

A geometric Approach to Inference in Set identified Entry Games

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Outline

- 1 Introduction
- 2 An example with three players
- 3 Testing a point in the identified set
- 4 Inference
- 5 Monte Carlo Experiments
- 6 Conclusion

Motivation

- Entry games are very popular in the empirical IO literature, mainly because they can estimate features of some industry while observing the decision of firms to enter or not in independent markets and their characteristics (see Berry and Reiss, 2007, for a survey).
- Entry games are games with multiple equilibria. There is no longer unicity of the model predictions. We can not estimate them with standard econometric procedures without imposing further assumptions.
- There are *incomplete* models because the selection mechanism is indeed unknown in the regions of multiple equilibria. Standard solutions often used are the following ones:
 - Postulating some selection mechanism (Bjorn and Vuong, 1984, Berry, 1992, Cleeren et al., 2010, among others ...),
 - Working from an outcome which is invariant, like the number of active firms at the equilibrium, e.g. Berry (1992),
 - Using the recent literature on moment inequalities like in Ciliberto and Tamer, 2009, Beresteanu et al., 2011, Galichon and Henry, 2011, Chesher and Rosen, 2019.

Remark that multiple equilibria does not necessary imply set identification.

This paper I

- In this paper, we consider the estimation of an entry game with simultaneous move, in the complete information setting. We consider a setting à la Berry (1992), with discrete covariates and Nash equilibrium concept.
- It is part of a broader agenda on the estimation of games with multiple equilibria.
- We estimate the model through moment inequalities (we invert a test, i.e. a set of values for the parameters is admissible if the moment inequalities are satisfied).
- The moment inequalities necessary to sharply characterize the identified set (see Beresteanu et al. 2011) are derived in closed form through the characterization of the set of predicted probabilities.

This paper II

- However, when the number of players/firms starts to be "big enough" (5, 6 or 7 players), the number of inequalities required to sharply define the identified set is very important (2114 for $N = 5$, 1 114 242 inequalities for $N = 6$ and 68 723 671 298 for $N = 7$ without expl. variables). Therefore, we propose a moment selection procedure based on the geometry of the problem and which does not require to evaluate all of them.
- We also propose a way to calculate the critical value of the test statistic, which is calculated once for all and we compare, in a Monte Carlo Section, our procedure with existing methods.
- Among the blossoming and huge literature on set identification and moment inequality models, here are four papers which are close to ours
 - Ciliberto and Tamer, Eca 2009, estimate a set identified entry game. They tackle the numerical issues that arise in a game with many players by estimating an outer set. It is not sharp.

This paper III

- Beresteanu et al. (2011) and Galichon and Henry (2011) propose an explicit characterization of the identified set by a collection of many moment inequalities. Galichon and Henry propose an algorithm that selects among this infinity of moment inequalities the sequence of non redundant ones.
- Chesher and Rosen (2019) use revealed preference arguments to derive moment inequalities and apply it to the case of entry with two types.

Set identification in Econometrics I

- In a (potentially) set identified model, observing the whole population is not sufficient to recover a unique value of the parameter. It generally arises when there is some missing information. In the Entry example, this is the selection mechanism in the regions of multiple equilibria.
- The literature on set identification (largely developed by Manski and his coauthors) provides a guideline to analyze such problems without adding any extra assumptions but collecting all values for which any acceptable arbitrary additional assumptions would obtain. The identifying power of different assumptions may be compared in terms of the size of the set which is identified.

Set identification in Econometrics II

- Estimation and inference are usually tackled by bounding this missing information and transforming the problem into moment inequalities (like in Ciliberto and Tamer, 2009, or Manski and Tamer, 2002). There is a huge literature on moment inequality models, among which Chernozhukov et al. (2007), Rosen (2008), Andrews and Soares (2010), Bugni (2010), Canay (2010), Romano and Shaikh (2008,10), Chernozhukov et al. (2012), Andrews and Shi (2013), Aradillas-Lopez and Rosen (2016) or Chesher and Rosen (2019).
- Another path consists of characterizing the geometry of the problem. The identified set may be convex (like in Beresteanu and Molinari, 2008, or Bontemps et al. 2012) or the model is a model with convex predictions, i.e. testing a point in the identified set is equivalent to testing that another point belongs to the convex set (Galichon and Henry, 2009, 2011, Beresteanu et al. 2011, Bontemps, Magnac and Pacini, 2020).
 - Possibility of using the tools of the convex literature (Rockafellar, 1970), in particular the support function.

