

A Passive Nonlinear Three-Port Junction Model For Fluid Mechanics and Acoustics.

Victor Wetzel (SU-CNRS-STMS)

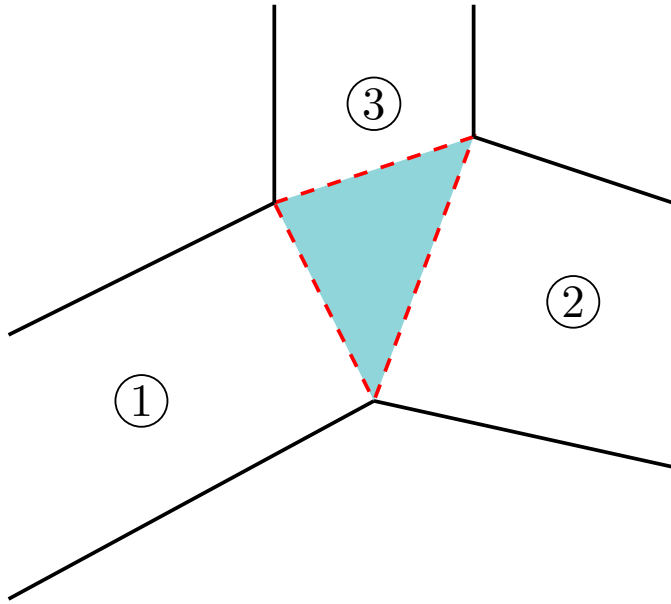
Thomas Hélie (SU-CNRS-STMS)

Fabrice Silva (LMA – Aix Marseille)



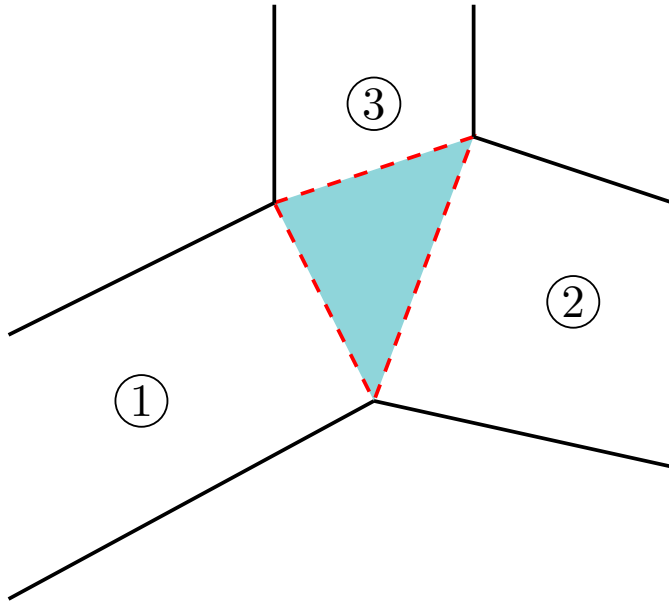
The **Three-port** Junction

The Three-port Junction



The **common volume** connecting three resonators.

The Three-port Junction

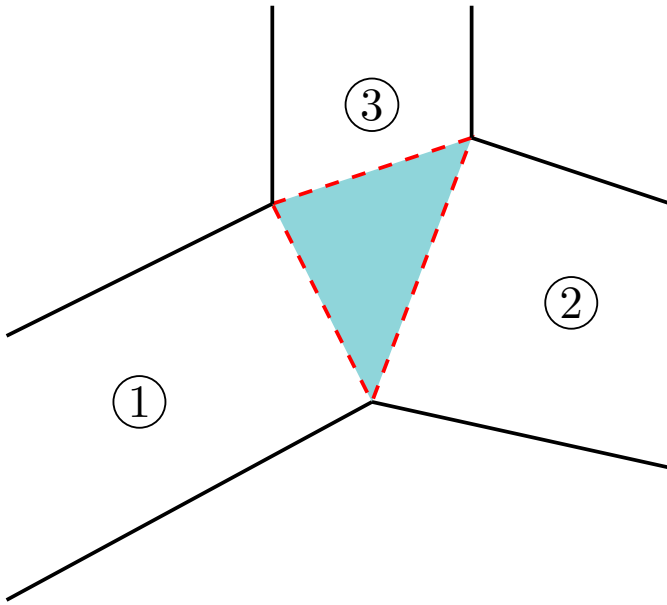


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Junctions in acoustics

- Lateral holes

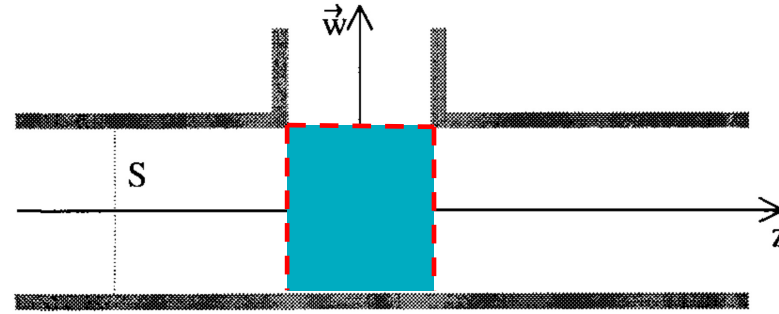
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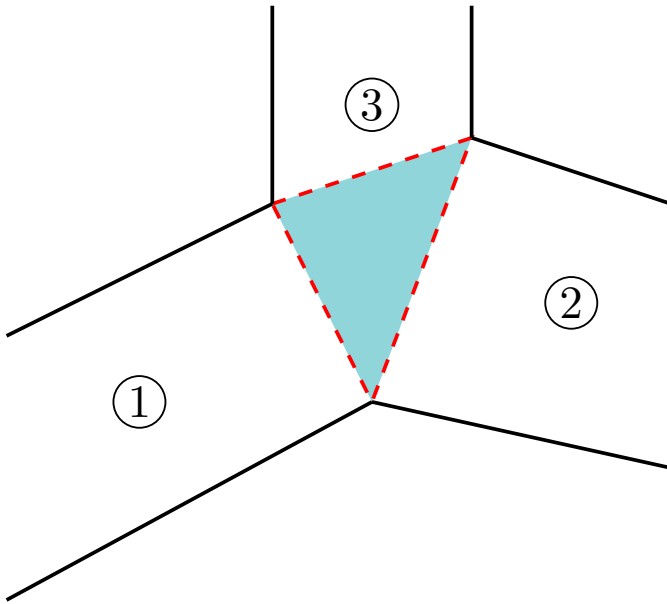
- Lateral holes



A lateral hole

Figure 1 from *Theory of Sound Propagation in a Duct with a Branched Tube Using Modal Decomposition*, Dubos et al. 1999, Acta Acustica.

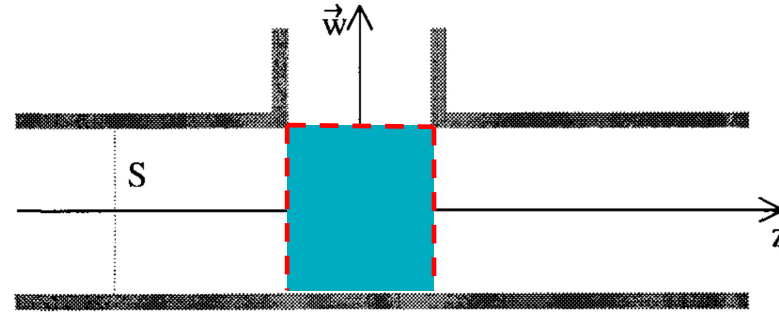
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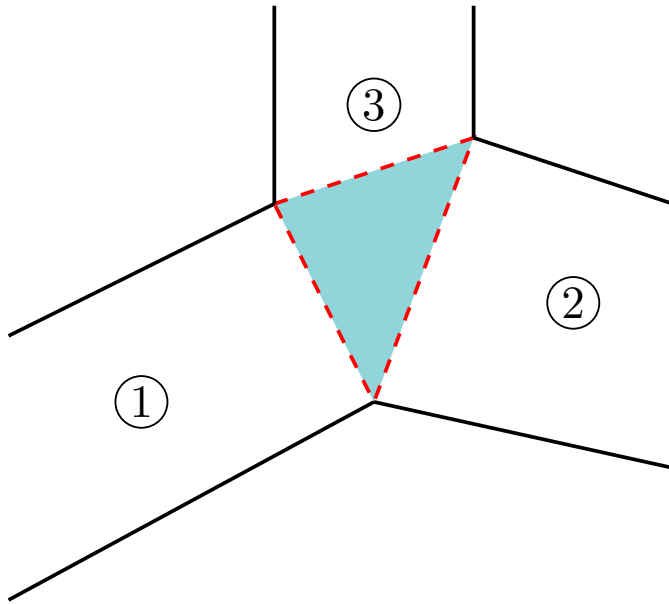
- Lateral holes
- Nasal derivation



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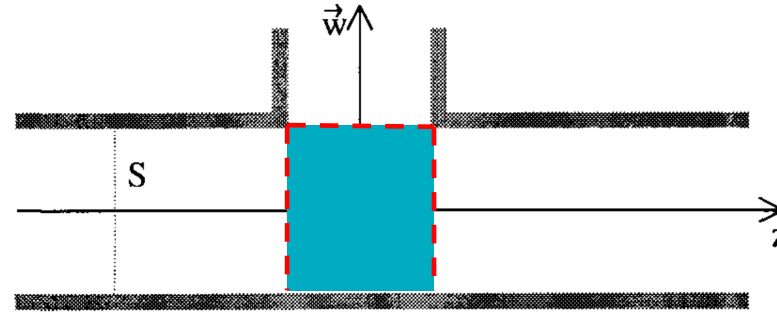
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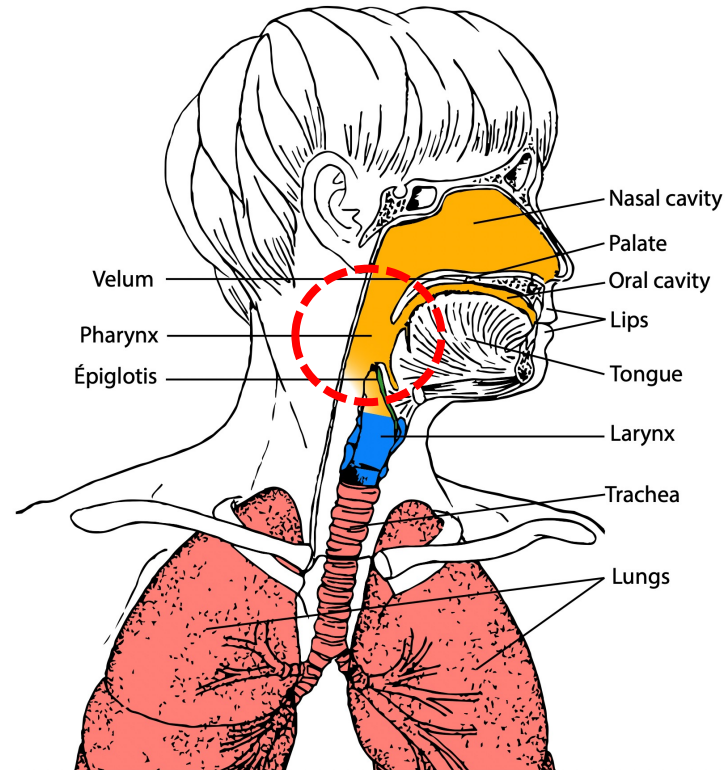
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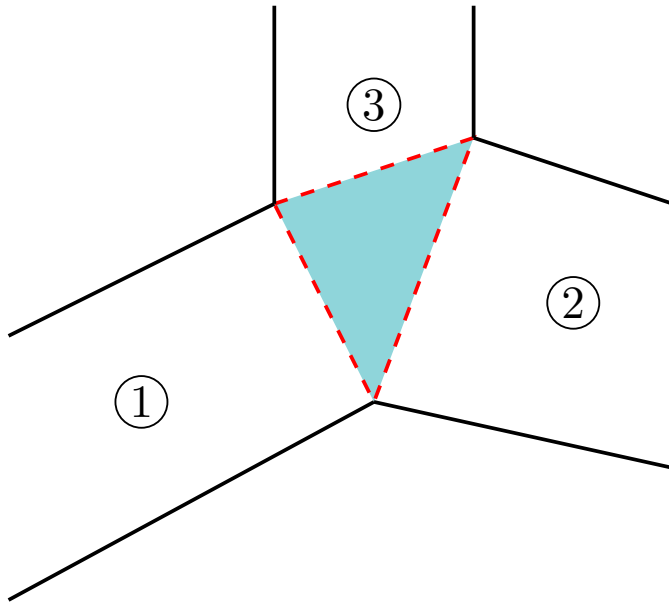
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Vocal apparatus (front cut)

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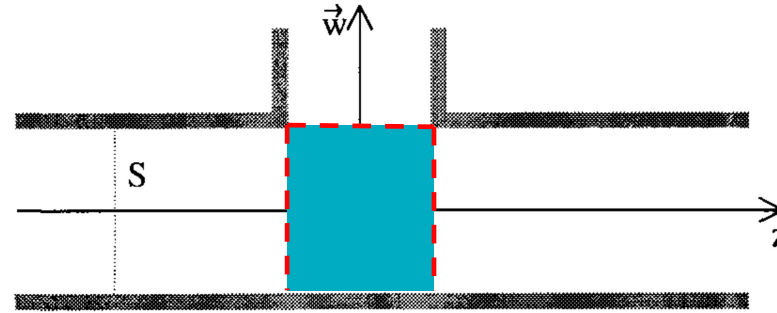
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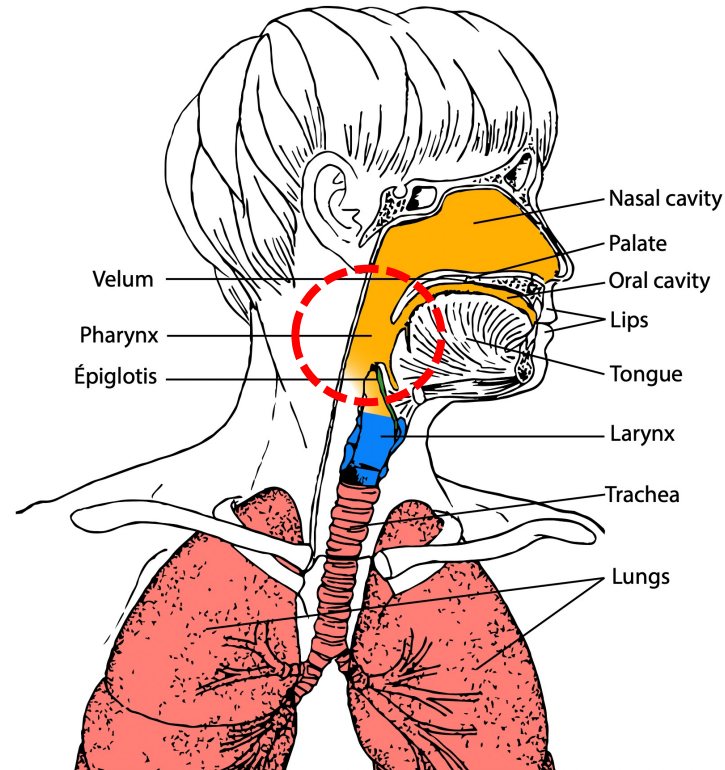
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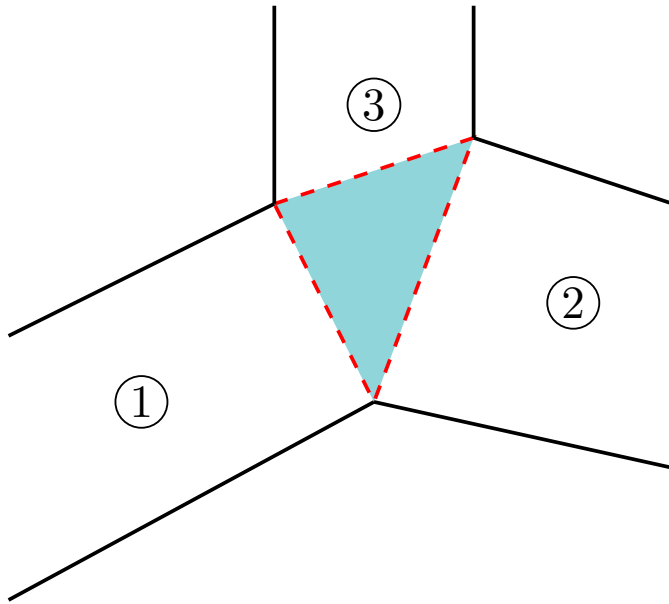
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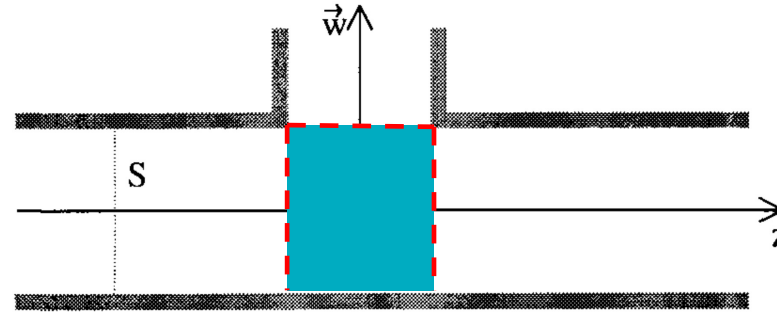
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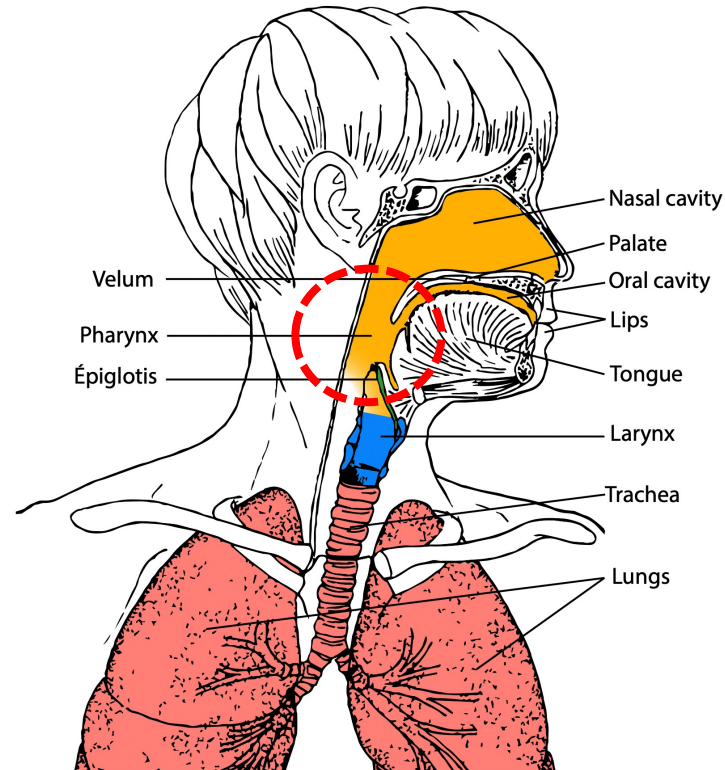
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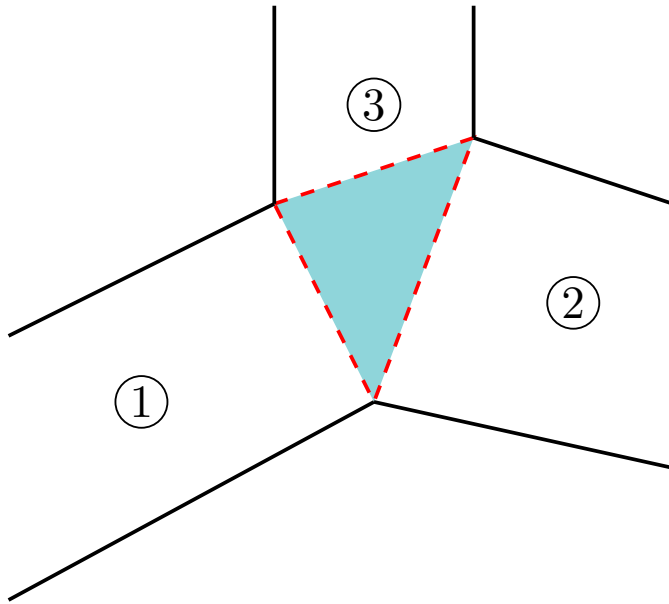
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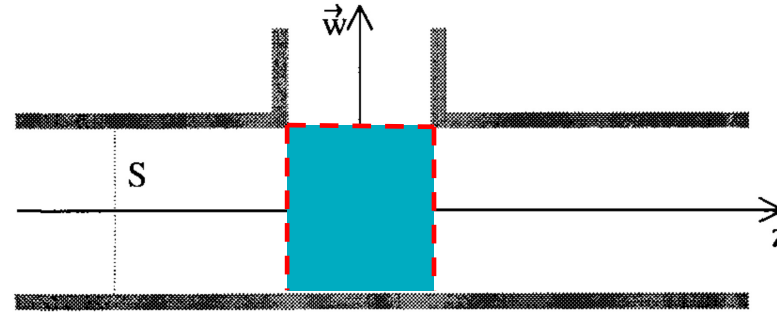
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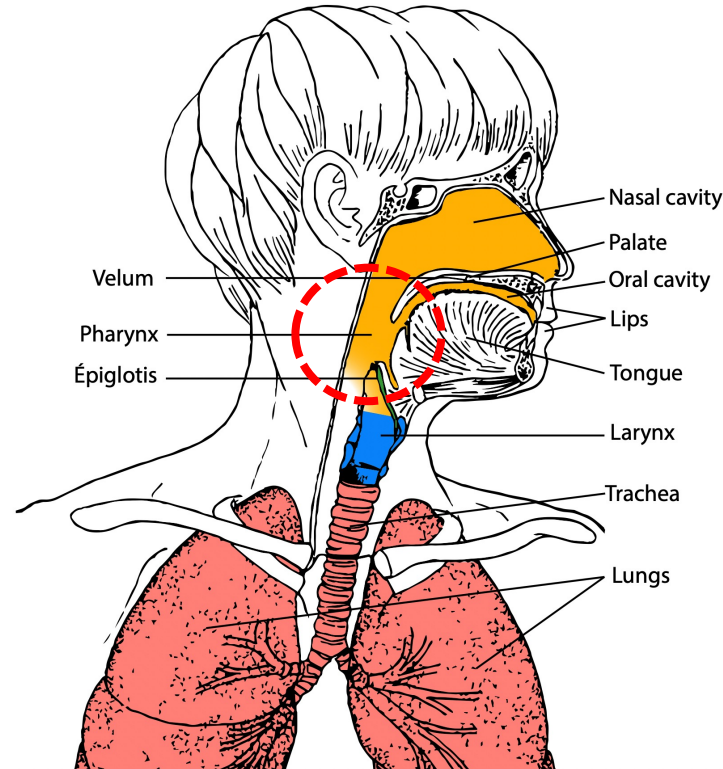
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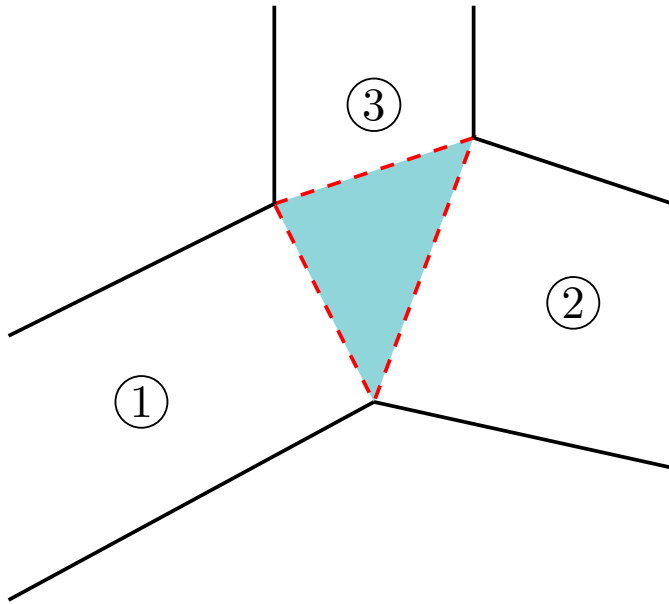
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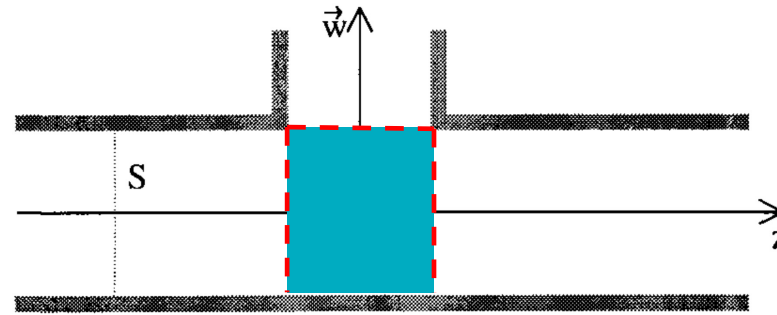
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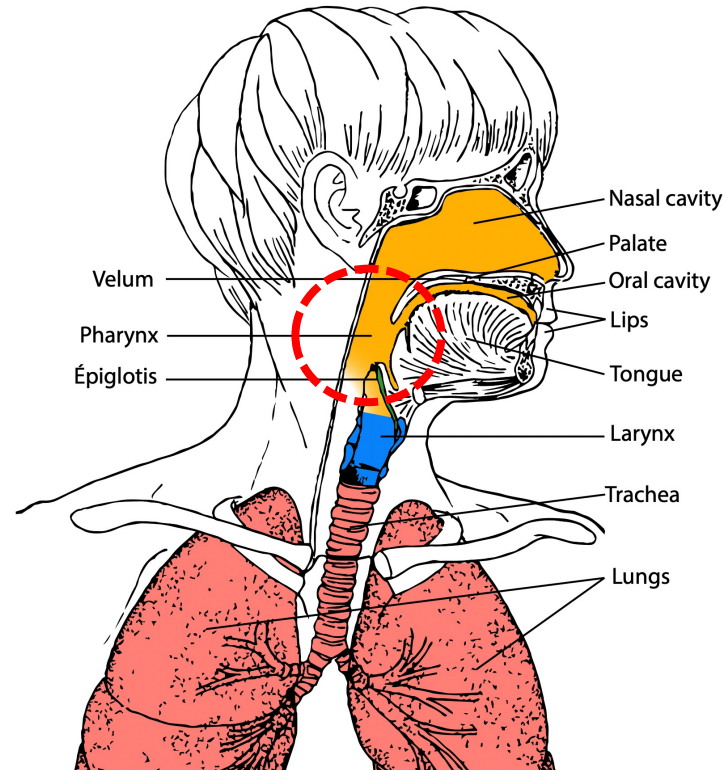
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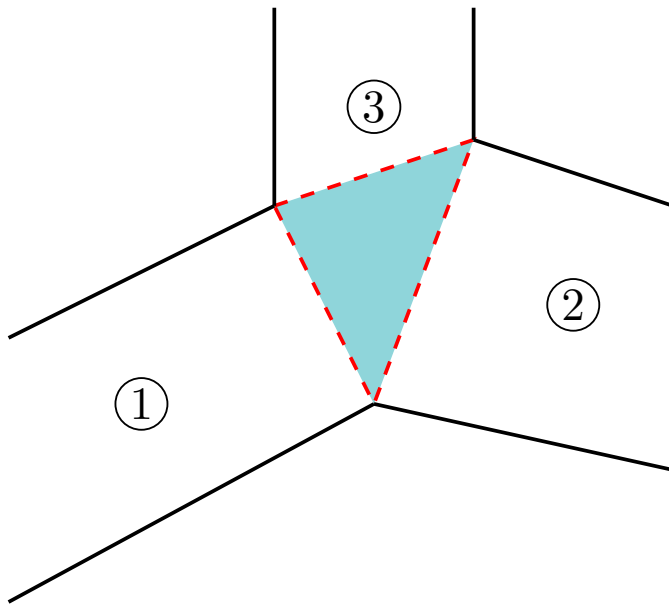
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We introduce 5 models:

