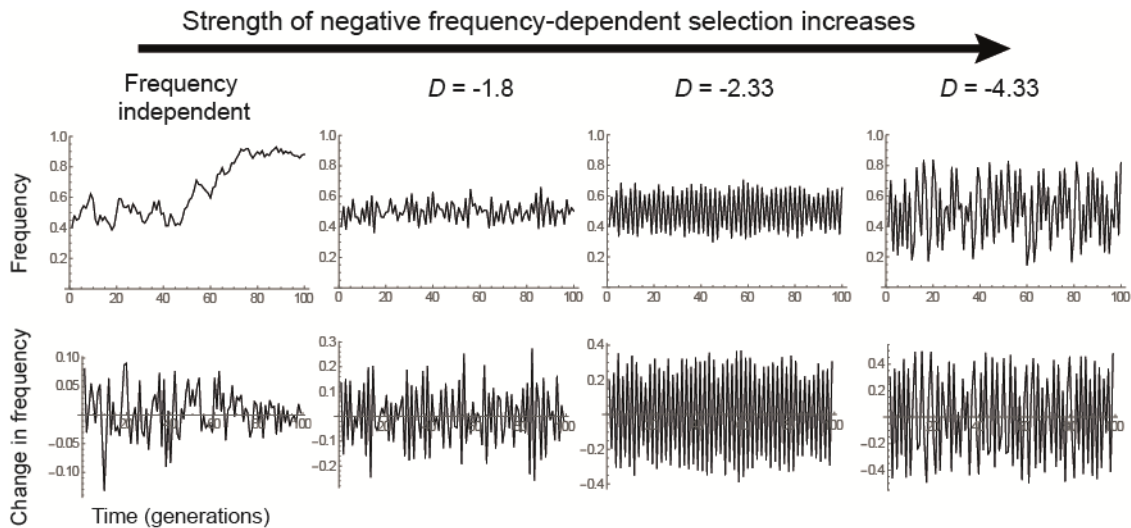


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704 **Figure 2: Fluctuations under non-linear frequency dependence.** Top row: The relationship between
 705 frequency change Δp and frequency p is shown for a diploid model of frequency dependence (adapted
 706 from Rice 2004). Green portions of the curve exhibit negative FD (downward slope), while orange
 707 portions have positive FD (upward slope). The slope D of the green portion it intersects the x axis
 708 increases from the left to the right panel. The dashed black line, with slope -2 , may intersect the Δp
 709 curve at the red dots. Evolutionary trajectories over 10 generations (away from initial conditions) are
 710 shown as black lines and dots. They appear as single dots in the two leftmost panels because a stable
 711 equilibrium is reached, while in the third panel they overlap with the dashed black line in between the
 712 red dots. Bottom row: the frequency dynamics are represented over the first 20 generations (inset: 200
 713 generations) for the same simulations. The sensitivity of heterozygote's fitness to their own frequency
 714 is s , while the sensitivity of each homozygote's fitness to the frequencies of the other genotypes is s_b
 715 (more detail in the Appendix). Parameter values are (from left to right): $s_b = 1.5$ and $s = 0.75$; $s_b =$
 716 2.5 and $s = 1.5$; $s_b = 3$ and $s = 1$; $s_b = 3$ and $s = 1.9$.

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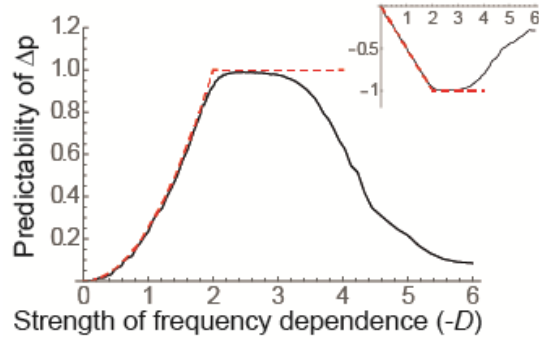


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720 **Figure 3: Frequency dependence in a randomly fluctuating environment.** The dynamics of allelic
 721 frequency (upper row) and frequency change (lower row) are shown in a diploid model of fluctuating
 722 selection caused by a random environment, combined with diploid frequency dependence as in Figure
 723 2. The fluctuating environment causes the selection coefficient of homozygotes to fluctuate randomly
 724 over time, with mean 0 and standard deviation 0.2. The frequency-dependent part of the model is as in
 725 Figure 2, with parameter values (from left to right panel): $s_b = 0$ and $s = 0$ (no frequency dependence);
 726 $s_b = 2.5$ and $s = 1.5$; $s_b = 3$ and $s = 1$; $s_b = 3$ and $s = 1.9$.