Set identification in Econometrics III

- Kaido and Santos (2014) prove that it is optimal to estimate the support function when the identified set is convex.
- Ultimately, moment inequalities are collected.

See surveys in *Annual Review of Economics*: Tamer (2010), Molchanov and Molinari (2015), Bontemps and Magnac (2017).

Outline

Organization of the talk

1. Introduction
2. The geometric characterization of an entry game
3. Testing a point in the identified set.
4. Inference.
5. Some Monte Carlo simulations.

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The specification I

A simple entry game with three firms and no explanatory variable.

The action of firm i to enter ($y_{i,m} = 1$) or not enter ($y_{i,m} = 0$) in market m , is depending on firm i 's profitability β_i , the other firms' actions and a profit shocks $\varepsilon_{i,m}$ known by both firms.

$$y_{i,m} = \mathbf{1}\{\beta_i + \alpha_i \sum_{j \neq i} y_{j,m} + \varepsilon_{i,m} > 0\}.$$

The profit when a firm does not enter is normalized to 0.

The joint distribution of $(\varepsilon_{1,m}, \varepsilon_{2,m}, \varepsilon_{3,m})$, $F(\cdot)$, is assumed to be known (up to a vector of parameters). We also assume that all α_i 's are negative.

The econometrician observes, in each market, one of the eight different outcomes (y_1, y_2, y_3) , from which he can estimate eight choice probabilities.

Regions of multiple equilibria with three players I

We first check what are the model predictions for the eight possible outcomes for a given value of the parameters.

First, the two outcomes with no entry or all players entering are uniquely determined as there is no multiple equilibria in this case:

$$P_{0,0,0} = P[\varepsilon_i < -\beta_i, i = 1, \dots, 3; \theta],$$

and

$$P_{1,1,1} = P[\varepsilon_i \geq -2\alpha_i - \beta_i, i = 1, \dots, 3; \theta],$$

Additionally, we have eight regions with multiple equilibria, four when there is one active firm at the equilibrium and four when there are two active firms at the equilibrium (see Berry, 1992). We now characterize the set of choice probabilities for one active firm.

Regions of multiple equilibria with three players II

- $\mathcal{R}_{1,2}^{(1)}(\theta)$ is the region where either firm 1 enter or firm 2 enters but the model does not predict which one enters. It is defined by $\varepsilon_i \in [-\beta_i; -\beta_i - \alpha_i]$, $i = 1, 2$ and $\varepsilon_3 < -\beta_3$. The probability of $\mathcal{R}_{1,2}^{(1)}(\theta)$ is

$$\Delta_{1,2}^{(1)}(\theta) = P(\varepsilon \in \mathcal{R}_{1,2}^{(1)}(\theta)).$$

- We can define similarly $\mathcal{R}_{2,3}^{(1)}(\theta)$, $\mathcal{R}_{1,3}^{(1)}(\theta)$, $\Delta_{2,3}^{(1)}(\theta)$ and $\Delta_{1,3}^{(1)}(\theta)$.
- $\mathcal{R}_{1,2,3}^{(1)}(\theta)$ is the region where any firm is profitable in the monopoly case but not in the duopoly case. This is the region $\varepsilon_i \in [-\beta_i; -\beta_i - \alpha_i]$, $i = 1, 2, 3$. $\Delta_{1,2,3}^{(1)}(\theta)$ is defined similarly.
- Remark that the values Δ can be either computed analytically or simulated (see Berry, 1992, or Ciliberto and Tamer, 2009).