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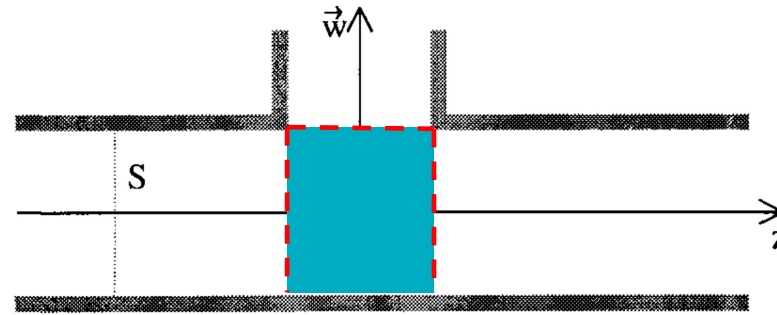
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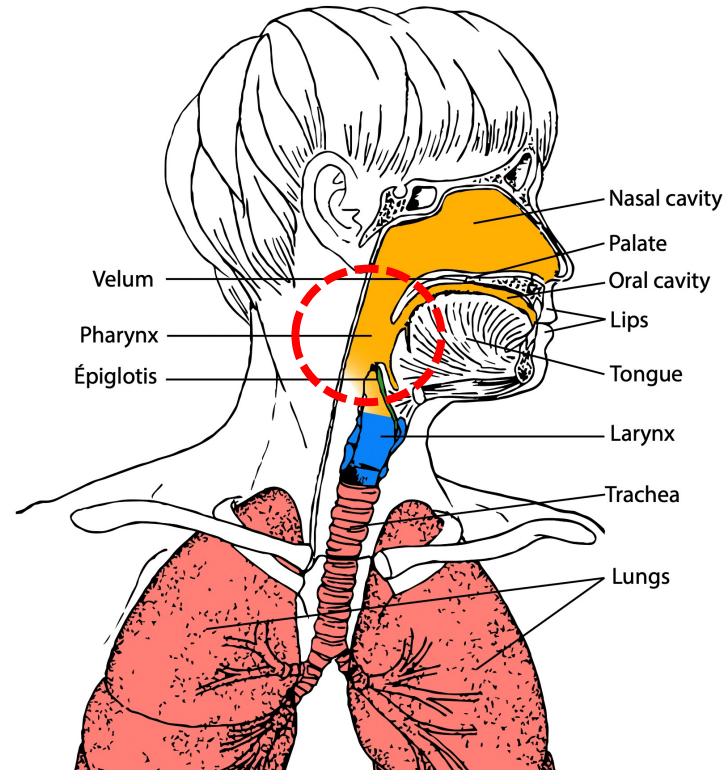
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We introduce 5 models:

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- written as **port-Hamiltonian systems** to ensure the **passivity**

Modelling process

Modelling process

1) Hypotheses

(H0) 2D Geometry

Boundary (port variables)

(H1-5) Physics

Modelling process

1) Hypotheses

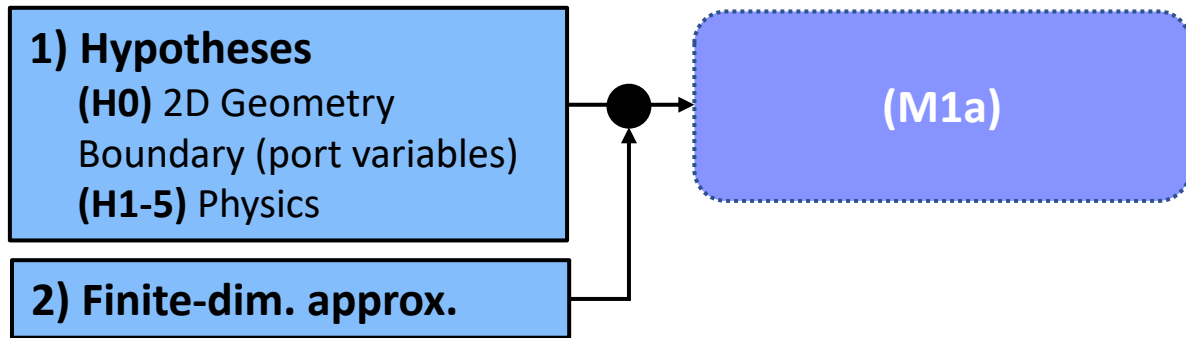
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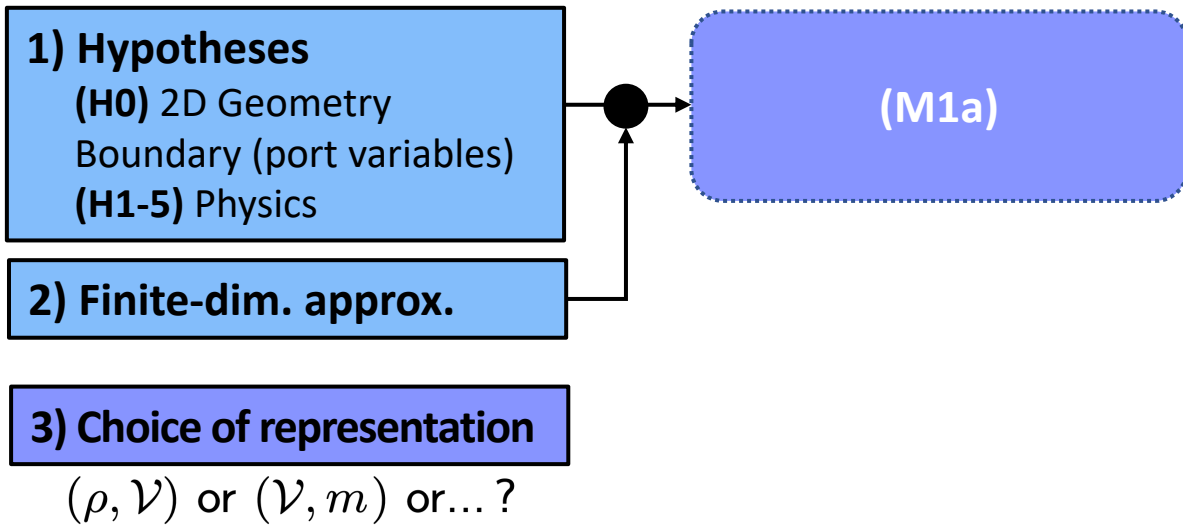
(H1-5) Physics

2) Finite-dim. approx.

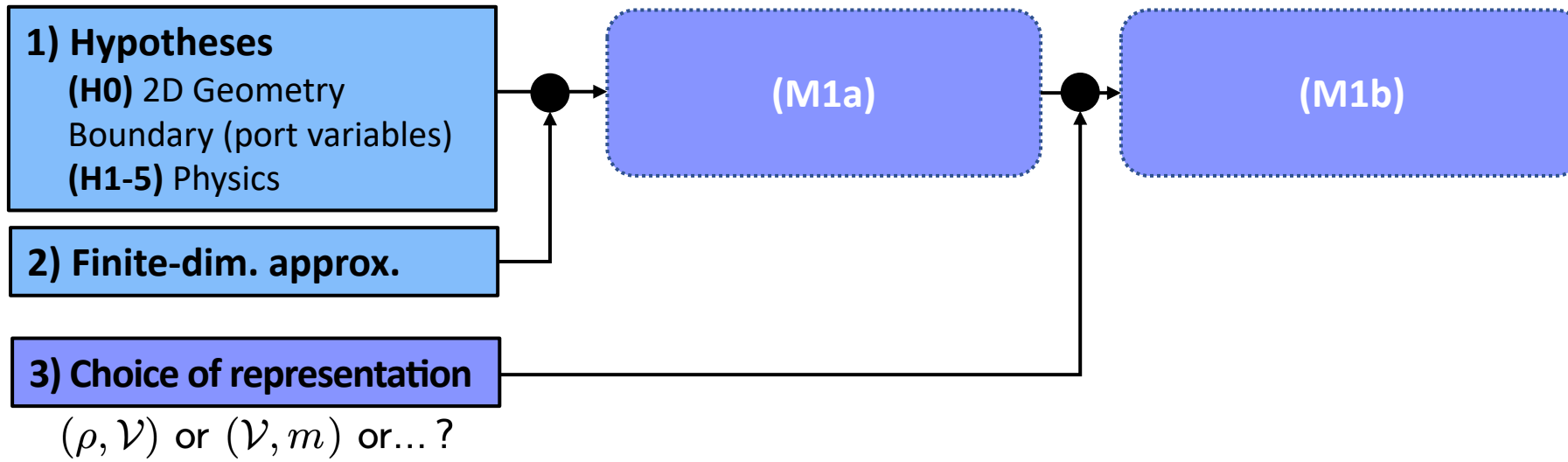
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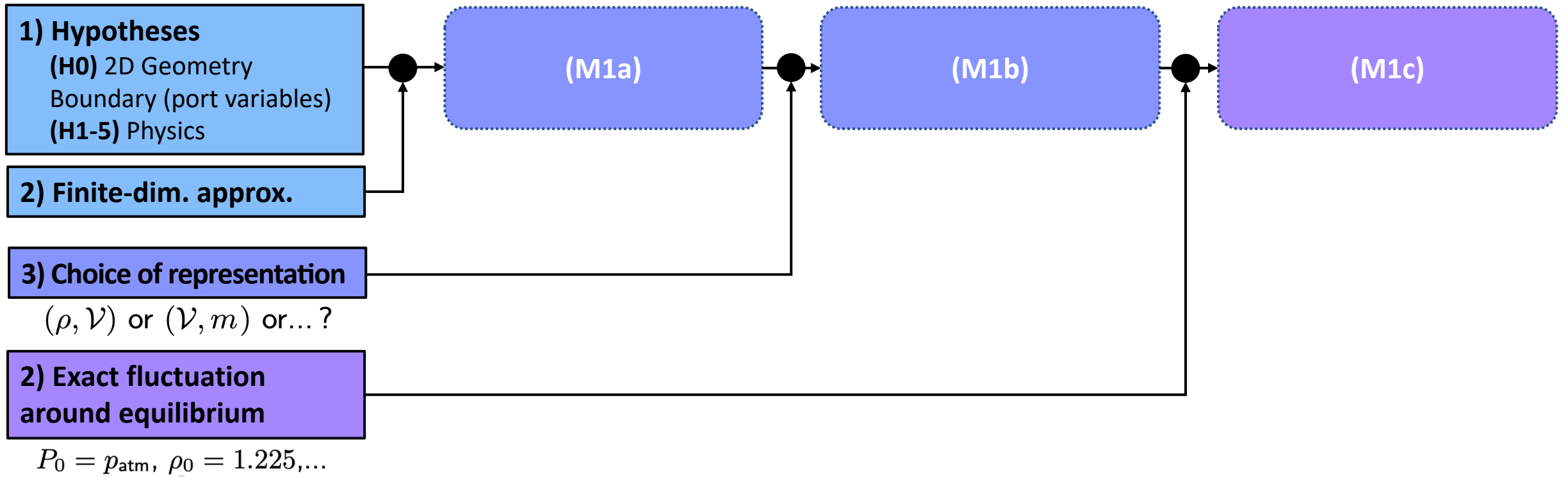
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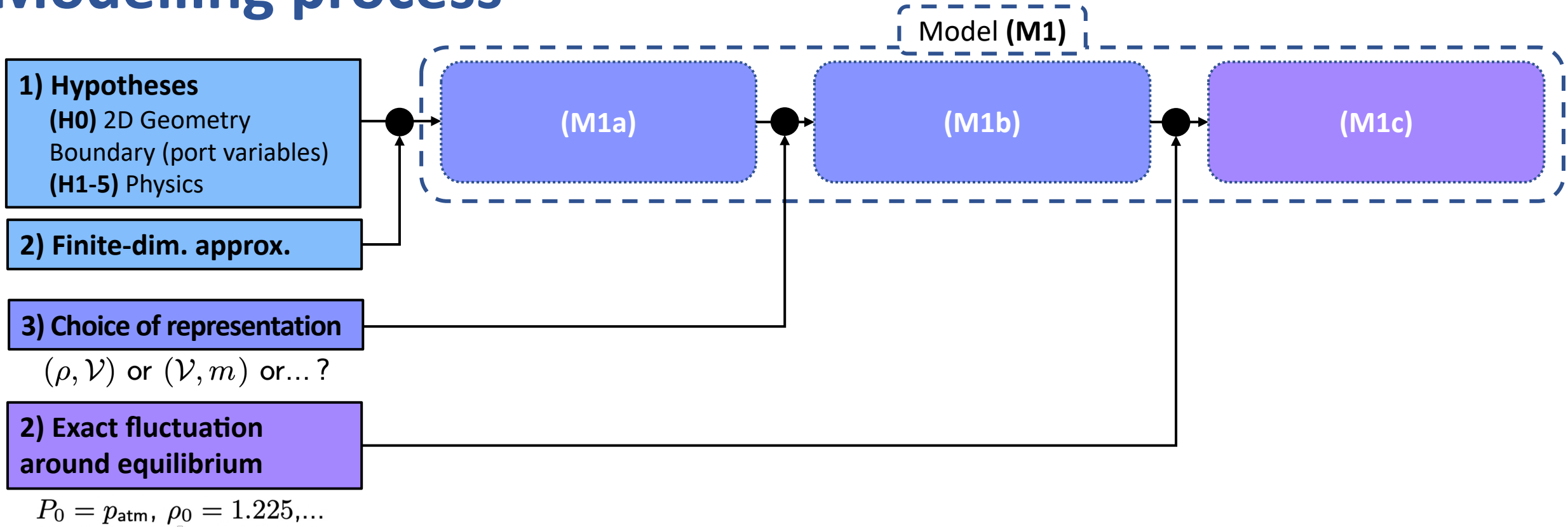
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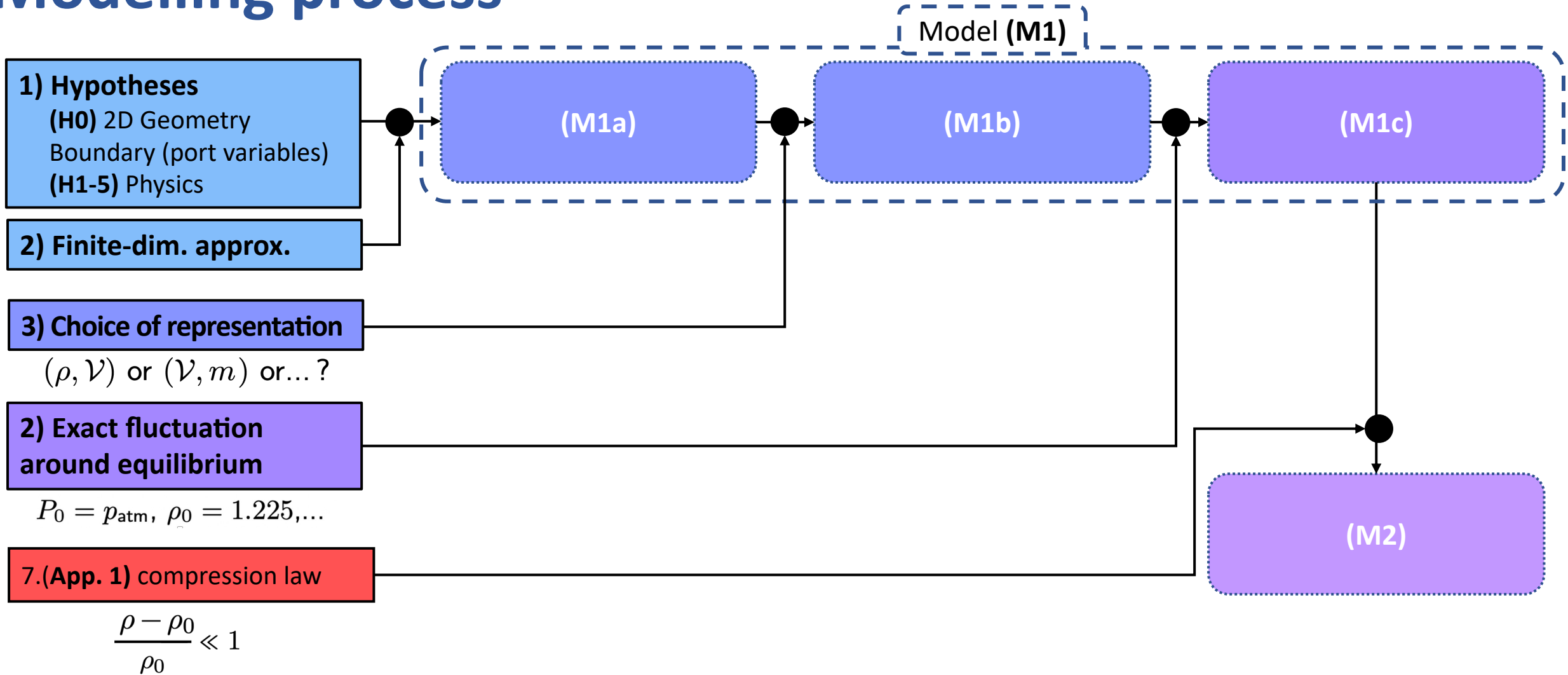
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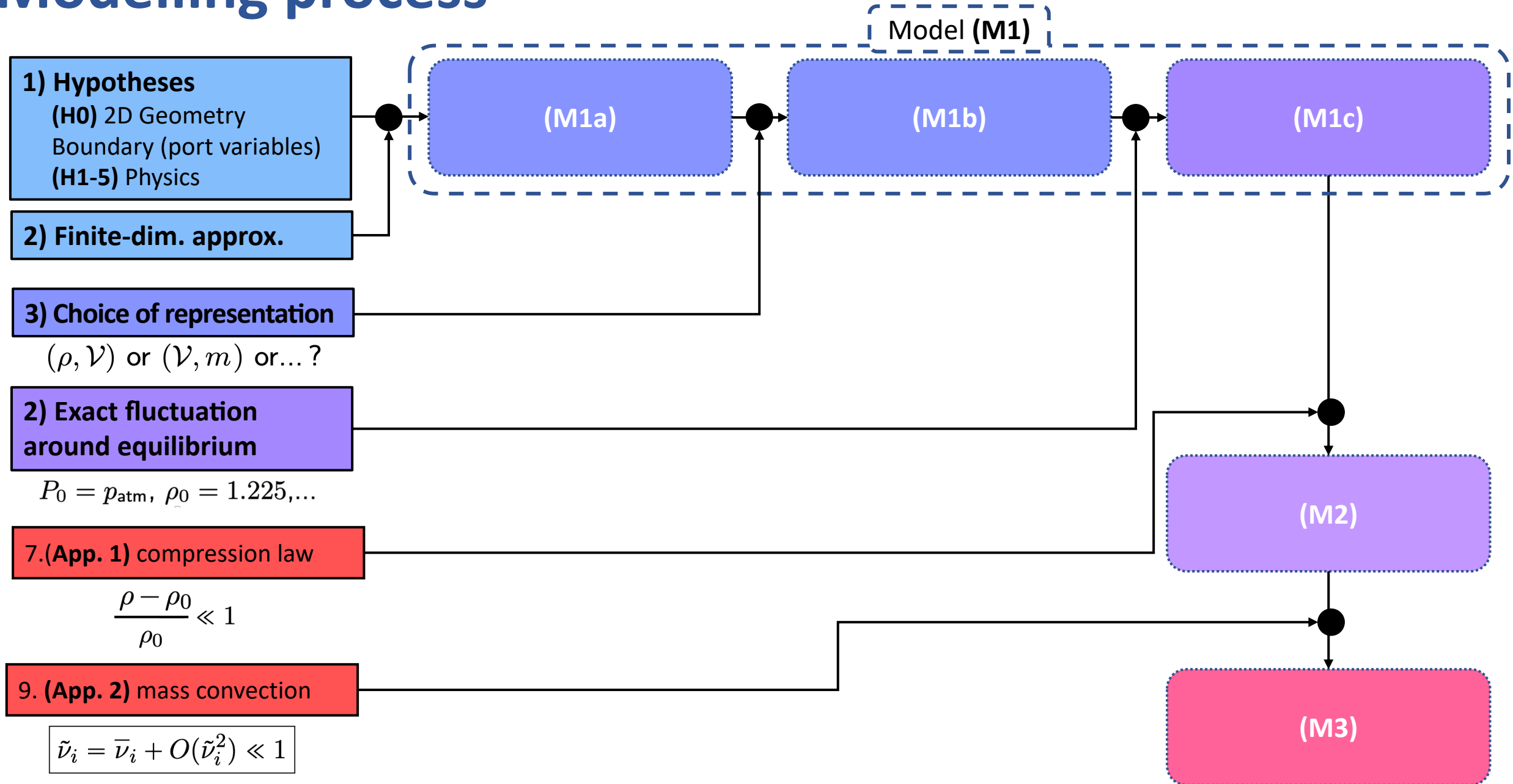
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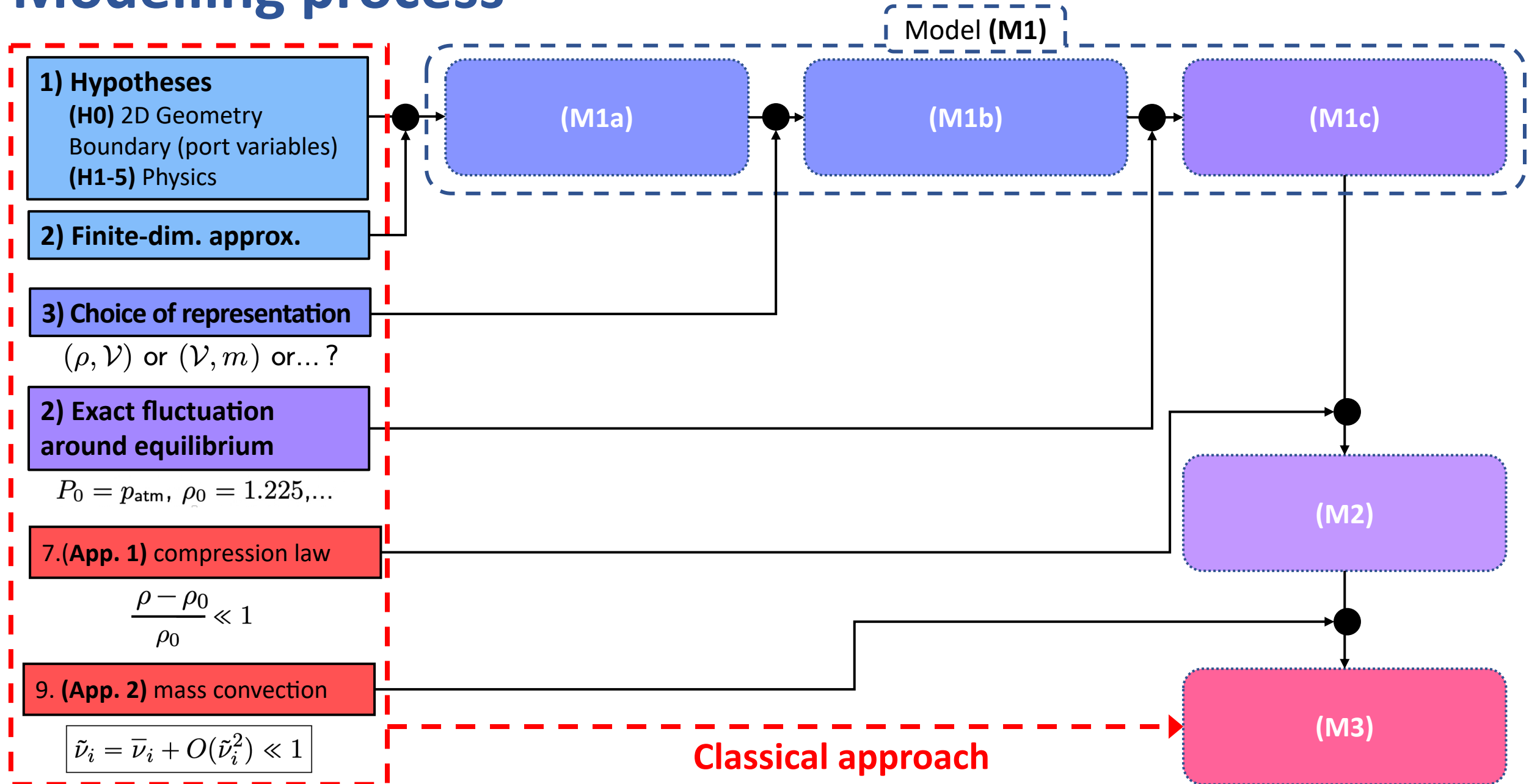
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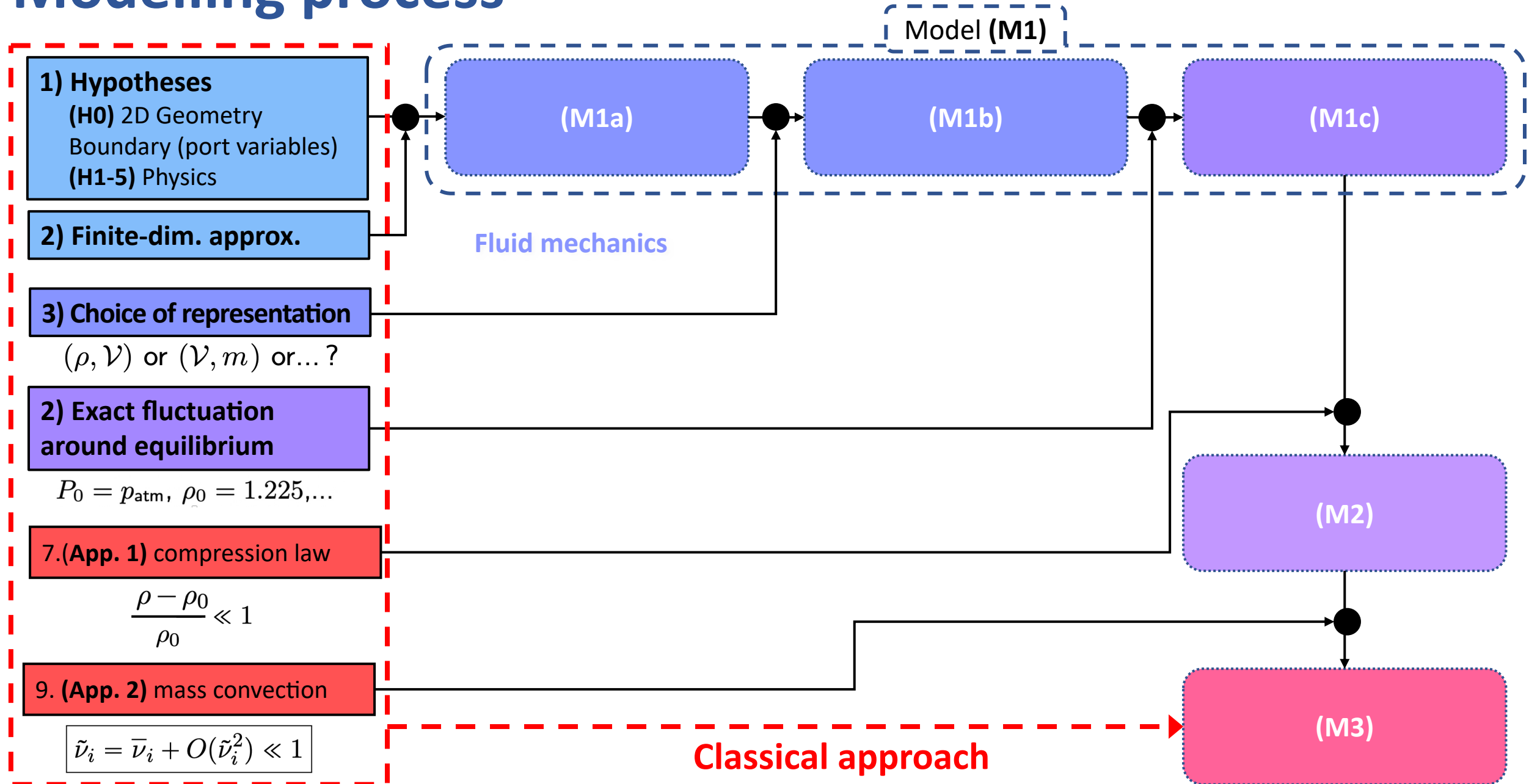
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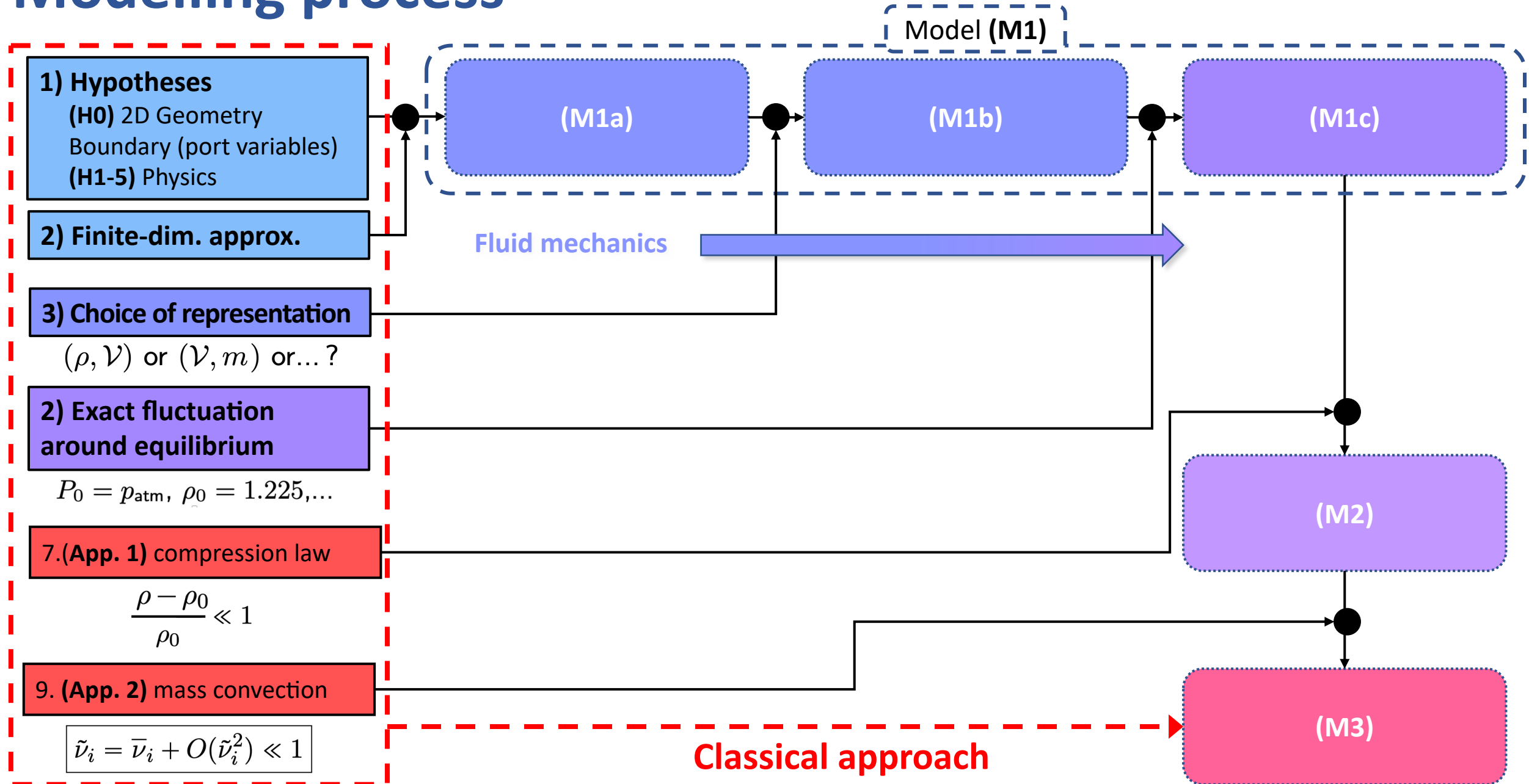
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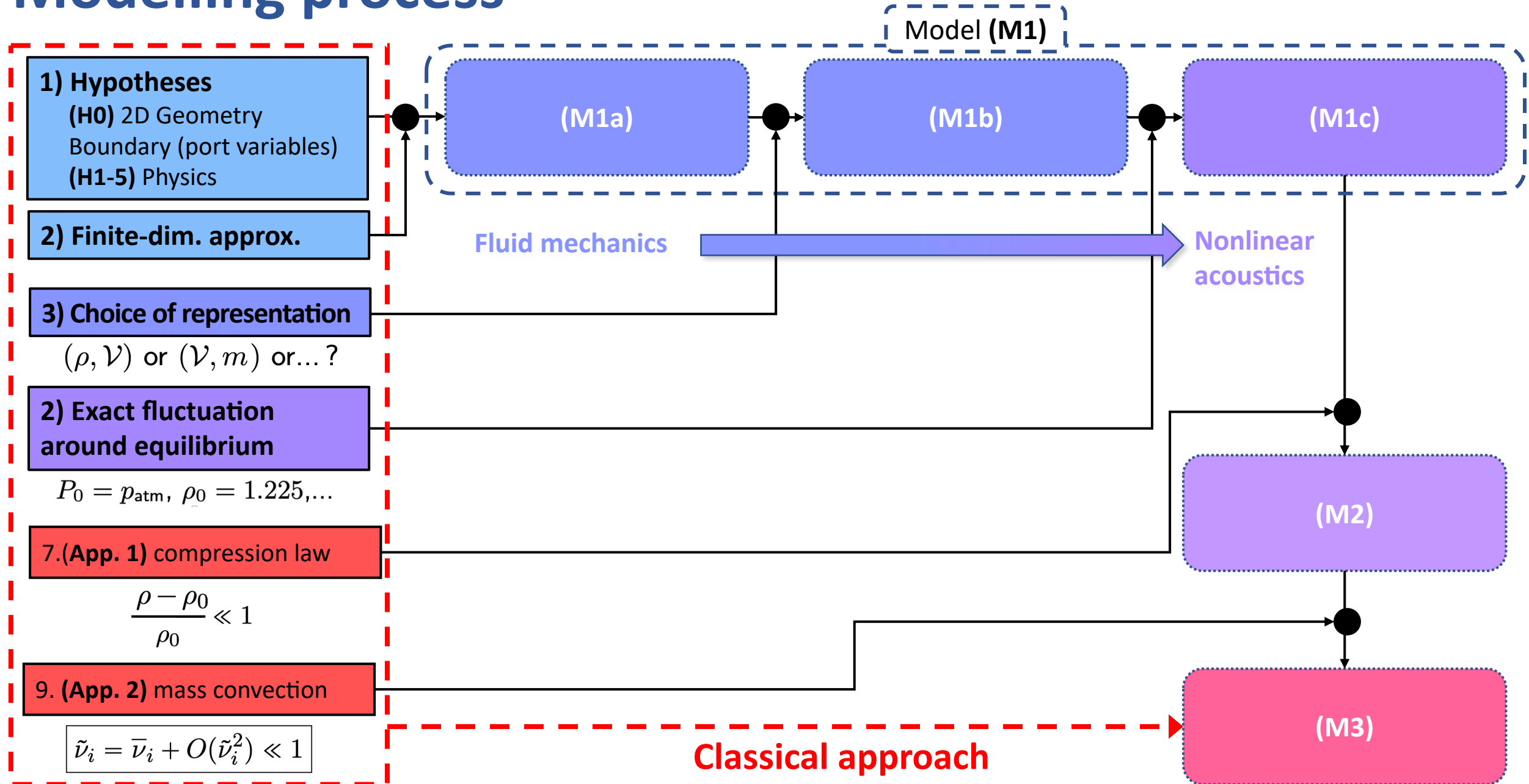
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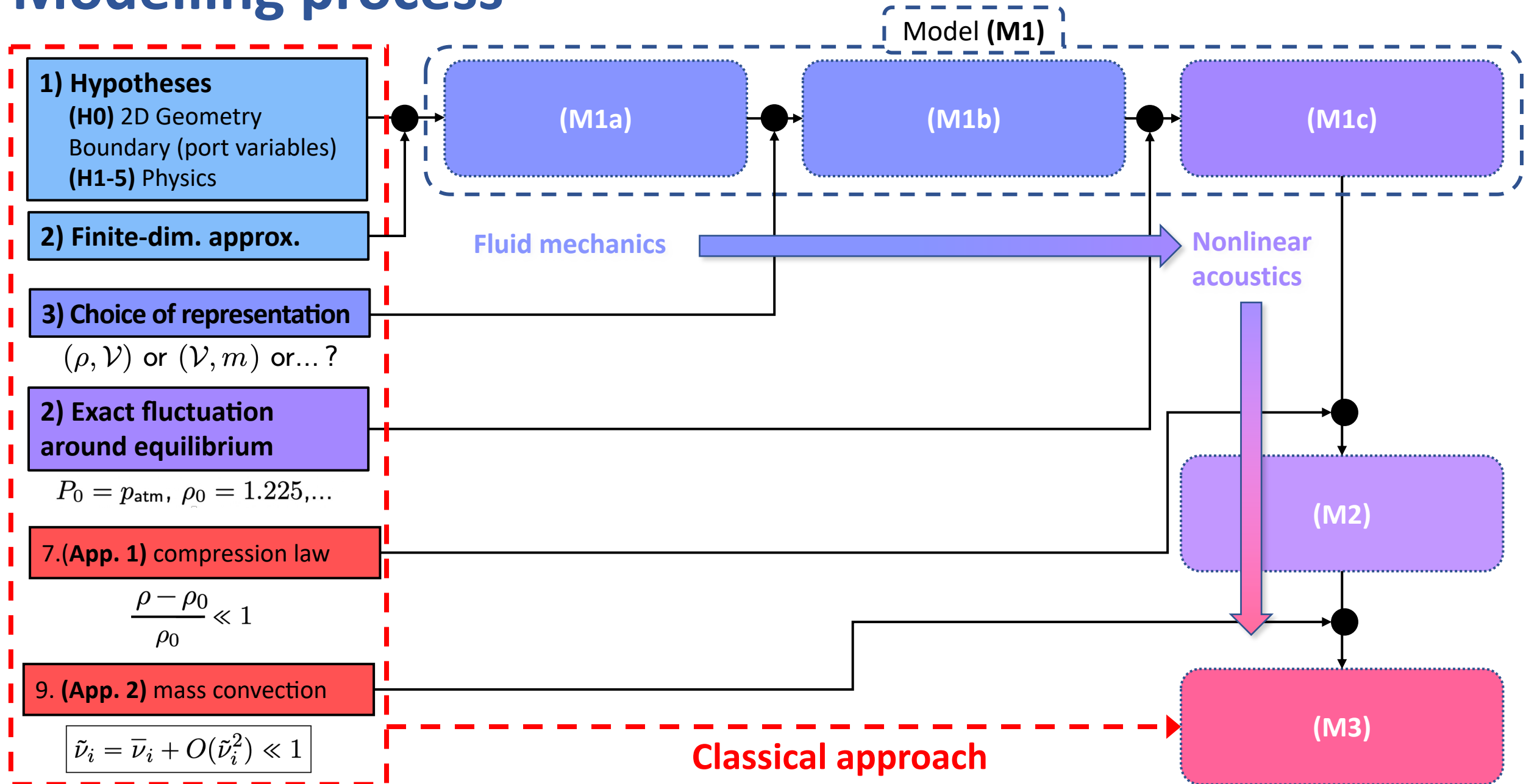
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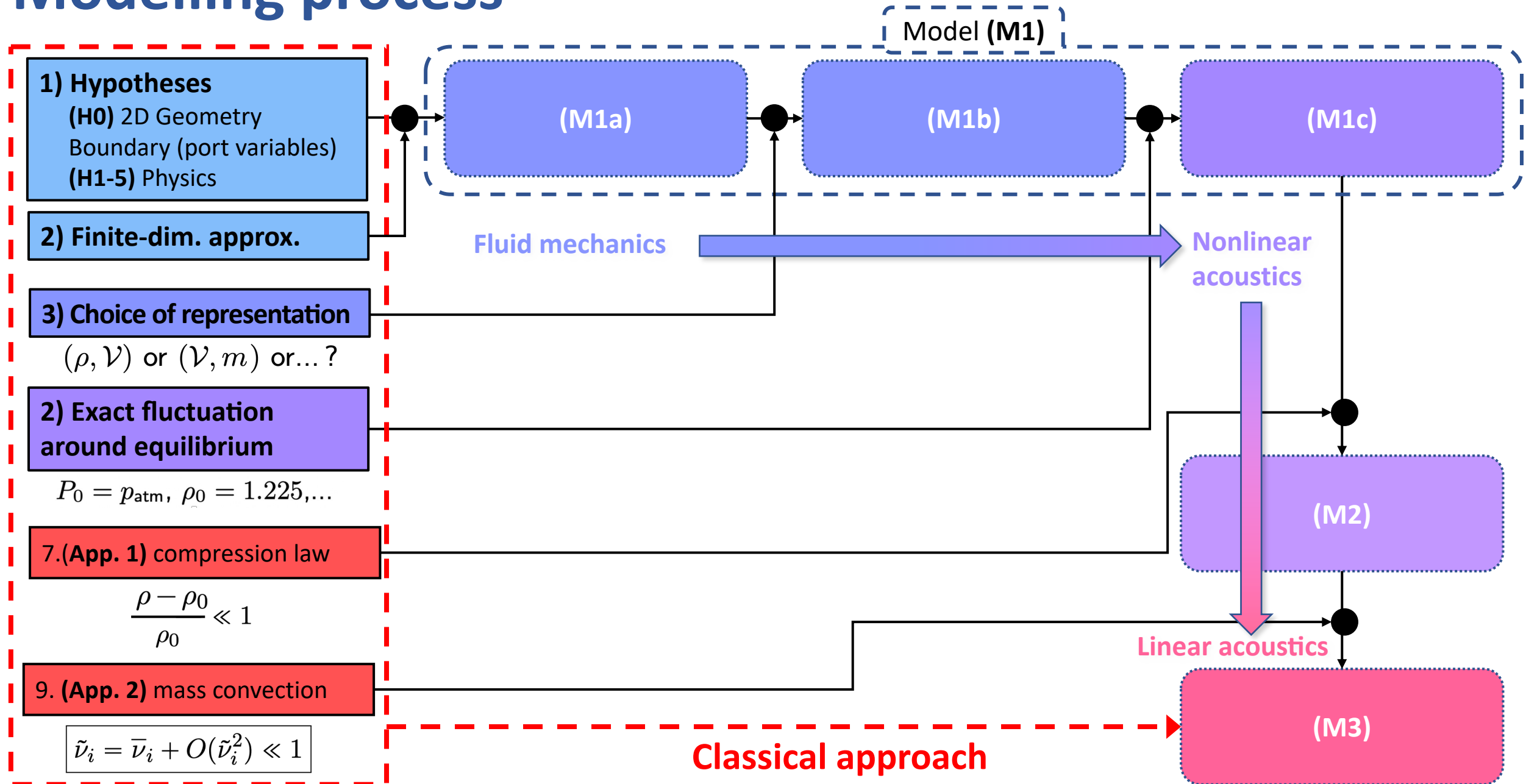
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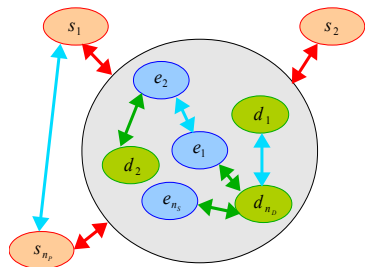
Modelling process