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730 **Figure 4: Predictability of evolution with frequency dependence in a randomly changing**

731 **environment.** The predictability of evolution, as measured by the squared autocorrelation of frequency

732 change over one time step, is represented against the strength of NFDS, for a population that undergoes

733 randomly fluctuating selection combined with diploid frequency dependence. The red dashed line shows

734 the analytical expectation under linear frequency dependence (eq. 5) for $0 \leq -D \leq 2$, followed by

735 saturation at 1 for $-D > 2$. The true predictability closely matches this prediction up to $-D \approx 3$, beyond

736 which chaotic dynamics reduce it. The inset shows the autocorrelation, with negative expectation $D/2$

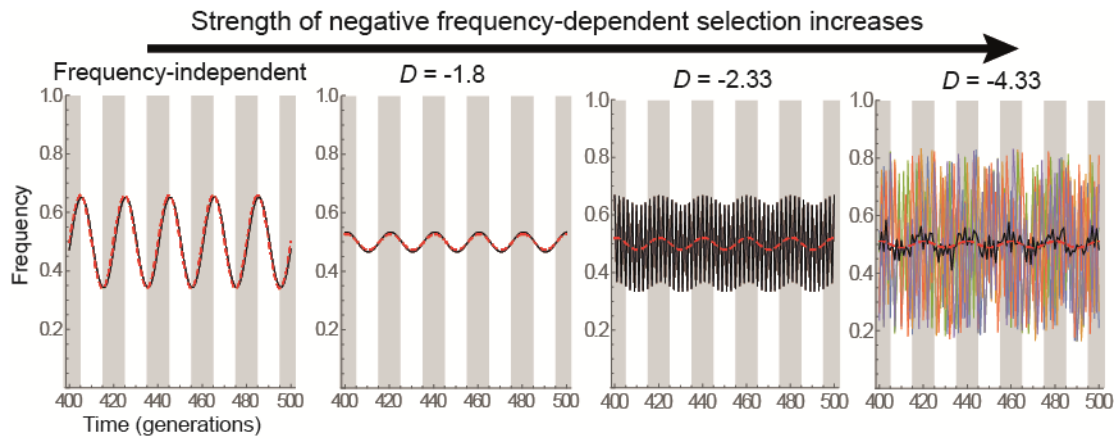
737 for $0 \leq -D \leq 2$. A single simulation was run for 5000 generations, of which the first 200 were removed

738 to compute the autocorrelation. The parameters for the randomly fluctuating environment are the same

739 as in Figure 3. For the frequency dependence, we used the same model as in previous figures, with $s_b =$

740 0 to 3 and $s = 0$ for all $0 \leq -D \leq 1.8$, and with $s_b = 3$ and $s = 0$ to 2.1 for all $1.8 \leq -D \leq 6$.

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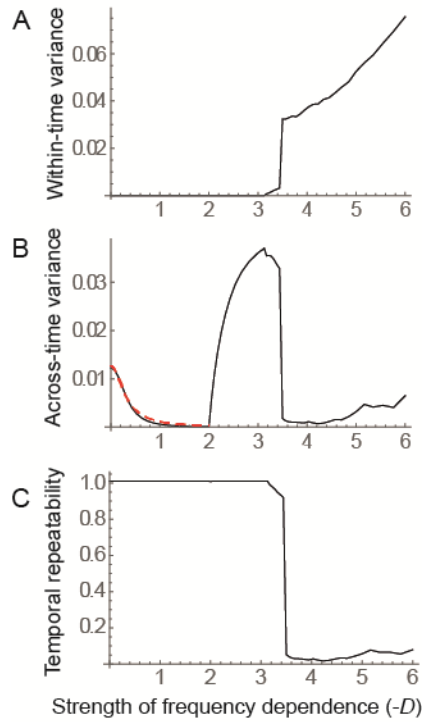


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744 **Figure 5: Frequency dependence in a periodic environment.** The dynamics of allelic frequency are
 745 represented for simulations starting from slightly different initial conditions at time 0 (SD of initial
 746 frequency: 10^{-3}). The black line shows the average over 100 replicates, and 5 individual replicates are
 747 also represented as colored lines (only visible in the rightmost panel). The red dashed lines show
 748 analytical predictions without (left) or with frequency dependence. The fluctuating environment causes
 749 the frequency-independent selection coefficient of homozygotes to undergo a deterministic sine wave
 750 (amplitude 0.2, period 20), materialized by the gray shadings when selection coefficients are positive.
 751 Generations 400 to 500 are represented to ensure that the stable cycles are reached where relevant, but
 752 the chaotic dynamics on the right appear in the first few generations, as in Figure 2. The FD part of the
 753 model is as in Figure 2, with parameter values (from left to right panel): $s_b = 0$ and $s = 0$ (no frequency
 754 dependence); $s_b = 2.5$ and $s = 1.5$; $s_b = 3$ and $s = 1$; $s_b = 3$ and $s = 1.9$.

