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Geophysics

Robust FWI with graph space optimal transport: application to 3D OBC Valhall data

Right running head: GSOT FWI: application to 3D Valhall

ARNAUD PLADYS*, ROMAIN BROSSIER*, NISHANT KAMATH[†] AND LUDOVIC MÉTIVIER^{‡*}

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*Univ. Grenoble Alpes, ISTERre, F-38000 Grenoble, France

[†]Formerly Univ. Grenoble Alpes, now CGG Crawley Research Center, Crawley, United Kingdom

[‡]Univ. Grenoble Alpes, CNRS, LJK, F-38000 Grenoble, France

Robust FWI with graph space optimal transport: application to 3D

OBC Valhall data

(January 20, 2022)

Running head: **GSOT FWI: application to 3D Valhall**

ABSTRACT

1 Improving full-waveform inversion to make it more robust to cycle-skipping has been the subject
2 of a large number of studies. From the several families of approaches developed, one of the most
3 documented consists in modifying the least-squares distance defining the discrepancy between ob-
4 served and calculated data. From all the propositions made to improve and replace the least-squares
5 distance, only a few of them have been applied to field data. One of the methods proposed recently,
6 the graph space optimal transport distance, presents appealing properties for field data applications.
7 We compare it with the least-squares distance in an analysis performed on the three-dimensional
8 ocean bottom cable data from the Valhall field. This data has already been at the heart of several
9 full-waveform inversion studies, making it an excellent candidate to evaluate the properties of this
10 new misfit function. We first perform this comparison starting the inversion from the reflection trav-
11 eltime tomography model used in previous studies. We then perform a second comparison from a
12 crude, linearly-varying-in-depth one-dimensional velocity model. Starting from this model, least-
13 squares-based full-waveform inversion fails to provide a meaningful estimate of the pressure-wave
14 velocity model due to cycle skipping. We illustrate how the graph-space optimal transport-based
15 full-waveform inversion mitigates this issue. A meaningful estimate of the pressure-wave velocity
16 model is obtained in the zone sampled by both diving and reflected waves, down to almost two kilo-
17 meters depth. To our knowledge, this is the first application of a graph space optimal transport-based
18 full-waveform inversion to three-dimensional field data.
19
20
21

INTRODUCTION

22 Full waveform inversion (FWI) is a seismic imaging method that aims to reconstruct high-resolution
23 models (up to half the shortest wavelength) of the mechanical properties of the subsurface (Devaney,
24 1984; Pratt and Shipp, 1999; Plessix and Perkins, 2010; Raknes et al., 2015; Górszczyk et al., 2017).
25 The method is an iterative process based on minimizing a misfit function between observed and
26 calculated data over a space of model parameters describing the subsurface. The improvement of
27 resolution that FWI provides over standard tomography methods makes it possible to significantly
28 improve depth-migration images or directly produce interpretable images of the subsurface physical
29 properties. This method is used at multiple scales; from global and regional scales (Fichtner et al.,
30 2010; Tape et al., 2010; Bozdağ et al., 2016) to seismic exploration targets for oil & gas industry
31 (Plessix and Perkins, 2010; Stopin et al., 2014; Operto et al., 2015) and even near-surface scale
32 (Bretaudeau et al., 2013; Groos et al., 2014; Schäfer et al., 2013; Irnaka et al., 2019). A thorough
33 review of FWI and its applications is given in Virieux et al. (2017).

34 As powerful as this method is, it suffers from a significant shortcoming in its classical formula-
35 tion: the non-convexity with respect to time-shifts of the least-squares (L^2) misfit function used to
36 calculate the distance between observed and synthetic data. This non-convexity of the misfit func-
37 tion is an issue as the iterative process used in FWI is based on local-optimization algorithms. This
38 leads to the so-called cycle-skipping issue. This limitation of FWI in its classical formulation has
39 been documented since FWI has been introduced (Gauthier et al., 1986), and it has been of great
40 interest to overcome it.

41 Numerous studies proposing different approaches have been published. One of the historical ap-
42 proaches to overcome this limitation in practical cases is to rely on a data hierarchy workflow. This
43 approach consists in interpreting first the lowest frequency available, generally 2 – 4 Hz for seismic

44 exploration targets, then progressively introducing higher frequency data following a multi-scale
45 approach (Pratt, 1999; Bunks et al., 1995; Sirgue and Pratt, 2004). A second level of data-hierarchy
46 can be defined by modifying the temporal and/or offset selection of the data used during inversion
47 (Shipp and Singh, 2002; Wang and Rao, 2009; Brossier et al., 2009). The idea is to reduce the
48 number of propagated wavelengths that are interpreted simultaneously. Current industrial applica-
49 tions generally rely on these two levels of data hierarchy, combined with a robust starting model,
50 obtained, for instance, through reflection traveltime tomography or stereotomography (Lambaré,
51 2008).

52 Nonetheless, the conditions to apply this workflow are not always satisfied. For instance, low-
53 frequency data around 2–4 Hz are not always available or of sufficient quality. Moreover, obtaining
54 low frequency increases the cost of the acquisition and can also compromise the quality of the high
55 frequency needed to obtain high-resolution model reconstructions. Accurate initial model building
56 can also be a time-consuming and challenging task requiring strong human expertise as it generally
57 requires accurate traveltime and/or reflected event picking. Besides, prior information coming from
58 geology and sonic logs are often needed. These constraints make FWI less robust and reduce its
59 potential range in terms of applications.

60 Several methods have been introduced to improve robustness to cycle-skipping. The first group
61 can be named as “extension strategies” and relies on introducing supplementary degrees of freedom
62 to the FWI problem (Symes, 2008, 2015; Huang et al., 2018; van Leeuwen and Herrmann, 2013;
63 Wang et al., 2016; Aghamiry et al., 2020), which can be used to artificially match the data at early
64 iterations of the FWI process, avoiding cycle-skipping.

65 The second group consists in reformulating the FWI problem using an alternative measure of the
66 distance between the observed and calculated data, namely a different misfit function. Numerous ap-

67 proaches have been proposed, such as cross-correlation (Luo and Schuster, 1991; van Leeuwen and
68 Mulder, 2010) and deconvolution based misfit function (Luo and Sava, 2011; Warner and Guasch,
69 2016), or by modifying the signal itself, making the L^2 norm between this new observable more
70 convex with, for instance, instantaneous envelope (Fichtner et al., 2008; Bozdağ et al., 2011). We
71 want to keep in mind that replacing the L^2 norm is not an easy task, as, despite its simplicity, the
72 L^2 misfit presents excellent and interesting properties. First, it is robust to Gaussian noise. Sec-
73 ond, it presents an excellent resolution power, translating into high-resolution reconstruction that
74 FWI is well known for. Third, it is straightforward to implement and the computational cost of the
75 misfit function evaluation is negligible compared to most of the proposed alternative misfit func-
76 tions. These advantages have made the L^2 misfit the “state of the art” for FWI at exploration scales
77 and could explain why L^2 is still widely used even if many alternative misfit functions have been
78 proposed to mitigate the cycle-skipping issue.

79 Indeed, there is a discrepancy between the many propositions for alternative misfit functions
80 compared to the number of actual field data applications. We think that this discrepancy could be
81 explained by the - often not deeply discussed - intrinsic limitations of these alternative formulations.
82 For instance, cross-correlation-based misfit functions (Luo and Schuster, 1991; van Leeuwen and
83 Mulder, 2010) might have difficulties handling complex data, when observed and synthetic traces
84 contain multiple arrivals (not necessarily the same number) with different time shifts, some being
85 in phase, other being out of phase. Deconvolution-based strategies (Luo and Sava, 2011; Warner
86 and Guasch, 2016) require a penalization/weighting function, which can be difficult to set. Such
87 settings are often case-dependent, making FWI less of an automated process. Instantaneous enve-
88 lope intrinsically modifies the signal shape and discards information coming from the phase (which
89 is essential to interpret the polarity of reflected events in the data correctly).

90 A new class of misfit functions based on optimal transport (OT) has been introduced recently.

91 The motivation is to benefit from the convexity of the optimal transport distance with respect to
92 translation and dilation, which provides a misfit function convex with respect to time and amplitude
93 shifts, a good proxy towards convexity with respect to velocities perturbations (Engquist and Froese,
94 2014; Métivier et al., 2018). Another important motivation to use OT as a misfit function is the
95 ability to take into account the coherency of the seismic signal in an adequate space, be it a common
96 shot or receivers gather. However, OT can only be applied to positive quantities and cannot be
97 directly applied to seismic traces. To circumvent this difficulty, three main strategies have been
98 developed.

99 The first one proposes to bring back the problem to the comparison of positive quantities by
100 modifying the signal before solving the OT problem (Engquist and Froese, 2014; Qiu et al., 2017;
101 Yang et al., 2018b; Yang and Engquist, 2018). A nonlinear transform is applied to the data in a
102 trace-by-trace framework to transform each of them as probability measures. However, modifying
103 the signal and altering the polarity information might be detrimental to a stable and satisfactory
104 reconstruction of the subsurface mechanical properties.

105 The second one relies on the dual formulation of a specific instance of optimal transport distance,
106 namely the 1-Wasserstein distance (Métivier et al., 2016a,b,c). This formulation can be naturally
107 extended to the comparison of signed data. It benefits from its ability to be applied directly to 2D
108 and 3D data, taking into account the coherency of the seismogram in the receiver and/or sources
109 direction. However, even if the attraction valley to the global minimum is enlarged compared with
110 the least-squares approach, the application of the 1-Wasserstein distance to signed data loses the
111 convexity with respect to time-shift, which was the original motivation to use OT in the framework
112 of FWI (see Métivier et al., 2018 for a review on different OT formulations). This strategy has been
113 successfully applied to several field datasets (Poncet et al., 2018; Messud and Sedova, 2019; Sedova
114 et al., 2019; Carotti et al., 2020; Hermant et al., 2020).

115 Finally, the third one considers each discrete seismic trace as point clouds and computes the
116 optimal transport distance between point clouds associated with synthetic and observed traces. This
117 method is called the graph space optimal transport (GSOT) and presents the main characteristic of
118 preserving the convexity with respect to time shifts (Métivier et al., 2018, 2019). GSOT has already
119 been successfully applied to 3D synthetic and field data (He et al., 2019; Pladys et al., 2019; Li
120 et al., 2019; Górszczyk et al., 2019).

121 In the following table 1 we summarize the pros and cons of the three discussed strategies, fol-
122 lowing three criteria: computational efficiency, data distortion (nonlinear transforms to make each
123 trace a probability distribution, which severely affect the data), and ability to be applied to multi-
124 dimensional data.

125

[Table 1 about here.]

126 As GSOT is a promising candidate to tackle cycle-skipping on field datasets, this study focuses
127 on applying the GSOT strategy against the classical L^2 misfit on a 3D OBC dataset from the North
128 Sea, the Valhall field data. This dataset has been one of the first used to make proof of concept of the
129 resolution power that FWI can bring on field data as shown in Sirgue et al. (2010). Since then, this
130 dataset has been used several times for FWI application (Prioux et al., 2011; Gholami et al., 2013;
131 Prioux et al., 2013; Operto and Miniussi, 2018; Kamath et al., 2021). This dataset can be seen
132 as a “calibrated reference” for testing FWI formulations, such as frequency-domain FWI in Operto
133 et al. (2015) or in time-domain with attenuation in Kamath et al. (2021). Here, we are using this
134 dataset to compare the GSOT misfit function to the conventional L^2 norm through a time-domain
135 3D visco-acoustic VTI FWI.

136 To make this comparison, we first consider a canonical case where the initial model is the
137 same as the one used in the aforementioned studies, which is derived from reflection tomography.
138 This initial model ensures FWI converges toward a plausible estimation of the subsurface using
139 conventional L^2 misfit function.

140 Then, we introduce a very “crude” initial model in which calculated data are shifted by more
141 than one cycle compared to observed data, which is the typical case scenario for cycle-skipping.
142 We show how L^2 -based FWI fails from this “crude” starting model, whereas GSOT-based FWI
143 manages to correctly interpret data to provide a plausible reconstruction of the subsurface (down to
144 almost 2 km depth, which corresponds to the zone of the data sampled by both diving and reflected
145 waves). This constitutes one of the first applications of the graph-space optimal transport misfit
146 function to 3D field data at the exploration scale.

147 In the next section, we present the modeling and inversion algorithm used for our FWI applica-
148 tion. Then we detail the global methodology of GSOT for FWI. This is followed by a presentation

149 of the Valhall field application, from the geological situation to the initial model and dataset pre-
 150 sentation. We detail our FWI workflow and analyze the results in two cases: from the reflection
 151 traveltome tomography initial model and then from a 1D initial model. Results are then discussed,
 152 followed by conclusion and perspectives, which are given in a final Section.

METHODOLOGY

153 Modeling

154 This study is performed in the frame of 3D time-domain FWI. We rely on the anisotropic visco-
 155 acoustic time-domain modeling and inversion algorithm developed by Yang et al. (2018a), based on
 156 the following partial differential equations:

$$\left\{ \begin{array}{l} \rho \partial_t v_x = \partial_x g \\ \rho \partial_t v_y = \partial_y g \\ \rho \partial_t v_z = \partial_z q \\ \partial_t g = c_{11}(\partial_x v_x + \partial_y v_y) + c_{13} \partial_z v_z - \sum_{\ell=1}^L Y_\ell [c_{11} \xi_\ell^g + c_{13} \xi_\ell^q] \\ \partial_t q = c_{13}(\partial_x v_x + \partial_y v_y) + c_{33} \partial_z v_z - \sum_{\ell=1}^L Y_\ell [c_{13} \xi_\ell^g + c_{33} \xi_\ell^q] \\ \partial_t \xi_\ell^g = -\omega_\ell \xi_\ell^g + \omega_\ell (\partial_x v_x + \partial_y v_y), \quad \ell = 1, 2, \dots, L \\ \partial_t \xi_\ell^q = -\omega_\ell \xi_\ell^q + \omega_\ell \partial_z v_z, \quad \ell = 1, 2, \dots, L. \end{array} \right. \quad (1)$$

157 In system 1, c_{11} , c_{13} , and c_{33} are the stiffness tensor coefficients, ρ is the density, v_x , v_y , v_z
 158 are the horizontal and vertical displacement velocities respectively, while g and q are related to the
 159 normal stress components σ_{xx} , σ_{yy} and σ_{zz} through

$$\begin{aligned} g &= \sigma_{xx} = \sigma_{yy} \\ q &= \sigma_{zz}. \end{aligned} \quad (2)$$

160 This simplification is due to the VTI approximation. Similarly, the memory variables ξ_ℓ^g and ξ_ℓ^g
 161 are related to the memory variables ξ^{xx} , ξ^{yy} and ξ^{zz} associated with the normal stress components
 162 through

$$\begin{aligned}\xi_\ell^g &= \xi^{xx} + \xi^{yy} \\ \xi_\ell^g &= \xi^{zz}.\end{aligned}\tag{3}$$

163 These memory variables are used to model the viscosity of the medium following the gener-
 164 alized Maxwell body theory. Each represents one relaxation mechanism. We use three relaxation
 165 mechanisms to approximate a constant attenuation within the considered frequency band (L=3).
 166 The variables Y_ℓ are therefore calibrated depending on the target quality factor representing the at-
 167 tenuation in the considered media. This calibration is done through the solution of a least-squares
 168 problem. The details of this calibration can be found in Yang et al. (2016a) for instance.

169 In the VTI approximation, the stiffness tensor coefficients are related to the vertical P-wave
 170 velocity, the density, and the Thomsen anisotropy parameters ϵ and δ through

$$\begin{aligned}c_{11} &= \rho V_P^2(1 + 2\epsilon) \\ c_{33} &= \rho V_P^2 \\ c_{13} &= \rho V_P^2(1 + 2\delta).\end{aligned}\tag{4}$$

171 The discretization of this system of partial differential equations is performed using a fourth-order
 172 in space and second-order in time staggered grid finite-difference method (Virieux, 1986; Levander,
 173 1988).

174 A flat free surface condition is applied on top of the model to represent the water/air inter-
 175 face. Sponge layers (Cerjan et al., 1985) are applied on the other faces of the model to mimic a
 176 medium of infinite extensions in these directions. This numerical method is used instead of per-
 177 fectly matched layers (PML, Bérenger, 1994) mainly to ensure stability, as anisotropy generates

178 instabilities (wavefield amplification) when PML are used. The combination of PML and attenua-
 179 tion through relaxation mechanisms is also not trivial in terms of implementation.

180 Finally, windowed sinc interpolation is used to simulate source and receivers off-grid points
 181 accurately (Hicks, 2002).

182 **Inversion**

183 *General formulation*

184 FWI is an iterative process which relies on the minimization of a misfit function. Classically, it is
 185 the L^2 misfit function defined as follows

$$f_{L^2}[m] = \sum_s \sum_r h(d_{cal}[m](x_r, t; x_s), d_{obs}(x_r, t; x_s)), \quad (5)$$

186 where

$$h(d_1(t), d_2(t)) = \frac{1}{2} \int_{t=0}^T |d_1(t) - d_2(t)|^2 dt. \quad (6)$$

187 The observed and synthetic trace calculated in model m , associated with source x_s and receiver x_r ,
 188 are denoted by $d_{obs}(x_r, t; x_s)$ and $d_{cal}[m](x_r, t; x_s)$ respectively.

