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Robust localized adaptive waveform inversion: A new regularization for Gabor deconvolution

Peng Yong^{1*}, Romain Brossier¹, and Ludovic Métivier^{2,1}

¹ Univ. Grenoble Alpes, ISTerre, F-38058 Grenoble, France

² Univ. Grenoble Alpes, CNRS, LJK, F-38058 Grenoble, France

SUMMARY

Recently, we have developed a localized adaptive waveform inversion method (LAWI) to tackle the cycle-skipping issue in velocity reconstruction by waveform inversion. In LAWI, the Gabor deconvolution is applied to compute a local matching filter, whose centroid time is used for measuring the instantaneous time shift between observed and calculated data. Different from AWI that is based on a stationary convolutional model, LAWI can take the non-stationarity nature of seismic data into account, therefore performs better in handling realistic cycle skipping problem. Numerical tests show that, compared with AWI, the application of LAWI seems to require a higher signal-to-noise ratio (SNR) of observed data. To make LAWI work for low-SNR data, a delta-type regularization is developed to deal with the noise problems inherent in the Gabor deconvolution. Despite a slight resolution loss and a “layer-stripping principle break” induced by this new regularization illustrated numerically, we present how this method can be useful to invert for low-SNR data on the Chevron benchmark dataset.

INTRODUCTION

Full-waveform inversion (FWI) is a powerful tool to build high-resolution velocity models (Lailly, 1983; Tarantola, 1984). The successful field data applications (Sirgue et al., 2010; Huang et al., 2021) makes it more and more attractive for academia and industry. However, velocity reconstruction by FWI is a highly non-linear inverse problem (Virieux and Operto, 2009; Virieux et al., 2017). When initial models are not accurate enough, FWI would fail to generate geologically meaningful velocity models. Studies indicate that enhancing travel-time information in misfit design can mitigate the non-linearity and relax the accuracy requirement of the initial model (Luo and Schuster, 1991; Woodward, 1992).

Measuring time shift between observed and calculated data is a difficult and long-standing task in exploration seismology. In the early work of wave-equation traveltome tomography (Luo and Schuster, 1991), time shift is taken as the time lag that maximizes the cross-correlation function. The adjoint source derivation relies on the assumption that observed and calculated data only differ by a time shift (the same amplitude). Inspired by the differential semblance optimization for migration velocity analysis (Symes and Carazzone, 1991), van Leeuwen and Mulder (2010) propose a penalization-based cross-correlation objective function in the data domain, whose adjoint source derivation does not need the pure time shift assumption. Following, Luo and Sava (2011) suggest using deconvolution instead of cross-correlation to handle realistic oscillatory signals, as deconvolution operation can make the matching filter more focused and lead it confined to zero lag with model update.

Warner and Guasch (2016) design a normalized form of penalization-based deconvolution misfit (AWI). With the normalization, the amplitude influence of the matching filter can be reduced and the misfit in AWI can be understood as an estimation of the centroid time of the matching filter (Yong et al., 2021). With this interpretation, Yong et al. (2021) propose a non-stationary extension of AWI through a time-frequency analysis of signals using a Gabor transform. This makes it possible to capture instantaneous time-shift for locally coherent events. Numerical tests show that the localized manipulation is more suitable for non-stationary data processing, and encouraging results on Chevron 2014 benchmark data have been obtained (Yong et al., 2022).

However, when we attempt to apply LAWI approach to field data, we find that, compared to AWI, LAWI seems to require a higher data quality. This has also been noticed in the test with low-SNR 0-3 Hz

Chevron benchmark. From numerical tests, we deduce that this issue is related to the computation of the Gabor deconvolution with noisy data. In this work, we shall analyze this issue from a point view of regularization of the Gabor deconvolution. An alternative regularization is proposed for the Gabor deconvolution, which is based on the fact that the local matching filter is confined to delta function with model update in LAWI. Two main features of this regularization are studied through simple signal analysis and numerical inversion results. With this regularization, we are able to apply LAWI in the low-SNR 0-3 Hz frequency band of the Chevron benchmark.