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758 **Figure 6: Variability and repeatability of evolution with frequency dependence in a predictable**

759 **environment.** For each strength of frequency dependence, 500 replicate simulations were run starting

760 from slightly different initial conditions (SD of initial frequency: 10^{-3}), and the variability of

761 evolutionary trajectories (allelic frequencies over time) was computed over the last 200 out of 300

762 generations. A shows the variance among replicates at each time point, averaged over time (within-time

763 variance). B shows the temporal variance in the mean trajectory (across-time variance), with the red

764 dashed line representing the analytical approximation assuming linear frequency dependence. C shows

765 the repeatability of trajectories, measured as the proportion of the total variability explained by the mean

766 evolutionary trajectory over time. Repeatability equals 1 when all replicates perfectly track the mean

767 trajectory, and tends to 0 when replicate trajectories fluctuate independently from each other. The

768 parameters for the periodic environment are the same as in Figure 5. For the frequency dependence, we

769 used the same model as in previous figures, with $s_b = 0$ to 3 and $s = 0$ for all $0 \leq -D \leq 1.8$, and with

770 $s_b = 3$ and $s = 0$ to 2.1 for all $1.8 \leq -D \leq 6$.

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Online Appendix to Chevin, Gompert & Nosil:

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Frequency dependence and the predictability of evolution in a changing environment.

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Published in *Evolution Letters*.

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Details of the models

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Diploid frequency dependence

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To investigate cycles or chaos caused by frequency-dependent selection, we relied on a model by Rice (2004), itself modified from previous models of frequency dependence in diploids (Altenberg 1991; Gavrillets and Hastings 1995; Cockerham et al. 2015), by letting selection coefficients (instead of fitnesses) be frequency dependent. The model focuses on a bi-allelic locus, where the relative fitnesses w_i of the three diploid genotypes AA, Aa and aa are linear combinations of their frequencies f_i ,

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$$\begin{pmatrix} w_{AA} \\ w_{Aa} \\ w_{Aa} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \mathbf{W} \cdot \begin{pmatrix} f_{AA} \\ f_{Aa} \\ f_{aa} \end{pmatrix}. \quad (\text{A1})$$

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The matrix \mathbf{W} describes the sensitivity of each genotype's fitness to the frequencies of all genotypes (including itself, along the diagonal). It emerges from how interactions between individuals depend on their genotypes and phenotypes. Previous work has made it clear that cycle and chaos can occur when the heterozygote has detrimental effects on all genotypes (Altenberg 1991; Gavrillets and Hastings 1995; Rice 2004). For simplicity we retain the symmetry assumptions from these previous studies (whereby the homozygotes are interchangeable in terms of fitness), and further simplify the model following Rice (2004) by only keeping two parameters: s for the dependence of heterozygote fitness on their own frequency, and s_b for the frequency dependence in homozygotes (s_b was fixed to 3 in Rice 2004). We thus have

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$$\mathbf{W} = \begin{pmatrix} 0 & -s_b & s_b \\ 0 & -s & 0 \\ s_b & -s_b & 0 \end{pmatrix}, \quad (\text{A2})$$

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such that the fitness of homozygotes not only decreases with increasing frequency of heterozygotes, but also increases with increasing frequency of the other homozygote, which also leads to a form of negative FD. From these frequency-dependent fitnesses, we obtained the genotype frequencies after selection classically as $f'_i = f_i w_i / \bar{w}$ (where $\bar{w} = \sum_i f_i w_i$ is the mean fitness), and frequencies in the next generations by assuming Hardy-Weinberg equilibrium under random mating. In this model, it can be

801 shown that the slope of frequency dependence defined eq. (1) is

$$802 \quad D = \frac{3s_b - 2s}{s_b + 2s - 8} \quad (\text{A3})$$

803 ***Environmental change in selection***

804 To model a variable environment causing persistent fluctuations in allelic frequencies, we used a model
 805 modified from Haldane and Jayakar (1963), where temporal change in the relative fitnesses of
 806 homozygotes leads to associative overdominance. More precisely, we assumed that the relative fitnesses
 807 of genotypes AA, Aa and aa are $1 + s_F(t)$, 1, and $1 - s_F(t)$, where $s_F(t)$ is a temporally changing
 808 selection coefficient responding to a changing environment. If $s_F(t)$ varies over time without bias (ie,
 809 if it averages to 0 over time), then polymorphism is maintained over time due to associative
 810 overdominance, but allelic frequencies may still fluctuate in response to fluctuating selection.