Regions of multiple equilibria with three players III

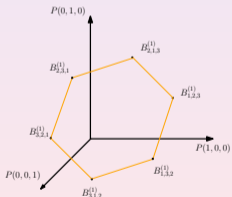
- Similarly $\mathcal{R}_i^{(1)}(\theta)$ are the regions of unique prediction. For $i = 1$ this is the region of ε that predicts unambiguously the outcome $(1, 0, 0)$. It is the region defined by the union of $(\varepsilon_1 > -\beta_1, \varepsilon_j \leq -\beta_j, j = 2, 3)$ and $(\varepsilon_1 > -\beta_1 - \alpha_1, \varepsilon_j \leq -\beta_j - \alpha_j, j = 2, 3)$.

A parametrization of the set of the predicted choice probabilities I

The set of predicted choice probabilities for the three outcomes $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$ can be written as function of these values:

$$\begin{bmatrix} P_{1,0,0} \\ P_{0,1,0} \\ P_{0,0,1} \end{bmatrix} = \begin{bmatrix} \Delta_1^{(1)}(\theta) + u_{1,3}^{(1,1)} \Delta_{1,3}^{(1)}(\theta) + u_{1,2}^{(1,1)} \Delta_{1,2}^{(1)}(\theta) & + u_{1,2,3}^{(1,1)} \Delta_{1,2,3}^{(1)}(\theta) \\ \Delta_2^{(1)}(\theta) + u_{2,3}^{(2,1)} \Delta_{2,3}^{(1)}(\theta) + (1 - u_{1,2}^{(1,1)}) \Delta_{1,2}^{(1)}(\theta) & + u_{1,2,3}^{(2,1)} \Delta_{1,2,3}^{(1)}(\theta) \\ \Delta_3^{(1)}(\theta) + (1 - u_{1,3}^{(1,1)}) \Delta_{1,3}^{(1)}(\theta) + (1 - u_{2,3}^{(2,1)}) \Delta_{2,3}^{(1)}(\theta) & + (1 - u_{1,2,3}^{(1,1)} - u_{1,2,3}^{(2,1)}) \Delta_{1,2,3}^{(1)}(\theta) \end{bmatrix}.$$

The u 's are bounded by 0 and 1. It is a parametrization of the following convex set, labeled $B_1(\theta)$:

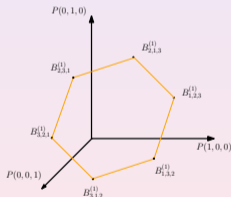


Similarly we obtain an equivalent characterization for the outcomes with two entering firms.

Testing that a point θ belongs to the identified set I

Let P^0 be the true probability vector generated by the true parameter θ^0 and the true selection mechanism η^0 which the econometrician does not know. The identified set Θ_I is the set of parameters θ such that the set of predicted choice probabilities $A(\theta)$ contains the observed choice probabilities P^0 . In other words, it exists some u 's between 0 and 1 such that

- 1 $P_{0,0,0}^0 = P[\varepsilon_i < -\beta_i, i = 1, \dots, 3; \theta]$
- 2 The point $(P_{1,0,0}^0, P_{0,1,0}^0, P_{0,0,1}^0)$ belongs to the set $B_1(\theta)$, detailed above:



- 3 The point $(P_{0,1,1}^0, P_{1,0,1}^0, P_{1,1,0}^0)$ belongs to a similar set, $B_2(\theta)$.
- 4 $P_{1,1,1}^0 = P[\varepsilon_i \geq -2\alpha_i - \beta_i, i = 1, \dots, 3; \theta]$.

Comparison with existing methods I

First the two outcomes with zero firm and all firms entering leads to two equalities.

- Ciliberto-Tamer (2009)

$$\Delta_1^{(1)}(\theta) \leq P_{1,0,0}^0 \leq \Delta_1^{(1)}(\theta) + \Delta_{1,3}^{(1)}(\theta) + \Delta_{1,2}^{(1)}(\theta) + \Delta_{1,2,3}^{(1)}(\theta)$$

$$\Delta_2^{(1)}(\theta) \leq P_{0,1,0}^0 \leq \Delta_2^{(1)}(\theta) + \Delta_{2,3}^{(1)}(\theta) + \Delta_{1,2}^{(1)}(\theta) + \Delta_{1,2,3}^{(1)}(\theta)$$

$$\Delta_3^{(1)}(\theta) \leq P_{0,0,1}^0 \leq \Delta_3^{(1)}(\theta) + \Delta_{2,3}^{(1)}(\theta) + \Delta_{1,3}^{(1)}(\theta) + \Delta_{1,2,3}^{(1)}(\theta)$$

- A revealed preference argument would give the upper bounds of CT 2009.