THEORY

Localized adaptive waveform inversion

With one-trace observed data $d(t)$ and predicted data $p(t)$, a local matching filter $\hat{w}(t, \omega)$ used in LAWI, can be defined by a non-stationary convolutional model in the time-frequency domain (Yong et al., 2021):

$$\hat{w}(t, \omega) \hat{d}(t, \omega) = \hat{p}(t, \omega), \quad (1)$$

where $\hat{d}(t, \omega)$ and $\hat{p}(t, \omega)$ are the time-frequency spectra of observed and predicted data. The Gabor transform (Strang and Nguyen, 1996; Fichtner et al., 2008) is applied to obtain the time-frequency spectrum, and the Gabor transform pair can be given by

$$\hat{f}(t, \omega) = \mathcal{G}[f](t, \omega) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} f(\xi) h_{\sigma}^{\dagger}(\xi - t) e^{-i\omega\xi} d\xi, \quad (2)$$

$$f(t) = \mathcal{G}^{-1}[\hat{f}](t) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}^2} \hat{f}(\xi, \omega) h_{\sigma}(t - \xi) e^{i\omega t} d\xi d\omega. \quad (3)$$

$h_{\sigma}(t) = (\pi\sigma^2)^{-\frac{1}{2}} e^{-\frac{t^2}{2\sigma^2}}$ denotes the window function, in which σ controls the radius.

With the time-varying local matching filter $\hat{w}(t, \omega)$ in the frequency domain, we can obtain its time-domain form $w(t, \tau)$ through

$$w(t, \tau) = \mathcal{F}_{\tau}^{-1}[\hat{w}(t, \omega)], \quad (4)$$

where \mathcal{F}_{τ}^{-1} denotes the inverse Fourier transform for the variable τ . The instantaneous centroid time shift can be estimated by

$$T(t) = \frac{\int_{\mathbb{R}} |\tau| w^2(t, \tau) d\tau}{\int_{\mathbb{R}} w^2(t, \tau) d\tau}. \quad (5)$$

Integrating all instantaneous time shift under L^2 norm, we can define the misfit function of LAWI as

$$\mathbb{J}_{LAWI} = \frac{1}{2} \int_{\mathbb{R}} T^2(t) dt. \quad (6)$$

Let us mention that the Gabor deconvolution itself is ill-posed for noisy data (Margrave et al., 2011). In practice, Tikhonov regularization is usually utilized to stabilize the solution. We shall discuss the importance of prior information in the regularization.

Regularization for Gabor deconvolution

The local matching filter, defined by the Gabor deconvolution (1), is commonly computed by

$$\hat{w}_{\varepsilon}(t, \omega) = \frac{\hat{d}^{\dagger}(t, \omega) \hat{p}(t, \omega)}{\hat{d}^{\dagger}(t, \omega) \hat{d}(t, \omega) + \varepsilon}, \quad (7)$$

where ε is a small positive number to prevent division by zero. From a point of view of regularization, the local matching filter, calculated by the equation (7), is the optimal solution of the regularized least-squares problem:

$$\min_{\hat{w}(t, \omega)} \frac{1}{2} \int_{\Omega} |\hat{p}(t, \omega) - \hat{d}(t, \omega) \hat{w}(t, \omega)|^2 + \varepsilon |\hat{w}(t, \omega)|^2 d\omega. \quad (8)$$

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Obviously, ε denotes the regularization parameter and the assumed prior information here is that the local matching filter $\hat{w}(t, \omega)$ should have a weak amplitude spectrum.

This commonly used regularization is designed for numerical stability. In practice, the regularization parameter ε selection depends on the SNR of observed data: low SNR data could require large ε . Note that, the local matching filter is supposed to converge into delta function with iteration in LAWI. However, this regularization attempts to make it approach zero, which constitutes a certain inconsistency between the expected convergence of the filter and the applied regularization.