189 To solve this minimization problem, we rely on a local optimization scheme. We use a pre-
 190 conditioned quasi-Newton l-BFGS algorithm (Nocedal, 1980), implemented in the SEISCOPE op-
 191 timization toolbox (Métivier and Brossier, 2016).

192 We compute the gradient following the adjoint state method (Plessix, 2006). This method makes
 193 it possible to easily replace the L^2 norm as a change of the misfit function only translates to a mod-
 194 ification of its associated adjoint-source to obtain the gradient. See Yang et al. (2018a) for a review.
 195 Since the method relies on reverse time propagation of the wavefield, which is numerically unstable
 196 with attenuation, our code relies on the checkpoint-assisted reverse forward simulation (CARFS)

197 strategy proposed by Yang et al. (2016b) to provide a stable and yet efficient implementation for
198 large scale problems with attenuation.

199 The preconditioner we use is the wavefield preconditioner presented in Kamath et al. (2021)
200 An anisotropic non-stationary Gaussian smoothing is also applied to the gradient. Inversions are
201 performed for P-wave velocities (V_P) only.

202 *Optimal transport for FWI*

203 Optimal transport (OT) distances are derived from the OT theory introduced by the French math-
204 ematician Gaspard Monge more than two centuries ago (Monge, 1781). More precisely, they rely
205 on the OT relaxation proposed by Kantorovich (1942). The distances, also called Wasserstein dis-
206 tances, have an intrinsic property of particular interest for the definition of inverse problems: they
207 are convex with respect to translation and dilation of the compared quantities. This convexity with
208 respect to translation has been, in particular, the motivation to introduce it in the framework of FWI
209 to obtain a distance measurement convex with respect to time-shifts (Engquist and Froese, 2014).
210 However, OT distances are defined for comparing probability distributions, which are by definition
211 positive and normalized. So this new distance cannot be directly applied to seismic data, which is
212 oscillatory (a generalization of OT to signed distribution is still an open question from a mathemat-
213 ical point of view, see Ambrosio et al. (2011) and Mainini (2012) for instance.

214 This study focuses on a recent proposition made to apply OT to seismic data: the graph space
215 optimal transport (GSOT) strategy, proposed in Métivier et al. (2019).

216 This formulation of OT distance should preserve the signal unmodified while also preserving
217 the convexity to shifted patterns. It relies on the idea of comparing the discrete graph of the data
218 rather than the data itself: each 1D trace in time becomes a point cloud of Dirac delta functions

219 (of amplitude 1) in a 2D space made of the time dimension, and a new amplitude dimension. This
 220 transformation does not affect the signal shape but makes it possible to deal with positive mass (the
 221 Dirac delta functions). Thus the OT distance can be applied directly.

222 The corresponding misfit function is formulated as

$$f_{GSOT}[m] = \sum_s \sum_r h(d_{cal}[m](x_r, t; x_s), d_{obs}(x_r, t; x_s)), \quad (7)$$

223 where this time

$$h(d_1, d_2) = \min_{\sigma \in S(N_t)} \sum_{i=1}^{N_t} c_{i\sigma(i)}(d_1, d_2). \quad (8)$$

224 $S(N_t)$ denotes the ensemble of permutations of $\{1, \dots, N_t\}$, and c_{ij} the L^2 distance between the
 225 discrete points of the graph $(t_i, d_1(t_i))$ and $(t_j, d_2(t_j))$:

$$c_{ij}(d_1, d_2) = |t_i - t_j|^2 + \psi^2 |d_1(t_i) - d_2(t_j)|^2. \quad (9)$$

226 The function h corresponds to the 2-Wasserstein distance between the discrete graph of the “calcu-
 227 lated” trace $d_1(t)$ and the “observed” trace $d_2(t)$.

228 The scaling parameter ψ controls the convexity of the misfit function f_{GSOT} with respect to
 229 time shifts. In practice, we define it as

$$\psi = \frac{\tau}{A}, \quad (10)$$

230 where τ is a user-defined parameter corresponding to the maximum expected time shift between
 231 observed and calculated data in the initial model, and A is the maximum peak amplitude difference
 232 between observed and calculated data in the initial model. It is automatically computed prior to the
 233 inversion. This ensures the convexity of the GSOT distance for time up to approximately τ .

234 A visual illustration of the GSOT concept is given in Figure 1 which we reproduce from Métivier
 235 et al. (2019). In this Figure, the optimal permutation (assignment) σ between two Ricker functions,
 236 interpreted as point clouds, is plotted for different τ parameters. A larger τ value induces an optimal

237 assignment coupling points along the time axis, rendering the misfit function convex to time shifts.
 238 A small τ value induces an optimal assignment coupling points along the amplitude axis, rendering
 239 the misfit function equivalent to a least-squares misfit function.

240 [Figure 1 about here.]

241 The final cost function we use for the purpose of FWI application with N_s shots containing N_r
 242 receivers is defined as:

$$\min_m f_{GSOT}[m] = \sum_{s=1}^{N_s} \sum_{r=1}^{N_r} w^{s,r} h(d_{cal}^{s,r}[m], d_{obs}^{s,r}), \quad (11)$$

243 where $w^{s,r}$ is a trace-by-trace weighting factor, typically used to restore the AVO trend in the data.
 244 This trend is removed from the trace-by-trace GSOT approach, as the amplitude of each trace is
 245 treated separately through the normalization factor ψ . In practice, we compute $w^{s,r}$ as the L^2
 246 energy of the corresponding observed trace

$$w^{s,r} = \left(\frac{1}{T} \int_0^T |d_{obs}^{s,r}(t)|^2 dt \right)^{1/2}. \quad (12)$$

247 The adjoint source of the misfit function $f_{GSOT}[m]$ is computed from $\frac{\partial h}{\partial_{cal}}$ using the adjoint-
 248 state strategy (Plessix, 2006). The following result is proved in Métivier et al. (2019). Denoting σ^*
 249 the minimizer in eq. 7, we have

$$\frac{\partial h}{\partial_{cal}} = 2 \left(d_{cal} - d_{obs}^{\sigma^*} \right), \quad (13)$$

250 where

$$d_{obs}^{\sigma^*}(t_i) = d_{obs}(t_{\sigma^*(i)}). \quad (14)$$

251 The GSOT approach can thus be viewed as a generalization of the L^2 distance: The adjoint source
 252 is equal to the difference between calculated and observed data at time samples connected by the

253 optimal assignment σ^* . The solution of the problem eq. 7 provides the information to compute both
254 the misfit function and the adjoint source.

255 To solve eq. 7 efficiently, we use the auction algorithm (Bertsekas and Castanon, 1989), ded-
256 icated to the solution of linear sum assignment problems such as eq. 7. Despite a relatively high
257 computational complexity in $O(N_t^3)$, it is quite efficient for small instances of such problems. Re-
258 sampling the data close to the Nyquist frequency (at the exploration scale, under acoustic approxi-
259 mation, traces are often around few hundreds of time steps after resampling) yields such small-scale
260 problems making the GSOT feasible for realistic scale FWI applications, as seen in the application
261 presented here.

FIELD DATA PRESENTATION AND FULL-WAVEFORM INVERSION

WORKFLOW

262 **Geological situation, dataset and initial models**

263 *Geological situation*

264 The Valhall field is located in the southern part of the Norwegian sector in the North Sea, ap-
265 proximately 300 km southwest of Stavanger (Fig. 2). It is a shallow environment with a nearly
266 constant water depth of 70 m. Valhall reservoir lies along the Lindesnes Ridge, which trends
267 NNW (Munns, 1985; Leonard and Munns, 1987). The field has been discovered in 1975 and is
268 used since then for oil production. It is characterized as an anticlinal in chalk in the Upper Cre-
269 taceous Hod and Tor formations, which form the reservoir at a depth of approximately 2400 m.
270 Trapped gas in Tertiary shale is present above the reservoir (Sirgue et al., 2010; Prieux et al.,
271 2011, 2013; Operto et al., 2015). The Tertiary overburden is relatively simple and free of com-

272 plex structure (Hall et al., 2002). Chalk compaction resulting from pressure depletion and water
273 weakening has led to seabed subsidence (Field: VALHALL - Norwegianpetroleum.no <https://www.norskpetroleum.no/en/facts/field/valhall>).

275 [Figure 2 about here.]

276 *Dataset presentation*

277 The seismic data are 4-components acquired by ocean-bottom cables (OBC), with wide aperture/azimuth
278 acquisition. The covered zone is a surface of 145 km². Twelve receiver cables are deployed on the
279 seabed, containing 2048 receivers with an inline spacing of 50 m and a cable spacing of 300 m. A
280 total of 50824 shots are available, located 5 m below the sea surface. The layout of this 3D acquisition
281 is presented in Figure 3. In this study, we use only the pressure component of one acquisition
282 performed in 2011 as part of the Valhall Life of Field Seismic (LoFS) project (Barkved et al., 2003).

283 [Figure 3 about here.]

284 From the raw data provided in SEG-Y format, only a simple pre-processing is applied. As
285 our FWI code relies on source parallelization, source-receiver reciprocity is applied to process the
286 hydrophone as explosive sources and the shots as hydrophones, hence sensibly reducing the impact
287 on computer resources. The data is then de-spiked before a quality control over the complete dataset:
288 the energy (RMS) of each gather is calculated to manually remove faulty gathers (the one with a
289 large variation of RMS amplitude). The last step is to create frequency bands for the inversion using
290 a minimum-phase band-pass filter in two distinct bands: 2.5 – 5 Hz and 2.5 – 7 Hz (referred to as
291 band 1 and band 2 in the following). This goes in hand with time-decimation; from $\Delta t = 4$ ms to
292 $\Delta t = 8$ ms on the first band, and from $\Delta t = 4$ ms to $\Delta t = 5$ ms on the second band (Fig. 4).

293 [Figure 4 about here.]

294 *Initial models*

295 An initial V_P model was made accessible to us thanks to AkerBP. It has been obtained through
 296 reflection traveltimes tomography. It is referred to as TOMO initial and is presented in Figure 5.
 297 This initial model has been used in several publications using this Valhall dataset and has proven its
 298 capacity to give satisfactory results with L^2 -based FWI as it predicts the arrival within half a period
 299 of the considered starting frequencies (2.5 – 5 Hz), avoiding cycle-skipping issue (Prioux et al.,
 300 2011; Operto et al., 2015; Operto and Miniussi, 2018). The associated density model is derived
 301 from V_P TOMO using Gardner’s law (Gardner et al., 1974), defined as $\rho = 309.6 * V_P^{0.25}$. This
 302 relation is a fair average for brine-saturated rock (excluding evaporites), which is coherent with the
 303 expected geology of the Valhall field. AkerBP also provided us the anisotropy model ϵ and δ ,
 304 and while their structure is not complicated, they are of significant influence in the modeling. The
 305 anisotropic parameter η defined by Alkhalifah (1998) as

$$\eta = \frac{\epsilon - \delta}{1 - 2\delta} \quad (15)$$

306 is shown in Figure 7. We can see that maximum anisotropy reach values around 15% near the
 307 reservoir. Finally, to introduce attenuation, a simple two-layer Q_P model, with 1000 in the water
 308 column and 200 in the sediments, is used (Operto et al., 2015). While it is a simple model, it
 309 has been proven to be of great importance to explain the data. Using a more complex Q_P model
 310 (for example derived from V_P) only results in marginal improvements. Moreover, it introduces
 311 complexity and uncertainties that we prefer not to deal with (Kamath et al., 2021).

312 As the TOMO initial model is good enough to match the data within half a period of the 2.5 –
 313 5 Hz frequency-band used to start FWI (Fig. 8), it does not represent any challenge regarding the

314 cycle-skipping issue. This is why we introduce a new “crude” V_P starting model called 1D initial
315 presented in Figure 6. It is a purely 1D vertical starting model, based on a linearly increasing
316 profile with one main interface around 2400 m depth. This 1D starting model generates strong
317 cycle-skipping, as made visible in Figure 9. As traditional L^2 -based FWI cannot tackle this cycle-
318 skipping in the data, it should be a good candidate to benchmark the capability of GSOT FWI.
319 Associated with this V_P 1D model, the density model is derived using the preceding Gardner’s law.
320 The anisotropy model ϵ and δ , as the attenuation model Q_P are kept similar to those used in the
321 TOMO setup.

322 [Figure 5 about here.]

323 [Figure 6 about here.]

324 [Figure 7 about here.]

325 [Figure 8 about here.]

326 [Figure 9 about here.]

327 **Full-waveform inversion workflow**

328 To obtain the final FWI reconstructed V_P model, we rely on several elements that compose our
329 complete workflow. Each part of it plays a critical role in obtaining the best results possible, from
330 wavelet estimation, data selection, to post-processing the FWI model. The generic workflow that
331 we use in this article is presented in Figure 10. We detail the different parts composing the workflow
332 in the following sections.

333 [Figure 10 about here.]

334 *Source wavelet estimation*

335 The first step before running FWI is to obtain a proper source wavelet that is used during the inver-
336 sion to generate synthetic data. This is a crucial step as an incorrect wavelet estimation could induce
337 artifacts into the reconstructed model.

338 The wavelet is estimated by solving a linear deconvolution problem in the frequency domain,
339 following the methodology described in Pratt (1999). The wavelet inversion is performed on a
340 single random subset of 240 shot-gathers. The sample of 240 wavelets (one per shot-gather) is
341 then averaged to produce a single wavelet, assuming that all hydrophones have the same coupling
342 response.

343 To minimize the cross-talk between the V_P model and the wavelet, we rely on a carefully de-
344 signed data weighting strategy, focusing on short-offset only as presented in Figure 11. The data
345 weighting consists in using 400 m of offset at full amplitude, then ramping down to zero at 1200 m
346 offset. A tail mute is applied to remove the Schölte waves. Before being used for inversion, the
347 wavelet is then manually checked and tapered to ensure its causality. With this methodology, the
348 wavelet is only estimated at the beginning of each frequency band and kept fixed during FWI steps.
349 The wavelet for the first frequency band and its associated spectrum is presented in Figure 11. We
350 can see that no oscillations are present after 2.7 s, and wavelets generated from TOMO or 1D initial
351 models are similar, which validates that this careful data selection mitigates the potential leakage
352 in the wavelet estimation, which could come from the P-wave velocity model. Finally, the wavelet
353 spectrum is coherent with the data (2.5 to 5 Hz).

354

[Figure 11 about here.]

355 *Random shot subsampling and frequency continuation*

356 The FWI workflow relies on a frequency continuation approach. Inversion is performed first on the
357 frequencies ranging between 2.5 – 5 Hz, then on a second band between 2.5 – 7 Hz. Respectively,
358 the model grid spacing is set to 70 m and 50 m in the three dimensions, ensuring at least five grid
359 points for the smallest wavelength. These parameters are the same as in Operto et al. (2015) and
360 Kamath et al. (2021).

361 As previously mentioned, the dataset contains 2048 shot-gathers. Since our FWI code relies on
362 source-parallelization, the complete dataset could not be reasonably fit in a single inversion on the
363 HPC facility we have access to. Therefore, we rely on a source-subsampling strategy, similar to the
364 one described in Warner et al. (2013). This makes it possible to divide the dataset into batches of
365 pseudo-randomly selected shot-gathers (120 in our case). The pseudo-random selection implies that
366 the previously used shot-gather could not be selected in the next subsample until all of the available
367 ones are used once. The subsample of shot-gathers is changed every time the memory limit for
368 l -BFGS is reached (3 iterations is the maximum memory of l -BFGS in this study). The source-
369 subsampling strategy makes it possible to fit the FWI problem on relatively small HPC facilities
370 while mitigating the acquisition foot-print in the reconstructed model.

371 *Hierarchical data weighting strategy*

372 The first data weighting strategy tried on the dataset for FWI is simple: only remove the Schölte
373 waves and muted the trace near-zero offset in a radius of 350 m (see the approach in Kamath et al.
374 2021). While this direct and straightforward approach (using all the data directly) could be justified
375 with TOMO initial models as it is not supposed to generate cycle-skipping, this approach could
376 not be applied to tackle a crude initial model as the 1D one, even with the GSOT misfit function.

377 Because of the strong cycle-skipping generated with the 1D initial model, we need to rely on a more
378 careful data weighting strategy to maximize the capability of the GSOT. This leads to introduce a
379 six-step data weighting strategy, presented in Figure 12. The three first steps only focus on diving
380 waves with a strict time windowing while restricting to offset to the first 4 km (later referred to as
381 **DW SO** for **D**iving **W**ave - **S**hort **O**ffsets), 8 km offset (later referred to as **DW MO** for **D**iving **W**ave
382 - **M**edium **O**ffsets), and full offset (later referred to as **DW FO** for **D**iving **W**ave - **F**ull **O**ffsets).
383 Then, the three next steps release the time windowing progressively, starting with 8 km offset (later
384 referred to as **RT1 MO** for **R**elease **T**ime 1 - **M**edium **O**ffsets), then on full offset (later referred
385 to as **RT1 FO** for **R**elease **T**ime 1 - **F**ull **O**ffsets); to finally finish with full offset and a complete
386 release of time windowing (later referred to as **RT2 FO** for **R**elease **T**ime 2 - **F**ull **O**ffsets). Please
387 also note that for each data weighting, the Schölte waves are additionally muted, as we cannot model
388 them in the acoustic approximation.