Comparison with existing methods II

- As for us,

$$P_{1,0,0}^0 \leq \Delta_1^{(1)}(\theta) + \Delta_{1,3}^{(1)}(\theta) + \Delta_{1,2}^{(1)}(\theta) + \Delta_{1,2,3}^{(1)}(\theta)$$

$$P_{0,1,0}^0 \leq \Delta_2^{(1)}(\theta) + \Delta_{2,3}^{(1)}(\theta) + \Delta_{1,2}^{(1)}(\theta) + \Delta_{1,2,3}^{(1)}(\theta)$$

$$P_{0,0,1}^0 \leq \Delta_3^{(1)}(\theta) + \Delta_{2,3}^{(1)}(\theta) + \Delta_{1,3}^{(1)}(\theta) + \Delta_{1,2,3}^{(1)}(\theta)$$

$$P_{1,0,0}^0 + P_{0,1,0}^0 \leq \Delta_1^{(1)}(\theta) + \Delta_2^{(1)}(\theta) + \Delta_{1,3}^{(1)}(\theta) + \Delta_{1,2}^{(1)}(\theta) + \Delta_{2,3}^{(1)}(\theta) + \Delta_{1,2,3}^{(1)}(\theta)$$

$$P_{1,0,0}^0 + P_{0,0,1}^0 \leq \Delta_1^{(1)}(\theta) + \Delta_3^{(1)}(\theta) + \Delta_{1,3}^{(1)}(\theta) + \Delta_{1,2}^{(1)}(\theta) + \Delta_{2,3}^{(1)}(\theta) + \Delta_{1,2,3}^{(1)}(\theta)$$

$$P_{0,1,0}^0 + P_{0,0,1}^0 \leq \Delta_2^{(1)}(\theta) + \Delta_3^{(1)}(\theta) + \Delta_{1,3}^{(1)}(\theta) + \Delta_{1,2}^{(1)}(\theta) + \Delta_{2,3}^{(1)}(\theta) + \Delta_{1,2,3}^{(1)}(\theta)$$

$$P_{1,0,0}^0 + P_{0,1,0}^0 + P_{0,0,1}^0 \leq \Delta_1^{(1)}(\theta) + \Delta_2^{(1)}(\theta) + \Delta_3^{(1)}(\theta) + \Delta_{1,3}^{(1)}(\theta) + \Delta_{1,2}^{(1)}(\theta) + \Delta_{2,3}^{(1)}(\theta) + \Delta_{1,2,3}^{(1)}(\theta)$$

General Entry Game

We now formalize the general case with N players.

$$y_{im} = \mathbb{1} \left\{ \beta_i + \alpha_i \left(\sum_{j \neq i} y_{jm} \right) + \varepsilon_{im} \geq 0 \right\}, \quad i = 1, \dots, N. \quad (1)$$

- Set-up: static game with complete information, pure strategy Nash Equilibria are assumed.
- The linear form is not important as long as we have a parametric form additive in ε
- What matters however is that the competitor effect only depends on $\sum_{j \neq i} y_{jm}$.
- ε_{im} can be correlated across i , but they are independent across m .
- Discrete covariates are considered in the paper.

Multiplicity Structure I

The set of predicted choice probabilities $A(\theta)$ has a specific structure:

- It's a convex polytope with a specific structure (as the number of active firms is unique in the multiple equilibria regions).
- Its number of vertices can be characterized.
- It's characterization depends, in particular, on the the number of multiple equilibria regions which we can characterize and count. There are much less than the maximum number of regions possible. For example, for $N = 4$ and $K = 2$, the two outcomes $(1, 1, 0, 0)$ and $(0, 0, 1, 1)$ can't be in multiplicity.
- We can count them. In our setting, the numbers are:

N	Number	max
$N = 4$	43	79
$N = 5$	194	2078
$N = 6$	793	1 114 173

Outline

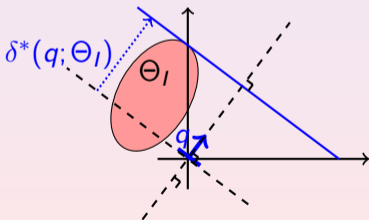
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The support function of a convex set I

A convex set can be uniquely characterized by many functions but the literature on set identification mainly uses the support function.