Since the local matching filter is confined to delta function with model update, thus we have

$$p(t) \rightarrow d(t), \quad \hat{w}(t, \omega) \rightarrow 1. \quad (9)$$

To incorporate this property, we can consider another form of the regularization problem:

$$\min_{\hat{w}(t, \omega)} \frac{1}{2} \int_{\Omega} |\hat{p}(t, \omega) - \hat{d}(t, \omega) \hat{w}(t, \omega)|^2 + \varepsilon |\hat{w}(t, \omega) - 1|^2 d\omega. \quad (10)$$

Now the solution of the regularized Gabor deconvolution problem can be written as

$$\hat{w}_d(t, \omega) = \frac{\hat{d}^\dagger(t, \omega) \hat{p}(t, \omega) + \varepsilon}{\hat{d}^\dagger(t, \omega) \hat{d}(t, \omega) + \varepsilon}. \quad (11)$$

We name this regularization delta-type regularization to make it distinct from the conventional zero-type regularization.

For two implementations of Gabor deconvolution, the adjoint sources share the same expression:

$$\mathbf{r} = 2\mathcal{G}^{-1} \left[\mathcal{F}_\tau \left[\frac{T(t)(|\tau - T(t)|w(t, \tau))}{\int_{\mathbb{R}} w^2(t, \tau) d\tau} \right] \frac{\hat{d}(t, \omega)}{\hat{d}(t, \omega) \hat{d}^\dagger(t, \omega) + \varepsilon} \right]. \quad (12)$$

Note that the adjoint source changes with the local matching filter.

TWO FEATURES OF DELTA-TYPE REGULARIZATION

Resolution loss

We start with one-event signal experiment to show how resolution could be downgraded with the delta-type regularization. Figure 1 (a) presents two 5 Hz Ricker wavelets with a 0.5 s time shift. The estimated instantaneous time shifts by two implementations of Gabor deconvolution are shown in Figure 1 (b), and both estimations are close to the true value (0.5 s). Comparing two local matching filters (Figure 1 (c-d)), we can find the difference that the delta-type regularization generates non-zero signals along the zero-lag axis, result from the injected prior information of delta function.

These non-zero energies could cause a slight resolution loss of inversion results. Let us illustrate it through comparing the objective function variation with respect to amplitude scaling and time shift, in which the relation between observed and calculated data is defined by

$$d_{obs}(t) = Ad_{cal}(t + \tau). \quad (13)$$

From Figure 2, one can notice that the objective function with the delta-type regularization becomes flat when approaching the global minimum, which makes inversion more difficult to converge to a high-resolution result. Besides, it can be seen that both misfit functions are nearly agnostic to amplitude scaling and mainly sensitive to time shifts, which reveals that LAWI focuses on interpreting the kinematic information.

Layer-stripping principle break

The classical zero-type regularization has one property: only making use of events in observed seismogram that are locally coherent to the events occurring in predicted data. The new regularization would change this property. Let us illustrate it with a case where observed data contains two events while only one event presents in the predicted data, as shown in Figure 3 (a). The adjoint sources presented in Figure 3 (e) clearly show that the delta-type regularization accounts for all events in the observed data.

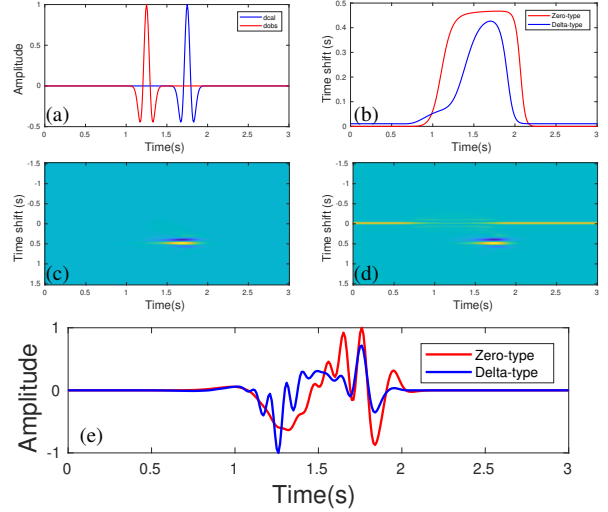


Figure 1: Single event study: observed and calculated data (a), the estimated time shift (b), the local matching filters $w(t, \tau)$ with zero-type regularization (c) and delta-type regularization (d), and the adjoint sources (e).