389 *Model post-processing*

390 The last step of the FWI workflow is a post-processing applied to the updated V_P model at the
391 end of each FWI step. It consists in cutting the model using a stencil (based on the shape of the
392 acquisition) and extrapolating the value outside the stencil with a nearest-neighbors algorithm. This
393 is performed for each depth slice. The stencil shaped is adapted for each depth slice to consider the
394 maximum illumination the acquisition can provide at depth. This aims to remove the area on the
395 edges of the model that are never updated during FWI due to the lack of illumination and remove
396 artificial low-velocity zones created on the border of the well-illuminated zone.

397 To be consistent and perform a fair comparison between our two starting models TOMO and 1D
398 , the complete workflow detailed before is applied to both starting models. By doing so, we validate

399 that our workflow can tackle the dataset properly and provide satisfactory results. Then, changing
400 only the misfit function from L^2 to GSOT, we reconduct the complete inversion process to check
401 how GSOT compares to L^2 in this controled environment.

402 [Figure 12 about here.]

FULL-WAVEFORM INVERSION RESULTS

403 **Starting from the reflection tomography model**

404 The first FWI results presented in this article are based on the TOMO initial model.

405 The final reconstructed V_P at 5 Hz is shown in Figure 13 using the L^2 misfit function, and in
406 Figure 14 using the GSOT misfit function. Respectively, the 7 Hz results are shown in Figure 15 and
407 Figure 16. Starting from the TOMO initial model, and as we do not expect cycle skipping, we keep
408 the τ parameter of GSOT to 0.2 s. This value is low enough to always allow for fast convergence
409 (similar to L^2) while preserving the potential improvement that GSOT can provide. Using a smaller
410 value of τ would make results similar to L^2 while making τ larger would degrade the capacity of
411 convergence of FWI due to a flatter attraction basin which is not needed in this case. The results are
412 almost identical between both formulations on this setup. Some minor differences can, however, be
413 observed. On the shallow depth slice (a) at 200 m, a small reduction of the acquisition imprint is
414 observable using the GSOT misfit function. On the depth slice (c) at 1 km, a slight improvement
415 of contrast between the low velocity anomaly and the sediment background is also visible. Finally,
416 on the vertical slices (d-g), more lateral coherency in the geological structures is visible using the
417 GSOT misfit function. The difference globally remains marginal, but this similarity between the
418 L^2 and GSOT misfit functions results is satisfying in itself. Indeed, alternative misfit functions

419 generally bring some drawbacks, such as loss of resolution power, which is not the case here. When
420 comparing the V_P profile extracted from the reconstructed model to the sonic log filtered in the
421 0 – 7 Hz frequency band (Fig. 17), we can see that the GSOT result is almost perfectly following L^2
422 result, which is consistent with the observation made directly on the V_P model. We can observe how
423 FWI improves the fit to the sonic log over the initial model. This validates that our FWI workflow
424 provides robust and reliable results.

425 [Figure 13 about here.]

426 [Figure 14 about here.]

427 [Figure 15 about here.]

428 [Figure 16 about here.]

429 [Figure 17 about here.]

430 The data fit presented in Figure 18 shows that GSOT can improve over the L^2 FWI data fit
431 (which is already very satisfying). On the *rec A cable A* 2D CRG (through the low velocity
432 anomaly), we can observe some data fit improvement with GSOT, for example, at –6.5 km off-
433 set and 7 s. On the *rec B cable B* 2D CRG, in an area with relatively mild variations in velocities
434 and well away from the “gas cloud”, we can observe that, globally, the data fit is almost the same for
435 L^2 and GSOT, with all phases correctly explained. This is not surprising as this CRG focuses on the
436 part of the model with sedimentary geology only, which the initial model better explains. However,
437 we can still see the advantages of GSOT this time with more arrivals present for late time, as visible
438 in the –7 to –4 km offset, from 5 to 7 s.

439 [Figure 18 about here.]

440 This “reference” result validates two essential aspects of this study. This first one is that our
441 FWI workflow is adapted to the dataset and can provide satisfactory results when using L^2 -based
442 FWI when a good enough initial model is used as the TOMO model. This point is of first impor-
443 tance as it validates that our FWI workflow is consistent with the literature on this dataset. The
444 second conclusion is that the GSOT misfit function can tackle field data problems without signif-
445 icant issues. Modifying the misfit function and keeping all other parameters similar, GSOT can
446 provide comparable results to those provided by the L^2 FWI. We can even see slight improvement
447 both in terms of the P-wave velocity model estimation and data fit with the GSOT misfit function.
448 These encouraging results make it possible to push the analysis forward: can GSOT help tackle the
449 cycle-skipping issue and improve FWI robustness compared to L^2 -based FWI?

450 **Starting from the 1D initial model**

451 To validate the capability of GSOT to tackle large cycle-skipping, we use the 1D initial model that
452 we introduced previously. This 1D initial model generates cycle-skipping, even on the mid-offset
453 diving waves (mainly at -4 and -8 km offset), as clearly illustrated in Figure 9.

454 First, we compare the data fit obtained with L^2 and GSOT at the 2nd step of our FWI workflow
455 as presented in Figure 19. For this first two steps of the FWI workflow, τ is set to 0.35 s, which is
456 enough to handle the cycle-skipping generated in the data. First-order observation may indicate a
457 similar data fit with GSOT and L^2 , but a more in-depth analysis shows several differences. At offset
458 higher than 5 km, we observe a degradation with out-of-phase arrivals for L^2 on the *rec A cable A*
459 CRG, while GSOT results present a significant improvement on this part. Then, late arrivals events
460 are better explained with GSOT. On the second *rec B cable B* CRG, we observe a more continuous

461 reconstruction of the first events while also reducing out of phase one. Late arrivals are also better
462 reconstructed with GSOT. On the second CRG, the data fit obtained with GSOT is already quite
463 good for such an early stage of the inversion. In Figure 20 is presented the same CRG, but without
464 applying the data weighting used at this early stage (DW MO data weighting), but instead the final
465 relaxed data weighting (RT2 FO). Here, we can see that we are starting to predict data at larger
466 offset when using GSOT compared to L^2 .

467 [Figure 19 about here.]

468 [Figure 20 about here.]

469 After the first two steps of FWI (DW SO and DW MO), we obtain the reconstructed model
470 presented in Figures 21 and 22. It is clear here that L^2 -based FWI result displays heavy artifacts
471 on the reconstructed V_P model for depth larger than 300 m. Even if the very shallow part of the
472 model is correctly reconstructed, the deeper part of the model is not (see shallow slice (a) compared
473 with deeper slices (b) and (c)). Starting from a crude 1D initial model, L^2 reaches its limit and
474 is likely affected by strong cycle-skipping. This is why we stop the L^2 inversion at this stage of
475 our workflow: pushing forward the inversion by introducing more data does not help. Conversely,
476 GSOT-based FWI can provide promising and meaningful V_P updates, with the recovery of correct
477 background velocities at depth and even key features such as the definition of strong low velocity
478 anomalies (slices (c) (d) (f)).

479 [Figure 21 about here.]

480 [Figure 22 about here.]

481 As GSOT results are encouraging after only two passes on the first frequency band, we apply
482 our complete workflow and perform the complete inversion similar to our reference inversion, with
483 six passes on 5 Hz data and 7 Hz data. Regarding the τ parameter for GSOT, as said earlier, the
484 first two steps on the first frequency band (2.5 to 5 Hz) were performed using $\tau = 0.35$ s. The
485 next four steps on the first band used a reduced $\tau = 0.2$ s as it is enough to tackle the shift present
486 in the data (which already illustrates the improvement that GSOT achieves in the first step of the
487 inversion). For the second frequency band (2.5 to 7 Hz), only the first step of the workflow uses a
488 slightly relaxed τ of 0.25 s, while the remaining five steps use $\tau = 0.2$ s. Again, τ is relaxed at
489 the beginning to mitigate cycle-skipping, while a smaller τ is used afterward to preserve resolution
490 power and speed up convergence.

491 We present the final data fit at 7 Hz for GSOT in Figure 23. We can see that a relatively good
492 data fit is obtained, with most of the arrivals correctly explained. Still, some late arrivals are out of
493 phase, mainly for large offsets (larger than 6 km)

494 [Figure 23 about here.]

495 We obtain the final reconstructed V_P presented in Figure 24. The results are promising and
496 show a clear improvement in resolution compared to the early results at 5 Hz, with the main target
497 structures retrieved above 2 km depth. The shape of the low velocity anomaly is correctly retrieved
498 (slice (c) (d) (f)). Lateral resolution is very significantly improved, allowing the definition of narrow
499 low-velocity (150 m wide) anomalies not resolved in the L^2 inversion (slice (e)).

500 To assess the quality of the final reconstructed V_P model, we can represent the value of the zero-
501 lag cross-correlation between the observed data and synthetic data for different common-receiver
502 gathers. When the data fit is converging toward a perfect match the (normalized) cross-correlation
503 value converges to 1. We have performed this analysis for two common-receiver gathers (rec A and

504 rec B) in Figure 25. The improvement in data-fit is clear after FWI (Figure 25 (b) and (d)) compared
505 to the data-fit obtained with the initial 1D model (Figure 25 (a) and (c)). This validates that final
506 FWI results are not likely affected by remaining cycle-skipping.

507 Comparing the sonic log filtered in the 0 – 7 Hz frequency band to the early L^2 and final GSOT
508 results (Fig. 26) clearly illustrates that only GSOT-based FWI performs meaningful updates of the
509 model that follows the trends of the sonic log 2 and 3. The case of sonic log 1 is interesting: as it
510 is close to the target, it exhibits the down-shift of approximately 150 m observed earlier in the V_P
511 model.

512 A comparison between the sonic log and GSOT-based FWI results for the two starting models
513 is shown in Figure 27. For the two logs outside the center target area (Log 2 & 3), a good agreement
514 of the FWI results is observed, with reconstructed V_P models following the same trend until 1.6 to
515 2 km. We want to remind that the illumination only constrains the model above 1.4 km depth when
516 starting from the 1D initial model. In the Log 1, near the target, even when starting from the TOMO
517 initial model, we can observe that the reconstruction is degrading under 1.6 km depth.

518 [Figure 24 about here.]

519 [Figure 25 about here.]

520 [Figure 26 about here.]

521 [Figure 27 about here.]

522 The results are still not perfect, and the main issue is the presence of a low-velocity update
523 around 500 m depth, as made visible on slice (b). This low-velocity update (not present in the
524 reference FWI results starting from TOMO initial model) introduces a down vertical shift of layer

525 under this perturbation. This is why slice (c) is extracted 100 m under (1.1 km instead of 1 km
526 depth). This vertical shift does not affect the shape of the low velocity anomaly but only its depth.
527 One possible way to avoid this artifact would be to modify the early stage of the inversion, for
528 example, with different data-selections or modifications of the initial model for a slightly better
529 one (for example, based on sonic log information). While this would have probably improved the
530 results, we decided to keep a crude 1D initial model to stay as generic as possible and assess the
531 capacity of GSOT in a setup without prior information.

532 **Computational costs**

533 Computational cost analysis is performed for one gradient estimation for both L^2 and GSOT misfit
534 functions. Computation is performed on Haswell E5-2690V3@2.6 GHz Intel nodes containing 24
535 cores and 64 Gb of memory per node. We use 4 cores per source as our finite difference scheme
536 uses an OpenMP parallelization.

537 The computational costs on the first frequency band (2.5 – 5 Hz) and second frequency band
538 (2.5 – 7 Hz) are summarized in the table 2.

539

[Table 2 about here.]

540 The gradient column corresponds to the time spent to calculate the gradient (containing all
541 required wave modelings), while the misfit column isolates the time spent computing the misfit
542 function. The total time is the sum of gradient and misfit, and the ratio compares L^2 (put at 100%)
543 with GSOT. This analysis shows that while the computational complexity of the solution of the
544 gradient estimation scales to $O(\omega^4)$, the computation complexity of the GSOT computation is in
545 $O(\omega^3)$, as noted in Métivier et al. (2019).

546 The overhead cost induced by GSOT is therefore reduced on the higher frequencies, which are
547 the most expensive ones computationally speaking (for example, with a maximum frequency of
548 5 Hz, the first frequency band is relatively fast to compute, making a 20% overhead acceptable).
549 This is one key feature that makes the GSOT misfit function able to tackle field data applications
550 as higher frequency drastically increases the computational cost, and GSOT overhead will become
551 smaller.

DISCUSSION

552 Facing challenging field data applications with alternative misfit functions is not widely documented
553 in the literature. One of the only other alternative misfit functions that has been applied successfully
554 to field data is adaptive waveform inversion (AWI) (Warner and Guasch, 2015; Ravaut et al., 2017;
555 Debens et al., 2017; Roth et al., 2018; Guasch et al., 2019; Warner et al., 2019) or Kantorovich-
556 Rubinstein optimal transport (KROT) (Poncet et al., 2018; Messud and Sedova, 2019; Sedova et al.,
557 2019).

558 Regarding the final results obtained with the GSOT misfit function, we first validate that it can
559 improve over L^2 -based FWI in a controlled environment (starting from a good enough V_P TOMO)
560 with improved datafit and improved lateral coherency and reduced acquisition imprint on the model

561 side. The computational overhead induced by GSOT stays limited thanks to the computational
562 complexity of GSOT being one order of magnitudes smaller than the complexity of the gradient
563 estimation for a given maximum frequency. This behavior translates into a good scaling property
564 when facing high-frequency data, which is now one trend for field data FWI applications. When
565 tackling a difficult 1D initial model, which generates cycle-skipping, we show that GSOT-based
566 FWI provides good model updates and good datafit where classical L^2 -based FWI fails. The control
567 of the convexity provided by the GSOT misfit function is performed through the choice of the
568 τ parameter. This parameter is directly linked to the observed time-shift between calculated and
569 observed data, making it easy to tune, depending on the initial fit of the data and the expected
570 maximum time shift. We adapt it from 0.2 s to 0.35 s in the initial stage of the workflow when we
571 switch from the initial tomography model to the initial 1D model. We emphasize the fact that the
572 τ parameter is simple to define and small modification (± 0.2 s) does not translate into drastically
573 different results.

574 We also emphasize that results under 2 km suffer from a substantial lack of illumination and are
575 therefore limited to the resolution of our initial model, explaining why no meaningful updates are
576 present for depth superior to 2 km. This limitation in terms of depth reconstruction (under 2 km) is
577 not surprising as the FWI alone is not expected to present enough illumination. In this case, it would
578 require the use of reflected wave inversion (RWI) or joint full-waveform inversion (JFWI). Some
579 preliminary but encouraging results were obtained combining GSOT and JFWI (Provenzano et al.,
580 2020): GSOT adds the convexity necessary to predict the reflected data, enabling robust velocity
581 updates of the model at depth.

CONCLUSION

582 In this work, we focus on the application of a new misfit function: graph space optimal transport.
583 This formulation shows a clear improvement over L^2 in a controlled environment, unleashing the
584 potential of FWI to perform meaningful updates when starting from a crude, 1D initial model. This
585 clear improvement in cycle-skipping robustness, combined with a simple setup (only one *physical*
586 parameter to define) and a reasonable computational overhead, illustrates that GSOT is a good can-
587 didate to improve FWI robustness and therefore make FWI more accessible and easily applicable.

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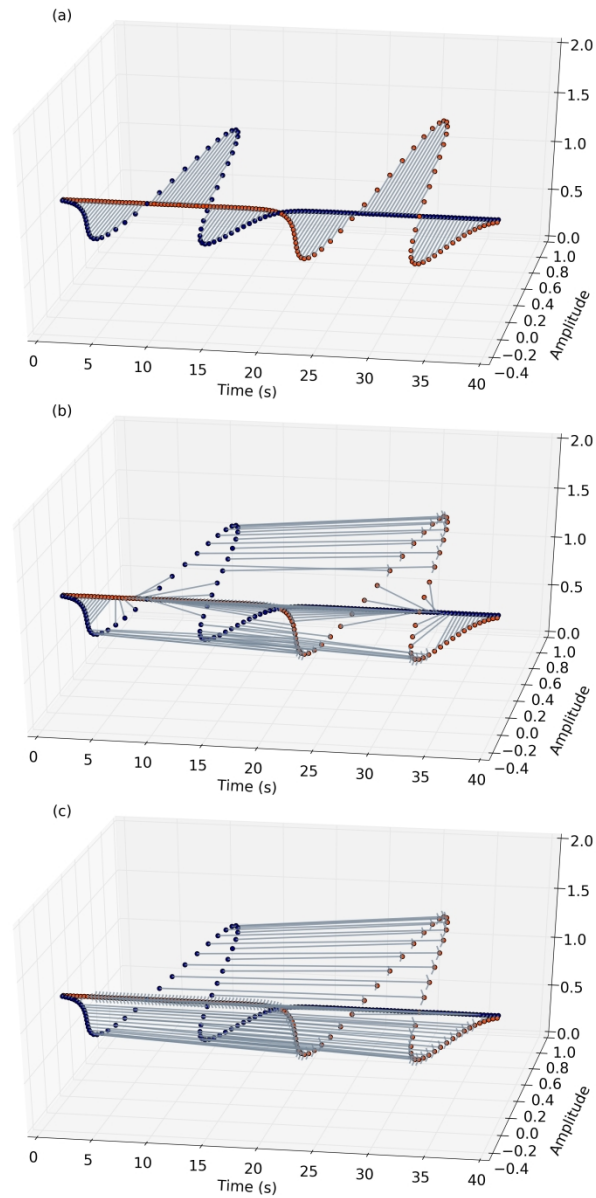


Figure 1. 3D representation of the discrete graph of a reference Ricker function (red points) and a shifted in time Ricker function (blue points) scaled in amplitude by a factor of 0.8. The gray arrows represent the assignment solution of the LSAP problem, which depends on the value of the parameter au . Top $au = 0.4$ s, middle $au = 4$ s, bottom $au = 20$ s.