The *support function* of a convex set $\Theta_I \subset \mathbb{R}^d$ is defined as:

$$\delta^*(q; \Theta_I) = \sup_{\theta \in \Theta_I} (q^\top \theta) \text{ for all directions, } q \in \mathbb{R}^d.$$



The support function of a convex set II

Following Rockafellar (1970), it uniquely characterizes set Θ_I :

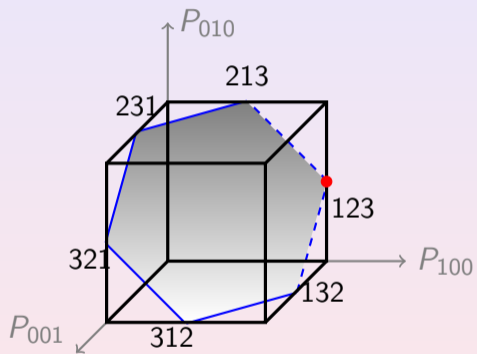
$$\theta \in \Theta_I \Leftrightarrow \forall q \in \mathbb{S}^{d-1}, q^\top \theta \leq \delta^*(q; \Theta_I). \quad (2)$$

The support function implicitly gathers all the inequalities which define the convex set into one single function. If the set is smooth, there is a continuum of such inequalities ; if it is a polytope, only a finite number of inequalities is necessary to characterize the set.

The Support Function

- We derive a closed-form expression for the support function which depends on the characterization of the multiple equilibria regions we mentioned above.
- Following Beresteanu et al. (2011), we give a collection of all moment inequalities which are necessary to sharply characterize the identified set.
- For small values of N , 2 to 5 (and the points of support of the discrete explanatory variables), it can be solved by brute force.
- However, this number increases exponentially. For $N = 6$, more than 1 million inequalities per point of support need to be evaluated, for $N = 7$, more than 60 billions, etc. The core determining class (see Galichon and Henry, 2011 or Beresteanu et al. 2011) does not help in reducing this number substantially.
- We propose an algorithm which selects, without having to evaluate them, the potentially binding moments. We call it a local moment selection algorithm. The main idea is to look for the closest vertex of the convex set $A(\theta)$ and checking the supporting hyperplanes in this vertex. This number is still increasing in N , but proportionally to 2^N . For $N = 3$ one has to test 10 inequalities instead of 18, for $N = 6$, 69 inequalities instead of 1 114 242.

Graphical illustration



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Test Statistic

- The support function provides an indirect characterization of the identified set whether it is point or set identified:

$$\begin{aligned}\theta \in \Theta_I &\iff P^* \in \mathcal{A}(\theta) \\ &\iff \xi_\infty(\theta) = \inf_{q \in \mathcal{C}} \delta^*(q; \mathcal{A}(\theta)) - q^\top P^* \geq 0.\end{aligned}$$

where \mathcal{C} is the core determining class for $\mathcal{A}(\theta)$.

- We observe sample from m markets and can consistently estimate P^* .
- Let \hat{P} be estimator for P^* and consider normalized test statistic

$$\xi_n(\theta) = \sqrt{n} \inf_{q \in \mathcal{C}} \frac{\delta^*(q; \mathcal{A}(\theta)) - q^\top \hat{P}}{\sqrt{q^\top \hat{\Sigma} q}}.$$

The Limiting Distribution:

- Let Q_θ , be the set of minimizers of $\delta^*(q; \mathcal{A}(\theta)) - q^\top P^*$ in \mathcal{C} , then

$$\begin{cases} \xi_n(\theta) \xrightarrow[n \rightarrow \infty]{d} \inf_{q \in Q_\theta} \frac{q^\top \mathcal{N}(0, \Sigma)}{\sqrt{q^\top \Sigma q}} & \text{if } P^* \in \mathcal{A}(\theta), \\ \xi_n(\theta) \xrightarrow[n \rightarrow \infty]{a.s.} -\infty, & \text{if } P^* \notin \mathcal{A}(\theta), \end{cases}$$

- It depends on θ only through the number of binding moments, i.e. Q_θ .
- Σ , usually, depend on θ in models where we have to estimate support function.
- However, more broadly in game theoretic models with complete information Σ doesn't depend on θ .