811 To model a random environment, we assumed white noise for simplicity, and drew $s_F(t)$ from a
 812 normal distribution with mean 0 and variance σ_F^2 . To model predictable, cyclical (eg seasonal) change
 813 in selection, we use a cosine function with amplitude A (the maximum selection coefficient) and period
 814 T ,

$$815 \quad s_F(t) = A \cos\left(\frac{2\pi t}{T}\right). \quad (\text{A4})$$

816 Finally, when combined with frequency-dependent selection, assumed to corresponds to an independent
 817 episode of viability selection, the overall fitnesses for the three diploid genotypes are

$$818 \quad \begin{pmatrix} w_{AA} \\ w_{Aa} \\ w_{Aa} \end{pmatrix} = \begin{pmatrix} 1 + s_F(t) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 - s_F(t) \end{pmatrix} \left[\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 & -s_b & s_b \\ 0 & -s & 0 \\ s_b & -s_b & 0 \end{pmatrix} \begin{pmatrix} f_{AA} \\ f_{Aa} \\ f_{aa} \end{pmatrix} \right]. \quad (\text{A5})$$

819 ***Analysis in a periodic environment***

820 In the periodic regime defined by eq. (A4), allelic frequencies eventually settle into cycles around their
 821 average of $p = \frac{1}{2}$. Without frequency dependence, combining the periodic selection coefficient above
 822 with the classic recursion for frequency change (eq. (3) in the main text), and approximating the
 823 dynamics in continuous time, leads to the differential equation

$$824 \quad \frac{dp}{dt} = p(1 - p)A \cos\left(\frac{2\pi t}{T}\right) \approx \frac{A}{4} \cos\left(\frac{2\pi t}{T}\right), \quad (\text{A6})$$

825 where the rightmost member in eq. (A6) comes from assuming that selection is weak enough that
 826 frequencies stay close to their average $\frac{1}{2}$, such that $p(1 - p) \approx \frac{1}{4}$. Solving for p leads to

$$827 \quad p \approx \frac{1}{2} + \frac{AT}{8\pi} \cos\left[2\pi\left(\frac{t}{T} - \frac{1}{4}\right)\right]. \quad (\text{A7})$$

828 Equation (A7) indicates that allelic frequencies approximately follow sinusoidal cycles with the same
 829 period as the selection coefficient, but lagging by a quarter of a cycle, and with amplitude multiplied by

830 $\frac{T}{8\pi}$. The exact solution to eq. (A6) (not assuming that $p(1 - p) \approx \frac{1}{4}$) is $p = \frac{1}{1 + \frac{1-p_0}{p_0} \text{Exp}\left\{-\frac{AT}{2\pi} \cos\left[2\pi\left(\frac{t}{T} - \frac{1}{4}\right)\right]\right\}}$,

831 *i.e.* a logistic function of a sine wave with same period and phase as in eq. (A7). This will be well
 832 approximated by the sine wave in eq. (A7) as long as the amplitude is not large (AT small), such that

833 frequencies do not expand much above and below 0.5. For larger AT , the exact solution is bounded to
 834 remain between 0 and 1, while the approximation in eq. (A7) is not.

835 If we also include frequency dependence, approximated as linear and with slope D near the
 836 equilibrium at $\hat{p} = 1/2$, then eq. (A6) becomes (still assuming that $p(1-p) \approx \frac{1}{4}$)

$$837 \quad \frac{dp}{dt} \approx \frac{A}{4} \cos\left(\frac{2\pi t}{T}\right) + D(p - 1/2). \quad (\text{A8})$$

838 The asymptotic solution for the dynamics of frequencies, after the influence of initial conditions has
 839 vanished (in non-chaotic regimes), is after some algebra,

$$840 \quad p \approx \frac{1}{2} + \frac{AT}{4\sqrt{4\pi^2 + D^2T^2}} \cos\left[2\pi\left(\frac{t}{T} - L\right)\right]. \quad (\text{A9})$$

841 The temporal variance of these cycles, obtained by averaging $(p - \frac{1}{2})^2$ over a period, is simply half the
 842 squared amplitude, leading to