Outline of the presentation

- 1) Introduction and modelling process
- 2) Port-Hamiltonian Systems
- 3) Base model M1
- 4) Deriving other models
- 5) Conclusion

Port-Hamiltonian Framework



Energy storing components

$$E = \sum_{n=1}^N e_n \geq 0 \quad (\text{energy})$$

Power dissipating components :

$$P_{diss} = \sum_{m=1}^M d_m \geq 0 \quad (\text{dissipated power})$$

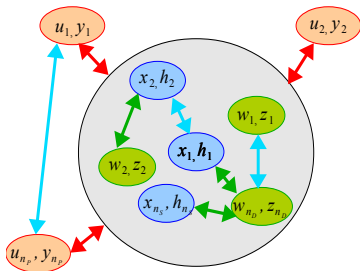
Ports (inputs/outputs) :

$$P_{ext} = \sum_{p=1}^P s_p \quad (\text{power flowing out of the system})$$

Conservative interconnection :

$$\dot{E} = -P_{diss} - P_{ext} \quad (\text{power balance})$$

Port-Hamiltonian Framework



Energy storing components

$$E = H(\mathbf{x}) = \sum_{n=1}^N H_n(x_n) \geq 0$$

Power dissipating components :

$$P_{diss} = \mathbf{z}(\mathbf{w})^T \mathbf{w} = \sum_{m=1}^M z_m(w_m)w_m \geq 0$$

Ports (inputs/outputs) :

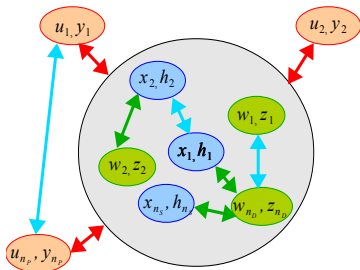
$$P_{ext} = \mathbf{u}^T \mathbf{y} = \sum_{p=1}^P u_p y_p$$

Conservative interconnection :

$$0 = \nabla H(\mathbf{x})^T \dot{\mathbf{x}} + \mathbf{z}(\mathbf{w})^T \mathbf{w} + \mathbf{u}^T \mathbf{y}$$

(effort \times flow : force \times velocity, voltage \times current, etc...)

Port-Hamiltonian Framework



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port-Hamiltonian formulation

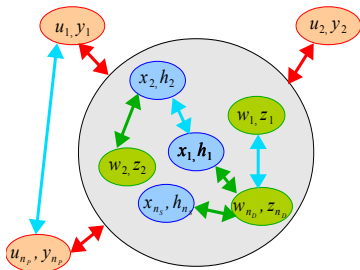
$$\underbrace{\begin{pmatrix} \dot{\mathbf{x}} \\ \mathbf{w} \\ \mathbf{y} \end{pmatrix}}_{\text{flow } \mathbf{F}} = S(\mathbf{x}) \cdot \underbrace{\begin{pmatrix} \nabla H(\mathbf{x}) = \mathbf{e} \\ \mathbf{z}(\mathbf{w}) \\ \mathbf{u} \end{pmatrix}}_{\text{efforts } \mathbf{E}}$$

Separation between :

- behaviour laws ($H(\mathbf{x})$ et $\mathbf{z}(\mathbf{w})$)
- conservative interconnection

$$\mathbf{E}^T \cdot \mathbf{F} = \mathbf{E}^T \cdot S \cdot \mathbf{E} = 0 \Leftrightarrow S \text{ skew-symmetric}$$

Port-Hamiltonian Framework



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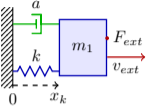
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[A. van der Schaft, "Port-Hamiltonian systems : an introductory survey," 2006.]

A Simple Example : The MKA

				
Variable				

A Simple Example : The MKA

	Mass	Spring	Damper	Action point
Parameter	m_1	k	a	.
Variable				

$$\begin{pmatrix} \text{---} \\ \text{---} \end{pmatrix} = \begin{bmatrix} | & | & | \\ \text{---} & \text{---} & \text{---} \\ | & | & | \end{bmatrix} \begin{pmatrix} \text{---} \\ \text{---} \end{pmatrix}$$

A Simple Example : The MKA

	Mass	Spring	Damper	Action point
Parameter	m_1	k	a	.
Variable	$\pi_1 = m_1 v_1$ momentum			

$$\begin{pmatrix} \text{---} \\ \text{---} \end{pmatrix} = \begin{bmatrix} | & | & | \\ \text{---} & \text{---} & \text{---} \\ | & | & | \end{bmatrix} \begin{pmatrix} \text{---} \\ \text{---} \end{pmatrix}$$

A Simple Example : The MKA

	Mass	Spring	Damper	Action point
Parameter	m_1	k	a	.
Variable	$\mathbf{x} = [\pi_1 = m_1 v_1, x_\ell]^\top$ momentum elongation			

$$\begin{pmatrix} \text{---} \\ \text{---} \end{pmatrix} = \begin{bmatrix} | & | & | \\ \text{---} & \text{---} & \text{---} \\ | & | & | \end{bmatrix} \begin{pmatrix} \text{---} \\ \text{---} \end{pmatrix}$$

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	Mass	Spring	Damper	Action point
Parameter	m_1	k	a	.
Variable	$\mathbf{x} = [\pi_1 = m_1 v_1, x_\ell]^\top$ momentum elongation		v_d elongation rate	

dashpot elongation-rate

$$\begin{pmatrix} \text{---} \\ v_d \\ \text{---} \end{pmatrix} = \begin{bmatrix} | & | & | & | \\ \text{---} & \text{---} & \text{---} & \text{---} \\ | & | & | & | \end{bmatrix} \begin{pmatrix} \text{---} \\ \text{---} \\ \text{---} \end{pmatrix}$$

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H(x) =				

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Variable	$\mathbf{x} = [\pi_1 = m_1 v_1, x_\ell]^\top$ momentum elongation		v_d elongation rate	v_{ext}, F_{ext} velocity/force
H(x) =	$\frac{1}{2m_1} \pi_1^2$			

dashpot elongation-rate

$$\begin{pmatrix} \underline{\quad} \\ v_d \\ \underline{\quad} \end{pmatrix} = \begin{bmatrix} | & | & | \\ \hline \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \hline \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \hline \end{bmatrix} \begin{pmatrix} \underline{\quad} \\ \underline{\quad} \\ \underline{\quad} \end{pmatrix}$$

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Flows F				

dashpot elongation-rate

$$\begin{pmatrix} \underline{\quad} \\ v_d \\ \underline{\quad} \end{pmatrix} = \begin{bmatrix} | & | & | & | \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \end{bmatrix} \begin{pmatrix} \underline{\quad} \\ \underline{\quad} \\ \underline{\quad} \\ \underline{\quad} \end{pmatrix}$$

A Simple Example : The MKA

	Mass	Spring	Damper	Action point
Parameter	m_1	k	a	.
Variable	$\mathbf{x} = [\pi_1 = m_1 v_1, x_\ell]^\top$ momentum elongation		v_d elongation rate	v_{ext}, F_{ext} velocity/force
H(x) =	$\frac{1}{2m_1} \pi_1^2 + \frac{1}{2} k x_\ell^2$.	.
Flows \mathbf{F}	$\dot{\pi}_1 = F_{inertia}$			

inertia force

dashpot elongation-rate

$$\begin{pmatrix} \dot{\pi}_1 \\ v_d \end{pmatrix} = \begin{bmatrix} | & | & | \\ \hline | & | & | \\ \hline | & | & | \end{bmatrix} \begin{pmatrix} \\ \\ \end{pmatrix}$$

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Flows F	$\dot{\pi}_1 = F_{inertia}$	$\dot{x}_\ell = v_k$		

inertia force
 spring elongation-rate
 dashpot elongation-rate

$$\begin{pmatrix} \dot{\pi}_1 \\ \dot{x}_\ell \\ v_d \end{pmatrix} = \begin{bmatrix} | & | & | \\ \hline | & | & | \\ \hline | & | & | \end{bmatrix} \begin{pmatrix} \text{---} \\ \text{---} \\ \text{---} \end{pmatrix}$$

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Parameter	m_1	k	a	.
Variable	$\mathbf{x} = [\pi_1 = m_1 v_1, x_\ell]^\top$ momentum elongation		v_d elongation rate	v_{ext}, F_{ext} velocity/force
$H(\mathbf{x}) =$	$\frac{1}{2m_1} \pi_1^2 + \frac{1}{2} k x_\ell^2$.	.
Flows \mathbf{F}	$\dot{\pi}_1 = F_{inertia}$	$\dot{x}_\ell = v_k$	v_d	v_{ext}

inertia force
 spring elongation-rate
 dashpot elongation-rate
 action point velocity

$$\begin{pmatrix} \dot{\pi}_1 \\ \dot{x}_\ell \\ v_d \\ v_{ext} \end{pmatrix} = \begin{bmatrix} | & | & | \\ \hline | & | & | \\ \hline | & | & | \\ \hline | & | & | \end{bmatrix} \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

A Simple Example : The MKA

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Parameter	m_1	k	a	.
Variable	$\mathbf{x} = [\pi_1 = m_1 v_1, x_\ell]^\top$ momentum elongation		v_d elongation rate	v_{ext}, F_{ext} velocity/force
H(x) =	$\frac{1}{2m_1} \pi_1^2 + \frac{1}{2} k x_\ell^2$.	.
Flows F	$\dot{\pi}_1 = F_{inertia}$	$\dot{x}_\ell = v_k$	v_d	v_{ext}
Efforts E				

inertia force
 spring elongation-rate
 dashpot elongation-rate
 action point velocity

$$\begin{pmatrix} \dot{\pi}_1 \\ \dot{x}_\ell \\ v_d \\ v_{ext} \end{pmatrix} = \begin{bmatrix} | & | & | \\ \hline & & \\ \hline & & \\ \hline & & \\ \hline & & \end{bmatrix} \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

A Simple Example : The MKA

	Mass	Spring	Damper	Action point
Parameter	m_1	k	a	.
Variable	$\mathbf{x} = [\pi_1 = m_1 v_1, x_\ell]^\top$ momentum elongation		v_d elongation rate	v_{ext}, F_{ext} velocity/force
H(x) =	$\frac{1}{2m_1} \pi_1^2 + \frac{1}{2} k x_\ell^2$.	.
Flows F	$\dot{\pi}_1 = F_{inertia}$	$\dot{x}_\ell = v_k$	v_d	v_{ext}
Efforts E	$\pi_1/m_1 = v_1$			

inertia force
 spring elongation-rate
 dashpot elongation-rate
 action point velocity

$$\begin{pmatrix} \dot{\pi}_1 \\ \dot{x}_\ell \\ v_d \\ v_{ext} \end{pmatrix} = \begin{bmatrix} | & | & | \\ \hline | & | & | \\ \hline | & | & | \\ \hline | & | & | \end{bmatrix} \begin{pmatrix} v_1 \\ \text{---} \\ \text{---} \end{pmatrix} \text{ velocity of the mass } m_1$$

A Simple Example : The MKA

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Parameter	m_1	k	a	.
Variable	$\mathbf{x} = [\pi_1 = m_1 v_1, x_\ell]^\top$ momentum elongation		v_d elongation rate	v_{ext}, F_{ext} velocity/force
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Efforts E	$\pi_1/m_1 = v_1$	$k x_\ell = F_k$		

inertia force
 spring elongation-rate
 dashpot elongation-rate
 action point velocity

$$\begin{pmatrix} \dot{\pi}_1 \\ \dot{x}_\ell \\ v_d \\ v_{ext} \end{pmatrix} = \begin{bmatrix} | & | & | \\ \hline | & | & | \\ \hline | & | & | \\ \hline | & | & | \end{bmatrix} \begin{pmatrix} v_1 \\ F_k \\ \\ \end{pmatrix}$$

velocity of the mass m_1
 reaction force of the spring

A Simple Example : The MKA

	Mass	Spring	Damper	Action point
Parameter	m_1	k	a	.
Variable	$\mathbf{x} = [\pi_1 = m_1 v_1, x_\ell]^\top$ momentum elongation		v_d elongation rate	v_{ext}, F_{ext} velocity/force
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Flows F	$\dot{\pi}_1 = F_{inertia}$	$\dot{x}_\ell = v_k$	v_d	v_{ext}
Efforts E	$\pi_1/m_1 = v_1$	$k x_\ell = F_k$	$z(v_d) = a v_d = F_d$	

inertia force	$\begin{pmatrix} \dot{\pi}_1 \\ \dot{x}_\ell \\ v_d \\ v_{ext} \end{pmatrix} = \begin{bmatrix} & & \\ \hline & & \\ \hline & & \\ \hline & & \\ \hline \end{bmatrix} \begin{pmatrix} v_1 \\ F_k \\ F_d \end{pmatrix}$	velocity of the mass m_1
spring elongation-rate		reaction force of the spring
dashpot elongation-rate		reaction force of the dashpot
action point velocity		

A Simple Example : The MKA

	Mass	Spring	Damper	Action point
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Flows F	$\dot{\pi}_1 = F_{inertia}$	$\dot{x}_\ell = v_k$	v_d	v_{ext}
Efforts E	$\pi_1/m_1 = v_1$	$k x_\ell = F_k$	$z(v_d) = a v_d = F_d$	$-F_{ext}$

	$\left(\begin{array}{c} \dot{\pi}_1 \\ \dot{x}_\ell \\ v_d \\ v_{ext} \end{array} \right)$	$=$	$\left[\begin{array}{ c c c } \hline & & \\ \hline & & \\ \hline & & \\ \hline & & \\ \hline \end{array} \right]$	$\left(\begin{array}{c} v_1 \\ F_k \\ F_d \\ -F_{ext} \end{array} \right)$	
inertia force					velocity of the mass m_1
spring elongation-rate					reaction force of the spring
dashpot elongation-rate					reaction force of the dashpot
action point velocity					force received by the exterior

A Simple Example : The MKA

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Efforts E	$\pi_1/m_1 = v_1$	$k x_\ell = F_k$	$z(v_d) = a v_d = F_d$	$-F_{ext}$

Rigid connection

inertia force	$\begin{pmatrix} \dot{\pi}_1 \\ \dot{x}_\ell \\ v_d \\ v_{ext} \end{pmatrix} = \begin{bmatrix} & & \\ \hline & & \\ \hline & & \\ \hline & & \\ \hline & & \end{bmatrix} \begin{pmatrix} v_1 \\ F_k \\ F_d \\ -F_{ext} \end{pmatrix}$					
spring elongation-rate						velocity of the mass m_1
dashpot elongation-rate						reaction force of the spring
action point velocity						reaction force of the dashpot
						force received by the exterior

A Simple Example : The MKA

	Mass	Spring	Damper	Action point
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Efforts E	$\pi_1/m_1 = v_1$	$k x_\ell = F_k$	$z(v_d) = a v_d = F_d$	$-F_{ext}$

Rigid connection

	inertia force	$\begin{pmatrix} \dot{\pi}_1 \\ \dot{x}_\ell \\ v_d \\ v_{ext} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \hline \hline \hline \hline \hline \end{bmatrix} \begin{pmatrix} v_1 \\ F_k \\ F_d \\ -F_{ext} \end{pmatrix}$	velocity of the mass m_1
	spring elongation-rate		reaction force of the spring
	dashpot elongation-rate		reaction force of the dashpot
	action point velocity		force received by the exterior

A Simple Example : The MKA

	Mass	Spring	Damper	Action point
Parameter	m_1	k	a	.
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Efforts E	$\pi_1/m_1 = v_1$	$k x_\ell = F_k$	$z(v_d) = a v_d = F_d$	$-F_{ext}$

Rigid connection

inertia force spring elongation-rate dashpot elongation-rate action point velocity	$\begin{pmatrix} \dot{\pi}_1 \\ \dot{x}_\ell \\ v_d \\ v_{ext} \end{pmatrix} =$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \hline & & & \end{bmatrix}$	$\begin{pmatrix} v_1 \\ F_k \\ F_d \\ -F_{ext} \end{pmatrix}$	velocity of the mass m_1 reaction force of the spring reaction force of the dashpot force received by the exterior
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A Simple Example : The MKA