Confidence Region for the Identified Set:

- For inference on the identified set, we need to know distribution of

$$\inf_{\theta \in \Theta_I} \xi_n(\theta).$$

- We know,

$$\xi_n(\theta) \xrightarrow[n \rightarrow \infty]{d} \inf_{q \in Q_\theta} \frac{q^\top \mathcal{N}(0, \Sigma)}{\sqrt{q^\top \Sigma q}}.$$

- But, we don't know Q_θ and Θ_I .
- But observe that

$$\bigcup_{\theta \in \Theta_I} Q_\theta \subseteq \mathcal{C}.$$

Confidence Region for the Identified Set:

- $\bigcup_{\theta \in \Theta_I} Q_\theta \subseteq \mathcal{C}$ implies that for large sample size

$$\inf_{\theta \in \Theta_I} \xi_n(\theta) \succeq \inf_{q \in \mathcal{C}} \frac{q^\top \mathcal{N}(0, \Sigma)}{\sqrt{q^\top \Sigma q}}.$$

- Lets define

$$\hat{c}_2(\mathcal{C}, \alpha) := 1 - \alpha \text{ quantile of } \min_{q \in \mathcal{C}} \frac{q^\top \mathcal{N}(0, \hat{\Sigma})}{\sqrt{q^\top \hat{\Sigma} q}}$$

where $\hat{\Sigma}$ is some consistent estimator of Σ . We can show

$$\liminf_{n \rightarrow \infty} \inf_{P \in \mathcal{P}} P(\Theta_I(P) \subseteq C(\hat{c}_2(\mathcal{C}, \alpha))) \geq 1 - \alpha.$$

where $C(c) = \{\theta \in \Theta : \xi_n(\theta) \geq c\}$.

Confidence Region for a Point in the Identified Set:

- For inference for a point in the identified set, we need to know distribution of

$$\xi_n(\theta)$$

- We know the limiting distribution of $\xi_n(\theta)$.
- We don't know Q_θ . But we know from local geometry

$$Q_\theta \subseteq \mathcal{L}(\theta)$$

- Which still $\mathcal{L}(\theta)$ depends on θ , but

$$|\mathcal{L}(\theta)| \leq \mathcal{L}^*$$

Confidence Region for a Point in the Identified Set:

- The idea is to develop inference method solely based on \mathcal{L}^* .
- Define

$$c_{\mathcal{L}^*}(\alpha) = \frac{-\Phi^{-1}(1 - \alpha/\mathcal{L}^*)}{\sqrt{1 - \Phi^{-1}(1 - \alpha/\mathcal{L}^*)^2/n}}$$

where Φ is the distribution function of the standard normal distribution, and Φ^{-1} is its quantile function.

- We can show that

$$\liminf_{n \rightarrow \infty} \inf_{P \in \mathbb{P}} \inf_{\theta \in \Theta_I(P)} P(\theta \in \mathcal{C}(c_{\mathcal{L}^*}(\alpha))) \geq 1 - \alpha.$$

Inference: Summary

- We propose simple methods for both notions of confidence region.
- Advantage: computationally very simple compare to existing methods.
- Simulation exercise show methods proposed perform reasonable well compare to existing methods for sample size up to 5000.
- We provide extension to include covariates in the paper.