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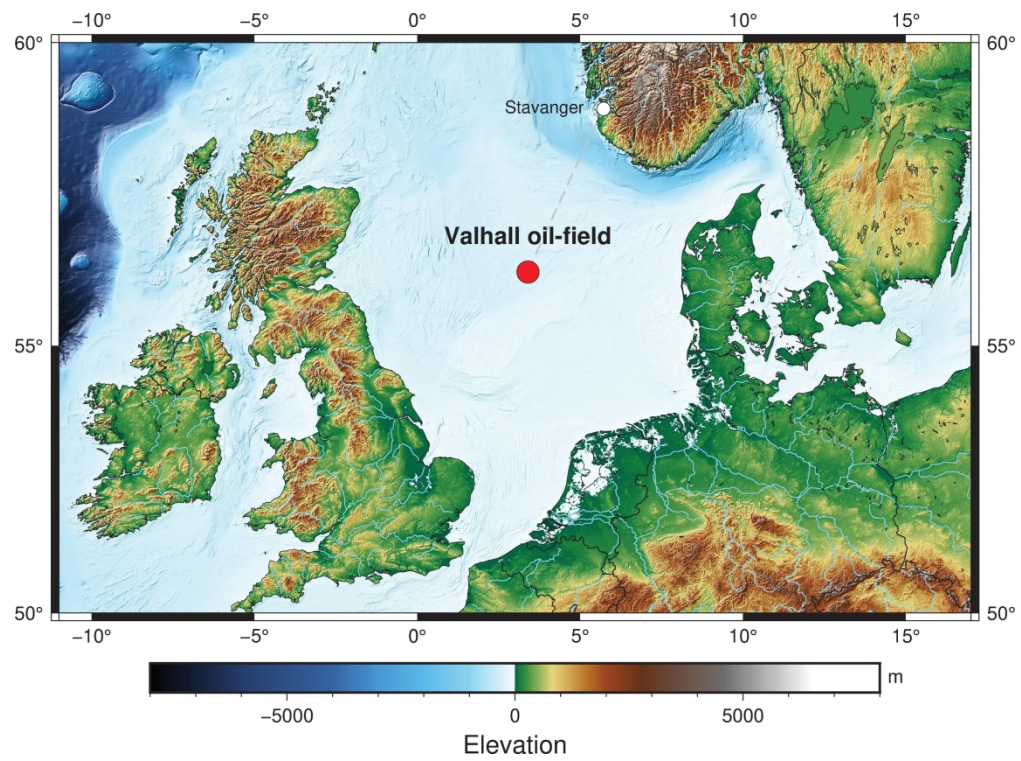


Figure 2: Location of the Valhall field on the North Sea (from \citealp{Thurin_2020_PhD})

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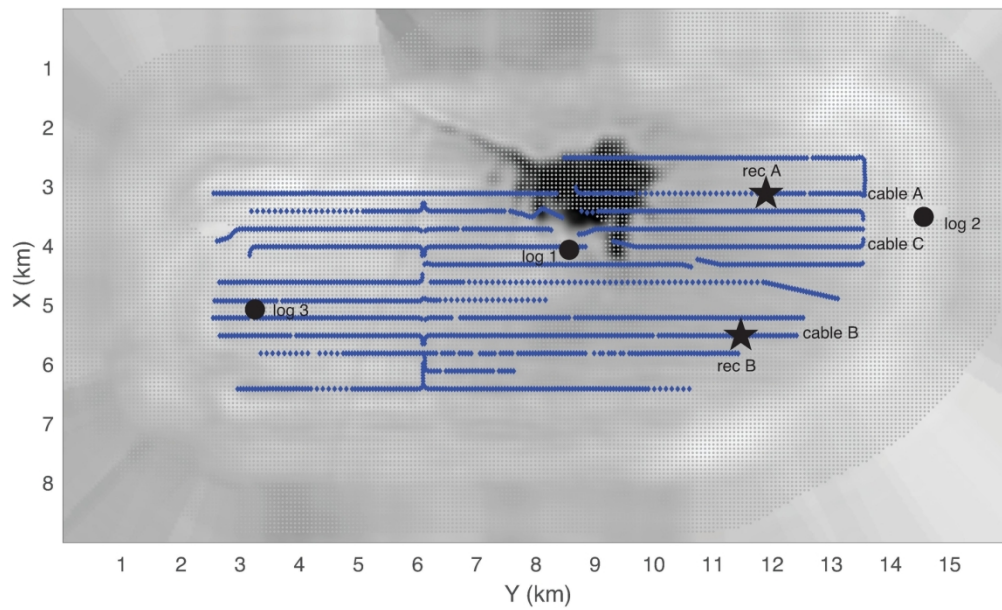


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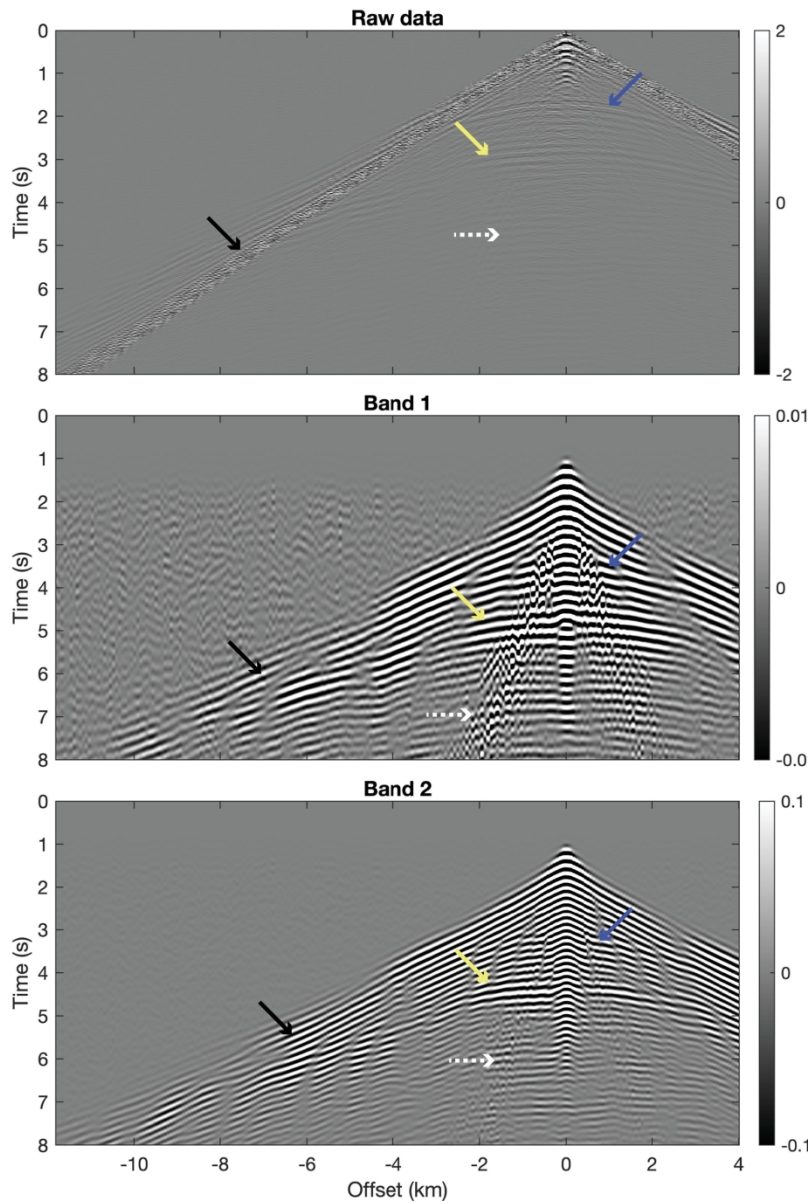


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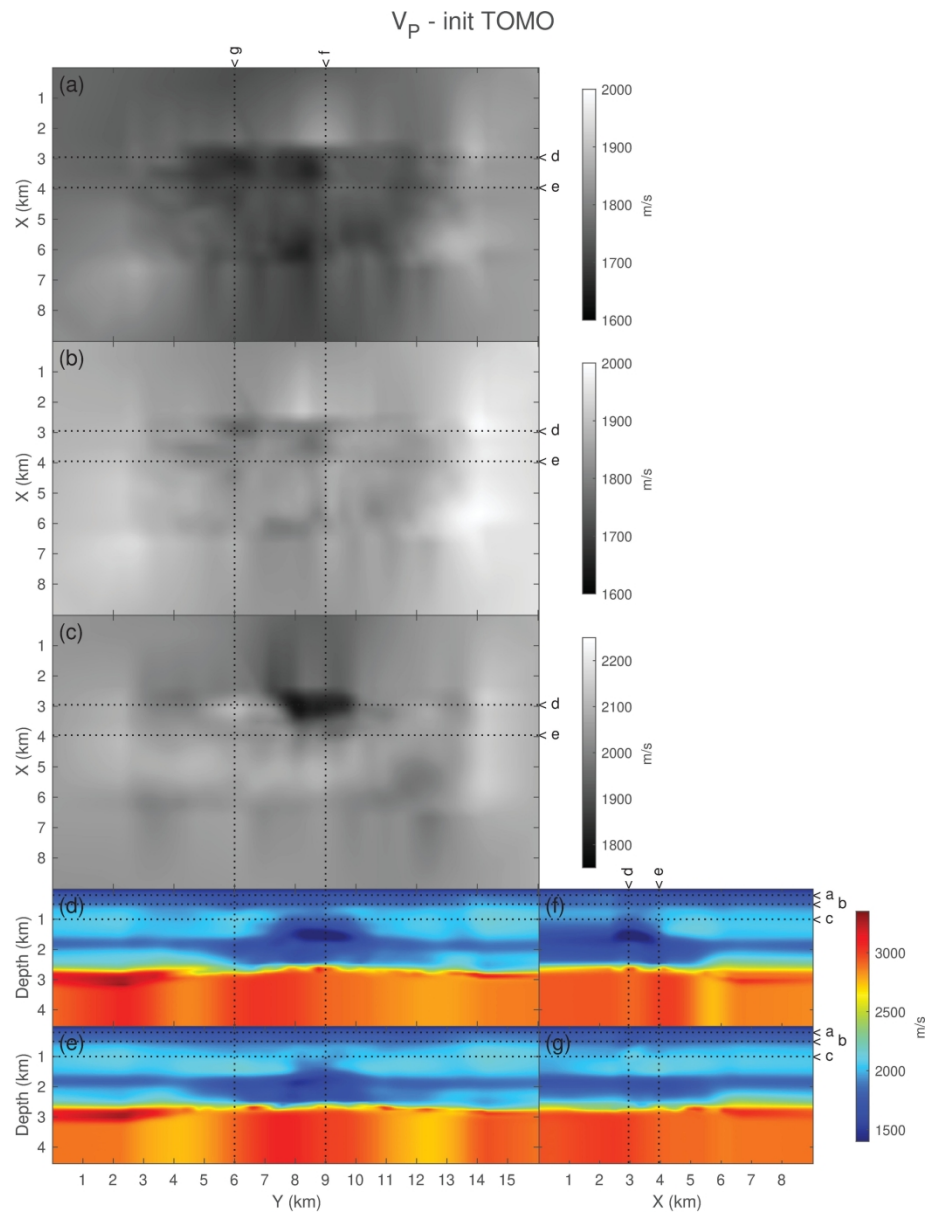


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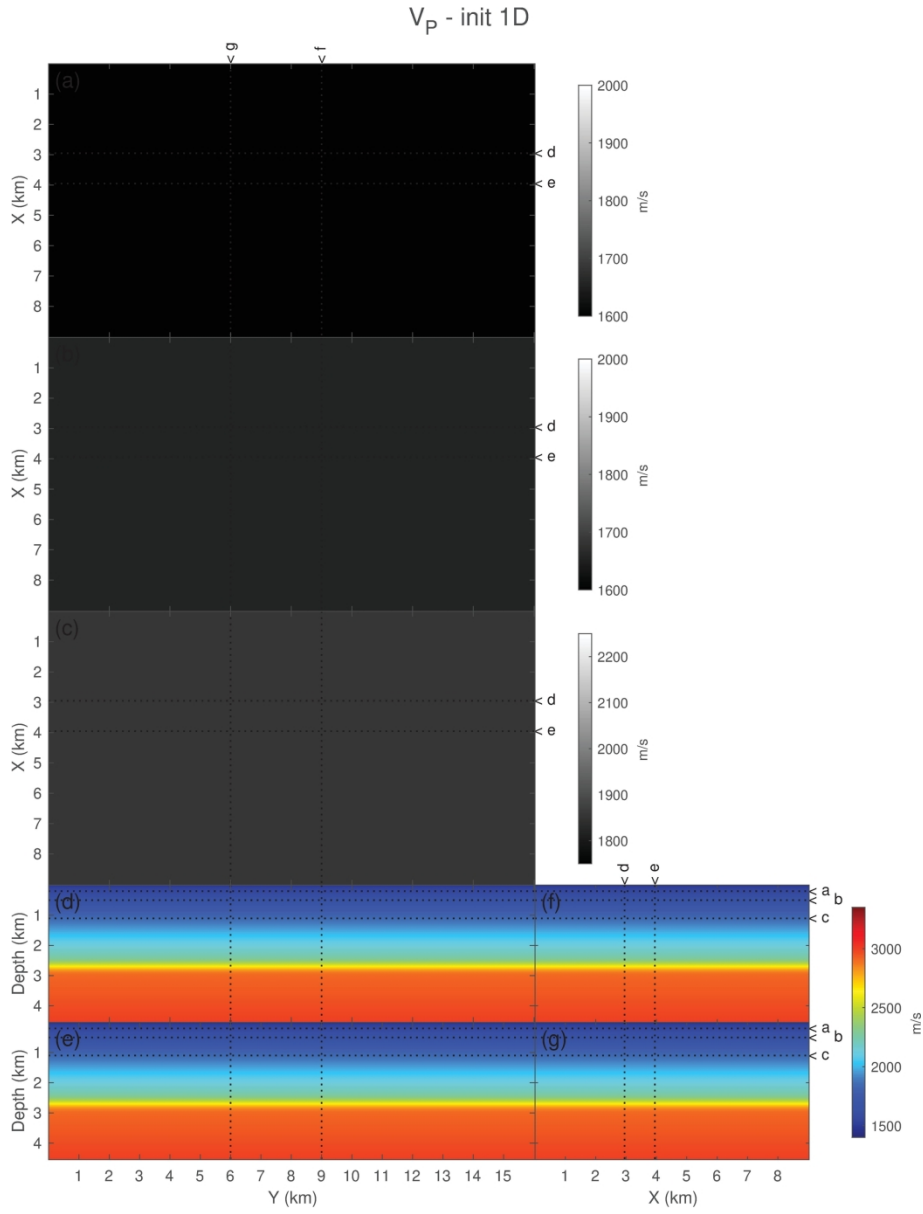


Figure 6: Same as $\text{\Cref{init:VP_initTOMO}}$ for initial model \vpb .