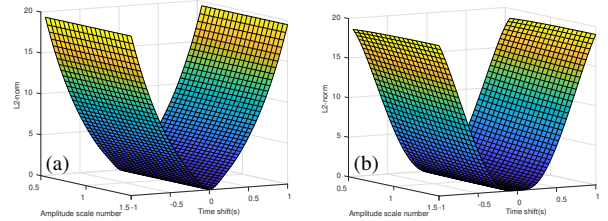


Figure 2: Objective functions with time shift and amplitude scaling: zero-type regularization (a) and delta-type regularization (b).

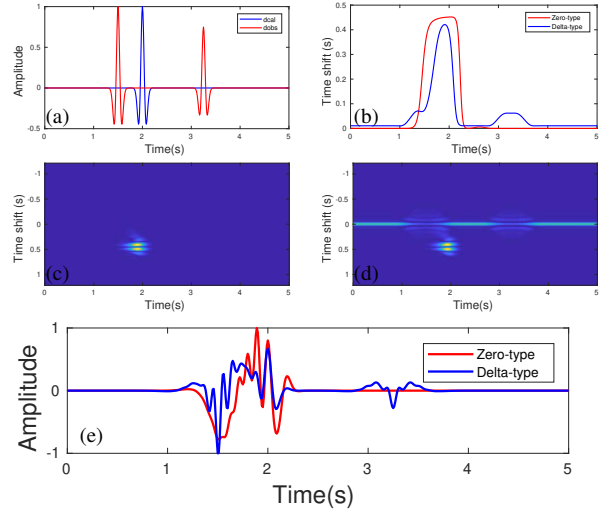


Figure 3: Layer-stripping principle break: the observed and calculated data (a), the estimated time shift (b), the local matching filter $|w(t, \tau)|$ with zero-type regularization (c) and delta-type regularization (d), and the adjoint sources (e).

From the mathematical formula, we know that, with the zero-type regularization, the local matching would be zero when predicted data is zero, no matter whether observed data is zero or not. Consequently, the zero-type regularization can not take the second event in the observed data into account. While the local matching filter, calculated by the delta-type regularization, has energy diffusively distributed around the horizontal coordinate of 3.25 s. This leads a non-physical time-shift estimation shown in Figure 3 (b). Due to the non-physical time shift, LAWI would bring the information of the second event in the observed data to the adjoint source (Figure 3 (e)) even though there is no locally

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coherent events in the predicted data.

In seismic exploration, the observed data usually contains direct, diving and reflected waves. When inversion starts from a smooth velocity model, the calculated data may only contain direct and diving waves. Thus the number of events in calculated data and observed data can not be matched. LAWI with the zero-type regularization focuses on the time shifts between diving and direct waves only, which might not provide sufficient information to match the observed reflections to the predicted data. When a significant part of the energy in the observed data is related to reflections, line-search issues can raise for LAWI with the zero-type regularization.

Let us recall that waveform inversion is based on small perturbation assumption (Tarantola, 1984). Thus, compared to direct projecting all information contained in observed data onto gradient, it could be more suitable if the gradient is constructed by only accounting for the events in the predicted data and gently including more information in observed data, especially when facing the cycle-skipping issue. This is the philosophy behind the layer-stripping strategy, which is effective to mitigate the cycle-skipping issue and widely used in practice (Wang and Rao, 2009; Pladys et al., 2022). The zero-type regularization naturally follows this principle. Although breaking this principle is beneficial to avoid the line-search issue, we recommend the zero-type regularization when it is applicable (high SNR data). In fact, a proper data-weighting function can be applied to fix the compared events, which allows the objective function of LAWI with the zero-type regularization decreasing with model update, even starting with a linear gradient velocity model. Please keep in mind that our motivation to develop the delta-type regularization is to make LAWI applicable for low SNR data. Besides, when the multi-scale strategy (Bunks et al., 1995) is adopted in LAWI, the data-weighting function becomes not mandatory, more discussions can be found in Yong et al. (2022).