	Mass	Spring	Damper	Action point
Parameter	m_1	k	a	.
Variable	$\mathbf{x} = [\pi_1 = m_1 v_1, x_\ell]^\top$ momentum elongation		v_d elongation rate	v_{ext}, F_{ext} velocity/force
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Rigid connection

	inertia force	$\begin{pmatrix} \dot{\pi}_1 \\ \dot{x}_\ell \\ v_d \\ v_{ext} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} v_1 \\ F_k \\ F_d \\ -F_{ext} \end{pmatrix}$	velocity of the mass m_1
	spring elongation-rate		reaction force of the spring
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Efforts E	$\pi_1/m_1 = v_1$	$k x_\ell = F_k$	$z(v_d) = a v_d = F_d$	$-F_{ext}$

Rigid connection

inertia force	$\begin{pmatrix} \dot{\pi}_1 \\ \dot{x}_\ell \\ v_d \\ v_{ext} \end{pmatrix} = \begin{bmatrix} 0 & -1 & -1 & -1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} v_1 \\ F_k \\ F_d \\ -F_{ext} \end{pmatrix}$	velocity of the mass m_1
spring elongation-rate		reaction force of the spring
dashpot elongation-rate		reaction force of the dashpot
action point velocity		force received by the exterior

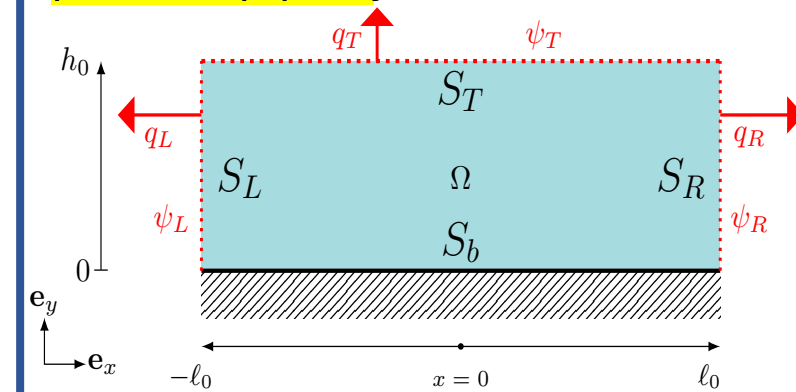
Hypotheses



Hypotheses

2D Geometry

parallelepiped junction

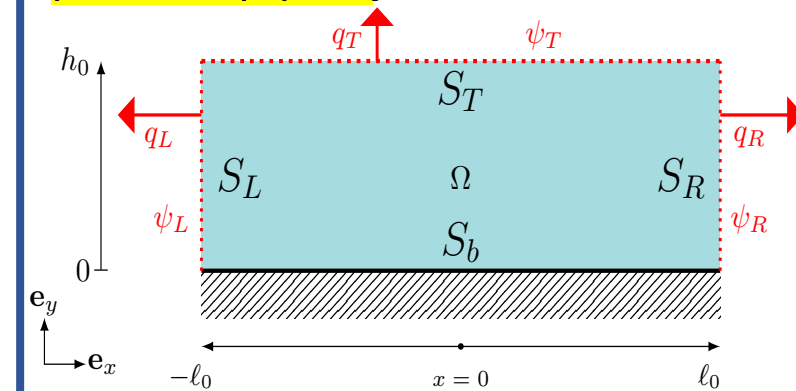


$$V_0 = 2\ell_0 L_0 h_0$$

Hypotheses

2D Geometry

parallelepiped junction



$$V_0 = 2\ell_0 L_0 h_0$$

Port variables

At open boundaries $S_L \cup S_T \cup S_R$

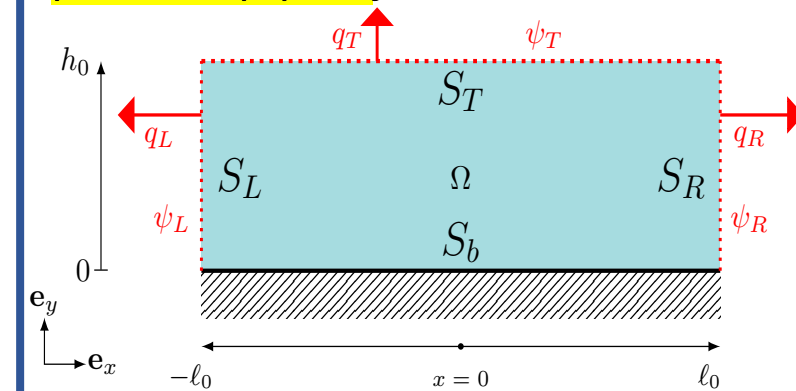
Mass rate $q_i = \iint_{\partial\Omega_i} \rho \mathbf{v} \cdot \mathbf{n} \, dS \quad i \in \{L, R, T\}$

Total enthalpy $\psi_i = \left\langle \frac{1}{2} \mathbf{v} \cdot \mathbf{v} + \frac{p}{\rho} + \epsilon \right\rangle_{S_i}$

Hypotheses

2D Geometry

parallelepiped junction



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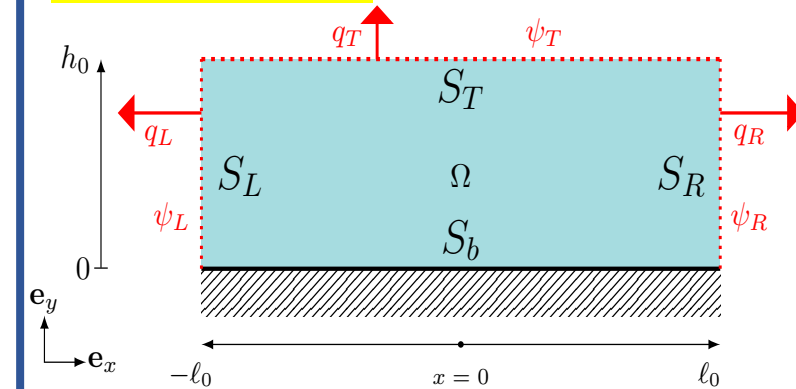
Mass rate: $\text{kg} \cdot \text{s}^{-1}$

Total enthalpy: $\text{m}^2 \cdot \text{s}^{-2}$

Hypotheses

2D Geometry

parallelepiped junction



$$V_0 = 2\ell_0 L_0 h_0$$

Port variables

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Mass rate: $\text{kg} \cdot \text{s}^{-1}$

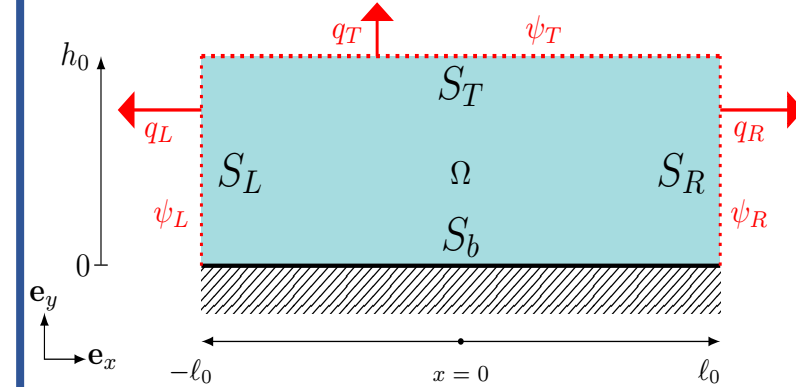
Total enthalpy: $\text{m}^2 \cdot \text{s}^{-2}$

} Power

Hypotheses

2D Geometry

parallelepiped junction



$$V_0 = 2\ell_0 L_0 h_0$$

Port variables

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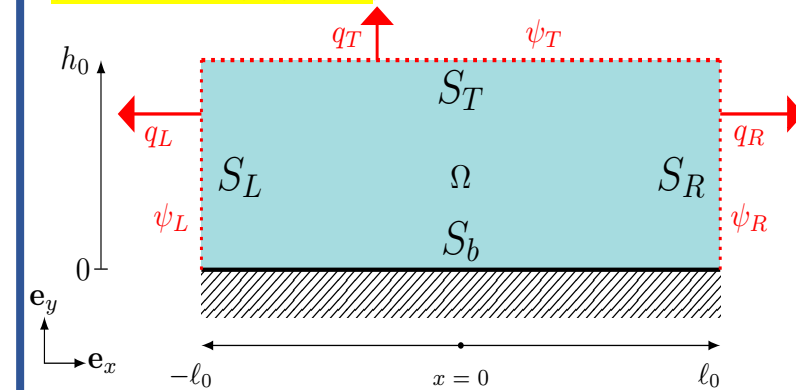
Total enthalpy: $\text{m}^2 \cdot \text{s}^{-2}$

} Power

Hypotheses

2D Geometry

parallelepiped junction



$$V_0 = 2\ell_0 L_0 h_0$$

Port variables

At open boundaries $S_L \cup S_T \cup S_R$

Mass rate $q_i = \iint_{\partial\Omega_i} \rho \mathbf{v} \cdot \mathbf{dS} \quad i \in \{L, R, T\}$

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Mass rate: $\text{kg} \cdot \text{s}^{-1}$

Total enthalpy: $\text{m}^2 \cdot \text{s}^{-2}$

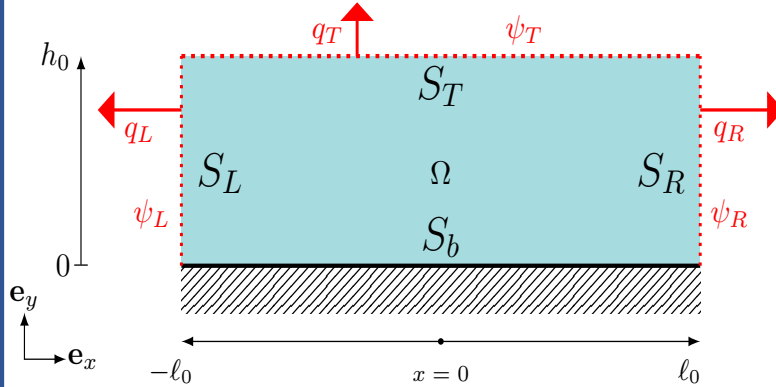
} Power

Physical Hypothesis

Hypotheses

2D Geometry

parallelepiped junction



$$V_0 = 2\ell_0 L_0 h_0$$

Physical Hypothesis

(H1) perfect fluid

Port variables

At open boundaries $S_L \cup S_T \cup S_R$

Mass rate $q_i = \iint_{\partial\Omega_i} \rho \mathbf{v} \cdot d\mathbf{S} \quad i \in \{L, R, T\}$

Total enthalpy $\psi_i = \langle \frac{1}{2} \mathbf{v} \cdot \mathbf{v} + \frac{p}{\rho} + \epsilon \rangle_{S_i}$

Mass rate: $\text{kg} \cdot \text{s}^{-1}$

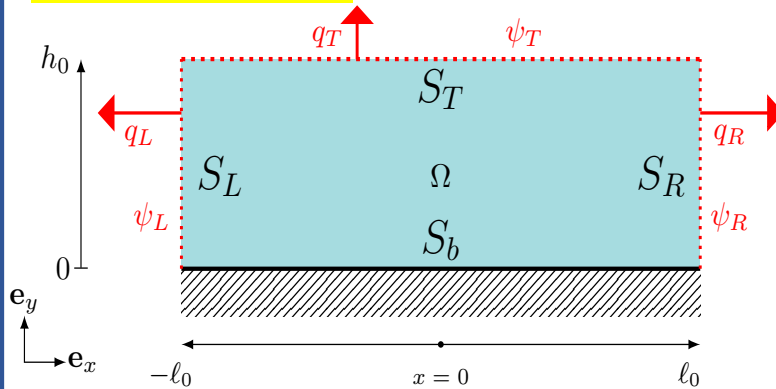
Total enthalpy: $\text{m}^2 \cdot \text{s}^{-2}$

} Power

Hypotheses

2D Geometry

parallelepiped junction



$$V_0 = 2\ell_0 L_0 h_0$$

Physical Hypothesis

(H1) perfect fluid

(H2) irrotational flow

Port variables

At open boundaries $S_L \cup S_T \cup S_R$

Mass rate $q_i = \iint_{\partial\Omega_i} \rho \mathbf{v} \cdot d\mathbf{S} \quad i \in \{L, R, T\}$

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Mass rate: $\text{kg} \cdot \text{s}^{-1}$

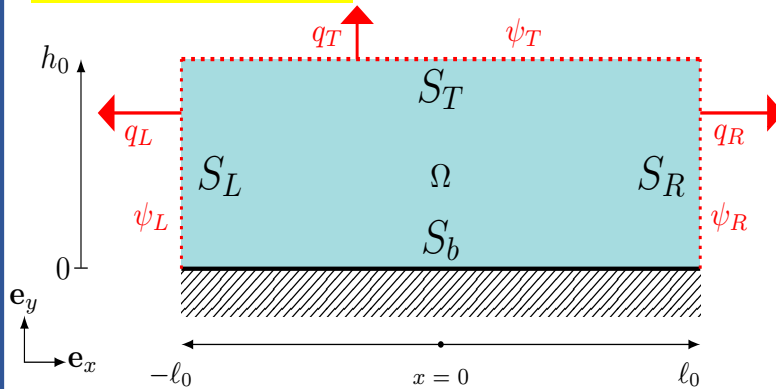
Total enthalpy: $\text{m}^2 \cdot \text{s}^{-2}$

} Power

Hypotheses

2D Geometry

parallelepiped junction



$$V_0 = 2\ell_0 L_0 h_0$$

Physical Hypothesis

- (H1) perfect fluid
- (H2) irrotational flow
- (H3) isentropic and adiabatic process
- (H4) constant heat capacity C_V
- (H5) perfect gas assumption

Port variables

At open boundaries $S_L \cup S_T \cup S_R$

Mass rate $q_i = \iint_{\partial\Omega_i} \rho \mathbf{v} \cdot d\mathbf{S} \quad i \in \{L, R, T\}$

Total enthalpy $\psi_i = \left\langle \frac{1}{2} \mathbf{v} \cdot \mathbf{v} + \frac{p}{\rho} + \epsilon \right\rangle_{S_i}$

Mass rate: $\text{kg} \cdot \text{s}^{-1}$

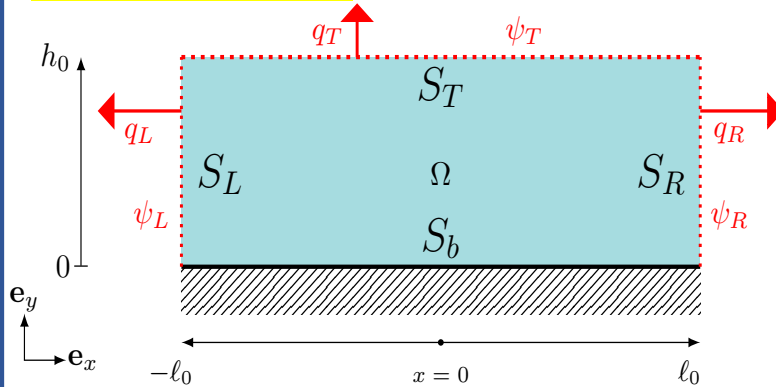
Total enthalpy: $\text{m}^2 \cdot \text{s}^{-2}$

} Power

Hypotheses

2D Geometry

parallelepiped junction



$$V_0 = 2\ell_0 L_0 h_0$$

Port variables

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Total enthalpy: $\text{m}^2 \cdot \text{s}^{-2}$

} Power

Physical Hypothesis

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- (H2) irrotational flow
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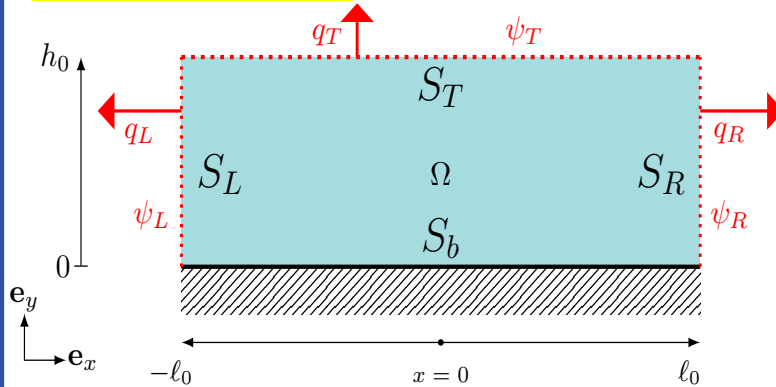
Lumped parameter hypothesis

- (H6) ρ is homogeneous within Ω

Hypotheses

2D Geometry

parallelepiped junction



$$V_0 = 2\ell_0 L_0 h_0$$

Port variables

At open boundaries $S_L \cup S_T \cup S_R$

Mass rate $q_i = \iint_{\partial\Omega_i} \rho \mathbf{v} \cdot d\mathbf{S} \quad i \in \{L, R, T\}$

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Mass rate: $\text{kg} \cdot \text{s}^{-1}$

Total enthalpy: $\text{m}^2 \cdot \text{s}^{-2}$

} Power

Physical Hypothesis

- (H1) perfect fluid
- (H2) irrotational flow
- (H3) isentropic and adiabatic process
- (H4) constant heat capacity C_V
- (H5) perfect gas assumption

Lumped parameter hypothesis

- (H6) ρ is homogeneous within Ω

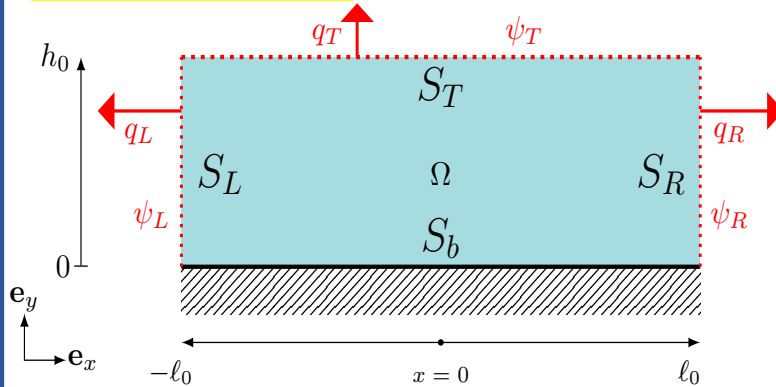
Mass and momentum conservation Eqs.

$$\frac{\partial \rho}{\partial t} = - \text{div}(\rho \mathbf{v}) \quad \frac{\partial \mathbf{v}}{\partial t} = - \text{grad}(\psi)$$

Hypotheses

2D Geometry

parallelepiped junction



$$V_0 = 2\ell_0 L_0 h_0$$

Port variables

At open boundaries $S_L \cup S_T \cup S_R$

Mass rate $q_i = \iint_{\partial\Omega_i} \rho \mathbf{v} \cdot d\mathbf{S} \quad i \in \{L, R, T\}$

Total enthalpy $\psi_i = \left\langle \frac{1}{2} \mathbf{v} \cdot \mathbf{v} + \frac{p}{\rho} + \epsilon \right\rangle_{S_i}$

Mass rate: $\text{kg} \cdot \text{s}^{-1}$

Total enthalpy: $\text{m}^2 \cdot \text{s}^{-2}$

} Power

Physical Hypothesis

(H1) perfect fluid

(H2) irrotational flow

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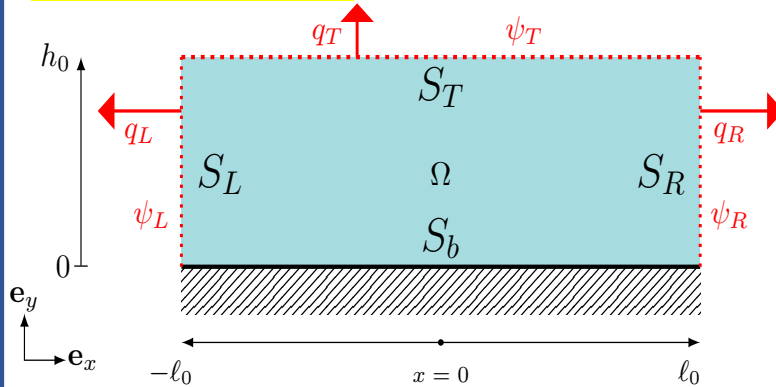
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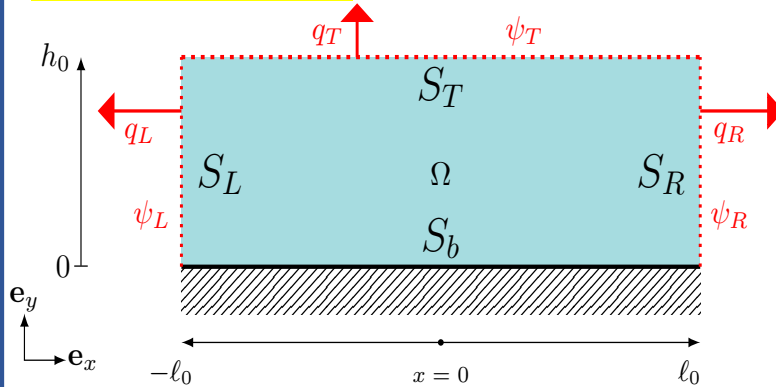
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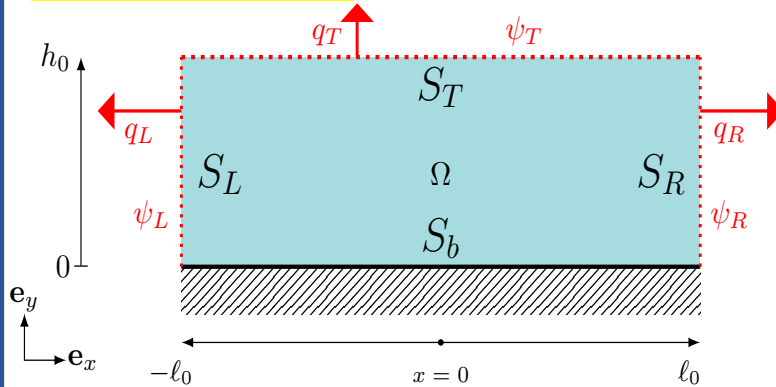
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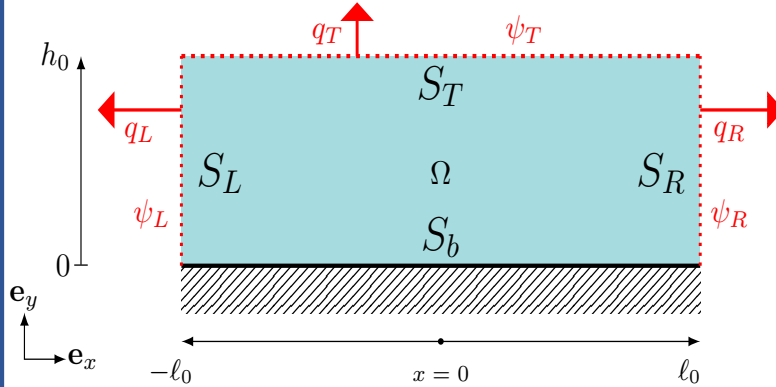
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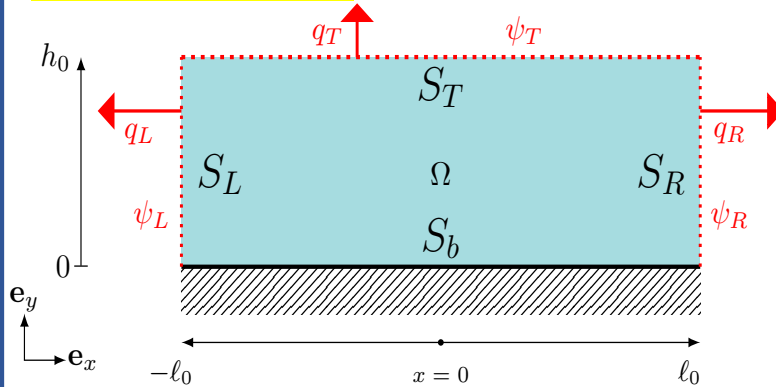
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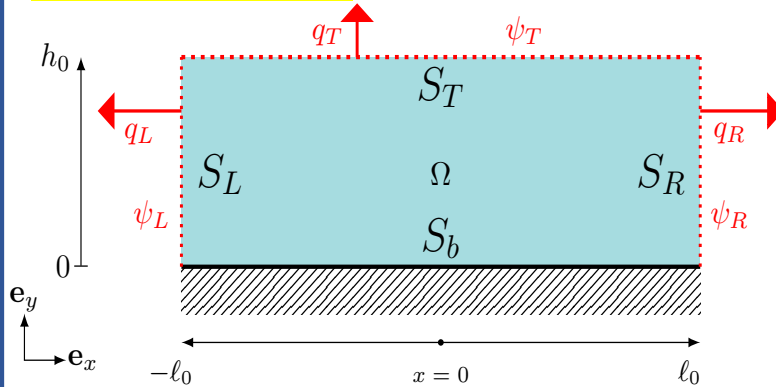
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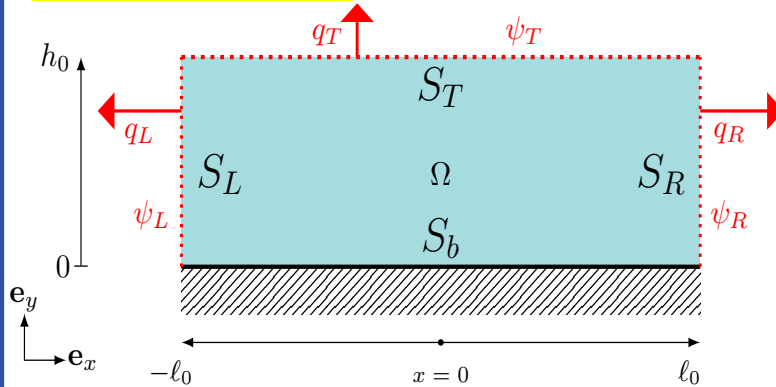
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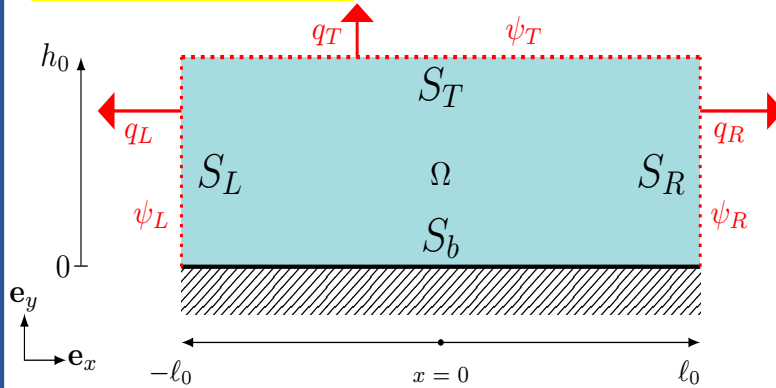
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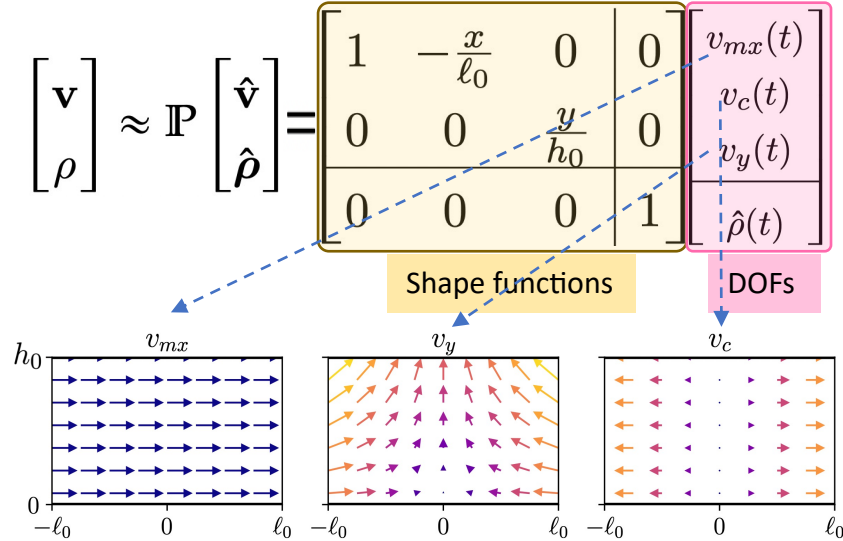
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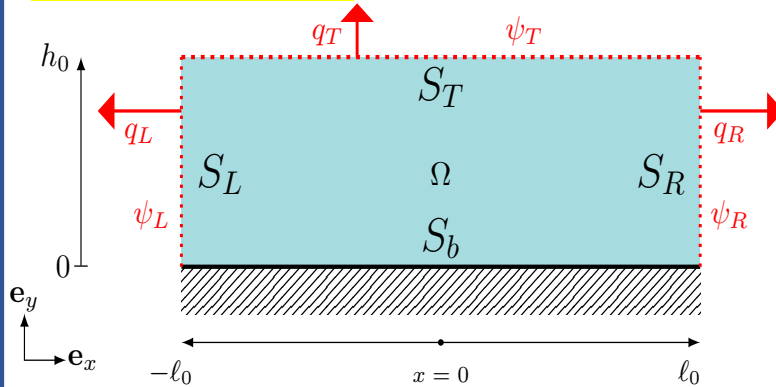
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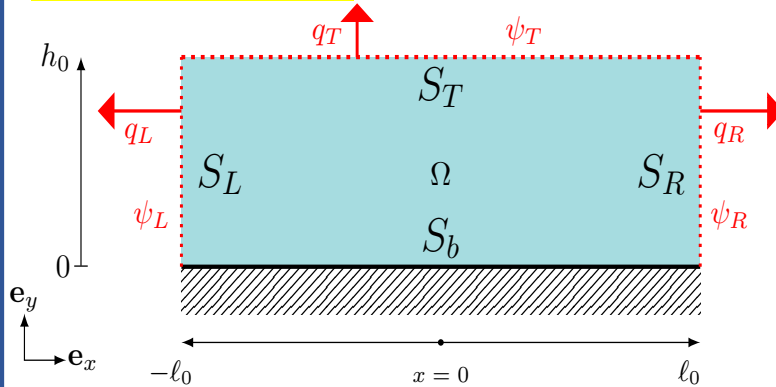
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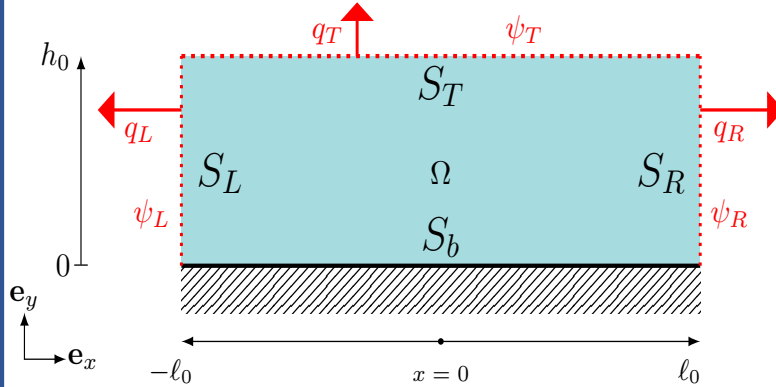
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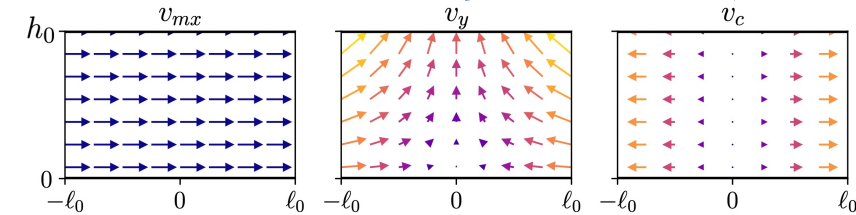
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$$2) \quad H_1(\mathbf{x}_1 = [\hat{\mathbf{v}}^T \quad \hat{\rho}]^T) = \frac{1}{2} \hat{\mathbf{v}}^T \begin{pmatrix} V_0 \hat{\rho} & 0 & 0 \\ 0 & \frac{V_0 \hat{\rho}}{3} & 0 \\ 0 & 0 & \frac{V_0 \hat{\rho}}{3} \end{pmatrix} \hat{\mathbf{v}} + \frac{V_0 P_0}{\gamma - 1} \left[\left(\frac{\hat{\rho}}{\rho_0} \right)^\gamma - \gamma \frac{\hat{\rho}}{\rho_0} \right]$$

$$3) \quad \mathbf{e}_1 = \nabla_{\mathbf{x}_1} H_1(\mathbf{x}_1) = \begin{bmatrix} e_{v_{mx}} \\ e_{v_c} \\ e_{v_y} \\ e_{\hat{\rho}} \end{bmatrix} = \begin{bmatrix} \hat{\rho} V_0 v_{mx} \\ \hat{\rho} V_0 \frac{v_c}{3} \\ \hat{\rho} V_0 \frac{v_y}{3} \\ \frac{1}{2} V_0 \left(v_{mx}^2 + \frac{v_c^2}{3} + \frac{v_y^2}{3} \right) + \frac{V_0 P_0 \gamma}{(\gamma - 1) \rho_0} \left[\left(\frac{\rho}{\rho_0} \right)^{\gamma - 1} - 1 \right] \end{bmatrix}$$