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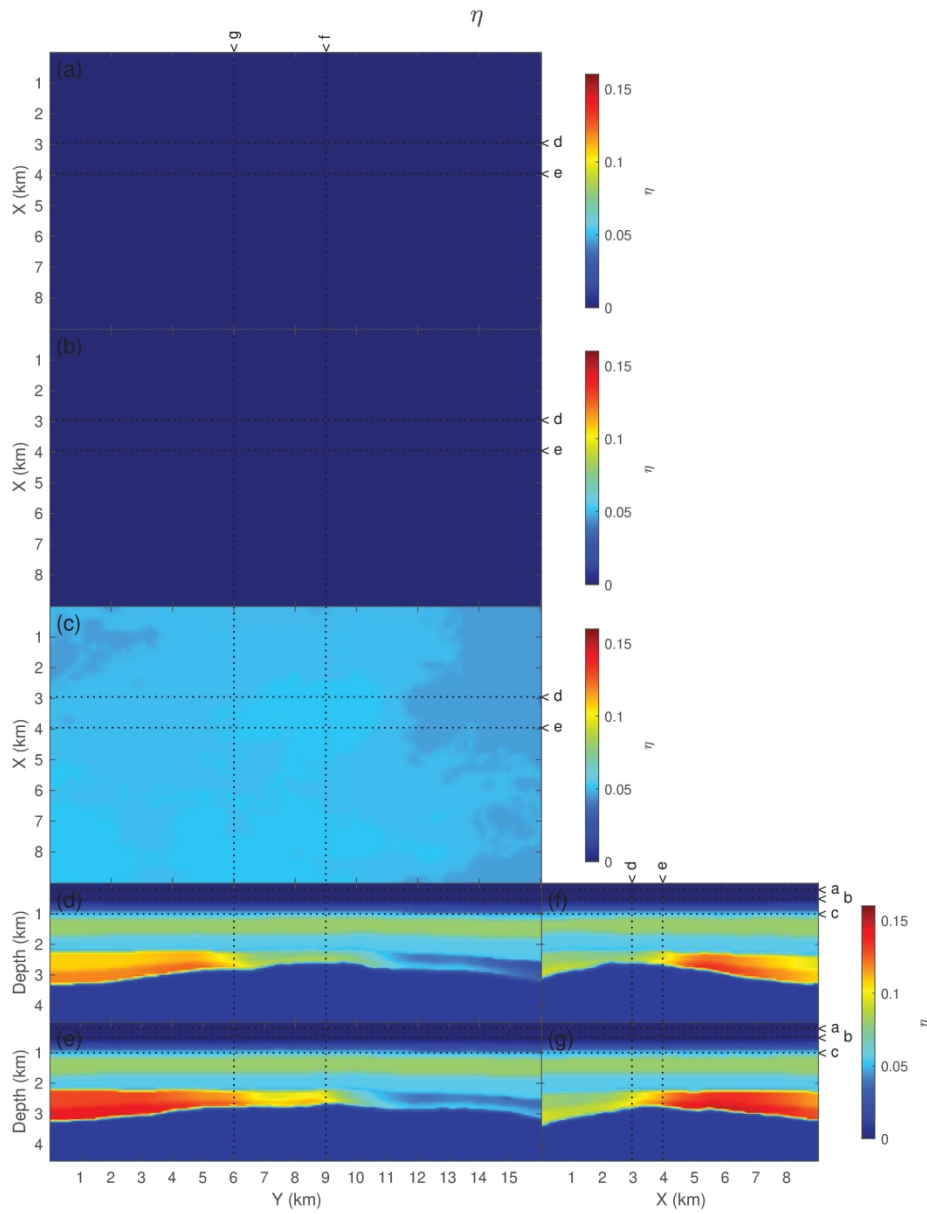


Figure 7: Same as \Cref{init:VP_initTOMO} for anisotropic parameter η .

209x273mm (300 x 300 DPI)

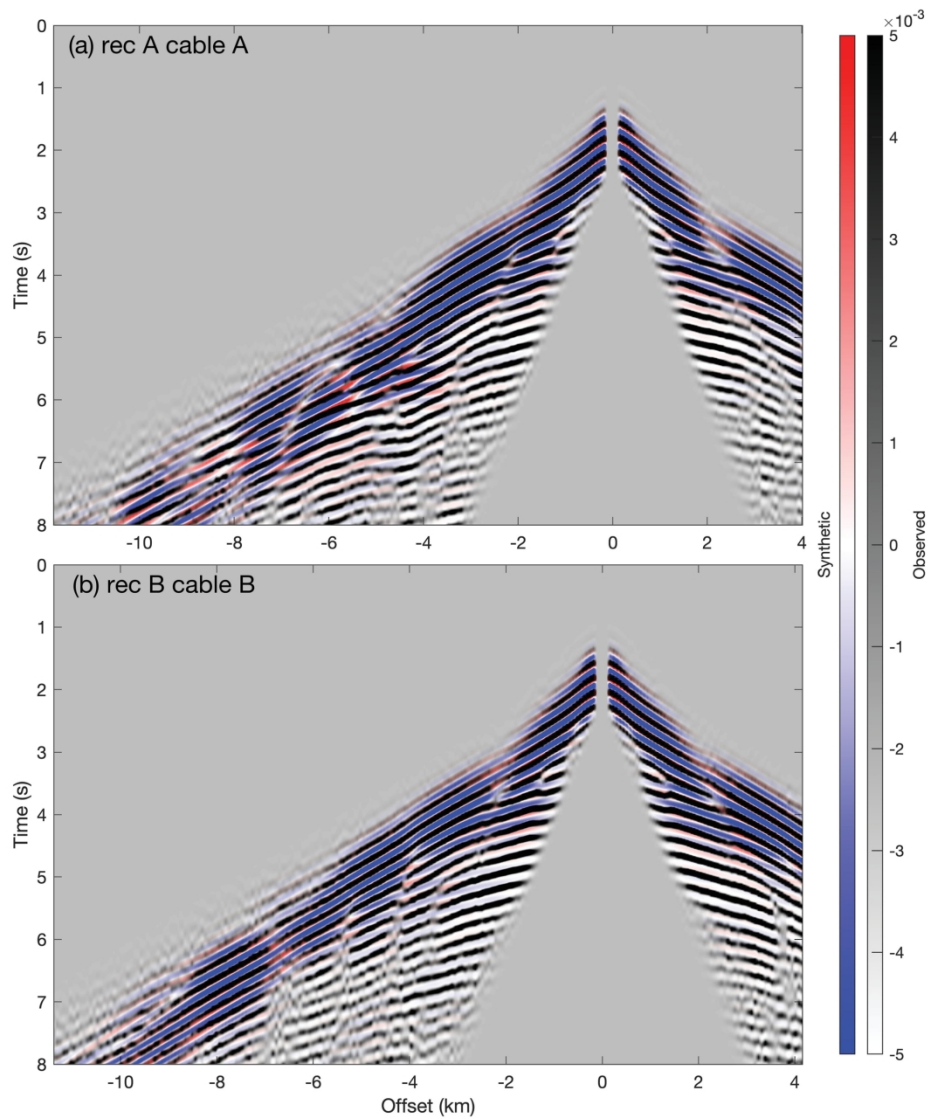


Figure 8: 2D common-receiver gathers at 5 Hz. Synthetic data generated into the initial model are displayed in a blue/white/red color scale, field data are overlapped in grayscale with transparency. The best result is achieved when black and blue are the only colors visible. Red and white are shown when data are not in phase. (a) receiver A along cable A (through the low velocity anomaly). (b) receiver B along cable B.

200x229mm (300 x 300 DPI)

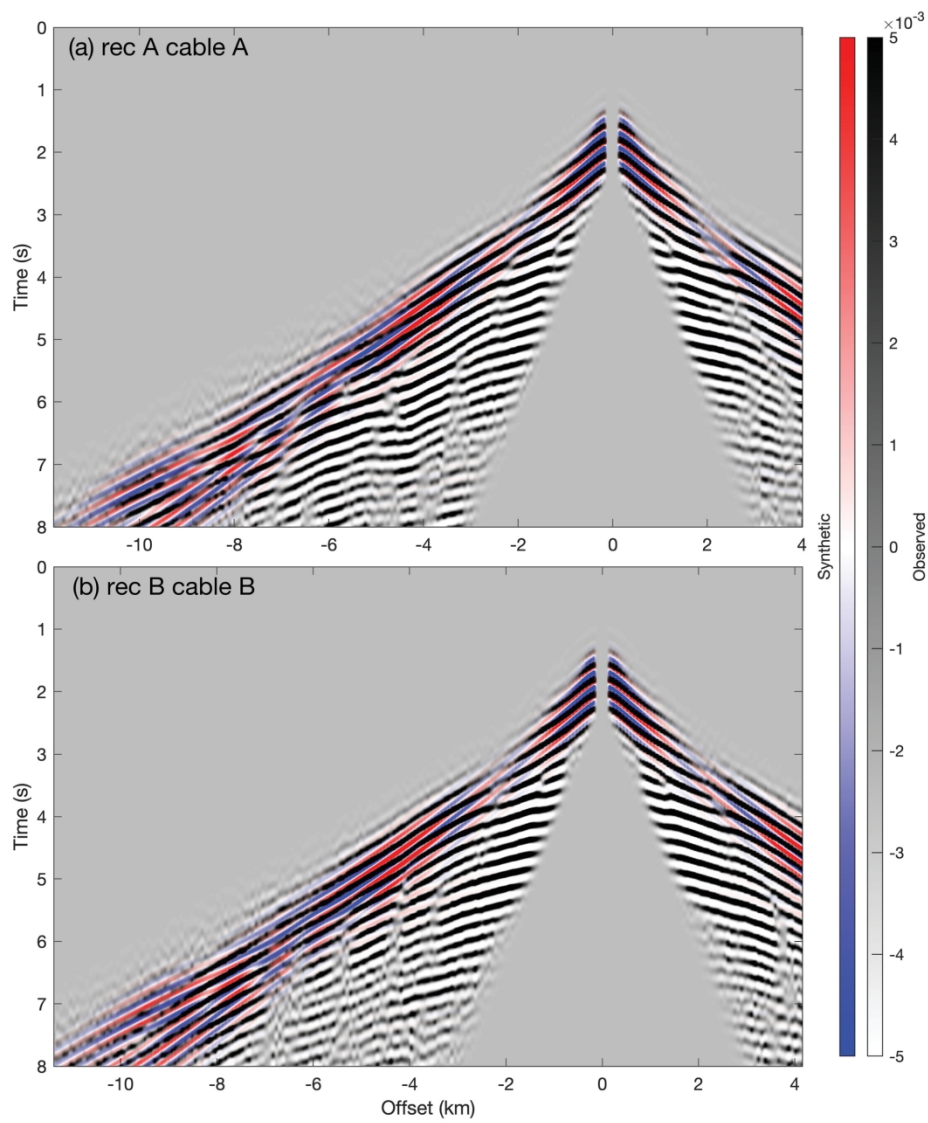


Figure 9: Same as [\Cref{crg:CRG_5Hz_initTOMO}](#) into the `\modb` initial model.

200x229mm (300 x 300 DPI)

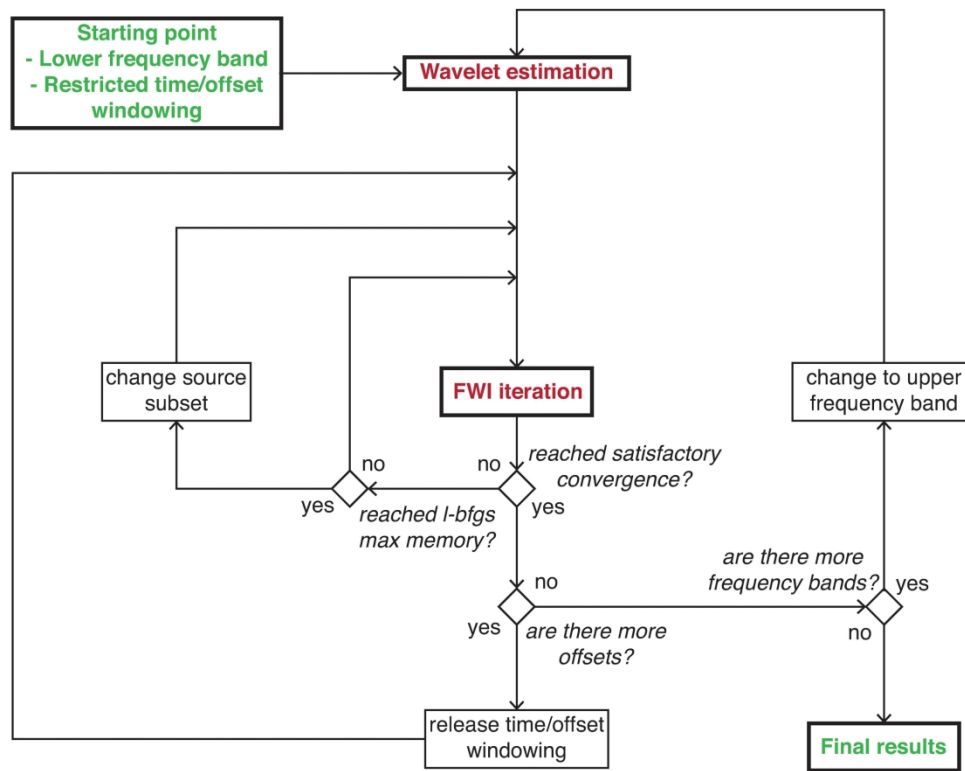


Figure 10: Complete FWI workflow used on the OBC Valhall dataset. At the core of the process lies the FWI iterations. Then several loops are nested one into another, from internal FWI iterations, source-subsampling, data selection, to finally the outer one of frequency continuation.

196x155mm (300 x 300 DPI)

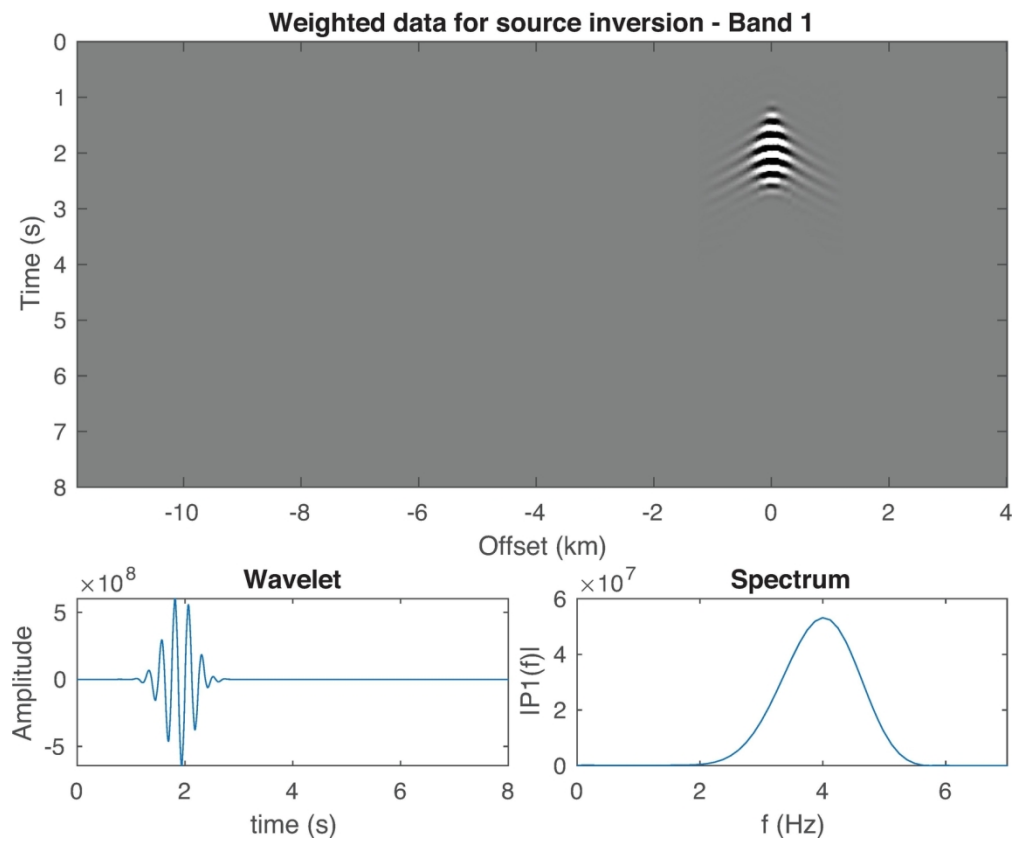


Figure 11: On top, weighted data for source inversion displayed on a 2D common-receiver gather (receiver A cable A). On the bottom, the estimated wavelet (left) and associated spectrum (right).

148x123mm (300 x 300 DPI)

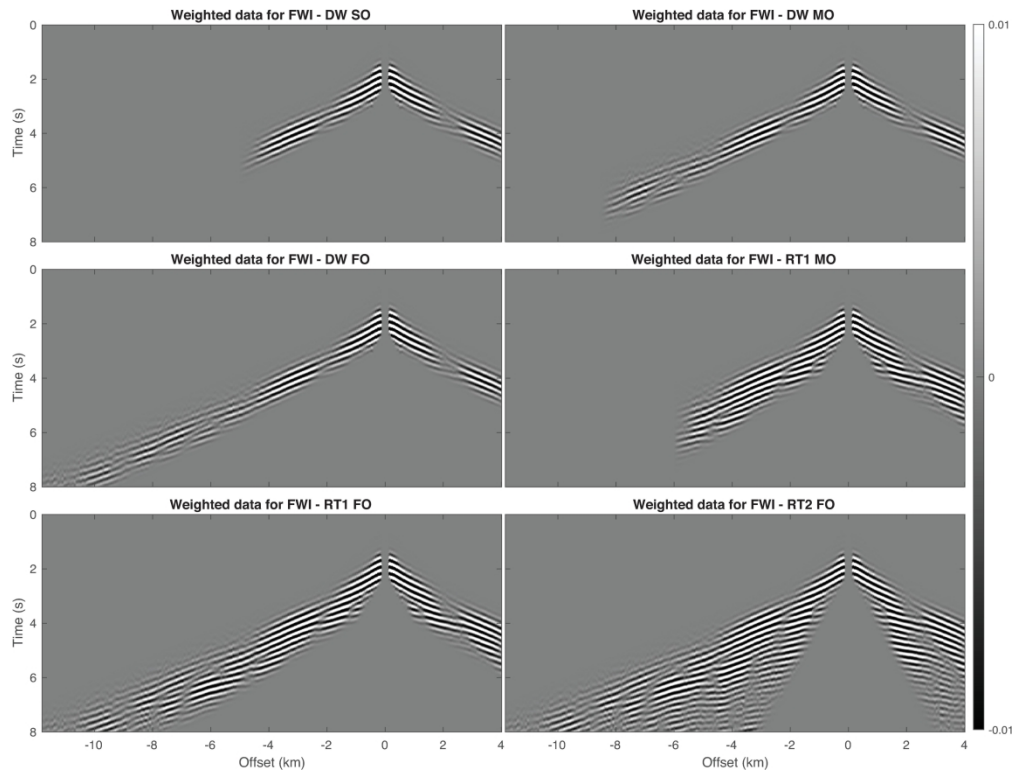


Figure 12: 2D common-receiver gathers extracted for receiver A along cable A with different data weighting applied on them. From top to bottom: first break & short offset (DW SO), first break & medium offset (DW MO), first break & full offset (DW FO), first time release & medium offset (RT1 MO), first time release & full offset (RT1 FO), second time release & full offset (RT2 FO).