2D VALHALL EXAMPLE

In this section, we use the 2D Valhall synthetics data to further illustrate the previous identified two features of LAWI. Figure 4 presents the true and initial velocity models. The density model, used to generate observed data, is calculated by Gardner's relation. The source function is a 5 Hz Ricker wavelet after a 2 Hz low-cut filter. The initial model is made by using the `sfsmooth` command in the open-source Madagascar software: repeatedly (15 times) applying a triangle filter with a radius of 250 m to the true model. The standard FWI suffers from cycle-skipping issue with this initial model. A fixed-spread acquisition is used, with 32 equally spaced sources and 352 equally spaced receivers with interval of 25 m placed on the surface. The ℓ -BFGS method ($\ell = 5$) is applied for model update (Métivier and Brossier, 2016). The maximum iteration is set as 50.

The observed data is presented in Figure 5 (a). Here, we would consider two cases that inversion starts with different density models: the true density model and a homogeneous density model. The two corresponding calculated data are shown in Figure 5 (b-c). When a homogeneous density model is used, the calculated data only contains direct waves and one reflection. From the adjoint sources shown in Figure 6, one can see that LAWI with the zero-type regularization only account for the events occurring in the predicted data. While LAWI with the delta-type regularization can bring all information in observed data to the gradient. Since AWI takes the whole information of each trace into account, it is also able to interpret all reflections in the observed data.

Figure 7 (a,c,e) display the inversion results starting with the true density model. One can see that AWI fails to reconstruct the low-velocity gas layers. LAWI with zero-type regularization is able to get rid of local minima and can converge to the true velocity model. The result provided by LAWI with delta-type regularization has a slightly lower resolution, consistent with the analysis above. When inversion begins with a homogeneous density model, AWI evolves towards local minima, leading to a spurious model (Figure 7 (b)). LAWI with the zero-type regularization would meet the line-search failure. As a result, the inversion gets stuck and the model cannot be updated (Figure 7 (d)). Meanwhile, the delta-type regularization can take all reflections into account at the beginning of inversion, and it does not suffer from

cycle-skipping issue. A comparable result can be produced by LAWI with delta-type regularization no matter true density or homogeneous density model is used.

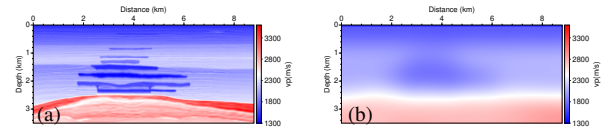


Figure 4: 2D Valhall models: true velocity (a) and initial velocity (b).

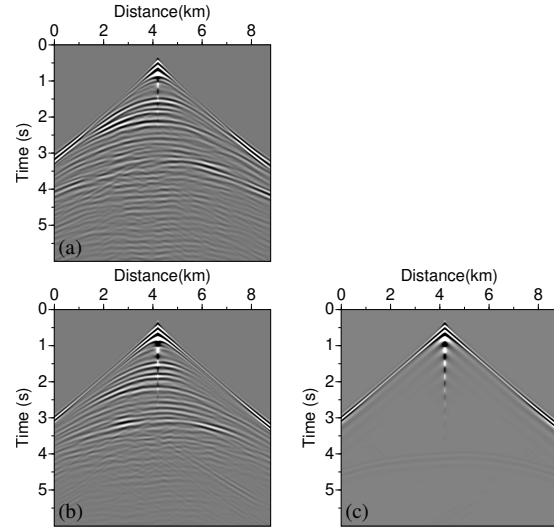


Figure 5: The observed data (a) generated with the true velocity and density model and the calculated data using the initial velocity model with the true density (b) and with homogeneous density model (c).