Model (M1a): fluid mechanics

$$1) \quad \mathbf{x}_1 = [v_{mx}, v_c, v_y, \hat{\rho}]^T$$

DOFs

$$2) \quad H_1(\mathbf{x}_1 = [\hat{\mathbf{v}}^T \quad \hat{\rho}]^T) = \frac{1}{2} \hat{\mathbf{v}}^T \begin{pmatrix} V_0 \hat{\rho} & 0 & 0 \\ 0 & \frac{V_0 \hat{\rho}}{3} & 0 \\ 0 & 0 & \frac{V_0 \hat{\rho}}{3} \end{pmatrix} \hat{\mathbf{v}} + \frac{V_0 P_0}{\gamma - 1} \left[\left(\frac{\hat{\rho}}{\rho_0} \right)^\gamma - \gamma \frac{\hat{\rho}}{\rho_0} \right]$$

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Momentum associated with each DOF

Model (M1a): fluid mechanics

1) $\mathbf{x}_1 = [v_{mx}, v_c, v_y, \hat{\rho}]^T$

DOFs

2) $H_1(\mathbf{x}_1 = [\hat{\mathbf{v}}^T \hat{\rho}]^T) = \frac{1}{2} \hat{\mathbf{v}}^T \begin{pmatrix} V_0 \hat{\rho} & 0 & 0 \\ 0 & \frac{V_0 \hat{\rho}}{3} & 0 \\ 0 & 0 & \frac{V_0 \hat{\rho}}{3} \end{pmatrix} \hat{\mathbf{v}} + \frac{V_0 P_0}{\gamma - 1} \left[\left(\frac{\hat{\rho}}{\rho_0} \right)^\gamma - \gamma \frac{\hat{\rho}}{\rho_0} \right]$

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1) $\mathbf{x}_1 = [v_{mx}, v_c, v_y, \hat{\rho}]^T$

DOFs

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4)

Model (M1a): fluid mechanics

1) $\mathbf{x}_1 = [v_{mx}, v_c, v_y, \hat{\rho}]^T$

DOFs

2) $H_1(\mathbf{x}_1 = [\hat{\mathbf{v}}^T \hat{\rho}]^T) = \frac{1}{2} \hat{\mathbf{v}}^T \begin{pmatrix} V_0 \hat{\rho} & 0 & 0 \\ 0 & \frac{V_0 \hat{\rho}}{3} & 0 \\ 0 & 0 & \frac{V_0 \hat{\rho}}{3} \end{pmatrix} \hat{\mathbf{v}} + \frac{V_0 P_0}{\gamma - 1} \left[\left(\frac{\hat{\rho}}{\rho_0} \right)^\gamma - \gamma \frac{\hat{\rho}}{\rho_0} \right]$

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Momentum associated with each DOF

4) Eqs. of dynamics: not detailed here (weak form of PDEs then use the chosen shape functions)

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Momentum associated with each DOF

$-V_0 < \hat{\psi} >_\Omega = e_{\hat{\rho}}$

4) Eqs. of dynamics: not detailed here (weak form of PDEs then use the chosen shape functions)

$\frac{d\mathbf{x}_1}{dt} = \mathbf{S}_1 \mathbf{u}_1$

$\frac{d\mathbf{x}_1}{dt}$	$\frac{dv_{mx}}{dt}$	$\frac{dv_c}{dt}$	$\frac{dv_y}{dt}$	$\frac{d\hat{\rho}}{dt}$	$\frac{d\hat{q}_L}{dt}$	$\frac{d\hat{q}_R}{dt}$	$\frac{d\hat{q}_T}{dt}$
		$-3/l_0 V_0$	$3/2l_0$	$3/2l_0$	$1/2l_0$	$-1/2l_0$	$-3/2l_0$
		$3/h_0 V_0$	$3/2l_0$	$3/2l_0$	$1/2l_0$	$-3/2l_0$	$-3/2l_0$
	$3/l_0 V_0$	$-3/h_0 V_0$			$3/h_0$		
\mathbf{u}_1	$e_{v_{mx}}$	e_{v_c}	e_{v_y}	$e_{\hat{\rho}}$	$< \hat{\psi} >_{S_L}$	$< \hat{\psi} >_{S_R}$	$< \hat{\psi} >_{S_T}$

Model (M1a): fluid mechanics

1) $\mathbf{x}_1 = [v_{mx}, v_c, v_y, \hat{\rho}]^T$

DOFs

2) $H_1(\mathbf{x}_1 = [\hat{\mathbf{v}}^T \hat{\rho}]^T) = \frac{1}{2} \hat{\mathbf{v}}^T \begin{pmatrix} V_0 \hat{\rho} & 0 & 0 \\ 0 & \frac{V_0 \hat{\rho}}{3} & 0 \\ 0 & 0 & \frac{V_0 \hat{\rho}}{3} \end{pmatrix} \hat{\mathbf{v}} + \frac{V_0 P_0}{\gamma - 1} \left[\left(\frac{\hat{\rho}}{\rho_0} \right)^\gamma - \gamma \frac{\hat{\rho}}{\rho_0} \right]$

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Momentum associated with each DOF

4) Eqs. of dynamics: not detailed here (weak form of PDEs then use the chosen shape functions)

$$\begin{matrix} \frac{d\mathbf{x}_1}{dt} \\ \\ \mathbf{y}_1 \end{matrix} \begin{pmatrix} \frac{dv_{mx}}{dt} \\ \frac{dv_c}{dt} \\ \frac{dv_y}{dt} \\ \frac{d\hat{\rho}}{dt} \\ \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \end{pmatrix} = \underbrace{\left[\begin{array}{cc|cc} & & 1/2l_0 & -1/2l_0 \\ & -3/l_0 V_0 & 3/2l_0 & 3/2l_0 \\ & 3/h_0 V_0 & & -3/h_0 \\ \hline & 3/l_0 V_0 & -3/h_0 V_0 & \\ \hline -1/2l_0 & -3/2l_0 & & \\ 1/2l_0 & -3/2l_0 & & \\ & & 3/h_0 & \end{array} \right]}_{\mathbf{S}_1} \begin{pmatrix} e_{v_{mx}} \\ e_{v_c} \\ e_{v_y} \\ e_{\rho} \\ \langle \hat{\psi} \rangle_{S_L} \\ \langle \hat{\psi} \rangle_{S_R} \\ \langle \hat{\psi} \rangle_{S_T} \end{pmatrix} \begin{matrix} \mathbf{e}_1 \\ \\ \mathbf{u}_1 \end{matrix}$$

Here:

- Fluid mechanical model
- pH formulation
- ➔ passive

However:

- Not fully interpretable

Can we do better?

➔ An other choice of state vector !

Model (M1a): fluid mechanics

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2) $H_1(\mathbf{x}_1 = [\hat{\mathbf{v}}^T \hat{\rho}]^T) = \frac{1}{2} \hat{\mathbf{v}}^T \begin{pmatrix} V_0 \hat{\rho} & 0 & 0 \\ 0 & \frac{V_0 \hat{\rho}}{3} & 0 \\ 0 & 0 & \frac{V_0 \hat{\rho}}{3} \end{pmatrix} \hat{\mathbf{v}} + \frac{V_0 P_0}{\gamma - 1} \left[\left(\frac{\hat{\rho}}{\rho_0} \right)^\gamma - \gamma \frac{\hat{\rho}}{\rho_0} \right]$

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$\frac{d\mathbf{x}_1}{dt}$	=	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; border-bottom: 1px solid black; padding: 5px;"></td> <td style="border-bottom: 1px solid black; padding: 5px; text-align: center;">$-3/l_0 V_0$</td> <td style="border-bottom: 1px solid black; padding: 5px; text-align: center;">$1/2l_0$ $-1/2l_0$</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;"></td> <td style="padding: 5px; text-align: center;">$3/h_0 V_0$</td> <td style="padding: 5px; text-align: center;">$3/2l_0$ $3/2l_0$ $-3/h_0$</td> </tr> <tr> <td style="border-right: 1px solid black; border-top: 1px solid black; padding: 5px;"></td> <td style="border-top: 1px solid black; padding: 5px; text-align: center;">$3/l_0 V_0$ $-3/h_0 V_0$</td> <td style="border-top: 1px solid black; padding: 5px;"></td> </tr> </table>		$-3/l_0 V_0$	$1/2l_0$ $-1/2l_0$		$3/h_0 V_0$	$3/2l_0$ $3/2l_0$ $-3/h_0$		$3/l_0 V_0$ $-3/h_0 V_0$		\mathbf{e}_1
			$-3/l_0 V_0$	$1/2l_0$ $-1/2l_0$								
	$3/h_0 V_0$	$3/2l_0$ $3/2l_0$ $-3/h_0$										
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	$-1/2l_0$ $-3/2l_0$											
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Model (M1b): a canonical interconnection



Model (M1b): a canonical interconnection

A new choice of representation

Model (M1b): a canonical interconnection

A new choice of representation

1)

Model (M1b): a canonical interconnection

A new choice of representation

$$\mathbf{1) } \mathbf{x}_2 = \mathbb{M}\mathbf{x}_1 = \begin{bmatrix} -\ell_0 & -\ell_0/3 & 0 & 0 \\ \ell_0 & -\ell_0/3 & 0 & 0 \\ 0 & 0 & h_0/3 & 0 \\ 0 & 0 & 0 & V_0 \end{bmatrix} \begin{pmatrix} v_{mx} \\ v_c \\ v_y \\ \hat{\rho} \end{pmatrix} =: \begin{pmatrix} \nu_L \\ \nu_R \\ \nu_T \\ m \end{pmatrix} = \begin{pmatrix} -\ell_0(v_{mx} + v_c/3) \\ \ell_0(v_{mx} - v_c/3) \\ v_y h_0/3 \\ \hat{\rho} V_0 \end{pmatrix}$$

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$$2) \quad H_2(\mathbf{x}_2) = \frac{m}{2\ell_0^2}(\nu_L^2 + \nu_R^2 + \nu_L\nu_R) + \frac{3m}{2h_0^2}\nu_T^2 + \frac{V_0 P_0}{\gamma - 1} \left[\left(\frac{m}{m_0} \right)^\gamma - \gamma \frac{m}{m_0} \right]$$

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$$3) \mathbf{e}_2 = \begin{pmatrix} \frac{m}{\ell_0^2}(\nu_L + \nu_R/2) \\ \frac{m}{\ell_0^2}(\nu_L/2 + \nu_R) \\ 3m\nu_T/h_0^2 \\ \frac{1}{2\ell_0^2}(\nu_L^2 + \nu_R^2 + \nu_L\nu_R) + \frac{3}{2h_0^2}\nu_T^2 + \frac{P_0\gamma}{\rho_0(\gamma-1)} \left((m/m_0)^{\gamma-1} - 1 \right) \end{pmatrix} = \begin{pmatrix} \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \\ \langle \psi \rangle_\Omega \end{pmatrix}$$

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$$1) \mathbf{x}_2 = \mathbb{M}\mathbf{x}_1 = \begin{bmatrix} -\ell_0 & -\ell_0/3 & 0 & 0 \\ \ell_0 & -\ell_0/3 & 0 & 0 \\ 0 & 0 & h_0/3 & 0 \\ 0 & 0 & 0 & V_0 \end{bmatrix} \begin{pmatrix} v_{mx} \\ v_c \\ v_y \\ \hat{\rho} \end{pmatrix} =: \begin{pmatrix} \nu_L \\ \nu_R \\ \nu_T \\ m \end{pmatrix} = \begin{pmatrix} -\ell_0(v_{mx} + v_c/3) \\ \ell_0(v_{mx} - v_c/3) \\ v_y h_0/3 \\ \hat{\rho} V_0 \end{pmatrix}$$

$$2) H_2(\mathbf{x}_2) = \frac{m}{2\ell_0^2}(\nu_L^2 + \nu_R^2 + \nu_L\nu_R) + \frac{3m}{2h_0^2}\nu_T^2 + \frac{V_0 P_0}{\gamma - 1} \left[\left(\frac{m}{m_0}\right)^\gamma - \gamma \frac{m}{m_0} \right]$$

$$3) \mathbf{e}_2 = \begin{pmatrix} \frac{m}{\ell_0^2}(\nu_L + \nu_R/2) \\ \frac{m}{\ell_0^2}(\nu_L/2 + \nu_R) \\ 3m\nu_T/h_0^2 \\ \frac{1}{2\ell_0^2}(\nu_L^2 + \nu_R^2 + \nu_L\nu_R) + \frac{3}{2h_0^2}\nu_T^2 + \frac{P_0\gamma}{\rho_0(\gamma-1)} \left((m/m_0)^{\gamma-1} - 1 \right) \end{pmatrix} = \begin{pmatrix} \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \\ \langle \psi \rangle_\Omega \end{pmatrix}$$

Port variables

Model (M1b): a canonical interconnection

A new choice of representation

$$1) \mathbf{x}_2 = \mathbb{M}\mathbf{x}_1 = \begin{bmatrix} -\ell_0 & -\ell_0/3 & 0 & 0 \\ \ell_0 & -\ell_0/3 & 0 & 0 \\ 0 & 0 & h_0/3 & 0 \\ 0 & 0 & 0 & V_0 \end{bmatrix} \begin{pmatrix} v_{mx} \\ v_c \\ v_y \\ \hat{\rho} \end{pmatrix} =: \begin{pmatrix} \nu_L \\ \nu_R \\ \nu_T \\ m \end{pmatrix} = \begin{pmatrix} -\ell_0(v_{mx} + v_c/3) \\ \ell_0(v_{mx} - v_c/3) \\ v_y h_0/3 \\ \hat{\rho} V_0 \end{pmatrix}$$

$$2) H_2(\mathbf{x}_2) = \frac{m}{2\ell_0^2}(\nu_L^2 + \nu_R^2 + \nu_L\nu_R) + \frac{3m}{2h_0^2}\nu_T^2 + \frac{V_0 P_0}{\gamma - 1} \left[\left(\frac{m}{m_0} \right)^\gamma - \gamma \frac{m}{m_0} \right]$$

$$3) \mathbf{e}_2 = \begin{pmatrix} \frac{m}{\ell_0^2}(\nu_L + \nu_R/2) \\ \frac{m}{\ell_0^2}(\nu_L/2 + \nu_R) \\ 3m\nu_T/h_0^2 \\ \frac{1}{2\ell_0^2}(\nu_L^2 + \nu_R^2 + \nu_L\nu_R) + \frac{3}{2h_0^2}\nu_T^2 + \frac{P_0\gamma}{\rho_0(\gamma-1)} \left((m/m_0)^{\gamma-1} - 1 \right) \end{pmatrix} = \begin{pmatrix} \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \\ \langle \psi \rangle_\Omega \end{pmatrix}$$

Port variables

4)

Model (M1b): a canonical interconnection

A new choice of representation

$$1) \mathbf{x}_2 = \mathbf{M}\mathbf{x}_1 = \begin{bmatrix} -\ell_0 & -\ell_0/3 & 0 & 0 \\ \ell_0 & -\ell_0/3 & 0 & 0 \\ 0 & 0 & h_0/3 & 0 \\ 0 & 0 & 0 & V_0 \end{bmatrix} \begin{pmatrix} v_{mx} \\ v_c \\ v_y \\ \hat{\rho} \end{pmatrix} =: \begin{pmatrix} \nu_L \\ \nu_R \\ \nu_T \\ m \end{pmatrix} = \begin{pmatrix} -\ell_0(v_{mx} + v_c/3) \\ \ell_0(v_{mx} - v_c/3) \\ v_y h_0/3 \\ \hat{\rho} V_0 \end{pmatrix}$$

$$2) H_2(\mathbf{x}_2) = \frac{m}{2\ell_0^2}(\nu_L^2 + \nu_R^2 + \nu_L\nu_R) + \frac{3m}{2h_0^2}\nu_T^2 + \frac{V_0 P_0}{\gamma - 1} \left[\left(\frac{m}{m_0}\right)^\gamma - \gamma \frac{m}{m_0} \right]$$

$$3) \mathbf{e}_2 = \begin{pmatrix} \frac{m}{\ell_0^2}(\nu_L + \nu_R/2) \\ \frac{m}{\ell_0^2}(\nu_L/2 + \nu_R) \\ 3m\nu_T/h_0^2 \\ \frac{1}{2\ell_0^2}(\nu_L^2 + \nu_R^2 + \nu_L\nu_R) + \frac{3}{2h_0^2}\nu_T^2 + \frac{P_0\gamma}{\rho_0(\gamma-1)} \left((m/m_0)^{\gamma-1} - 1 \right) \end{pmatrix} = \begin{pmatrix} \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \\ \langle \psi \rangle_\Omega \end{pmatrix}$$

Port variables

$$4) \begin{pmatrix} \frac{d\nu_L}{dt} \\ \frac{d\nu_R}{dt} \\ \frac{d\nu_T}{dt} \\ \frac{dm}{dt} \\ \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \end{pmatrix} = \begin{array}{c|c|c} & \begin{array}{c} 1 \\ 1 \\ 1 \end{array} & \begin{array}{c} -1 \\ \\ -1 \end{array} \\ \hline \begin{array}{c} -1 \\ -1 \\ -1 \end{array} & \begin{array}{c} \\ \\ \\ \end{array} & \begin{array}{c} \\ \\ \\ \end{array} \\ \hline \begin{array}{c} 1 \\ \\ \\ \end{array} & \begin{array}{c} \\ \\ \\ \end{array} & \begin{array}{c} \\ \\ \\ \end{array} \\ \hline & \begin{array}{c} \\ \\ 1 \end{array} & \begin{array}{c} \\ \\ \\ \end{array} \end{array} \begin{pmatrix} e_{\nu_L} \\ e_{\nu_R} \\ e_{\nu_T} \\ e_m \\ \langle \hat{\psi} \rangle_{S_L} \\ \langle \hat{\psi} \rangle_{S_R} \\ \langle \hat{\psi} \rangle_{S_T} \end{pmatrix}$$

S_2

Model (M1b): a canonical interconnection

A new choice of representation

$$1) \mathbf{x}_2 = \mathbf{M}\mathbf{x}_1 = \begin{bmatrix} -\ell_0 & -\ell_0/3 & 0 & 0 \\ \ell_0 & -\ell_0/3 & 0 & 0 \\ 0 & 0 & h_0/3 & 0 \\ 0 & 0 & 0 & V_0 \end{bmatrix} \begin{pmatrix} v_{mx} \\ v_c \\ v_y \\ \hat{\rho} \end{pmatrix} =: \begin{pmatrix} \nu_L \\ \nu_R \\ \nu_T \\ m \end{pmatrix} = \begin{pmatrix} -\ell_0(v_{mx} + v_c/3) \\ \ell_0(v_{mx} - v_c/3) \\ v_y h_0/3 \\ \hat{\rho} V_0 \end{pmatrix}$$

$$2) H_2(\mathbf{x}_2) = \frac{m}{2\ell_0^2}(\nu_L^2 + \nu_R^2 + \nu_L\nu_R) + \frac{3m}{2h_0^2}\nu_T^2 + \frac{V_0 P_0}{\gamma - 1} \left[\left(\frac{m}{m_0} \right)^\gamma - \gamma \frac{m}{m_0} \right]$$

$$3) \mathbf{e}_2 = \begin{pmatrix} \frac{m}{\ell_0^2}(\nu_L + \nu_R/2) \\ \frac{m}{\ell_0^2}(\nu_L/2 + \nu_R) \\ 3m\nu_T/h_0^2 \\ \frac{1}{2\ell_0^2}(\nu_L^2 + \nu_R^2 + \nu_L\nu_R) + \frac{3}{2h_0^2}\nu_T^2 + \frac{P_0\gamma}{\rho_0(\gamma-1)} \left(\left(\frac{m}{m_0} \right)^{\gamma-1} - 1 \right) \end{pmatrix} = \begin{pmatrix} \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \\ \langle \psi \rangle_\Omega \end{pmatrix}$$

Port
variables

$$4) \begin{pmatrix} \frac{d\nu_L}{dt} \\ \frac{d\nu_R}{dt} \\ \frac{d\nu_T}{dt} \\ \frac{dm}{dt} \\ \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \end{pmatrix} = \begin{array}{c|ccc|ccc} & & & 1 & -1 & & & \\ & & & 1 & & -1 & & \\ & & & 1 & & & -1 & \\ \hline & -1 & -1 & -1 & & & & \\ \hline & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ \hline \end{array} \begin{pmatrix} e_{\nu_L} \\ e_{\nu_R} \\ e_{\nu_T} \\ e_m \\ \langle \hat{\psi} \rangle_{S_L} \\ \langle \hat{\psi} \rangle_{S_R} \\ \langle \hat{\psi} \rangle_{S_T} \end{pmatrix}$$

S_2

Model (M1b): a canonical interconnection

A new choice of representation

→ Physical interpretation: equivalent electrical circuits

$$1) \mathbf{x}_2 = \mathbf{M}\mathbf{x}_1 = \begin{bmatrix} -\ell_0 & -\ell_0/3 & 0 & 0 \\ \ell_0 & -\ell_0/3 & 0 & 0 \\ 0 & 0 & h_0/3 & 0 \\ 0 & 0 & 0 & V_0 \end{bmatrix} \begin{pmatrix} v_{mx} \\ v_c \\ v_y \\ \hat{\rho} \end{pmatrix} =: \begin{pmatrix} \nu_L \\ \nu_R \\ \nu_T \\ m \end{pmatrix} = \begin{pmatrix} -\ell_0(v_{mx} + v_c/3) \\ \ell_0(v_{mx} - v_c/3) \\ v_y h_0/3 \\ \hat{\rho} V_0 \end{pmatrix}$$

$$2) H_2(\mathbf{x}_2) = \frac{m}{2\ell_0^2}(\nu_L^2 + \nu_R^2 + \nu_L\nu_R) + \frac{3m}{2h_0^2}\nu_T^2 + \frac{V_0 P_0}{\gamma - 1} \left[\left(\frac{m}{m_0}\right)^\gamma - \gamma \frac{m}{m_0} \right]$$

$$3) \mathbf{e}_2 = \begin{pmatrix} \frac{m}{\ell_0^2}(\nu_L + \nu_R/2) \\ \frac{m}{\ell_0^2}(\nu_L/2 + \nu_R) \\ 3m\nu_T/h_0^2 \\ \frac{1}{2\ell_0^2}(\nu_L^2 + \nu_R^2 + \nu_L\nu_R) + \frac{3}{2h_0^2}\nu_T^2 + \frac{P_0\gamma}{\rho_0(\gamma-1)} \left((m/m_0)^{\gamma-1} - 1 \right) \end{pmatrix} = \begin{pmatrix} \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \\ \langle \psi \rangle_\Omega \end{pmatrix}$$

Port variables

$$4) \begin{pmatrix} \frac{d\nu_L}{dt} \\ \frac{d\nu_R}{dt} \\ \frac{d\nu_T}{dt} \\ \frac{dm}{dt} \\ \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \end{pmatrix} = \begin{array}{c|c|c} \begin{array}{ccc} 1 & -1 & \\ 1 & & -1 \\ 1 & & -1 \end{array} & & \begin{array}{c} e_{\nu_L} \\ e_{\nu_R} \\ e_{\nu_T} \end{array} \\ \hline \begin{array}{ccc} -1 & -1 & -1 \end{array} & & e_m \\ \hline \begin{array}{ccc} 1 & & \\ & 1 & \\ & & 1 \end{array} & & \begin{array}{c} \langle \hat{\psi} \rangle_{S_L} \\ \langle \hat{\psi} \rangle_{S_R} \\ \langle \hat{\psi} \rangle_{S_T} \end{array} \end{array}$$

S_2

Model (M1b): a canonical interconnection

A new choice of representation

$$1) \mathbf{x}_2 = \mathbf{M}\mathbf{x}_1 = \begin{bmatrix} -\ell_0 & -\ell_0/3 & 0 & 0 \\ \ell_0 & -\ell_0/3 & 0 & 0 \\ 0 & 0 & h_0/3 & 0 \\ 0 & 0 & 0 & V_0 \end{bmatrix} \begin{pmatrix} v_{mx} \\ v_c \\ v_y \\ \hat{\rho} \end{pmatrix} =: \begin{pmatrix} \nu_L \\ \nu_R \\ \nu_T \\ m \end{pmatrix} = \begin{pmatrix} -\ell_0(v_{mx} + v_c/3) \\ \ell_0(v_{mx} - v_c/3) \\ v_y h_0/3 \\ \hat{\rho} V_0 \end{pmatrix}$$

$$2) H_2(\mathbf{x}_2) = \frac{m}{2\ell_0^2}(\nu_L^2 + \nu_R^2 + \nu_L\nu_R) + \frac{3m}{2h_0^2}\nu_T^2 + \frac{V_0 P_0}{\gamma - 1} \left[\left(\frac{m}{m_0}\right)^\gamma - \gamma \frac{m}{m_0} \right]$$

$$3) \mathbf{e}_2 = \begin{pmatrix} \frac{m}{\ell_0^2}(\nu_L + \nu_R/2) \\ \frac{m}{\ell_0^2}(\nu_L/2 + \nu_R) \\ 3m\nu_T/h_0^2 \\ \frac{1}{2\ell_0^2}(\nu_L^2 + \nu_R^2 + \nu_L\nu_R) + \frac{3}{2h_0^2}\nu_T^2 + \frac{P_0\gamma}{\rho_0(\gamma-1)} \left((m/m_0)^{\gamma-1} - 1 \right) \end{pmatrix} = \begin{pmatrix} \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \\ \langle \psi \rangle_\Omega \end{pmatrix}$$