288x219mm (300 x 300 DPI)

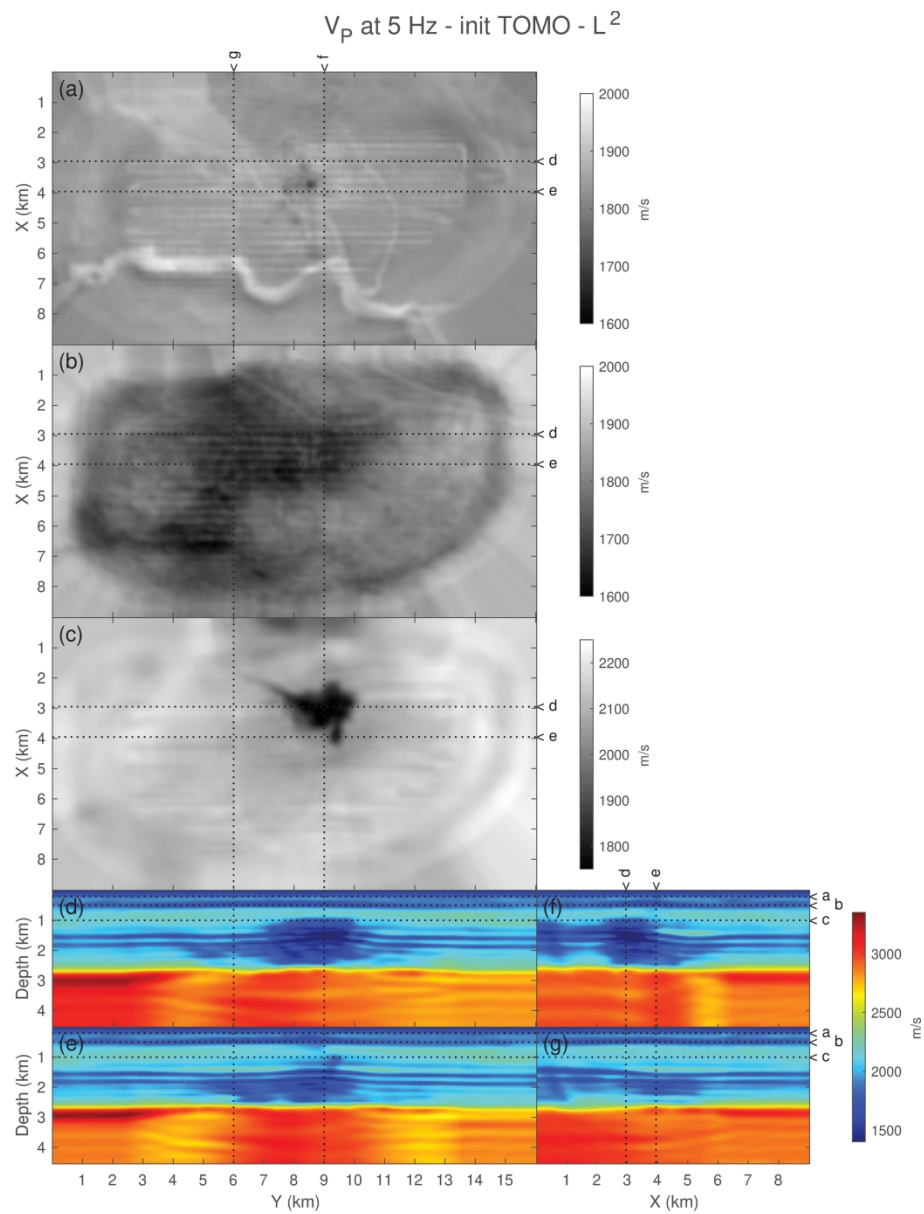


Figure 13: Slices of the 5 Hz FWI reconstructed v_p using L^2 misfit function starting from initial model. (a-c) Horizontal slices at (a) 200 m depth, (b) 500 m depth and (c) 1 km depth. (d-e) Inline vertical slices for (d) $x=2.95$ km and (e) $x=3.95$ km. (f-g) Cross-line vertical slices at (f) $y=9$ km and (g) $y=6$ km.

209x277mm (300 x 300 DPI)

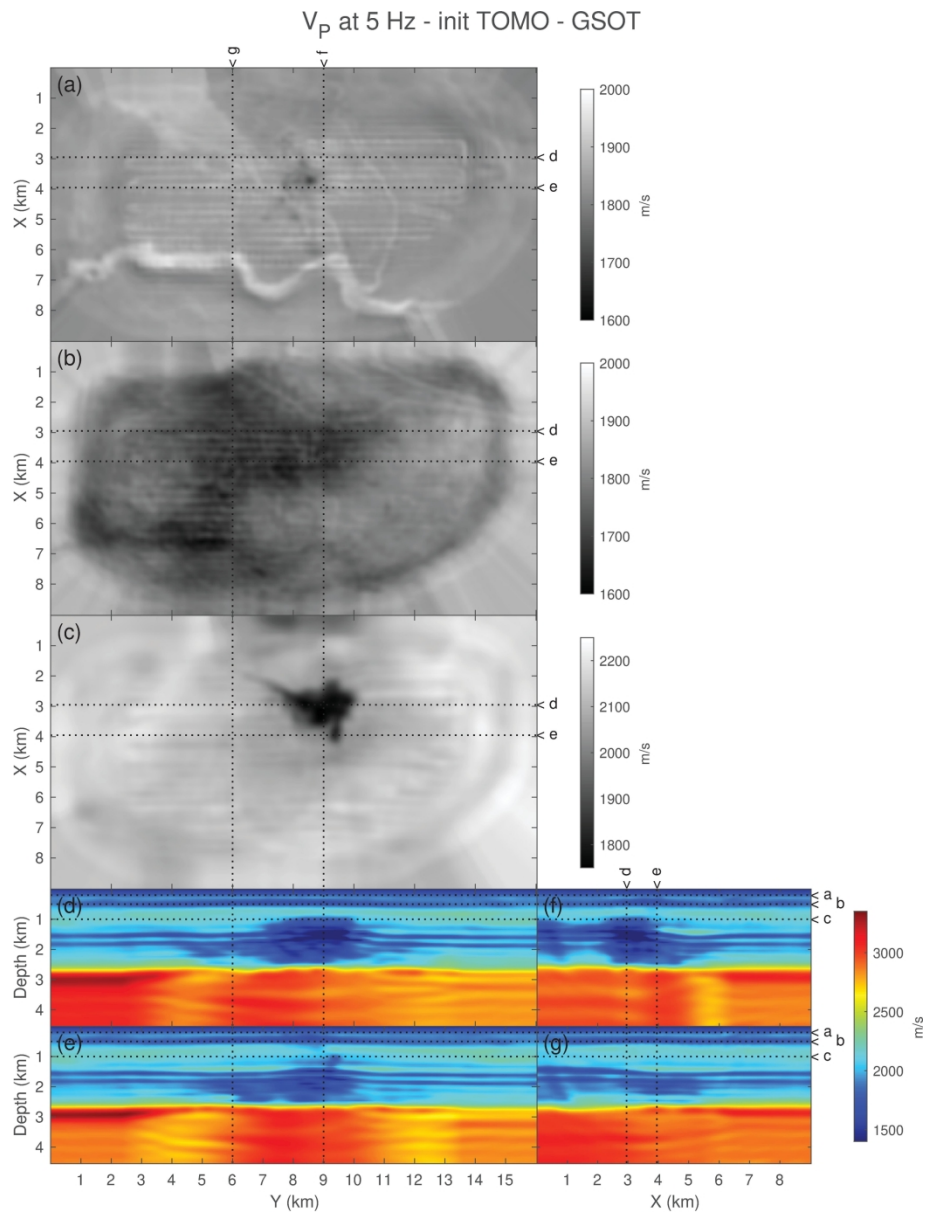


Figure 14: Same as $\backslash\text{Cref}\{vp_VP_initTOMO_5Hz_L2\}$ using GSOT misfit function.

209x276mm (300 x 300 DPI)

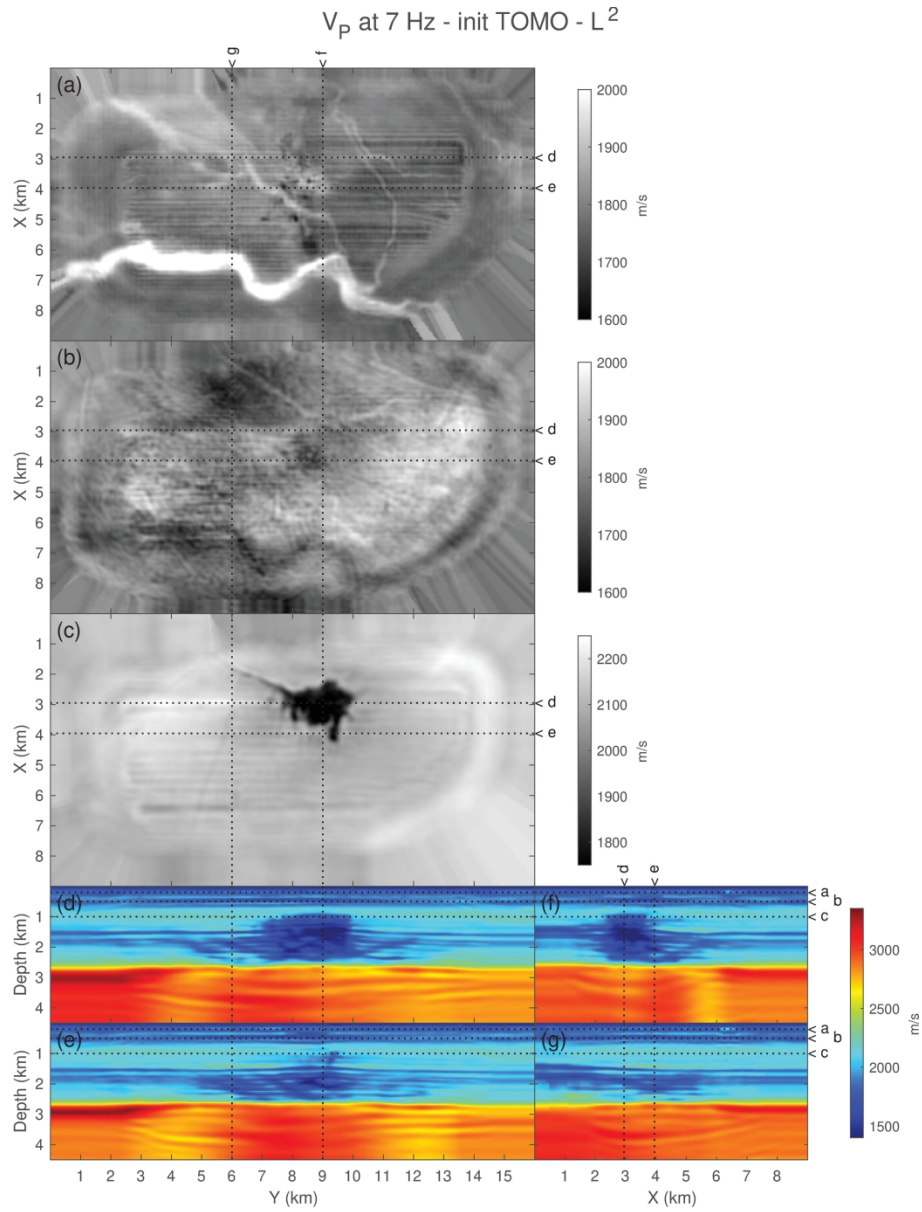


Figure 15: Same as $\text{VP_initTOMO_5Hz_L2}$ for 7 Hz results.

209x277mm (300 x 300 DPI)

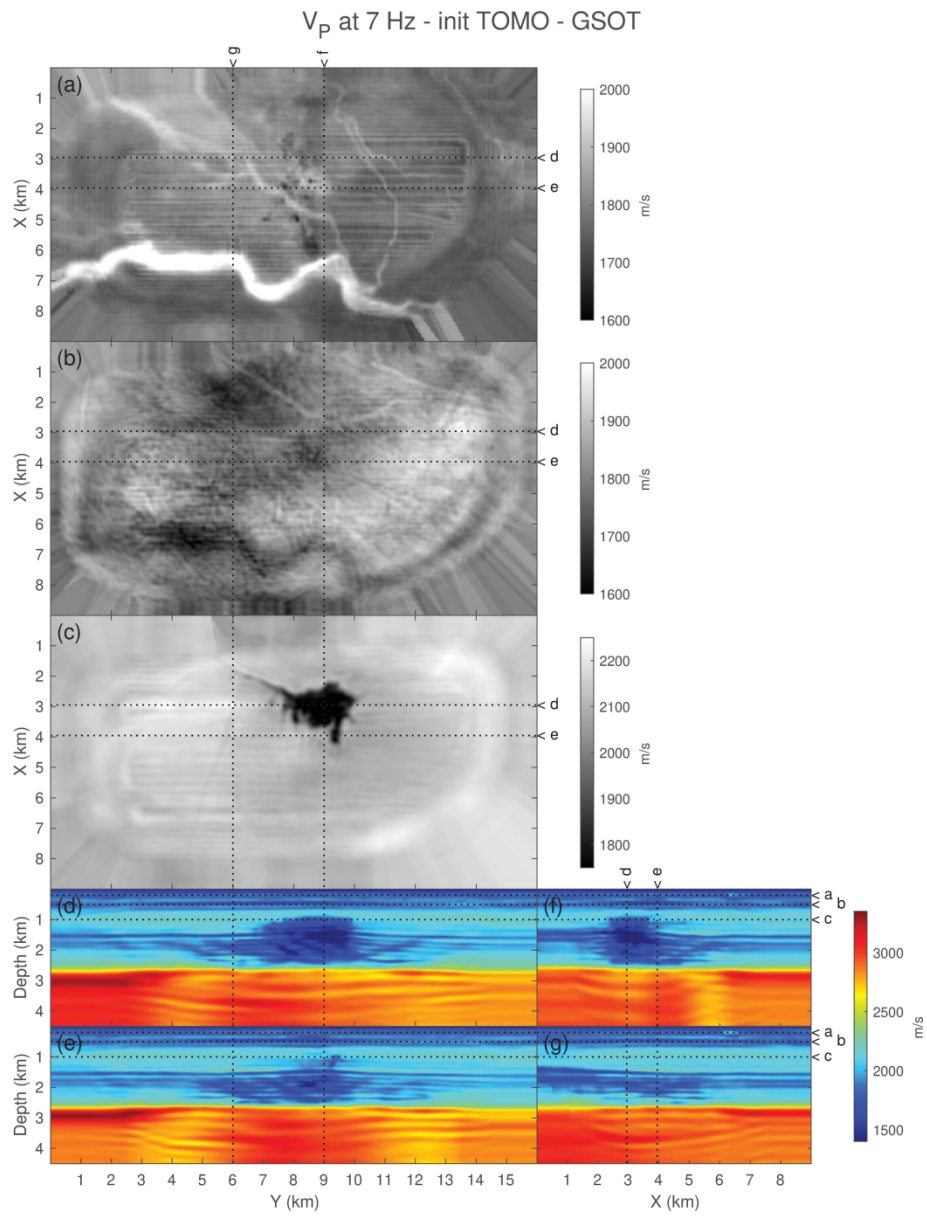


Figure 16: Same as $\backslash\text{Cref}\{vp:VP_initTOMO_5Hz_GSOT\}$ for 7 Hz results.