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Simulation Designs:

Consider the simple game with 3 players, no explanatory variable,

$$y_i = 1 \left\{ \beta + \alpha_i \sum_{j \neq i} y_j + \varepsilon_i \geq 0 \right\}$$

Design	ε	Parameter
1	$\mathcal{N}(0, 1)$	$\beta = .35$ $\alpha_1 = \alpha_3 = -.4, \alpha_2 = -.4$

Results I

Point-identified case

Test	Crit. value	Normalized Tests Volume CR	Non-normalized Tests Volume CR
Bound	G	1.50	9.40
	L	1.14	4.80
Ineq ₁	G	1.80	9.23
	L	1.50	3.96
Ineq ₄	G	1.27	5.55
	L	1.03	3.11
Local	L	1.00	1.83

$n = 1000$.

Results II

Ignoring $\alpha_1 = \alpha_3$. Set identified case.

Test	Crit. value	α_1 min	α_1 max	α_2 min	α_2 max	α_3 min	α_3 max	β min	β max	Vol. CR
Bound	G	-0.657	-0.166	-0.715	-0.227	-0.684	-0.197	0.253	0.492	1.49
	L	-0.631	-0.184	-0.688	-0.243	-0.658	-0.214	0.263	0.479	1.06
Ineq ₁	G	-0.714	-0.118	-0.755	-0.165	-0.743	-0.152	0.120	0.488	2.02
	L	-0.693	-0.134	-0.731	-0.180	-0.721	-0.167	0.132	0.478	1.59
Ineq ₄	G	-0.655	-0.165	-0.716	-0.228	-0.682	-0.196	0.252	0.495	1.36
	L	-0.634	-0.179	-0.695	-0.240	-0.661	-0.209	0.258	0.483	1.05
Local	L	-0.630	-0.181	-0.692	-0.242	-0.658	-0.212	0.259	0.481	1.00

Results III

Another DGP with higher probabilities of multiple equilibria

Type	Test	Crit. value	α_1 min	α_1 max	α_2 min	α_2 max	α_3 min	α_3 max	β min	β max	Vol. CR
	Bound	<i>G</i>	-1.146	-0.429	-0.973	-0.233	-1.150	-0.434	0.484	0.843	1.92
		<i>L</i>	-1.097	-0.446	-0.922	-0.250	-1.101	-0.450	0.492	0.809	1.36
	Ineq ₄	<i>G</i>	-1.133	-0.431	-0.943	-0.238	-1.137	-0.435	0.483	0.838	1.33
		<i>L</i>	-1.099	-0.443	-0.908	-0.252	-1.102	-0.448	0.489	0.816	1.04
Local	<i>L</i>		-1.099	-0.443	-0.908	-0.252	-1.102	-0.448	0.489	0.816	1.04
	Exact		-1.093	-0.446	-0.904	-0.254	-1.097	-0.451	0.491	0.814	1.00

Results IV

A four player case.

Test	Crit. value	α_1 min	α_1 max	α_2 min	α_2 max	β min	β max	Vol. CR
Bound	<i>G</i>	-0.602	-0.232	-0.455	-0.062	0.253	0.682	2.60
	<i>L</i>	-0.576	-0.240	-0.430	-0.072	0.260	0.648	2.04
Ineq ₁	<i>G</i>	-0.573	-0.242	-0.421	-0.095	0.176	0.631	2.21
	<i>L</i>	-0.542	-0.254	-0.394	-0.106	0.197	0.593	1.55
Ineq ₃	<i>G</i>	-0.575	-0.247	-0.423	-0.097	0.259	0.633	1.80
	<i>L</i>	-0.547	-0.258	-0.399	-0.108	0.271	0.600	1.30
Ineq ₄	<i>G</i>	-0.561	-0.243	-0.417	-0.083	0.250	0.645	1.55
	<i>L</i>	-0.525	-0.259	-0.381	-0.101	0.267	0.598	1.00
Local	<i>L</i>	-0.526	-0.258	-0.381	-0.100	0.267	0.598	1.00

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Conclusion and perspectives

- A geometric approach of the entry game with complete information
- Provides both the collection of non redundant moments and a geometric selection of the (potentially) binding moments without evaluating them
- Improvements with respect to existing methods even in cases where all the moment inequalities can be evaluated.
- Empirical applications are now considered on airlines while relaxing the assumption that the impact of one competitor's entry does not depend on its identity.
- Continuous covariates are also considered.
- Full paper is available on the TSE website.