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Active exterior cloaking and mimicking for the heat equation

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Abstract – We present two methods for cloaking objects from thermal measurements that use active sources instead of metamaterials. One method deals with the parabolic heat equation [1], in which case active sources need to completely surround the object to cloak or mimic. Another method [2], for which the sources do not completely surround the object to cloak, relies on the frequency domain formulation of the heat equation as a Helmholtz equation with complex wave numbers. This second method extends the active exterior cloaking for waves modelled with the Helmholtz equation in [3] from positive wavenumbers to complex ones [2]. We point out the second method can be applied to waves in dispersive media.

I. INTRODUCTION

The temperature $u(x, t)$ of a homogeneous isotropic body satisfies the parabolic heat equation

$$\frac{\partial u}{\partial t} = \sigma \Delta u + h, \text{ for } t > 0, \quad (1)$$

where σ is the thermal diffusivity (a scalar constant), h is a source term and Δ is the Laplace operator in 2D.

If one makes a change of variables in (1), $\sigma \Delta u$ becomes $\nabla' \cdot \sigma' \nabla'$, with σ' an anisotropic heterogeneous parameter and ∇' the gradient in the transformed coordinate system. Cloaking of heat in the transient regime was originally introduced in this way that led to an anisotropic heterogeneous diffusivity [4]. An experimental validation of this concept with a metamaterial cloak engineered by the group of Martin Wegener at the Karlsruhe Institute for Technology can be found in [5]. Changing the shape of the cloak and reducing its thickness leads to extreme material parameters (except for diamond shape cloaks). Many research groups have contributed to the design of thermal metamaterials, notably to make them more amenable to fabrication and this field of transformation thermodynamics is now quite mature. We recommend the research monograph for a state of the art [6]. However, managing thermal fields does not necessarily require the use of metamaterials, as we shall now see.

II. EXTERNAL CLOAKING FOR THE HEAT EQUATION

We wish to find sources for the heat equation (1) in a homogeneous medium so that they generate a cloaking field $u_c(x, t)$ that cancels out a probing field $u_i(x, t)$ inside a region Ω without perturbing the total field outside Ω . By linearity of the heat equation the total field $u_c(x, t) + u_i(x, t)$ vanishes inside Ω allowing us to hide e.g. a homogeneous Dirichlet inclusion, while it is identical to $u_i(x, t)$ outside of Ω , preventing an observer outside of Ω from detecting the inclusion solely based on thermal measurements. Of course, our approach is not limited to Dirichlet inclusions, it applies to any linear passive object characterized by a boundary condition (Dirichlet, Neumann, Robin, ...) or by transmission conditions (e.g. penetrable inclusions) or even to some active objects. It is based on the Green identities to show that a distribution of monopoles and dipoles supported on $\partial\Omega$ can provide the desired cloaking field [1]. In practise, these monopoles and dipoles can be realized using heat pumps such as Peltier devices, see Figure 1. This cloaking method inspired by [7] in the context of propagative phenomena comes with two main drawbacks: (a) the incident field $u_i(x, t)$ needs to be known ahead of time and (b) the sources needed to generate the cloaking field completely surround the object, as shown in Figure 1. However, this method provides also an efficient way to create thermal illusions, as shown in Figure 2, without dealing with complex metamaterial parameters as in passive mimicking methods [8].

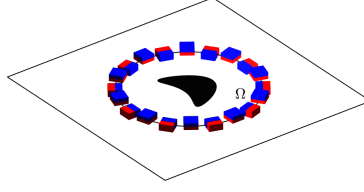


Fig. 1: Example of an arrangement of Peltier elements that could be used to cloak objects (e.g. a kite) inside a two-dimensional region Ω , illustrated here by a disk within a heat conducting plate. Each Peltier device is represented by two adjacent red and blue boxes, where the heat flux it can create is oriented in the direction normal to their interface. We have represented both Peltier elements that transport heat within the plate (across the boundary $\partial\Omega$ of Ω) and also between the exterior and the plate. This figure comes from the article [1].

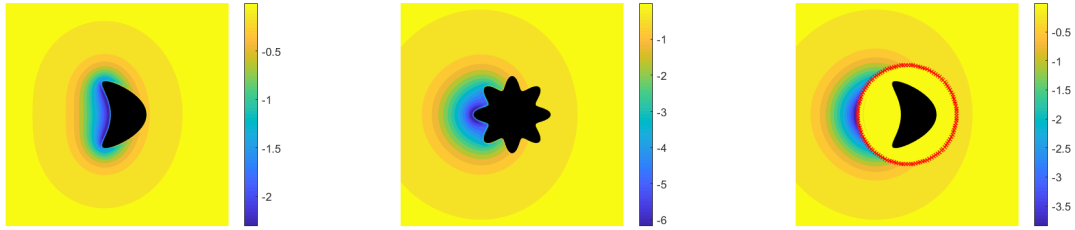


Fig. 2: Numerical example of a kite mimicking a flower object, both with homogeneous Dirichlet boundary conditions. A snapshot the scattered field from the original object at a given time ts appears in (a). In (c), the scattered field from the original object is made to appear as the scattered field from the object in (b) from the perspective of thermal measurements outside of the cloaking region $\bar{\Omega}$. This figure comes from the article [1].