209x276mm (300 x 300 DPI)

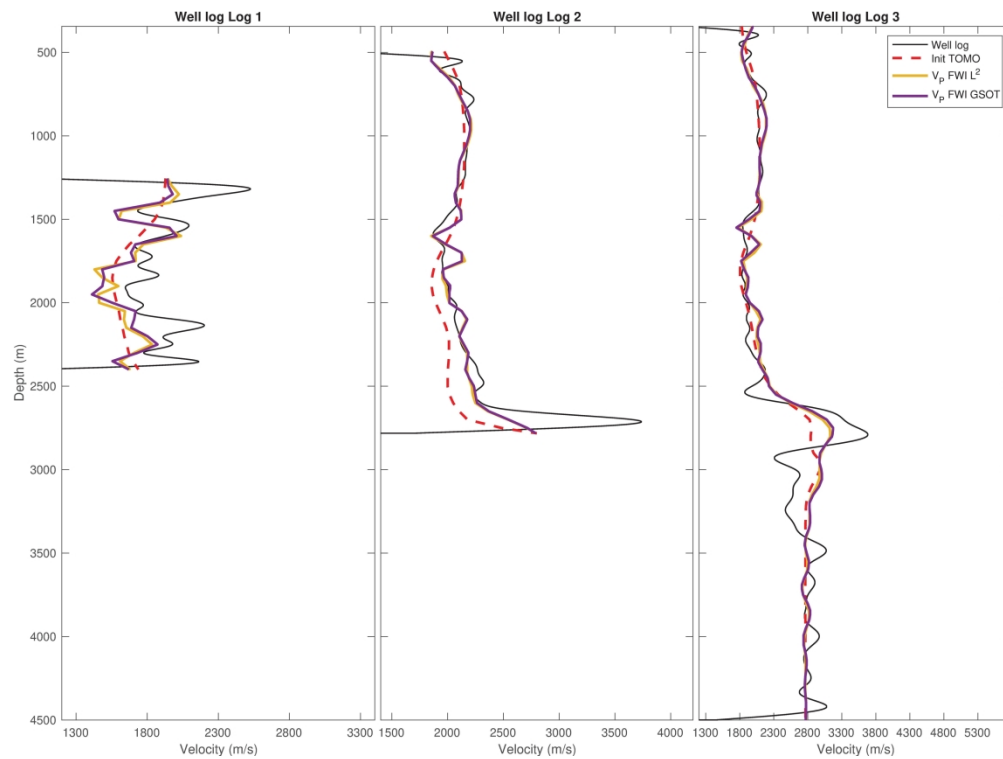


Figure 17: Comparison of v_p profiles extracted from the initial model (dashed red), 0-7 Hz FWI models using L^2 (solid yellow) and 7-7 Hz FWI model using GSOT (solid purple) with sonic log filtered in the 0-7 Hz frequency band (solid black). Left subfigure corresponds to Log 1 at the center of the target. Middle subfigure to Log 2, and right subfigure to Log 3 (far away from the target).

299x223mm (300 x 300 DPI)

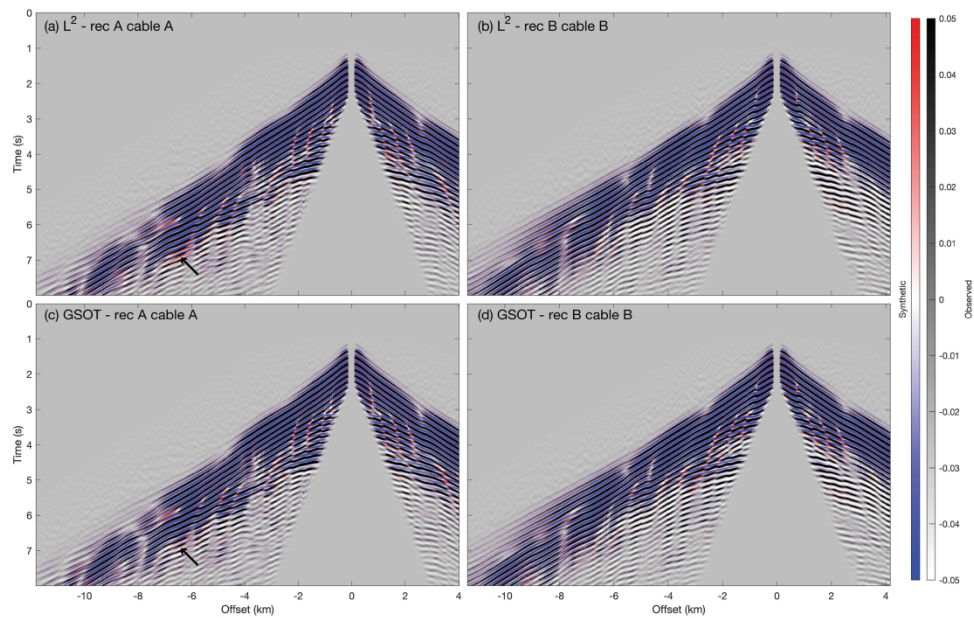


Figure 18: 2D common-receiver gathers at 7 Hz starting from the \moda initial model. Synthetic data (blue/white/red color scale) generated into the final reconstructed v_p using (a,b) L^2 misfit function, (c,d) GSOT misfit function. (a,c) receiver A along cable A (through the low velocity anomaly). (b,d) receiver B along cable B. Field data are overlapped in grayscale with transparency. Black arrows point to improvement obtained with GSOT.

354x229mm (300 x 300 DPI)

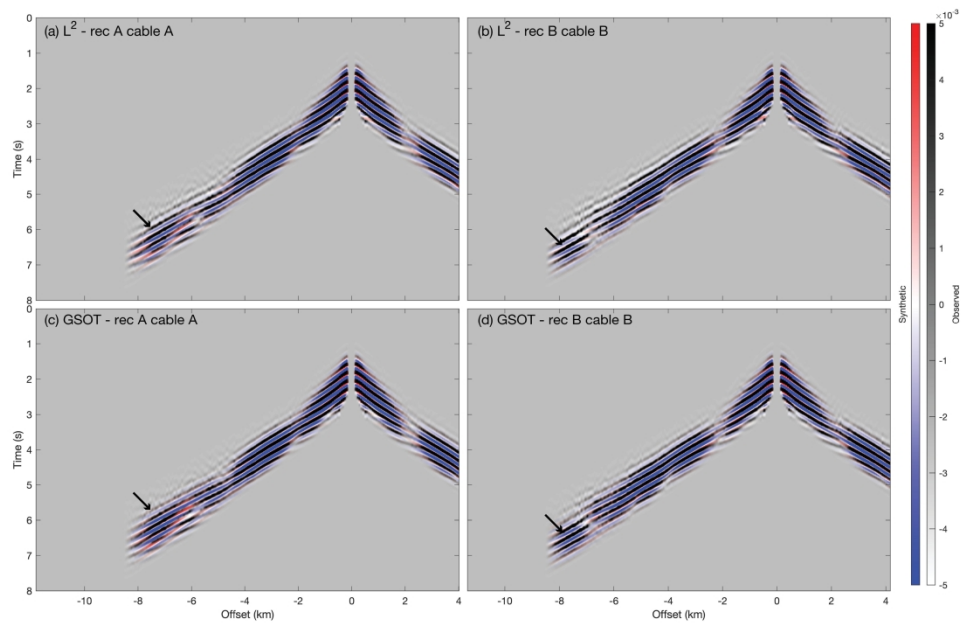


Figure 19: 2D common-receiver gathers at 5 Hz starting from the modb initial model with data weighting apply (DW MO). Synthetic data (blue/white/red color scale) generated into the final reconstructed v_p at 2nd workflow step using: (a,b) L^2 misfit function, (c,d) GSOT misfit function. (a,c) receiver A along cable A (through the low velocity anomaly). (b,d) receiver B along cable B. Field data are overlapped in grayscale with transparency. Black arrows point to area where GSOT improves the datafit.

354x229mm (300 x 300 DPI)

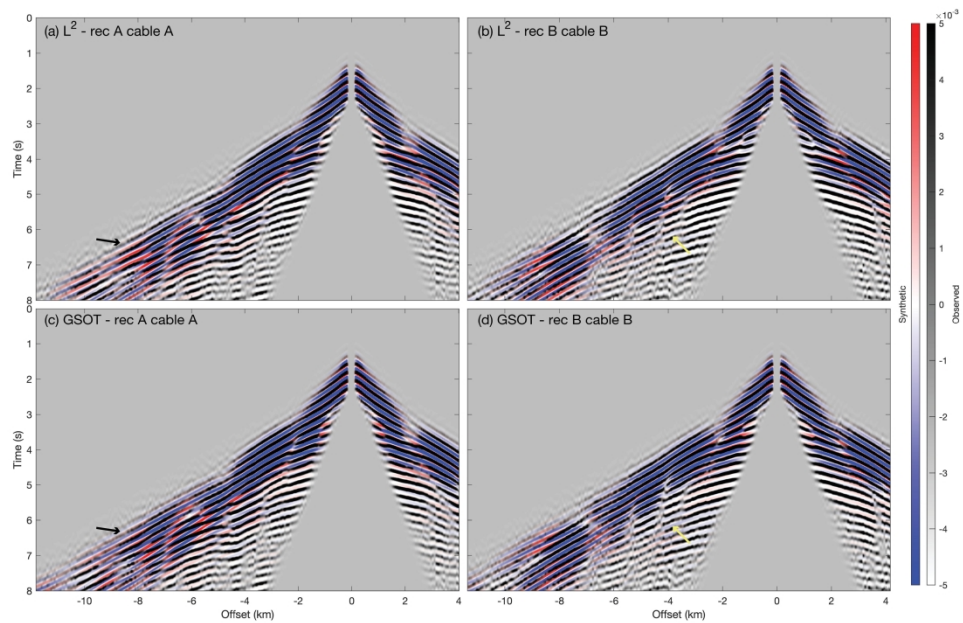


Figure 20: Same as `\Cref{crg:CRG_5Hz_init1D_L2-GS_muted}` but with final relaxed data weighting (RT2 FO) for display. This exhibits the improvement of datafit obtained in area which are not yet inverted. Here, black arrows point to improve fit and coherency of the diving wave with GSOT, whereas yellow arrows point to improved data-fit of reflected events with GSOT.

354x229mm (300 x 300 DPI)

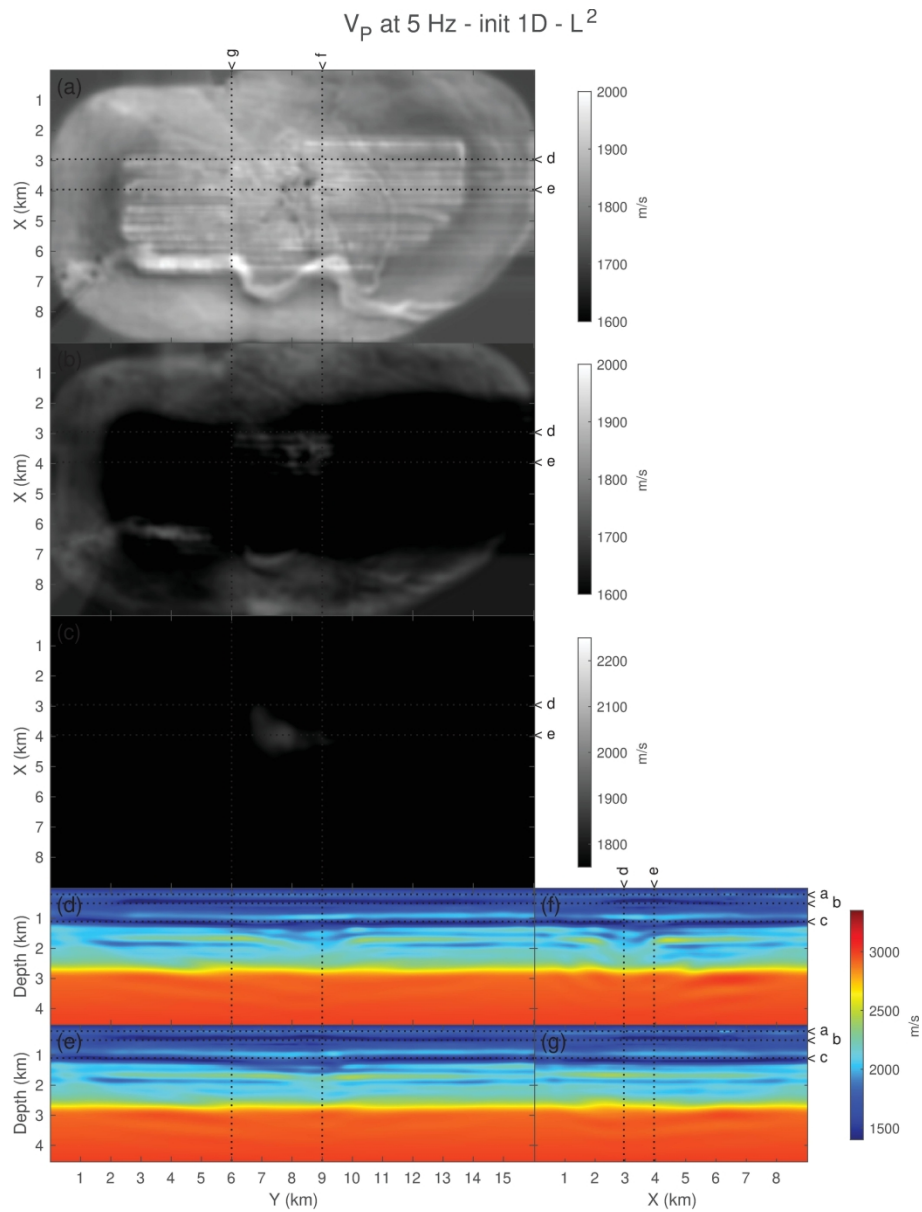


Figure 21: Slices of the 5 Hz FWI reconstructed v_p using ls misfit starting from $modb$ initial model. (a-c) Horizontal slices at (a) 200 m depth, (b) 500 m depth and (c) 1.1 km depth. (d-e) Inline vertical slices for (d) $x=2.95$ km and (e) $x=3.95$ km. (f-g) Cross-line vertical slices at (f) $y=9$ km and (g) $y=6$ km.

209x277mm (300 x 300 DPI)

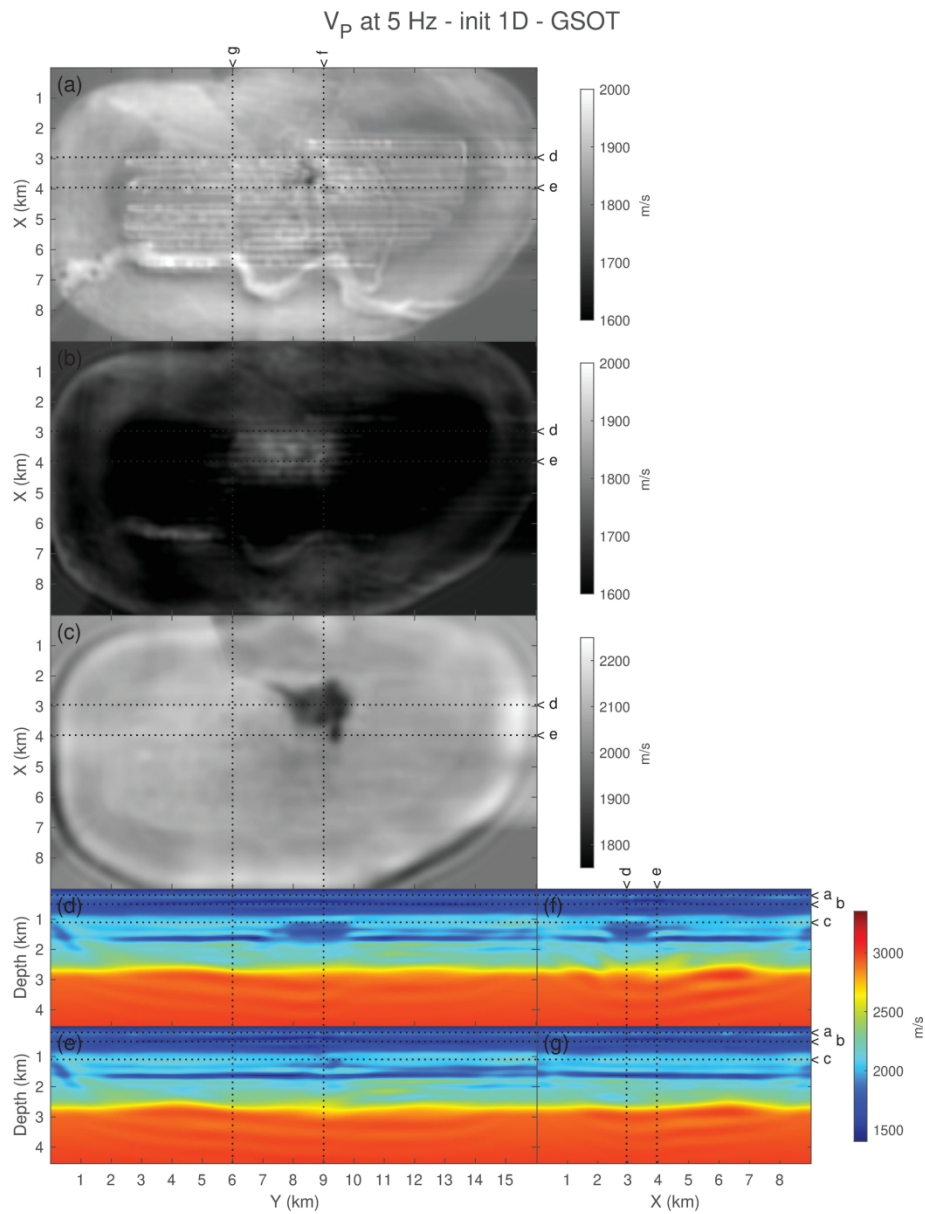


Figure 22: Same as $\text{\Cref{vp:VP_init1D_5Hz_L2}}$ using GSOT. Here, results are constant until 1.5 km to 2 km depth compared to FWI. Characteristic structures of the Valhall field are recovered.