Port variables

$$4) \begin{pmatrix} \frac{d\nu_L}{dt} \\ \frac{d\nu_R}{dt} \\ \frac{d\nu_T}{dt} \\ \frac{dm}{dt} \\ \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \end{pmatrix} = \begin{matrix} \begin{bmatrix} & & & 1 & & & -1 \\ & & & 1 & & & -1 \\ & & & 1 & & & -1 \\ -1 & -1 & -1 & & & & \\ 1 & & & & & & \\ & 1 & & & & & \\ & & 1 & & & & \\ & & & 1 & & & \end{bmatrix} & \begin{matrix} e_{\nu_L} \\ e_{\nu_R} \\ e_{\nu_T} \\ e_m \\ \langle \hat{\psi} \rangle_{S_L} \\ \langle \hat{\psi} \rangle_{S_R} \\ \langle \hat{\psi} \rangle_{S_T} \end{matrix} \end{matrix}$$

S_2

→ Physical interpretation: equivalent electrical circuits

Domains	Electronics	Acoustics	Fluid mechanics
Quantities (effort/flux)	voltage U	pressure p	specific total enthalpy ψ
	current i	flow rate u	mass rate q
Component	inductor	acoustic mass	fluid mass
	capacitor	compliance	compressibility

Model (M1b): a canonical interconnection

A new choice of representation

$$1) \mathbf{x}_2 = \mathbf{M}\mathbf{x}_1 = \begin{bmatrix} -\ell_0 & -\ell_0/3 & 0 & 0 \\ \ell_0 & -\ell_0/3 & 0 & 0 \\ 0 & 0 & h_0/3 & 0 \\ 0 & 0 & 0 & V_0 \end{bmatrix} \begin{pmatrix} v_{mx} \\ v_c \\ v_y \\ \hat{\rho} \end{pmatrix} =: \begin{pmatrix} \nu_L \\ \nu_R \\ \nu_T \\ m \end{pmatrix} = \begin{pmatrix} -\ell_0(v_{mx} + v_c/3) \\ \ell_0(v_{mx} - v_c/3) \\ v_y h_0/3 \\ \hat{\rho} V_0 \end{pmatrix}$$

$$2) H_2(\mathbf{x}_2) = \frac{m}{2\ell_0^2}(\nu_L^2 + \nu_R^2 + \nu_L\nu_R) + \frac{3m}{2h_0^2}\nu_T^2 + \frac{V_0 P_0}{\gamma - 1} \left[\left(\frac{m}{m_0}\right)^\gamma - \gamma \frac{m}{m_0} \right]$$

$$3) \mathbf{e}_2 = \begin{pmatrix} \frac{m}{\ell_0^2}(\nu_L + \nu_R/2) \\ \frac{m}{\ell_0^2}(\nu_L/2 + \nu_R) \\ 3m\nu_T/h_0^2 \\ \frac{1}{2\ell_0^2}(\nu_L^2 + \nu_R^2 + \nu_L\nu_R) + \frac{3}{2h_0^2}\nu_T^2 + \frac{P_0\gamma}{\rho_0(\gamma-1)} \left((m/m_0)^{\gamma-1} - 1 \right) \end{pmatrix} = \begin{pmatrix} \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \\ \langle \psi \rangle_\Omega \end{pmatrix}$$

Port variables

$$4) \begin{pmatrix} \frac{d\nu_L}{dt} \\ \frac{d\nu_R}{dt} \\ \frac{d\nu_T}{dt} \\ \frac{dm}{dt} \\ \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \end{pmatrix} = \begin{matrix} \begin{bmatrix} & & 1 & -1 & & & \\ & & 1 & & -1 & & \\ & & 1 & & & -1 & \\ -1 & -1 & -1 & & & & \\ 1 & & & & & & \\ & 1 & & & & & \\ & & 1 & & & & \\ & & & 1 & & & \end{bmatrix} & \begin{matrix} e_{\nu_L} \\ e_{\nu_R} \\ e_{\nu_T} \\ e_m \\ \langle \hat{\psi} \rangle_{S_L} \\ \langle \hat{\psi} \rangle_{S_R} \\ \langle \hat{\psi} \rangle_{S_T} \end{matrix} \end{matrix}$$

S_2

→ Physical interpretation: equivalent electrical circuits

Domains	Electronics	Acoustics	Fluid mechanics
Quantities (effort/flux)	voltage U	pressure p	specific total enthalpy ψ
	current i	flow rate u	mass rate q
Component	inductor	acoustic mass	fluid mass
	capacitor	compliance	compressibility

Model (M1b): a canonical interconnection

A new choice of representation

$$1) \mathbf{x}_2 = \mathbf{M}\mathbf{x}_1 = \begin{bmatrix} -l_0 & -l_0/3 & 0 & 0 \\ l_0 & -l_0/3 & 0 & 0 \\ 0 & 0 & h_0/3 & 0 \\ 0 & 0 & 0 & V_0 \end{bmatrix} \begin{pmatrix} v_{mx} \\ v_c \\ v_y \\ \hat{\rho} \end{pmatrix} =: \begin{pmatrix} \nu_L \\ \nu_R \\ \nu_T \\ m \end{pmatrix} = \begin{pmatrix} -l_0(v_{mx} + v_c/3) \\ l_0(v_{mx} - v_c/3) \\ v_y h_0/3 \\ \hat{\rho} V_0 \end{pmatrix}$$

$$2) H_2(\mathbf{x}_2) = \frac{m}{2\ell_0^2}(\nu_L^2 + \nu_R^2 + \nu_L\nu_R) + \frac{3m}{2h_0^2}\nu_T^2 + \frac{V_0 P_0}{\gamma - 1} \left[\left(\frac{m}{m_0}\right)^\gamma - \gamma \frac{m}{m_0} \right]$$

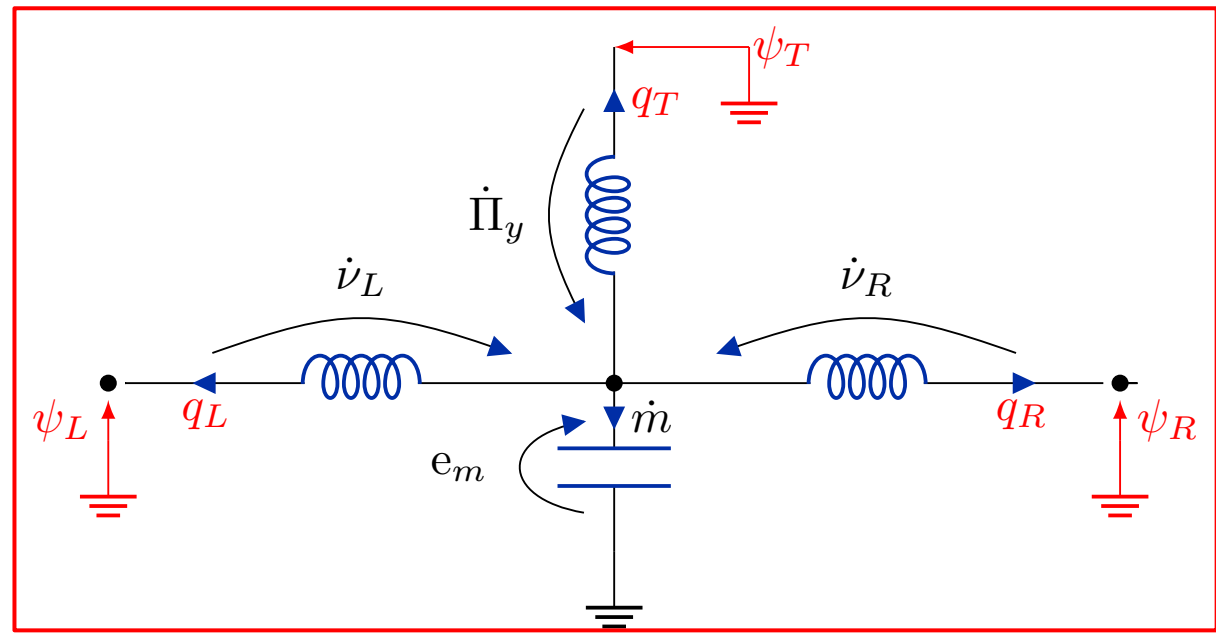
$$3) \mathbf{e}_2 = \begin{pmatrix} \frac{m}{\ell_0^2}(\nu_L + \nu_R/2) \\ \frac{m}{\ell_0^2}(\nu_L/2 + \nu_R) \\ 3m\nu_T/h_0^2 \\ \frac{1}{2\ell_0^2}(\nu_L^2 + \nu_R^2 + \nu_L\nu_R) + \frac{3}{2h_0^2}\nu_T^2 + \frac{P_0\gamma}{\rho_0(\gamma-1)} \left(\left(\frac{m}{m_0}\right)^{\gamma-1} - 1 \right) \end{pmatrix} = \begin{pmatrix} \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \\ \langle \psi \rangle_\Omega \end{pmatrix}$$

Port variables

$$4) \begin{pmatrix} \frac{d\nu_L}{dt} \\ \frac{d\nu_R}{dt} \\ \frac{d\nu_T}{dt} \\ \frac{dm}{dt} \\ \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \end{pmatrix} = \begin{bmatrix} & & & 1 & -1 & & \\ & & & 1 & & -1 & \\ & & & 1 & & & -1 \\ -1 & -1 & -1 & & & & \\ 1 & & & & & & \\ & 1 & & & & & \\ & & 1 & & & & \\ & & & 1 & & & \end{bmatrix} \begin{pmatrix} e_{\nu_L} \\ e_{\nu_R} \\ e_{\nu_T} \\ e_m \\ \langle \hat{\psi} \rangle_{S_L} \\ \langle \hat{\psi} \rangle_{S_R} \\ \langle \hat{\psi} \rangle_{S_T} \end{pmatrix}$$

→ Physical interpretation: equivalent electrical circuits

Domains	Electronics	Acoustics	Fluid mechanics
Quantities (effort/flux)	voltage U	pressure p	specific total enthalpy ψ
	current i	flow rate u	mass rate q
Component	inductor	acoustic mass	fluid mass
	capacitor	compliance	compressibility



Model (M1b): a canonical interconnection

A new choice of representation

$$1) \mathbf{x}_2 = \mathbf{M}\mathbf{x}_1 = \begin{bmatrix} -l_0 & -l_0/3 & 0 & 0 \\ l_0 & -l_0/3 & 0 & 0 \\ 0 & 0 & h_0/3 & 0 \\ 0 & 0 & 0 & V_0 \end{bmatrix} \begin{pmatrix} v_{mx} \\ v_c \\ v_y \\ \hat{\rho} \end{pmatrix} =: \begin{pmatrix} \nu_L \\ \nu_R \\ \nu_T \\ m \end{pmatrix} = \begin{pmatrix} -l_0(v_{mx} + v_c/3) \\ l_0(v_{mx} - v_c/3) \\ v_y h_0/3 \\ \hat{\rho} V_0 \end{pmatrix}$$

$$2) H_2(\mathbf{x}_2) = \frac{m}{2\ell_0^2}(\nu_L^2 + \nu_R^2 + \nu_L\nu_R) + \frac{3m}{2h_0^2}\nu_T^2 + \frac{V_0 P_0}{\gamma - 1} \left[\left(\frac{m}{m_0}\right)^\gamma - \gamma \frac{m}{m_0} \right]$$

$$3) \mathbf{e}_2 = \begin{pmatrix} \frac{m}{\ell_0^2}(\nu_L + \nu_R/2) \\ \frac{m}{\ell_0^2}(\nu_L/2 + \nu_R) \\ 3m\nu_T/h_0^2 \\ \frac{1}{2\ell_0^2}(\nu_L^2 + \nu_R^2 + \nu_L\nu_R) + \frac{3}{2h_0^2}\nu_T^2 + \frac{P_0\gamma}{\rho_0(\gamma-1)} \left(\left(\frac{m}{m_0}\right)^{\gamma-1} - 1 \right) \end{pmatrix} = \begin{pmatrix} \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \\ \langle \psi \rangle_\Omega \end{pmatrix}$$

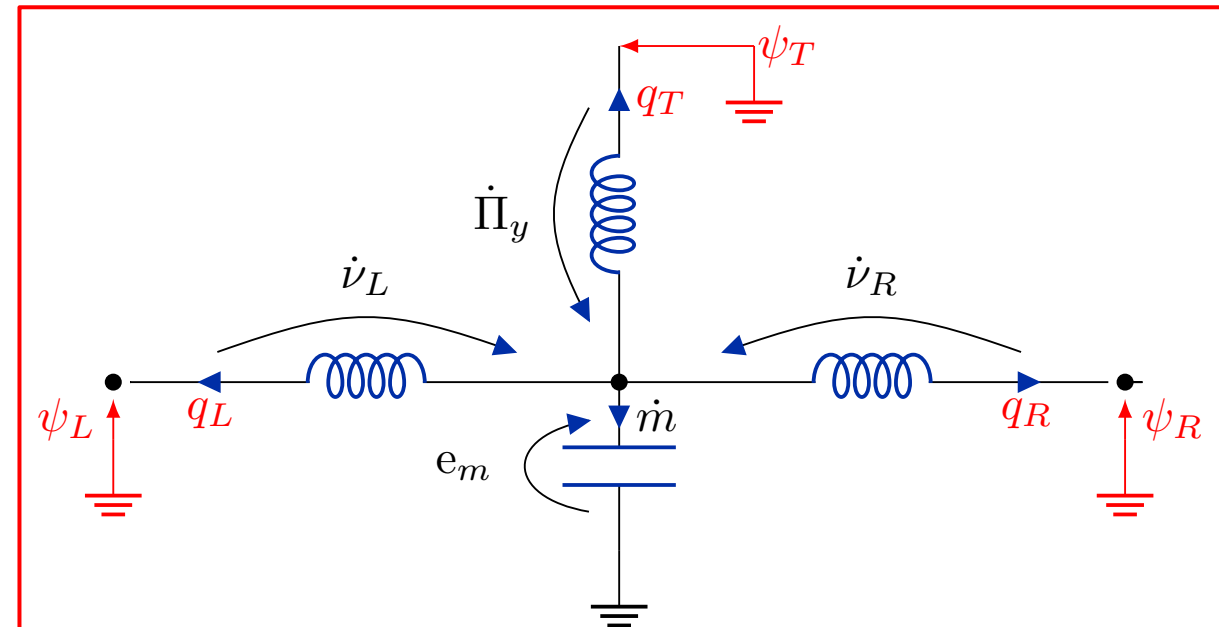
Port variables

$$4) \begin{pmatrix} \frac{d\nu_L}{dt} \\ \frac{d\nu_R}{dt} \\ \frac{d\nu_T}{dt} \\ \frac{dm}{dt} \\ \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \end{pmatrix} = \begin{matrix} \begin{bmatrix} & & 1 & -1 & & & \\ & & 1 & & -1 & & \\ & & 1 & & & -1 & \\ -1 & -1 & -1 & & & & \\ 1 & & & & & & \\ & 1 & & & & & \\ & & 1 & & & & \\ & & & 1 & & & \end{bmatrix} & \begin{matrix} e_{\nu_L} \\ e_{\nu_R} \\ e_{\nu_T} \\ e_m \\ \langle \hat{\psi} \rangle_{S_L} \\ \langle \hat{\psi} \rangle_{S_R} \\ \langle \hat{\psi} \rangle_{S_T} \end{matrix} \end{matrix}$$

S_2

→ Physical interpretation: equivalent electrical circuits

Domains	Electronics	Acoustics	Fluid mechanics
Quantities (effort/flux)	voltage U	pressure p	specific total enthalpy ψ
	current i	flow rate u	mass rate q
Component	inductor	acoustic mass	fluid mass
	capacitor	compliance	compressibility



With a simple **variable change**, we now have:

➤ a fully interpretable model (with a canonical interconnection)

Model (M1c): nonlinear acoustics

Model (M1c): nonlinear acoustics

Goal: rewrite the system as **exact fluctuations** around the atmospheric conditions

Model (M1c): nonlinear acoustics

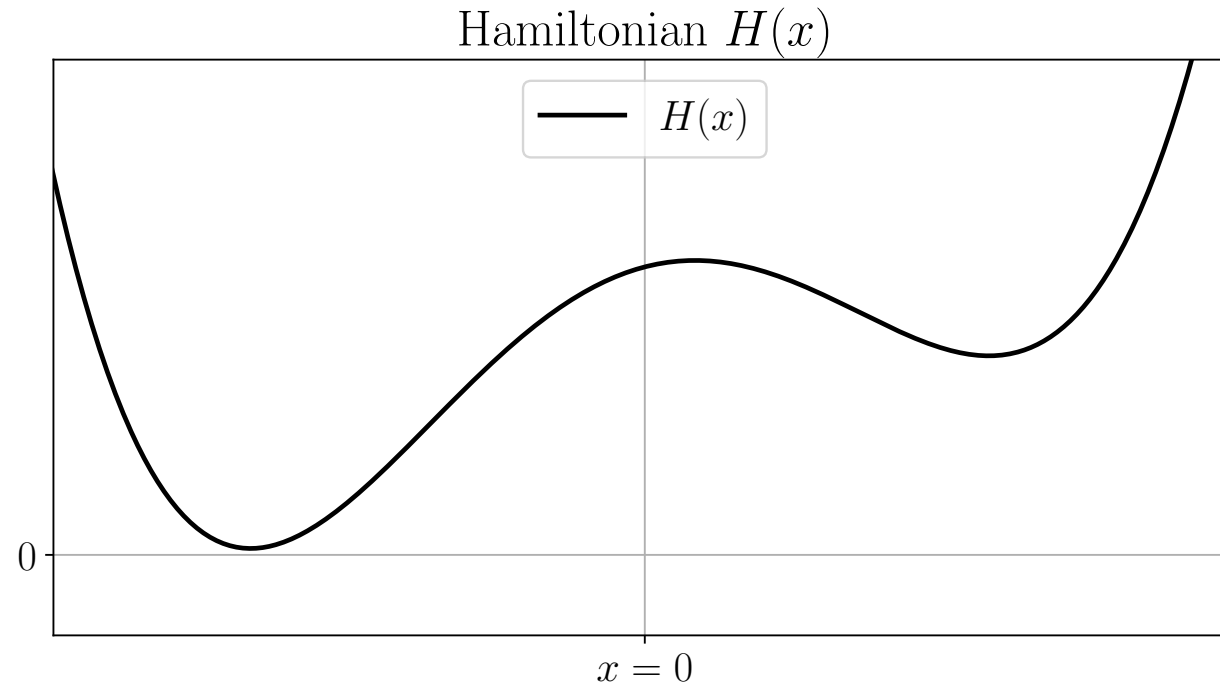
Goal: rewrite the system as **exact fluctuations** around the atmospheric conditions

How? → Shift the pHs (*Monshizadeh et al., 2019*)

Model (M1c): nonlinear acoustics

Goal: rewrite the system as **exact fluctuations** around the atmospheric conditions

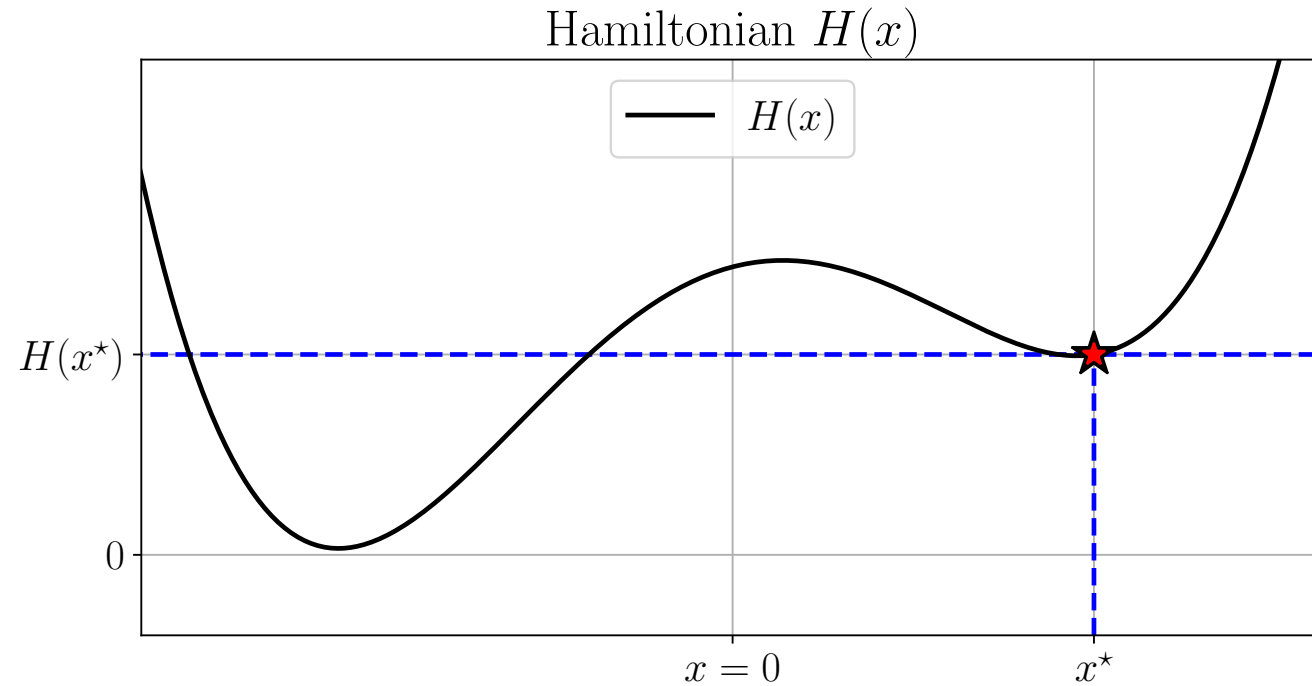
How? → Shift the pHs (*Monshizadeh et al., 2019*)



Model (M1c): nonlinear acoustics

Goal: rewrite the system as **exact fluctuations** around the atmospheric conditions

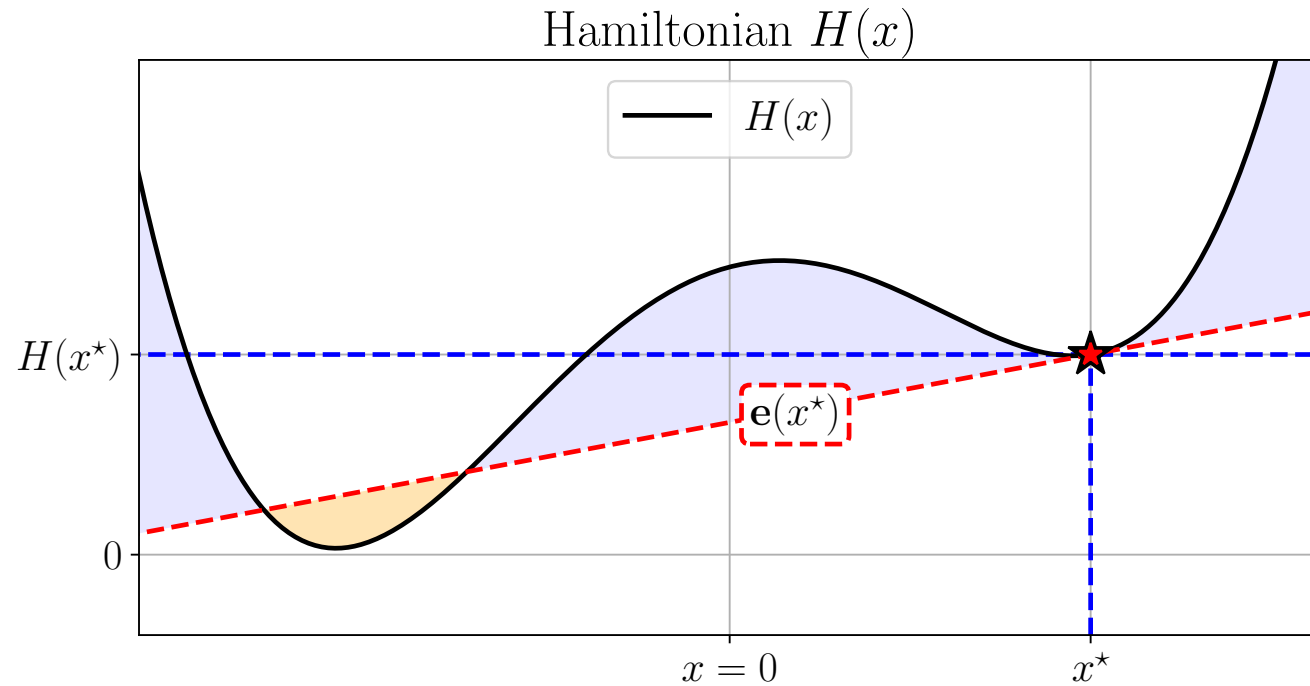
How? → Shift the pHs (*Monshizadeh et al., 2019*)



Model (M1c): nonlinear acoustics

Goal: rewrite the system as **exact fluctuations** around the atmospheric conditions

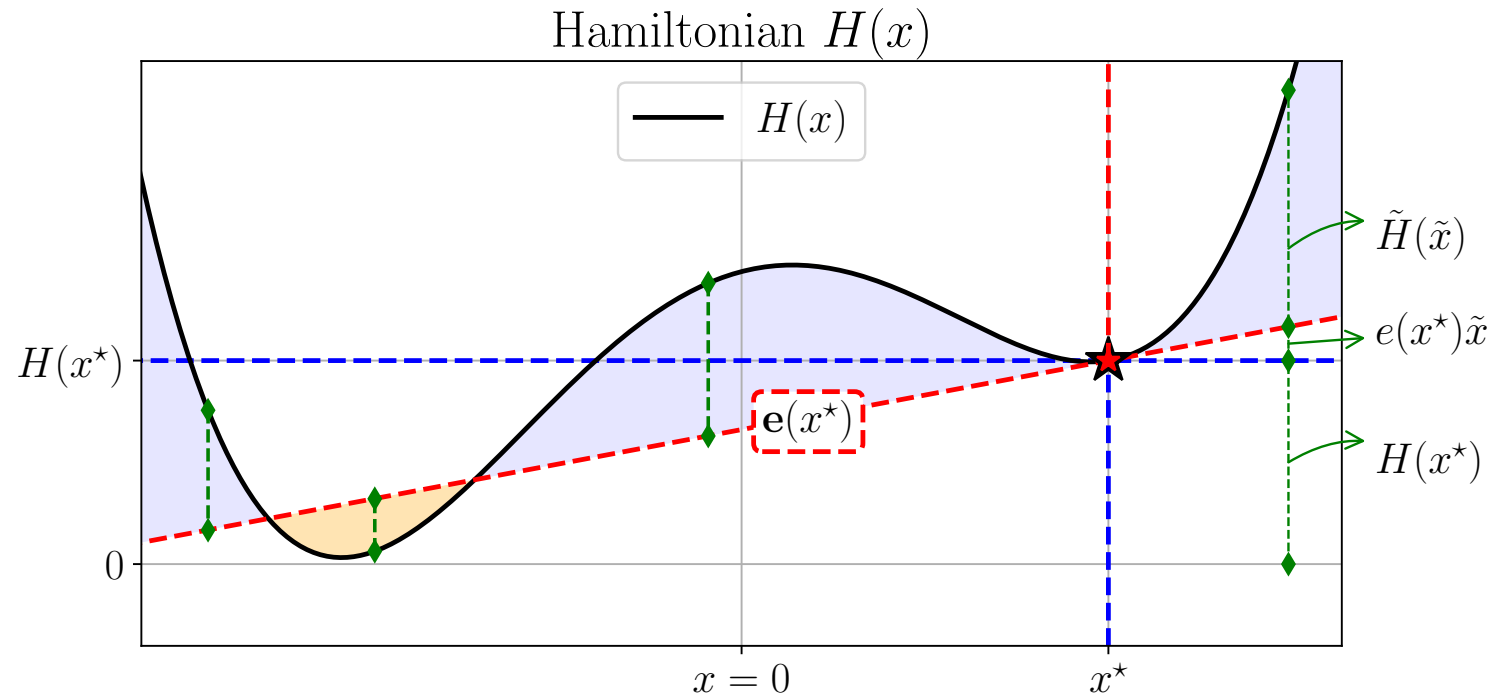
How? → Shift the pHs (*Monshizadeh et al., 2019*)



Model (M1c): nonlinear acoustics

Goal: rewrite the system as **exact fluctuations** around the atmospheric conditions