III. ACTIVE EXTERIOR CLOAKING FOR THE HELMHOLTZ EQUATION WITH COMPLEX WAVENUMBERS

To work around drawback (b), we notice that the Fourier-Laplace transform of a solution to the heat equation must solve the Helmholtz equation with a complex wavenumber. To be more precise, let $u(x, t)$ be a solution to the heat equation (1). Under mild-assumptions, the Fourier-Laplace transform $u(x, \omega)$ of $u(x, t)$ is given by

$$u(x, \omega) = \int_0^{\infty} dt \exp[i\omega t]u(x, t), \quad (2)$$

and satisfies the Helmholtz equation with a wavenumber k given by the relation $k^2 = i\omega/\sigma$:

$$\Delta u(x, \omega) + (i\omega/\sigma)u(x; \omega) = -h(x, \omega), \quad (3)$$

with ω the angular frequency.

Once in the frequency domain we can achieve active exterior cloaking for the Helmholtz equation by generalizing the approach in [3] from real positive wavenumbers to complex wavenumbers and then take the inverse Fourier-Laplace transform to recover time domain sources for the heat equation that satisfy our requirements. The approach in [3] consists in using the Green identities for the Helmholtz equation to determine a density of monopoles and dipoles on $\partial\Omega$ that achieves cloaking. Then the boundary $\partial\Omega$ is split into N_{dev} segments $\partial\Omega_1, \dots, \partial\Omega_{N_{dev}}$ and the monopole and dipole density in segment $\partial\Omega_j$ is “moved” to a new location x_j . This can be done using the Graf addition formula and what we obtain are N_{dev} multipolar sources supported at the new locations x_j . In the case $k > 0$, the series defining the multipolar sources are proven to converge [3] to the desired fields outside of the union of the gray disks in figure 3.

A numerical result of thermal cloaking is given in figure 4. We proved [2] that the same result holds for complex wavenumbers (excluding the negative real axis) by establishing that the tail of the series can be bounded by the tail of a geometric series with ratio depending only on the point of interest and its position relative to the new source locations x_k and the boundary segments $\partial\Omega_j$. Such bounds on the truncation error allow us to predict

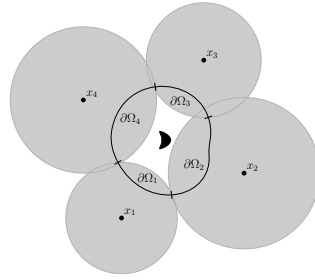


Fig. 3: Example of surface $\partial\Omega$ split into four sections $\partial\Omega_1, \dots, \partial\Omega_4$. where monopoles and dipoles on $\partial\Omega_j$ are moved to a new location x_j . The disks in gray denote the region where the cloaking field u_c diverges.

quantitatively the cloaking effect. We also apply the maximum principle to equation (3) to guarantee that the error from truncating the multipolar sources series on a region reaches its maximum on the boundary of the region.

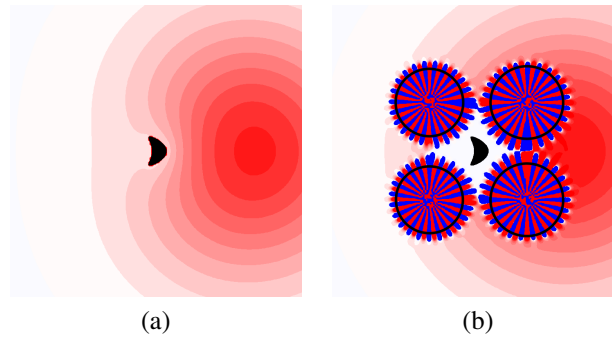


Fig. 4: Temperature distribution at a given time t resulting from (a) a point source in the presence of a kite object with homogeneous Dirichlet boundary condition and (b) same as in (a), but with the cloaking devices activated. The linear color scale ranges from blue to red, with zero being represented in white.

IV. CONCLUSION

We have proposed a new path to thermal cloaking, based on active sources. Our approach, when performed in the frequency domain, relies upon a Helmholtz equation with a complex wavenumber, also valid for wave problems in dispersive media. Our proposal is a source of inspiration for other groups [9].

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