209x276mm (300 x 300 DPI)

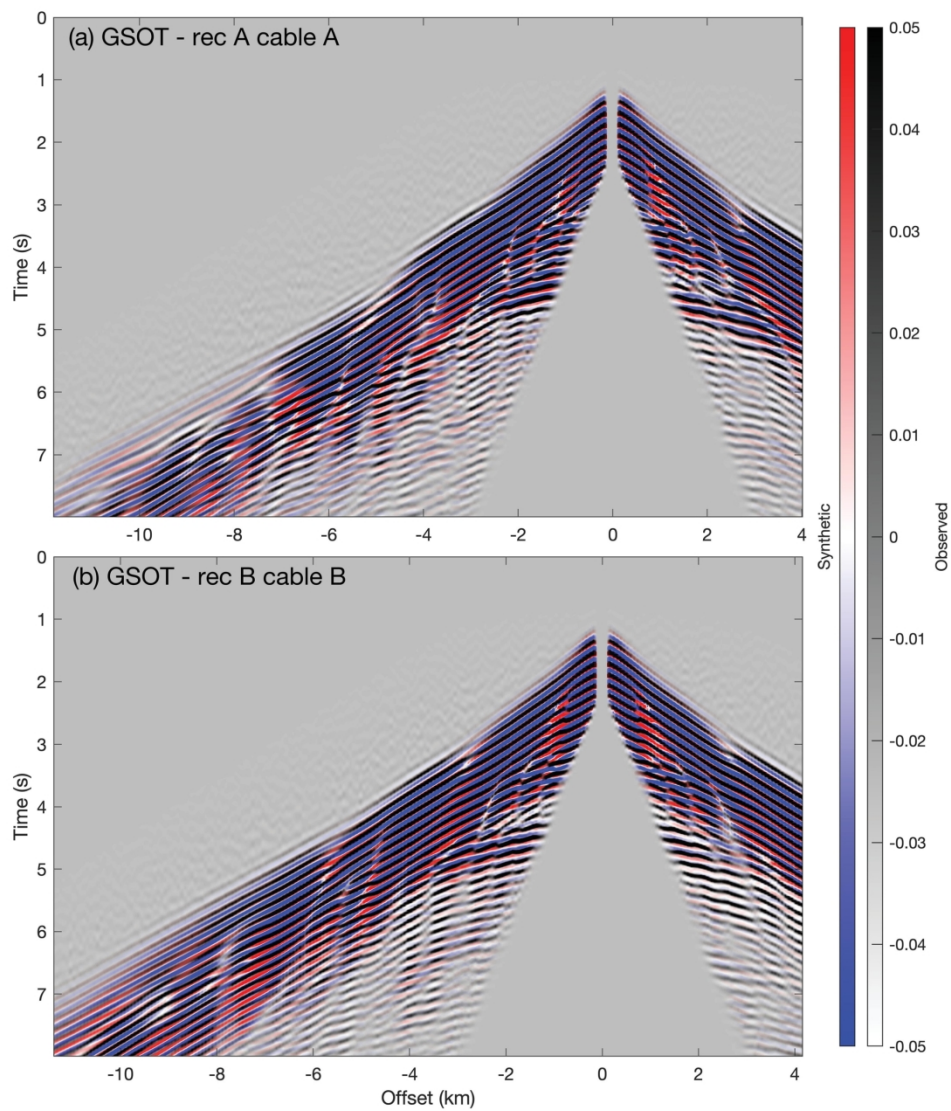


Figure 23: 2D common-receiver gathers at 7 Hz starting from the modb initial model. Synthetic data (blue/white/red color scale) generated into the final reconstructed v_p using GSOT. (a) receiver A along cable A (through the low velocity anomaly). (b) receiver B along cable B. Field data are overlapped in grayscale with transparency.

200x229mm (300 x 300 DPI)

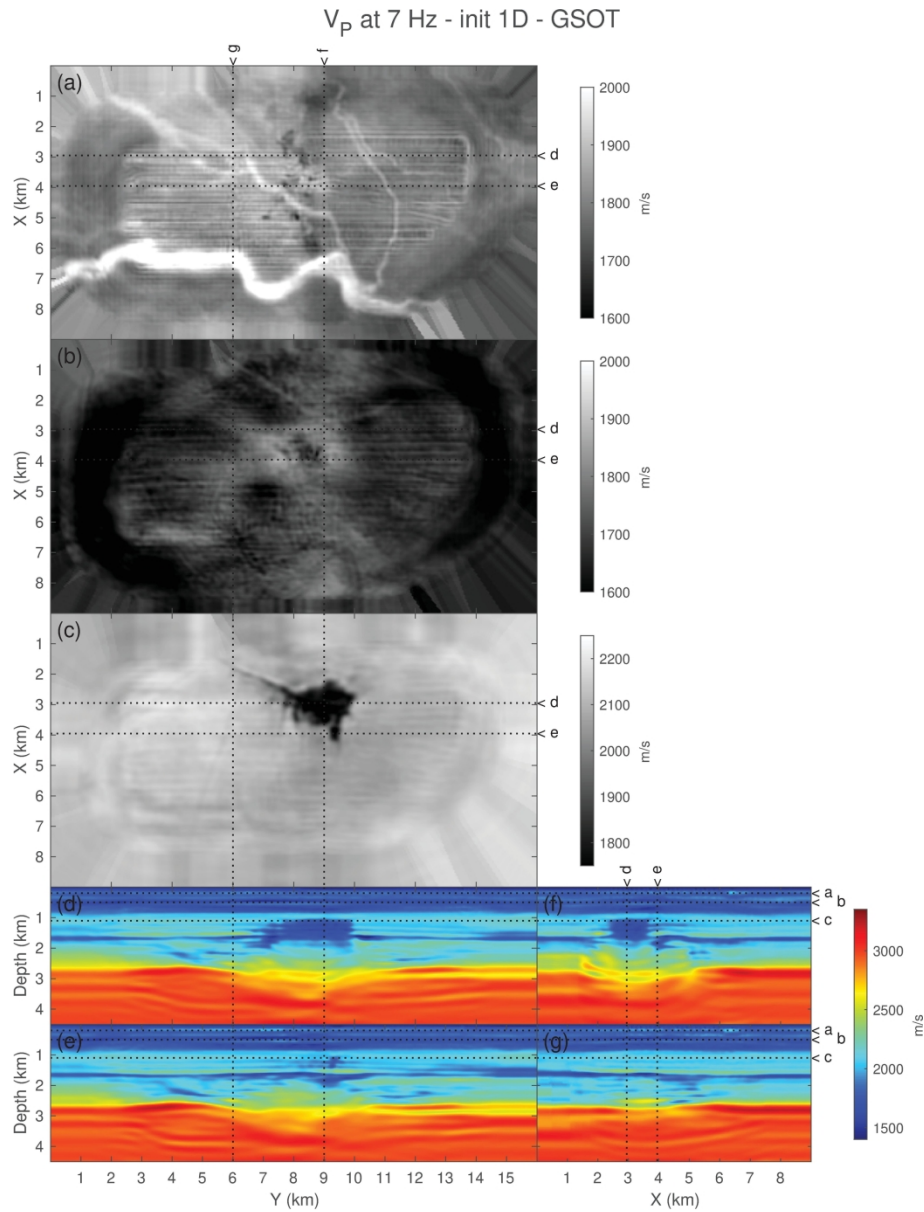


Figure 24: Slices of the 7 Hz FWI reconstructed v_p using GSOT misfit starting from `modb` initial model. (a-c) Horizontal slices at (a) 200 m depth, (b) 500 m depth and (c) 1.1 km depth. (d-e) Inline vertical slices for (d) $x=2.95$ km and (e) $x=3.95$ km. (f-g) Cross-line vertical slices at (f) $y=9$ km and (g) $y=6$ km.

209x276mm (300 x 300 DPI)

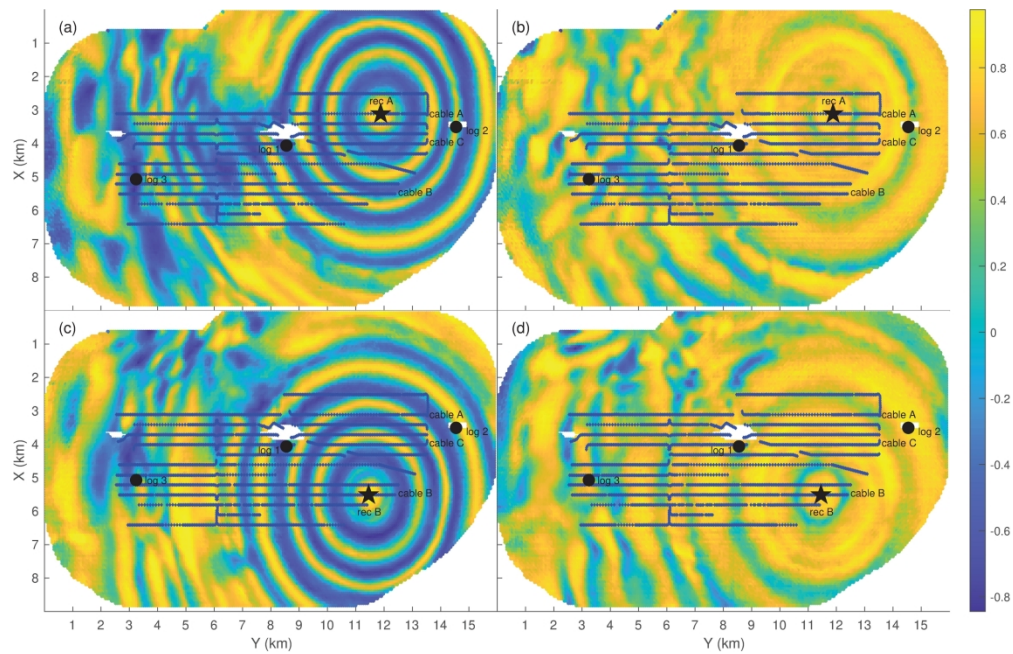


Figure 25: Cross-correlation analysis between field data and synthetic data. (a) common-receiver gather A and synthetic data in the \modb initial model. (b) common-receiver gather A and synthetic data in the final reconstructed GSOT-based FWI model. (c) common-receiver gather B and synthetic data in the \modb initial model. (d) common-receiver gather B and synthetic data in the final reconstructed GSOT-based FWI model.

274x176mm (300 x 300 DPI)

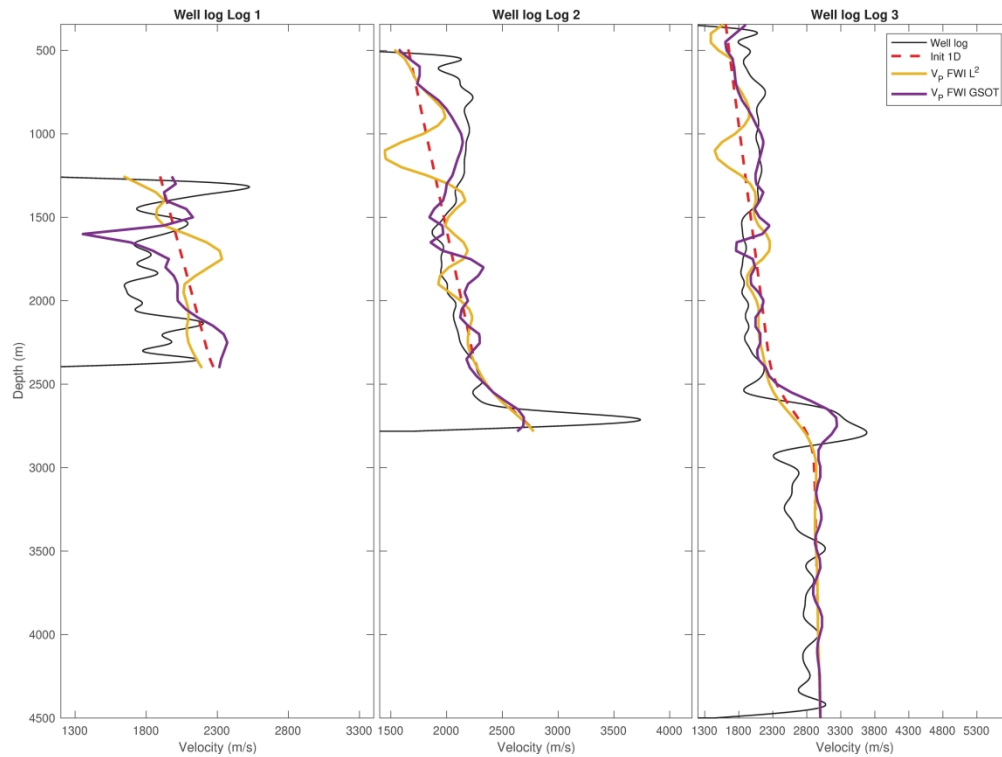


Figure 26: Comparison of v_p profiles extracted from the `modb` initial model (dashed red), early FWI models using `L1` (solid yellow) and $7\text{--}7\text{ Hz}$ FWI model using GSOT (solid purple) with sonic log filtered in the $0\text{--}7\text{ Hz}$ frequency band (solid black). Left subfigure corresponds to Log 1 at the center of the target. Middle subfigure to Log 2, and right subfigure to Log 3 (far away from the target). Updates of velocity model obtained with GSOT are following the sonic logs trend until $\approx 2\text{ km}$ depth.

299x223mm (300 x 300 DPI)

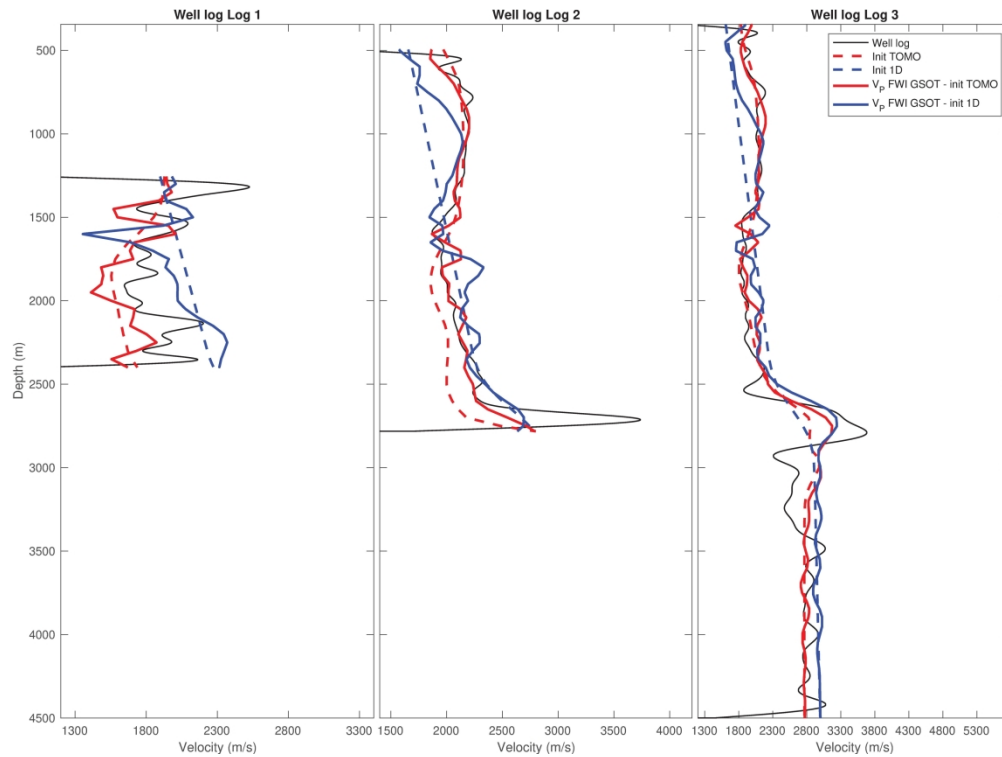


Figure 27: Comparison of v_p profiles extracted from the \moda and \modb initial models (respectively dashed red and dashed blue), GSOT-based FWI reconstructed models at 7 Hz starting from \moda and \modb initial models (respectively solid red and solid blue), with sonic log filtered in the $0\text{-}7\text{ Hz}$ frequency band (solid black). Logs 2 and 3 show that results from the two different starting models are globally following the same trend. Results from Log 1 passing through the target are following the same trend until 1.4 km depth.

299x223mm (300 x 300 DPI)

| | computational efficiency | data distorsion | convexity | multi-D |
|------------------------|--------------------------|-----------------|-----------|---------|
| Nonlinear transform OT | ++ | - | + | - |
| 1-Wasserstein | - | + | - | + |
| GSOT | + | + | + | - |

Table 1

| | Gradient | Misfit | Total time | Ratio |
|--------------|----------|--------------|------------|-------|
| L^2 - 5 Hz | 243 s | 1 s | 254 s | 100 % |
| GSOT - 5 Hz | 243 s | 55 s | 308 s | 121 % |
| L^2 - 7 Hz | 898 s | 1 s | 912 s | 100 % |
| GSOT - 7 Hz | 898 s | 101 s | 1012 s | 111 % |

Table 2