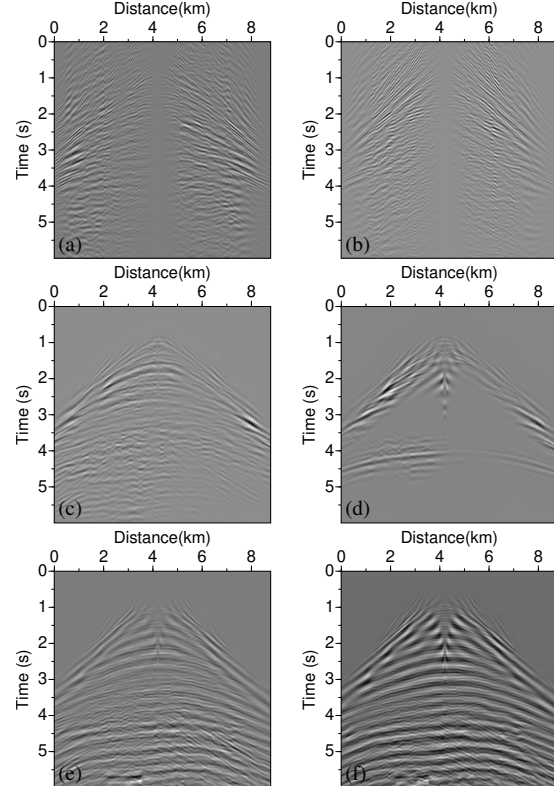


Figure 6: The adjoint sources: true density model (a,c,and e), homogeneous density model (b,d, and f). AWI with the zero type regularization (a-b), LAWI with the zero type regularization (c-d), and LAWI with the delta type regularization (e-f).

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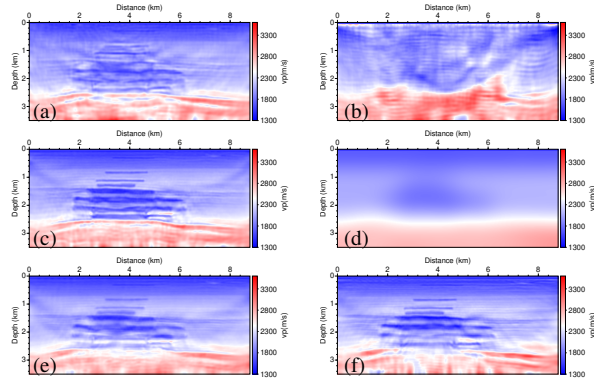


Figure 7: Inversion results with (left) and without (right) true density model: AWI (a-b), LAWI with zero-type regularization (c-d), and LAWI with delta-type regularization (e-f).

APPLICATION TO LOW-SNR DATA

Figure 8 (a) displays the Chevron benchmark in the 0-3 Hz frequency band. The weak-amplitude reflected waves are not usable due to strong noise, and only diving and direct waves are available in the low-frequency band. The predicted data, generated by the initial model (Figure 9 (a)), is shown in Figure 8 (b). The two adjoint sources of LAWI at the first iteration, corresponding to two kinds of regularization, are presented in Figure 8 (c-d). It can be observed again that, with the zero-type regularization, LAWI only accounts for the information around available predicted data, while the delta-type regularization can take all information in the observed data into account.

Figure 9 shows the inversion evolution of LAWI using the zero-type regularization for Gabor deconvolution. From Figure 9 (d), we see that inversion meets over-updating issue. Although the computation of adjoint source with the zero-type regularization is numerically stable, the final inversion result goes to a wrong direction. With the new implementation of Gabor deconvolution, the solution can go into a correct direction (Figure 10 (a)). As a comparison, we also give the result by AWI with the zero-type regularization, shown in Figure 10 (b). One may question: AWI adopts the zero-type regularization for deconvolution, but why can it generate a stable result in this low SNR data?