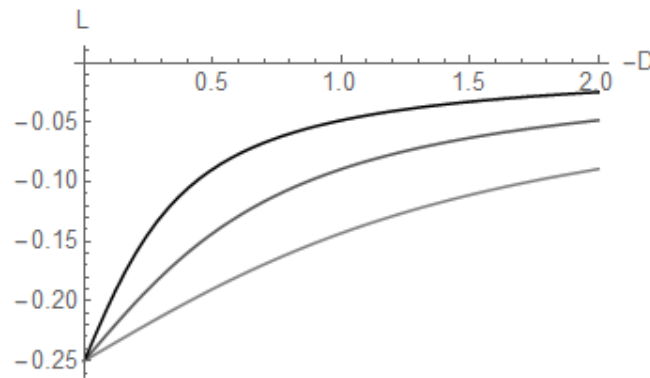
$$843 \quad V(p|cycle) = \frac{A^2T^2}{32(4\pi^2 + D^2T^2)}. \quad (\text{A10})$$

844 The periodic lag, or phase shift, between the dynamics of allelic frequencies and that of selection is

$$845 \quad L = \frac{\text{ArcTan}\left[-\frac{2\pi}{DT}\right]}{2\pi}. \quad (\text{A11})$$

846 From the properties of the arctangent function, under weak FD ($-DT \ll 2\pi$) the lag L tends towards 1/4
 847 of a cycle, as without FD. Under strong frequency dependence ($-DT \gg 2\pi$), L tends to 0, such that the
 848 cycles in allelic frequencies are synchronized with those of selection coefficients. The overall behavior
 849 of the lag behind the selection coefficient is represented below as a function of the strength of FD, for
 850 different periods of the cycling environment (5, 10, and 20, as curves become darker).

851



852

853 **Empirical estimates of the strength of frequency dependence.**

854 Wright and Dobzhansky (1946) estimated two parameters a and b , such that (from their eqs. 23-24)

$$855 \quad \Delta p = p(1-p) \frac{a-bp}{1-(a-bp)(1-2p)}. \quad (\text{A12})$$

856 The equilibrium occurs for $\hat{p} = b/a$. Taking the derivative of Δp with respect to p at \hat{p} then yields

$$857 \quad D = \frac{a(a-b)}{b}, \quad (\text{A13})$$

858 which combined with the reported estimates for $a = .902$ and $b = 1.288$ leads to $D = -0.27$. Nosil *et*
859 *al.* (2018) reported frequencies of the striped morph of *Timema cristinae* among all green morphs
860 (striped or unstriped), across host plants over 18 years. We used these frequencies (computed anew from
861 the dryad repository associated with the paper) to fit a linear model for the relationship between change
862 in frequency Δp and frequency p . We did the same for the datasets from le Rouzic *et al.* (2015), also
863 including a fixed effect for the population of origin on the intercept.

864 **Literature cited**

- 865 Altenberg, L. 1991. Chaos from linear frequency-dependent selection. *Am. Nat.*, doi: 10.1086/285204.
- 866 Cockerham, C. C., P. M. Burrows, S. S. Young, and T. Prout. 2015. Frequency-Dependent Selection in
867 Randomly Mating Populations. <https://doi-org.inee.bib.cnrs.fr/10.1086/282790> 106:493–515.
868 University of Chicago Press .
- 869 Gavrillets, S., and A. Hastings. 1995. Intermittency and transient chaos from simple frequency-
870 dependent selection. *Proc. R. Soc. B Biol. Sci.* 261:233–238.
- 871 Haldane, J. B. S., and S. D. Jayakar. 1963. Polymorphism Due to Selection of Varying Direction. *J.*
872 *Genet.* 58:237–242.
- 873 Nosil, P., R. Villoutreix, C. F. de Carvalho, T. E. Farkas, V. Soria-Carrasco, J. L. Feder, B. J. Crespi,
874 and Z. Gompert. 2018. Natural selection and the predictability of evolution in *Timema* stick
875 insects. *Science* 359:765–770.
- 876 Rice, S. H. 2004. *Evolutionary theory: mathematical and conceptual foundations*. Sinauer, Sunderland,
877 Massachussets.
- 878 Wright, S., and T. Dobzhansky. 1946. *Genetics of Natural Populations*. Xii. Experimental Reproduction
879 of Some of the Changes Caused by Natural Selection in Certain Populations of *Drosophila*
880 *Pseudoobscura*. *Genetics* 31:125. Oxford University Press.
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