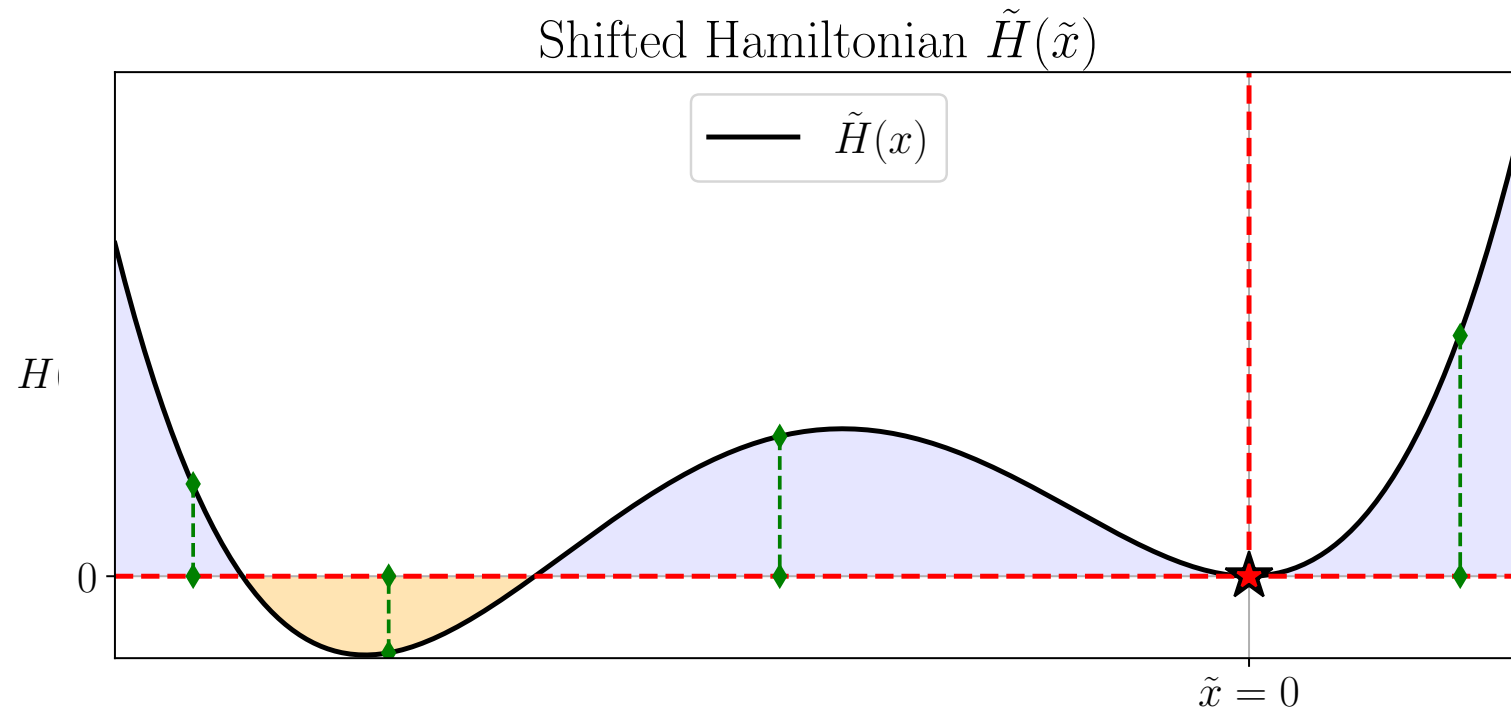
How? → Shift the pHs (*Monshizadeh et al., 2019*)



Model (M1c): nonlinear acoustics

Goal: rewrite the system as **exact fluctuations** around the atmospheric conditions

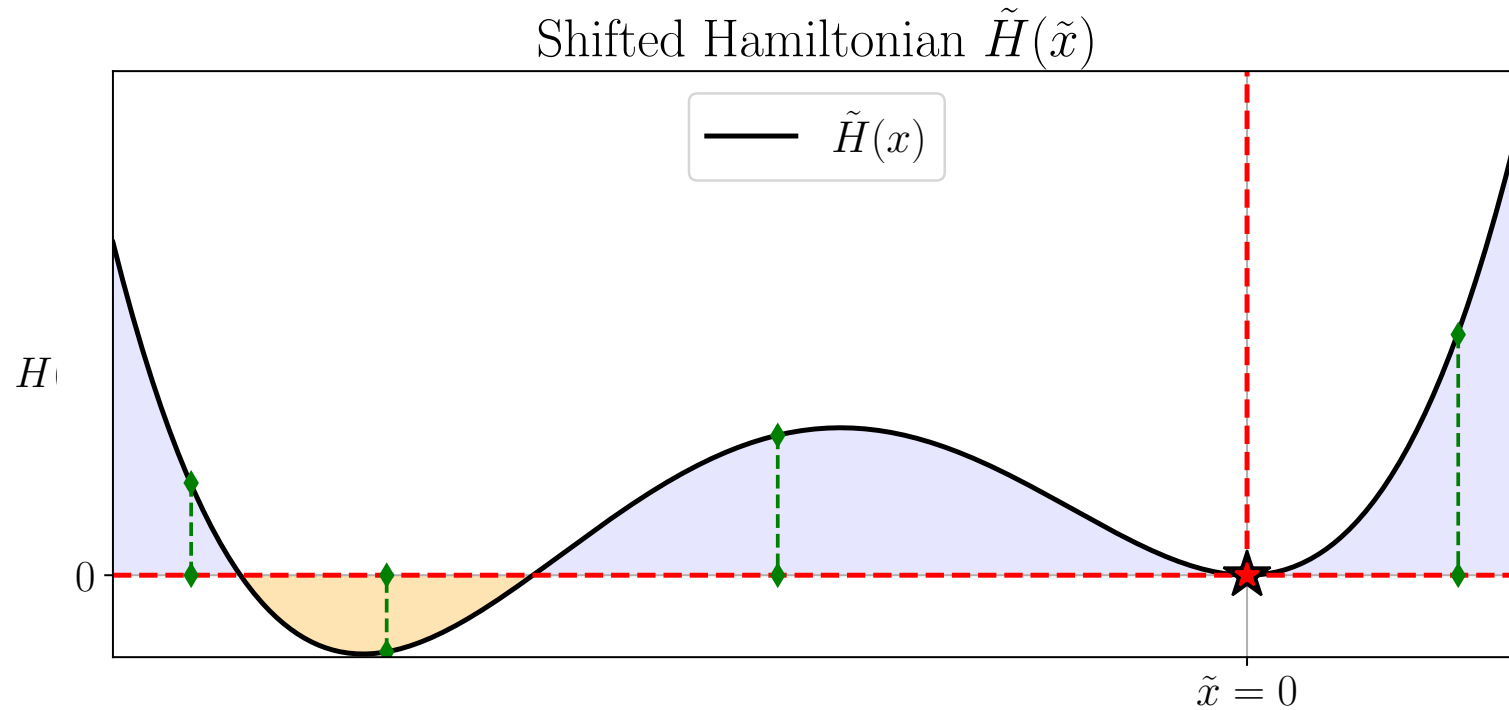
How? → Shift the pHs (*Monshizadeh et al., 2019*)



Model (M1c): nonlinear acoustics

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How? → Shift the pHs (*Monshizadeh et al., 2019*)

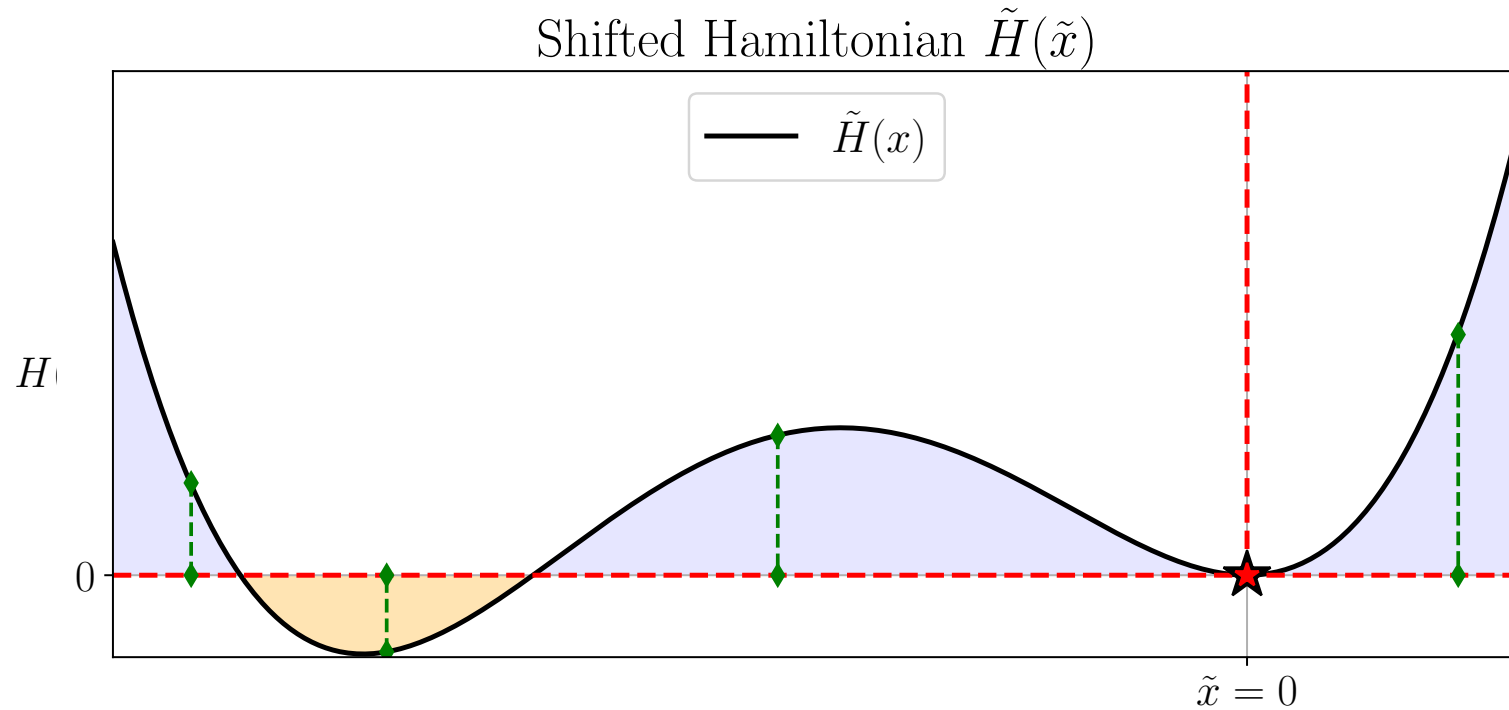


Here

Model (M1c): nonlinear acoustics

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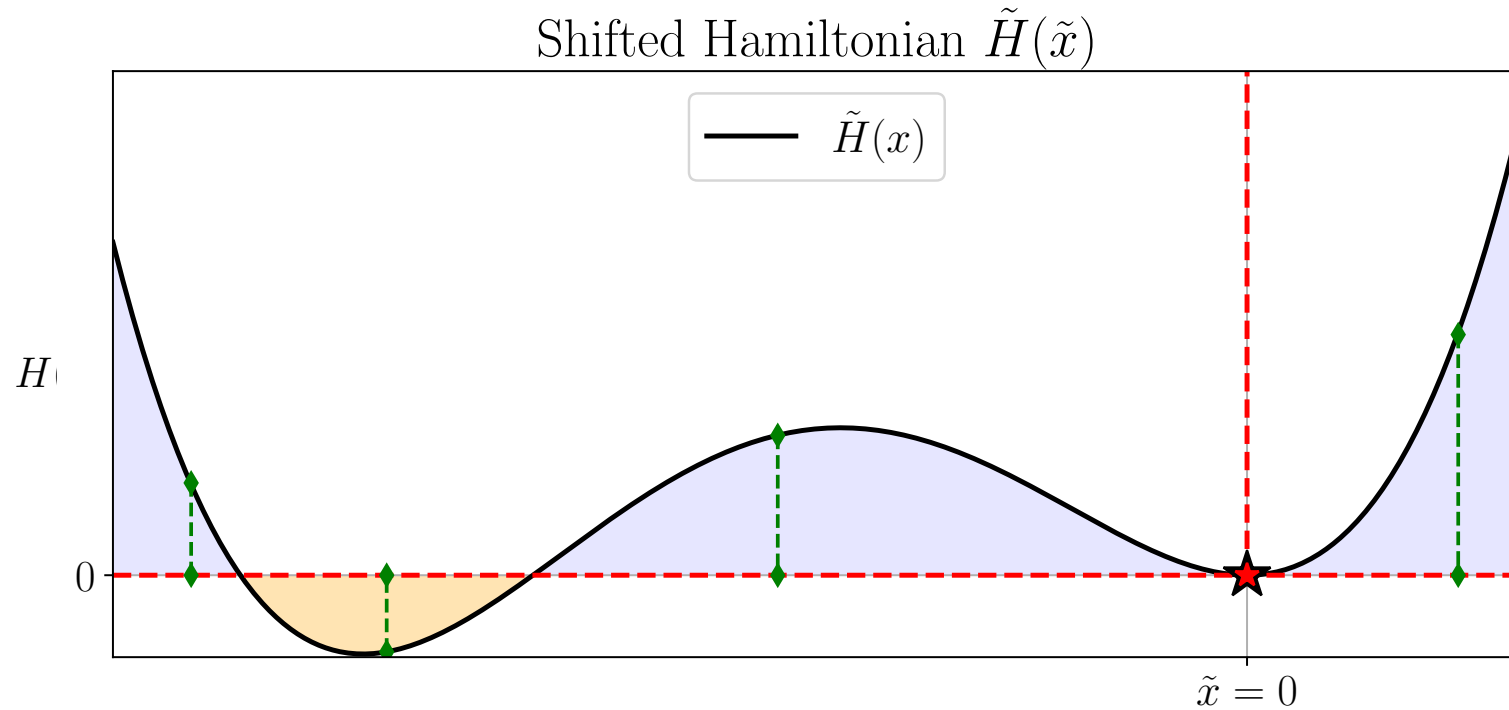
Here

a) Atmospheric pressure

Model (M1c): nonlinear acoustics

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How? → Shift the pHs (*Monshizadeh et al., 2019*)



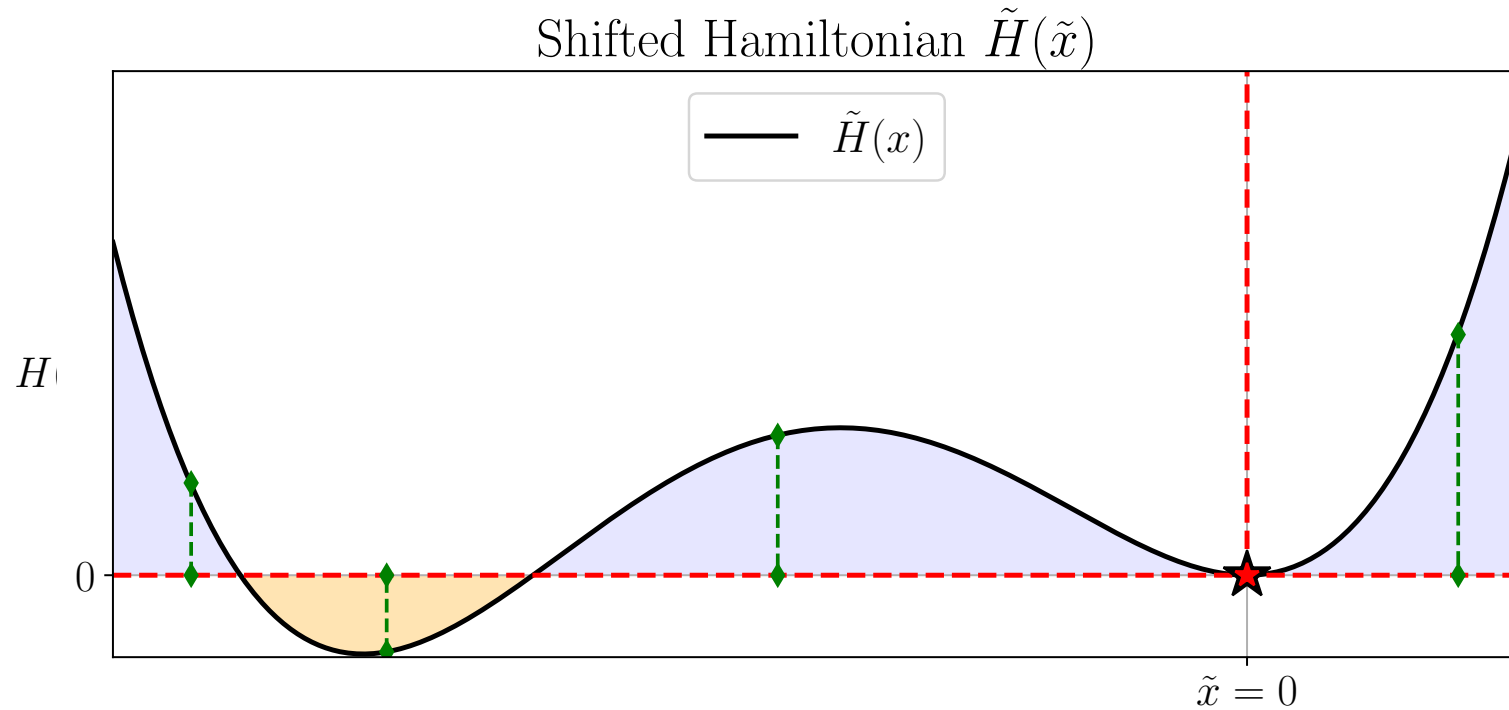
Here

- a) Atmospheric pressure
- b) State at rest $\mathbf{x}_2^* = [0, 0, 0, m_0]^\top$

Model (M1c): nonlinear acoustics

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Here

a) Atmospheric pressure

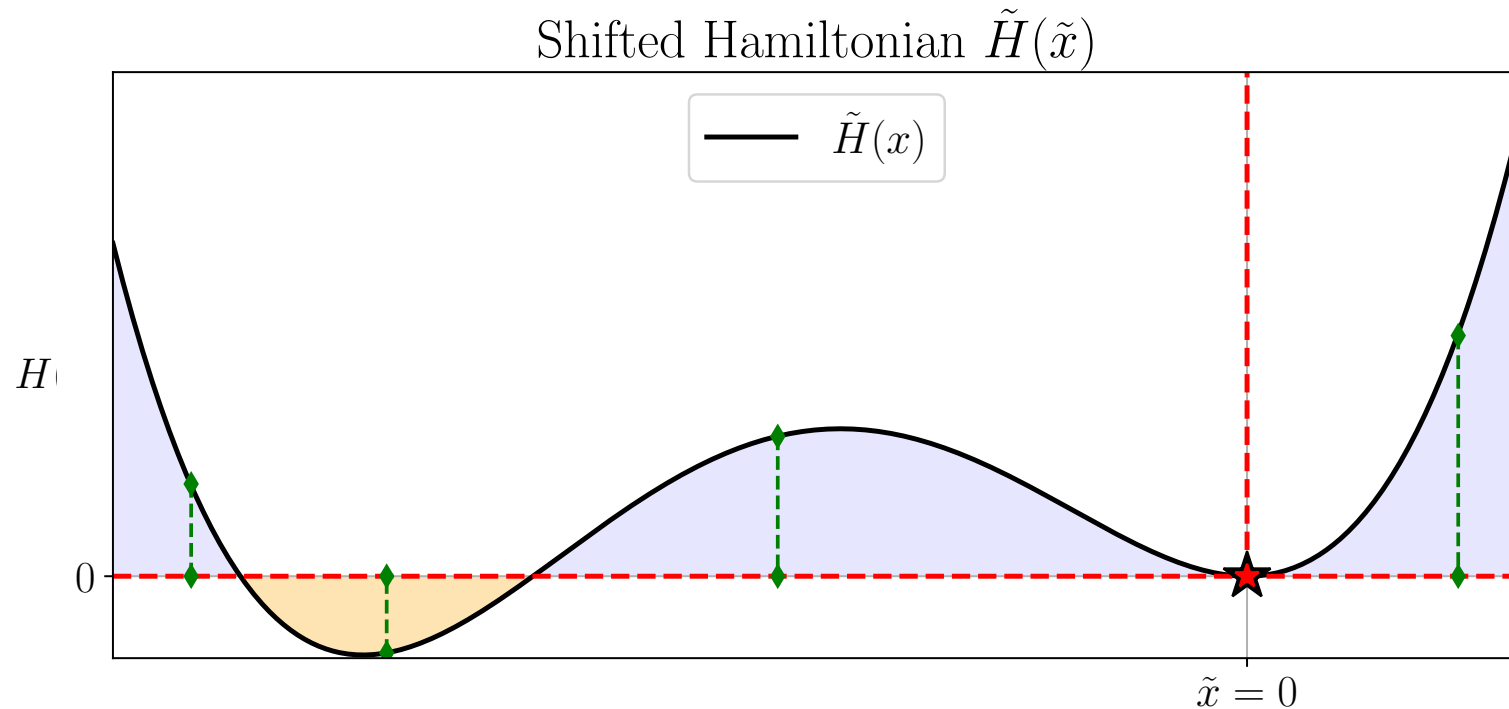
No velocity

b) State at rest $\mathbf{x}_2^* = [0, 0, 0, m_0]^T \rightarrow \rho = \rho_0$

Model (M1c): nonlinear acoustics

Goal: rewrite the system as **exact fluctuations** around the atmospheric conditions

How? → Shift the pHs (Monshizadeh et al., 2019)



Here

No velocity

a) Atmospheric pressure

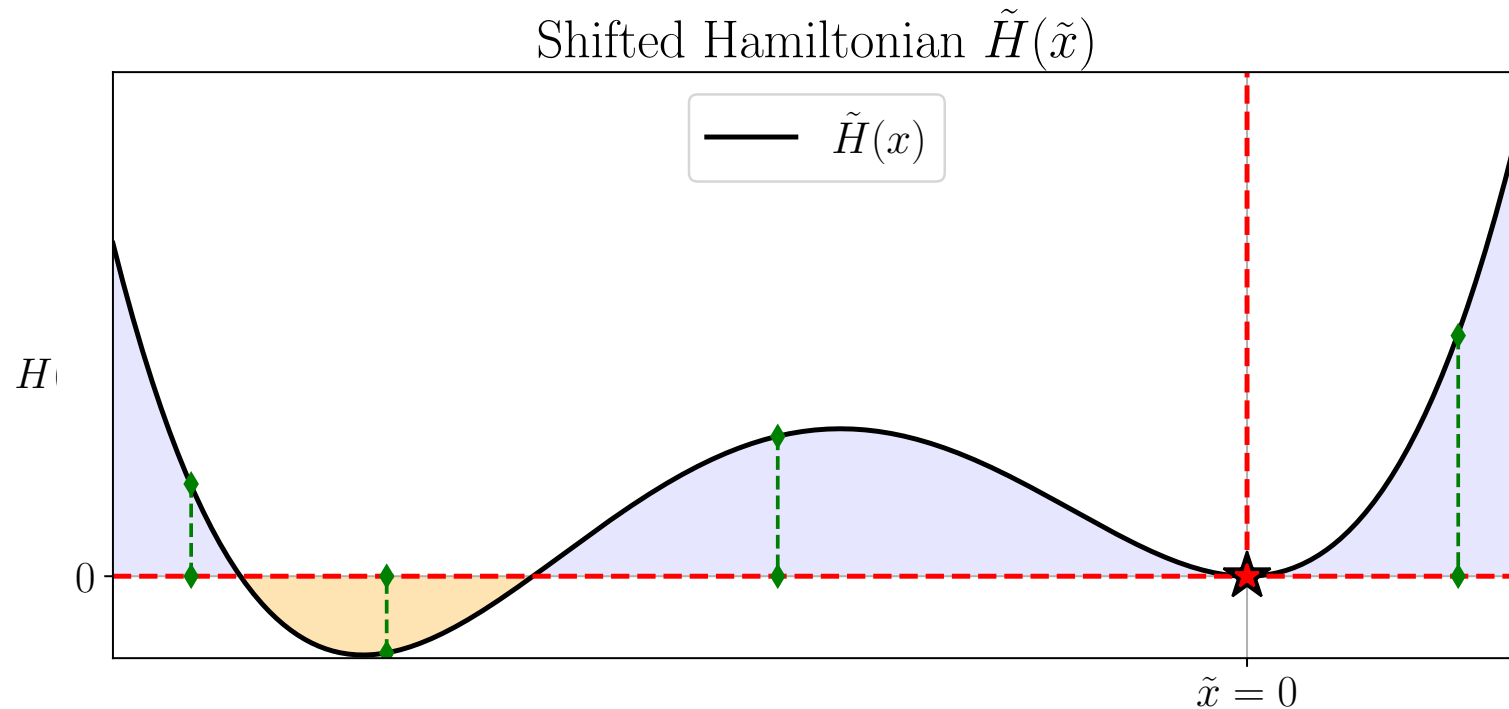
b) State at rest $\mathbf{x}_2^* = [0, 0, 0, m_0]^\top \rightarrow \rho = \rho_0$

c) Fluctuating state: $\tilde{\mathbf{x}} = \mathbf{x}_3 = \mathbf{x}_2 - \mathbf{x}_2^* = [\tilde{v}_L, \tilde{v}_R, \tilde{v}_T, \tilde{m}]^\top$

Model (M1c): nonlinear acoustics

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Here

No velocity

a) Atmospheric pressure

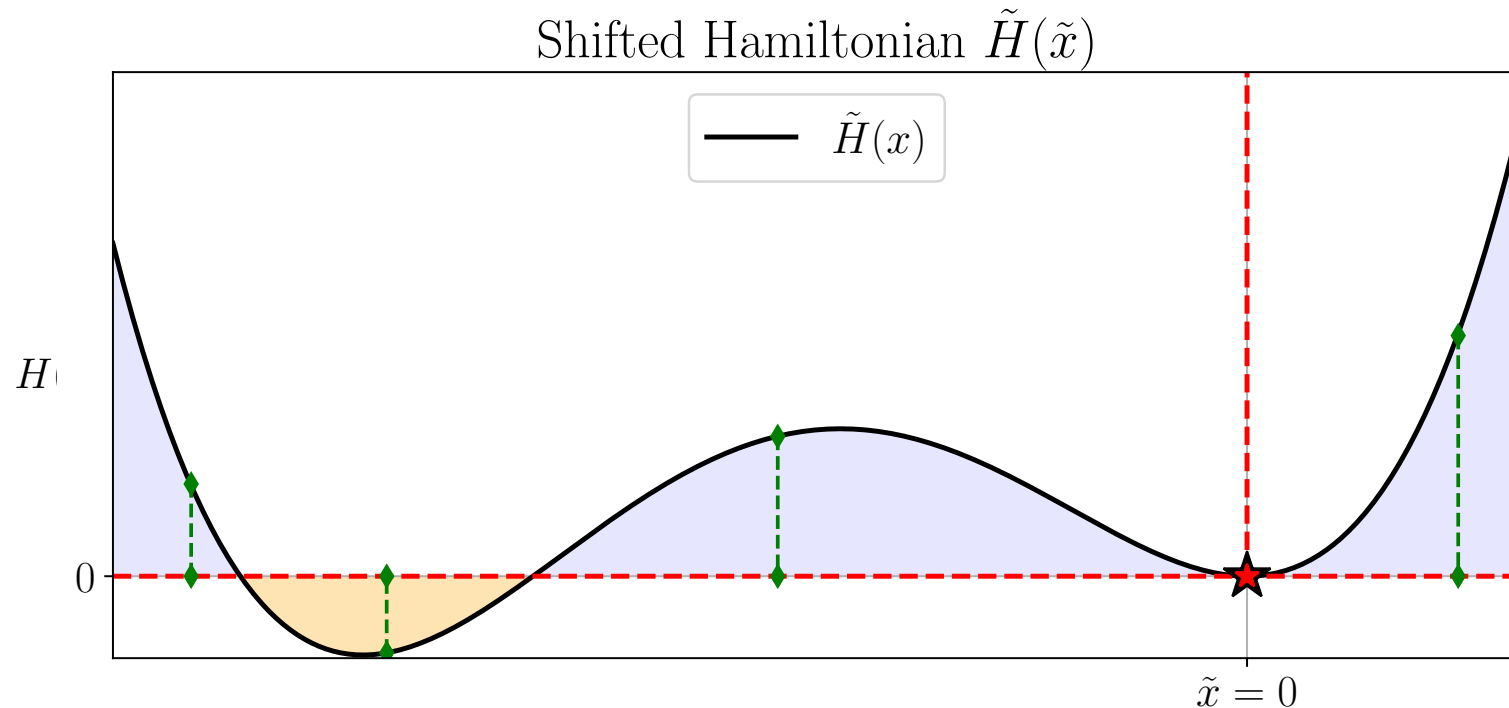
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No velocity