Let us mention that, in this test, the value of ε is selected as the average of the power spectrum of the observed data scaled by 1% for these three methods. Note that ε is not time-varying in the Gabor deconvolution, thus the regularization term in the misfit function (8) makes a large contribution for small-amplitude signal. Differently, in AWI, the whole information of each trace is taken into consideration together, which makes the stationary deconvolution less affected by the regularization term. It is necessary to point out that, with the increase of SNR, inversion becomes more robust and LAWI with the zero-type regularization can converge to a correct solution. In fact, it works well for 0-4 Hz Chevron benchmark data (Yong et al., 2022).

One may consider using a time-varying ε for the Gabor deconvolution to mitigate the inconsistency issue. However, it is quite difficult to set universal rules to determine time-varying ε , making inversion as stable as the delta-type regularization from our numerical experience.

CONCLUSION

Generally, the prior knowledge plays a significant role in regularization applications. A new regularization is proposed for Gabor deconvolution to stabilize LAWI for low-SNR data. The success of the new regularization relies on the fact that the matching filter should converge towards delta function with model update in LAWI. Compared with the classical zero-type regularization, the developed delta-type regularization will cause a slight resolution loss. Besides, the new regularization would break the layer-stripping principle, which somehow extends the application domains of LAWI. Overall, the two regularizations can be used for field data processing in a hybrid way: we shall

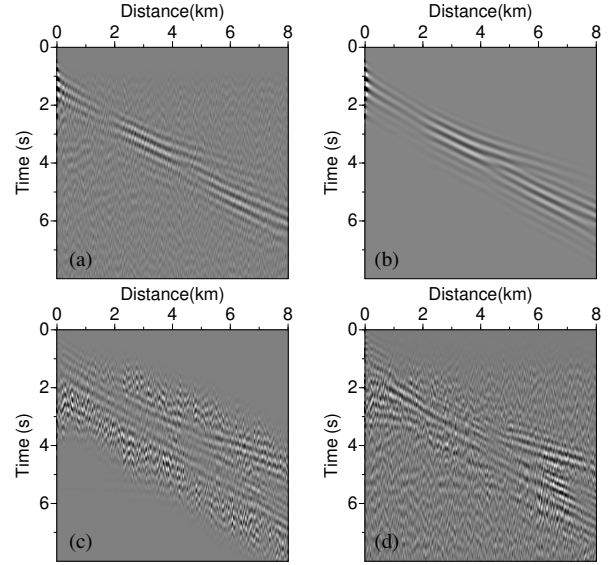


Figure 8: Chevron benchmark data in 0-3 Hz frequency band: observed data (a), predicted data (b), and the adjoint sources of LAWI with zero-type regularization (c) and delta-type regularization (d).

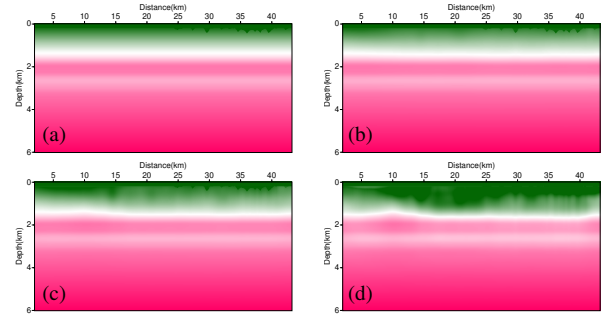


Figure 9: LAWI using zero-type regularization: initial model (a), reconstructed models after the 1st iteration (b), 2nd iteration (c), 3rd iteration (d).

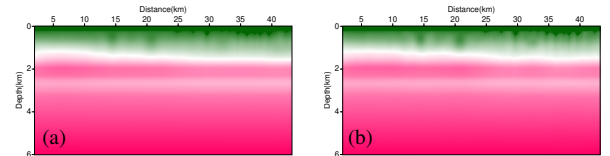


Figure 10: Reconstructed models by LAWI using delta-type regularization (a) and AWI using zero-type regularization (b) after the 15th iteration. The resolution loss is not obvious due to the limited low-frequency band.

recommend the delta-type regularization for low SNR data or at the early stage of inversion when reflection events are absent in the predicted data. With the increase of SNR and model update, the zero-type regularization becomes preferable.

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