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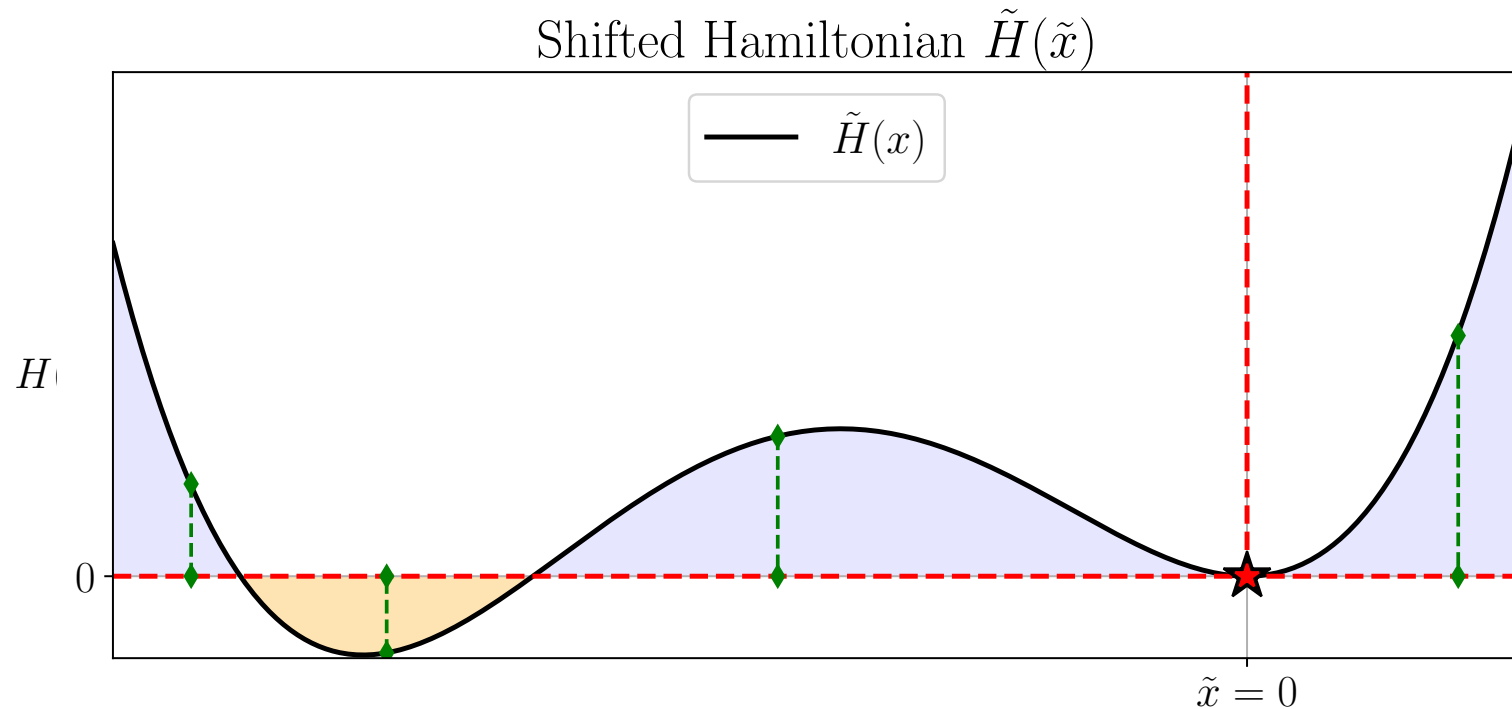
d) Shifted Hamiltonian:

$$H_3(\mathbf{x}_3 = [\tilde{v}_L, \tilde{v}_R, \tilde{v}_T, \tilde{m}]^\top) = \frac{\tilde{m} + m_0}{2\ell_0^2} (\tilde{v}_L^2 + \tilde{v}_R^2 + \tilde{v}_L\tilde{v}_R) + \frac{3(\tilde{m} + m_0)}{2h_0^2} \tilde{v}_T^2 + \frac{P_0 V_0}{\gamma - 1} \left[\left(1 + \frac{\tilde{m}}{m_0}\right)^\gamma - \gamma \frac{\tilde{m}}{m_0} - 1 \right],$$

Model (M1c): nonlinear acoustics

Goal: rewrite the system as **exact fluctuations** around the atmospheric conditions

How? → Shift the pHs (Monshizadeh et al., 2019)



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No velocity

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Model (M1C) is

- A nonlinear acoustical model
- Written as exact fluctuations around the atmospheric conditions

Model (M2): linearised thermodynamical behaviour



Model (M2): linearised thermodynamical behaviour

Approximation



Model (M2): linearised thermodynamical behaviour

Approximation

The fluctuations of mass density are small compared to its state at rest value

Model (M2): linearised thermodynamical behaviour

Approximation

The fluctuations of mass density are small compared to its state at rest value

$$\text{(App 1) : } \frac{\tilde{m}}{m_0} = \frac{\tilde{\rho}}{\rho_0} \ll 1$$

Model (M2): linearised thermodynamical behaviour

Approximation

The fluctuations of mass density are small compared to its state at rest value

$$\text{(App 1) : } \frac{\tilde{m}}{m_0} = \frac{\tilde{\rho}}{\rho_0} \ll 1$$

1)

Model (M2): linearised thermodynamical behaviour

Approximation

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$$\text{(App 1) : } \frac{\tilde{m}}{m_0} = \frac{\tilde{\rho}}{\rho_0} \ll 1$$

$$\mathbf{1) } \mathbf{x}_4 = [\tilde{v}_L, \tilde{v}_R, \tilde{v}_T, \overline{m}]^T$$

Model (M2): linearised thermodynamical behaviour

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Model (M2): linearised thermodynamical behaviour

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$$\text{(App 1) : } \frac{\tilde{m}}{m_0} = \frac{\tilde{\rho}}{\rho_0} \ll 1$$

1) $\mathbf{x}_4 = [\tilde{\nu}_L, \tilde{\nu}_R, \tilde{\nu}_T, \overline{m}]^T \rightarrow$ Small mass fluctuations

Model (M2): linearised thermodynamical behaviour

Approximation

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$$\text{(App 1) : } \frac{\tilde{m}}{m_0} = \frac{\tilde{\rho}}{\rho_0} \ll 1$$

1) $\mathbf{x}_4 = [\underbrace{\tilde{\nu}_L, \tilde{\nu}_R, \tilde{\nu}_T}_{\text{unchanged}}, \tilde{m}]^T \rightarrow$ Small mass fluctuations

Model (M2): linearised thermodynamical behaviour

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2)

Model (M2): linearised thermodynamical behaviour

Approximation

The fluctuations of mass density are small compared to its state at rest value

$$\text{(App 1) : } \quad \frac{\tilde{m}}{m_0} = \frac{\tilde{\rho}}{\rho_0} \ll 1$$

1) $\mathbf{x}_4 = [\tilde{\nu}_L, \tilde{\nu}_R, \tilde{\nu}_T, \tilde{m}]^T$ → Small mass fluctuations
unchanged

2) $E_c(\tilde{m}) = \frac{P_0 V_0}{\gamma - 1} \left[\left(1 + \frac{\tilde{m}}{m_0} \right)^\gamma - \gamma \frac{\tilde{m}}{m_0} - 1 \right]$

Model (M2): linearised thermodynamical behaviour

Approximation

The fluctuations of mass density are small compared to its state at rest value

$$\text{(App 1) : } \frac{\tilde{m}}{m_0} = \frac{\tilde{\rho}}{\rho_0} \ll 1$$

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unchanged

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Model (M2): linearised thermodynamical behaviour

Approximation

The fluctuations of mass density are small compared to its state at rest value

$$\text{(App 1) : } \boxed{\frac{\tilde{m}}{m_0} = \frac{\tilde{\rho}}{\rho_0} \ll 1}$$

1) $\mathbf{x}_4 = [\underbrace{\tilde{v}_L, \tilde{v}_R, \tilde{v}_T}_{\text{unchanged}}, \tilde{m}]^T \rightarrow$ Small mass fluctuations

2) $E_c(\tilde{m}) = \frac{P_0 V_0}{\gamma - 1} \left[\left(1 + \frac{\tilde{m}}{m_0} \right)^\gamma - \gamma \frac{\tilde{m}}{m_0} - 1 \right] \stackrel{\text{(App 1)}}{\approx} \frac{P_0 V_0 \gamma}{2} \left(\frac{\tilde{m}}{m_0} \right)^2$

Model (M2): linearised thermodynamical behaviour

Approximation

The fluctuations of mass density are small compared to its state at rest value

$$\text{(App 1) : } \frac{\tilde{m}}{m_0} = \frac{\tilde{\rho}}{\rho_0} \ll 1$$

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Model (M2): linearised thermodynamical behaviour

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$$\text{(App 1) : } \frac{\tilde{m}}{m_0} = \frac{\tilde{\rho}}{\rho_0} \ll 1$$

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$$H_4(\mathbf{x}_4 = [\tilde{\nu}_L, \tilde{\nu}_R, \tilde{\nu}_T, \tilde{m}]) = \underbrace{\frac{\tilde{m} + m_0}{2\ell_0^2} (\tilde{\nu}_L^2 + \tilde{\nu}_R^2 + \tilde{\nu}_L \tilde{\nu}_R) + \frac{3(\tilde{m} + m_0)}{2h_0^2} \tilde{\nu}_T^2}_{\text{Nonlinear}} + \underbrace{\frac{P_0 V_0 \gamma}{2} \left(\frac{\tilde{m}}{m_0}\right)^2}_{\text{Quadratic}}$$

Model (M2): linearised thermodynamical behaviour

Approximation

The fluctuations of mass density are small compared to its state at rest value

$$\text{(App 1) : } \frac{\tilde{m}}{m_0} = \frac{\tilde{\rho}}{\rho_0} \ll 1$$

3)

1) $\mathbf{x}_4 = [\underbrace{\tilde{\nu}_L, \tilde{\nu}_R, \tilde{\nu}_T}_{\text{unchanged}}, \overline{m}]^T \rightarrow$ Small mass fluctuations

2) $E_c(\tilde{m}) = \frac{P_0 V_0}{\gamma - 1} \left[\left(1 + \frac{\tilde{m}}{m_0} \right)^\gamma - \gamma \frac{\tilde{m}}{m_0} - 1 \right] \stackrel{\text{(App 1)}}{\approx} \underbrace{\frac{P_0 V_0 \gamma}{2} \left(\frac{\overline{m}}{m_0} \right)^2}_{\text{Compression}}$

$$H_4(\mathbf{x}_4 = [\tilde{\nu}_L, \tilde{\nu}_R, \tilde{\nu}_T, \overline{m}]) = \underbrace{\frac{\overline{m} + m_0}{2\ell_0^2} (\tilde{\nu}_L^2 + \tilde{\nu}_R^2 + \tilde{\nu}_L \tilde{\nu}_R) + \frac{3(\overline{m} + m_0)}{2h_0^2} \tilde{\nu}_T^2}_{\text{Nonlinear Mass convection}} + \underbrace{\frac{P_0 V_0 \gamma}{2} \left(\frac{\overline{m}}{m_0} \right)^2}_{\text{Quadratic Compression}}$$

Model (M2): linearised thermodynamical behaviour

Approximation

The fluctuations of mass density are small compared to its state at rest value

$$\text{(App 1) : } \quad \frac{\tilde{m}}{m_0} = \frac{\tilde{\rho}}{\rho_0} \ll 1$$

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3) $\mathbf{e}_4 = \begin{pmatrix} \frac{(\tilde{m} + m_0)}{\ell_0^2} (\tilde{\nu}_L + \tilde{\nu}_R/2) \\ \frac{(\tilde{m} + m_0)}{\ell_0^2} (\tilde{\nu}_L/2 + \tilde{\nu}_R) \\ 3(\tilde{m} + m_0) \tilde{\nu}_T / h_0^2 \\ \frac{1}{2\ell_0^2} (\tilde{\nu}_L^2 + \tilde{\nu}_R^2 + \tilde{\nu}_L \tilde{\nu}_R) + \frac{3}{2h_0^2} \tilde{\nu}_T^2 + \frac{P_0 V_0 \gamma \tilde{m}}{m_0^2} \end{pmatrix} = \begin{pmatrix} \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \\ \langle \tilde{\psi} \rangle_\Omega \end{pmatrix}$

Model (M2): linearised thermodynamical behaviour

Approximation

The fluctuations of mass density are small compared to its state at rest value

$$\text{(App 1) : } \quad \frac{\tilde{m}}{m_0} = \frac{\tilde{\rho}}{\rho_0} \ll 1$$

1) $\mathbf{x}_4 = [\underbrace{\tilde{\nu}_L, \tilde{\nu}_R, \tilde{\nu}_T}_{\text{unchanged}}, \overline{m}]^T \rightarrow$ Small mass fluctuations

2) $E_c(\tilde{m}) = \frac{P_0 V_0}{\gamma - 1} \left[\left(1 + \frac{\tilde{m}}{m_0} \right)^\gamma - \gamma \frac{\tilde{m}}{m_0} - 1 \right] \stackrel{\text{(App 1)}}{\approx} \underbrace{\frac{P_0 V_0 \gamma}{2} \left(\frac{\overline{m}}{m_0} \right)^2}_{\text{Compression}}$

$$H_4(\mathbf{x}_4 = [\tilde{\nu}_L, \tilde{\nu}_R, \tilde{\nu}_T, \overline{m}]) = \underbrace{\frac{\overline{m} + m_0}{2\ell_0^2} (\tilde{\nu}_L^2 + \tilde{\nu}_R^2 + \tilde{\nu}_L \tilde{\nu}_R) + \frac{3(\overline{m} + m_0)}{2h_0^2} \tilde{\nu}_T^2}_{\text{Mass convection, Nonlinear}} + \underbrace{\frac{P_0 V_0 \gamma}{2} \left(\frac{\overline{m}}{m_0} \right)^2}_{\text{Compression, Quadratic}}$$

3) $\mathbf{e}_4 = \begin{pmatrix} \frac{(\overline{m} + m_0)}{\ell_0^2} (\tilde{\nu}_L + \tilde{\nu}_R/2) \\ \frac{(\overline{m} + m_0)}{\ell_0^2} (\tilde{\nu}_L/2 + \tilde{\nu}_R) \\ 3(\overline{m} + m_0) \tilde{\nu}_T / h_0^2 \\ \frac{1}{2\ell_0^2} (\tilde{\nu}_L^2 + \tilde{\nu}_R^2 + \tilde{\nu}_L \tilde{\nu}_R) + \frac{3}{2h_0^2} \tilde{\nu}_T^2 + \frac{P_0 V_0 \gamma}{m_0^2} \overline{m} \end{pmatrix} = \begin{pmatrix} \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \\ \langle \overline{\psi} \rangle_\Omega \end{pmatrix}$

Model (M2): linearised thermodynamical behaviour

Approximation

The fluctuations of mass density are small compared to its state at rest value

$$\text{(App 1) : } \quad \frac{\tilde{m}}{m_0} = \frac{\tilde{\rho}}{\rho_0} \ll 1$$

1) $\mathbf{x}_4 = [\underbrace{\tilde{\nu}_L, \tilde{\nu}_R, \tilde{\nu}_T}_{\text{unchanged}}, \overline{m}]^T \rightarrow$ Small mass fluctuations

2) $E_c(\tilde{m}) = \frac{P_0 V_0}{\gamma - 1} \left[\left(1 + \frac{\tilde{m}}{m_0} \right)^\gamma - \gamma \frac{\tilde{m}}{m_0} - 1 \right] \stackrel{\text{(App 1)}}{\approx} \underbrace{\frac{P_0 V_0 \gamma}{2} \left(\frac{\overline{m}}{m_0} \right)^2}_{\text{Compression}}$

$$H_4(\mathbf{x}_4 = [\tilde{\nu}_L, \tilde{\nu}_R, \tilde{\nu}_T, \overline{m}]) = \underbrace{\frac{\overline{m} + m_0}{2\ell_0^2} (\tilde{\nu}_L^2 + \tilde{\nu}_R^2 + \tilde{\nu}_L \tilde{\nu}_R) + \frac{3(\overline{m} + m_0)}{2h_0^2} \tilde{\nu}_T^2}_{\text{Mass convection, Nonlinear}} + \underbrace{\frac{P_0 V_0 \gamma}{2} \left(\frac{\overline{m}}{m_0} \right)^2}_{\text{Compression, Quadratic}}$$

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4)

Model (M2): linearised thermodynamical behaviour

Approximation

The fluctuations of mass density are small compared to its state at rest value

$$\text{(App 1) : } \quad \frac{\tilde{m}}{m_0} = \frac{\tilde{\rho}}{\rho_0} \ll 1$$

1) $\mathbf{x}_4 = [\underbrace{\tilde{\nu}_L, \tilde{\nu}_R, \tilde{\nu}_T}_{\text{unchanged}}, \overline{m}]^T \rightarrow$ Small mass fluctuations

2) $E_c(\tilde{m}) = \frac{P_0 V_0}{\gamma - 1} \left[\left(1 + \frac{\tilde{m}}{m_0} \right)^\gamma - \gamma \frac{\tilde{m}}{m_0} - 1 \right] \stackrel{\text{(App 1)}}{\approx} \underbrace{\frac{P_0 V_0 \gamma}{2} \left(\frac{\overline{m}}{m_0} \right)^2}_{\text{Compression}}$

$$H_4(\mathbf{x}_4 = [\tilde{\nu}_L, \tilde{\nu}_R, \tilde{\nu}_T, \overline{m}]) = \underbrace{\frac{\overline{m} + m_0}{2\ell_0^2} (\tilde{\nu}_L^2 + \tilde{\nu}_R^2 + \tilde{\nu}_L \tilde{\nu}_R) + \frac{3(\overline{m} + m_0)}{2h_0^2} \tilde{\nu}_T^2}_{\text{Mass convection, Nonlinear}} + \underbrace{\frac{P_0 V_0 \gamma}{2} \left(\frac{\overline{m}}{m_0} \right)^2}_{\text{Compression, Quadratic}}$$

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4) $\begin{pmatrix} \frac{d\mathbf{x}_4}{dt} \\ \mathbf{y}_4 \end{pmatrix} = \mathbf{S}_4 \begin{pmatrix} \mathbf{e}_4 \\ \mathbf{u}_4 \end{pmatrix}$

Model (M2): linearised thermodynamical behaviour

Approximation

The fluctuations of mass density are small compared to its state at rest value

$$\text{(App 1) : } \quad \frac{\tilde{m}}{m_0} = \frac{\tilde{\rho}}{\rho_0} \ll 1$$

1) $\mathbf{x}_4 = [\underbrace{\tilde{\nu}_L, \tilde{\nu}_R, \tilde{\nu}_T}_{\text{unchanged}}, \overline{m}]^T \rightarrow$ Small mass fluctuations

2) $E_c(\tilde{m}) = \frac{P_0 V_0}{\gamma - 1} \left[\left(1 + \frac{\tilde{m}}{m_0} \right)^\gamma - \gamma \frac{\tilde{m}}{m_0} - 1 \right] \stackrel{\text{(App 1)}}{\approx} \underbrace{\frac{P_0 V_0 \gamma}{2} \left(\frac{\overline{m}}{m_0} \right)^2}_{\text{Compression}}$

$$H_4(\mathbf{x}_4 = [\tilde{\nu}_L, \tilde{\nu}_R, \tilde{\nu}_T, \overline{m}]) = \underbrace{\frac{\overline{m} + m_0}{2\ell_0^2} (\tilde{\nu}_L^2 + \tilde{\nu}_R^2 + \tilde{\nu}_L \tilde{\nu}_R) + \frac{3(\overline{m} + m_0)}{2h_0^2} \tilde{\nu}_T^2}_{\text{Mass convection, Nonlinear}} + \underbrace{\frac{P_0 V_0 \gamma}{2} \left(\frac{\overline{m}}{m_0} \right)^2}_{\text{Compression, Quadratic}}$$

3) $\mathbf{e}_4 = \begin{pmatrix} \frac{(\overline{m} + m_0)}{\ell_0^2} (\tilde{\nu}_L + \tilde{\nu}_R/2) \\ \frac{(\overline{m} + m_0)}{\ell_0^2} (\tilde{\nu}_L/2 + \tilde{\nu}_R) \\ 3(\overline{m} + m_0) \tilde{\nu}_T / h_0^2 \\ \frac{1}{2\ell_0^2} (\tilde{\nu}_L^2 + \tilde{\nu}_R^2 + \tilde{\nu}_L \tilde{\nu}_R) + \frac{3}{2h_0^2} \tilde{\nu}_T^2 + \frac{P_0 V_0 \gamma}{m_0^2} \overline{m} \end{pmatrix} = \begin{pmatrix} \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \\ \langle \overline{\psi} \rangle_\Omega \end{pmatrix}$

4) $\begin{pmatrix} \frac{d\mathbf{x}_4}{dt} \\ \mathbf{y}_4 \end{pmatrix} = \mathbf{S}_4 \begin{pmatrix} \mathbf{e}_4 \\ \mathbf{u}_4 \end{pmatrix}$
Same interconnection

Model (M2): linearised thermodynamical behaviour

Approximation

The fluctuations of mass density are small compared to its state at rest value

$$\text{(App 1) : } \frac{\tilde{m}}{m_0} = \frac{\tilde{\rho}}{\rho_0} \ll 1$$

1) $\mathbf{x}_4 = [\underbrace{\tilde{\nu}_L, \tilde{\nu}_R, \tilde{\nu}_T}_{\text{unchanged}}, \tilde{m}]^T \rightarrow$ Small mass fluctuations

2) $E_c(\tilde{m}) = \frac{P_0 V_0}{\gamma - 1} \left[\left(1 + \frac{\tilde{m}}{m_0} \right)^\gamma - \gamma \frac{\tilde{m}}{m_0} - 1 \right] \stackrel{\text{(App 1)}}{\approx} \frac{P_0 V_0 \gamma}{2} \left(\frac{\tilde{m}}{m_0} \right)^2$

$$H_4(\mathbf{x}_4 = [\tilde{\nu}_L, \tilde{\nu}_R, \tilde{\nu}_T, \tilde{m}]) = \underbrace{\frac{\bar{m} + m_0}{2\ell_0^2} (\tilde{\nu}_L^2 + \tilde{\nu}_R^2 + \tilde{\nu}_L \tilde{\nu}_R) + \frac{3(\bar{m} + m_0)}{2h_0^2} \tilde{\nu}_T^2}_{\text{Nonlinear}} + \underbrace{\frac{P_0 V_0 \gamma}{2} \left(\frac{\tilde{m}}{m_0} \right)^2}_{\text{Quadratic}}$$

Mass convection Compression

3) $\mathbf{e}_4 = \begin{pmatrix} \frac{(\bar{m} + m_0)}{\ell_0^2} (\tilde{\nu}_L + \tilde{\nu}_R/2) \\ \frac{(\bar{m} + m_0)}{\ell_0^2} (\tilde{\nu}_L/2 + \tilde{\nu}_R) \\ 3(\bar{m} + m_0) \tilde{\nu}_T / h_0^2 \\ \frac{1}{2\ell_0^2} (\tilde{\nu}_L^2 + \tilde{\nu}_R^2 + \tilde{\nu}_L \tilde{\nu}_R) + \frac{3}{2h_0^2} \tilde{\nu}_T^2 + \frac{P_0 V_0 \gamma}{m_0^2} \bar{m} \end{pmatrix} = \begin{pmatrix} \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \\ \langle \tilde{\psi} \rangle_\Omega \end{pmatrix}$

4) $\begin{pmatrix} \frac{d\mathbf{x}_4}{dt} \\ \mathbf{y}_4 \end{pmatrix} = \mathbf{S}_4 \begin{pmatrix} \mathbf{e}_4 \\ \mathbf{u}_4 \end{pmatrix}$

Same interconnection

Model (M2): linearised thermodynamical behaviour

Approximation

The fluctuations of mass density are small compared to its state at rest value

$$\text{(App 1) : } \frac{\tilde{m}}{m_0} = \frac{\tilde{\rho}}{\rho_0} \ll 1$$

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2) $E_c(\tilde{m}) = \frac{P_0 V_0}{\gamma - 1} \left[\left(1 + \frac{\tilde{m}}{m_0}\right)^\gamma - \gamma \frac{\tilde{m}}{m_0} - 1 \right] \stackrel{\text{(App 1)}}{\approx} \underbrace{\frac{P_0 V_0 \gamma}{2} \left(\frac{\tilde{m}}{m_0}\right)^2}_{\text{Compression}}$

$$H_4(\mathbf{x}_4 = [\tilde{\nu}_L, \tilde{\nu}_R, \tilde{\nu}_T, \tilde{m}]) = \underbrace{\frac{\tilde{m} + m_0}{2\ell_0^2} (\tilde{\nu}_L^2 + \tilde{\nu}_R^2 + \tilde{\nu}_L \tilde{\nu}_R) + \frac{3(\tilde{m} + m_0)}{2h_0^2} \tilde{\nu}_T^2}_{\text{Mass convection, Nonlinear}} + \underbrace{\frac{P_0 V_0 \gamma}{2} \left(\frac{\tilde{m}}{m_0}\right)^2}_{\text{Compression, Quadratic}}$$

3) $\mathbf{e}_4 = \begin{pmatrix} \frac{(\tilde{m} + m_0)}{\ell_0^2} (\tilde{\nu}_L + \tilde{\nu}_R/2) \\ \frac{(\tilde{m} + m_0)}{\ell_0^2} (\tilde{\nu}_L/2 + \tilde{\nu}_R) \\ 3(\tilde{m} + m_0)\tilde{\nu}_T/h_0^2 \\ \frac{1}{2\ell_0^2} (\tilde{\nu}_L^2 + \tilde{\nu}_R^2 + \tilde{\nu}_L \tilde{\nu}_R) + \frac{3}{2h_0^2} \tilde{\nu}_T^2 + \frac{P_0 V_0 \gamma}{m_0^2} \tilde{m} \end{pmatrix} = \begin{pmatrix} \hat{q}_L \\ \hat{q}_R \\ \hat{q}_T \\ \langle \tilde{\psi} \rangle_\Omega \end{pmatrix}$

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Same interconnection

(M2) exhibits:

- a **linear** thermodynamical behaviour
- **nonlinear** mass convection phenomenon

Model (M3): linear acoustics



Model (M3): linear acoustics

Approximation 2

Model (M3): linear acoustics

Approximation 2

The fluctuations of the kinetic variables are small:

Model (M3): linear acoustics

Approximation 2

The fluctuations of the kinetic variables are small:

(App 2) : $\tilde{\nu}_i = \bar{\nu}_i + O(\tilde{\nu}_i^2) \ll 1 \quad \forall i \in \{L, R, T\}.$

Model (M3): linear acoustics

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Model (M3): linear acoustics

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1) $\mathbf{x}_5 = [\bar{\nu}_L, \bar{\nu}_R, \bar{\nu}_T, \bar{m}]^\top$
Small fluctuations

Model (M3): linear acoustics

Approximation 2

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Small fluctuations

2)

Model (M3): linear acoustics

Approximation 2

The fluctuations of the kinetic variables are small:

$$\text{(App 2): } \quad \tilde{v}_i = \bar{v}_i + O(\tilde{v}_i^2) \ll 1 \quad \forall i \in \{L, R, T\}.$$

$$1) \quad \mathbf{x}_5 = [\bar{v}_L, \bar{v}_R, \bar{v}_T, \bar{m}]^\top$$

Small fluctuations

$$2) \quad H_5(\mathbf{x}_5) = \frac{m_0}{2\ell_0^2} (\bar{v}_L^2 + \bar{v}_R^2 + \bar{v}_L \bar{v}_R) + \frac{3m_0}{2h_0^2} \bar{v}_T^2 + \frac{P_0 V_0 \gamma}{2} \left(\frac{\bar{m}}{m_0} \right)^2$$

Model (M3): linear acoustics

Approximation 2

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$$= \frac{1}{2} \mathbf{x}_5^\top \underbrace{\begin{bmatrix} m_0/\ell_0^2 & m_0/2\ell_0^2 & 0 & 0 \\ m_0/2\ell_0^2 & m_0/\ell_0^2 & 0 & 0 \\ 0 & 0 & 3m_0/h_0^2 & 0 \\ 0 & 0 & 0 & P_0V_0\gamma/m_0^2 \end{bmatrix}}_{\mathbf{Q}} \mathbf{x}_5$$

Model (M3): linear acoustics

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$$= \frac{1}{2}\mathbf{x}_5^\top \underbrace{\begin{bmatrix} m_0/\ell_0^2 & m_0/2\ell_0^2 & 0 & 0 \\ m_0/2\ell_0^2 & m_0/\ell_0^2 & 0 & 0 \\ 0 & 0 & 3m_0/h_0^2 & 0 \\ 0 & 0 & 0 & P_0V_0\gamma/m_0^2 \end{bmatrix}}_{\mathbf{Q}} \mathbf{x}_5$$

$$3) \text{ Quadratic form } \rightarrow \boxed{\mathbf{e}_5 = \mathbf{Q}\mathbf{x}_5}$$

Model (M3): linear acoustics

Approximation 2

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Model (M3): linear acoustics

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3) Quadratic form $\rightarrow \mathbf{e}_5 = \mathbf{Q}\mathbf{x}_5$

4) $\left(\frac{d\mathbf{x}_5}{dt}\right) = \mathbf{S}_5 \left(\frac{\mathbf{e}_5}{\mathbf{u}_5}\right)$ where

Model (M3): linear acoustics

Approximation 2

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and $\mathbf{y}_5 = \begin{pmatrix} \bar{q}_L \\ \bar{q}_R \\ \bar{q}_T \end{pmatrix} = \begin{pmatrix} \frac{m_0}{\bar{v}_0^2}(\bar{v}_L + \bar{v}_R/2) \\ \frac{m_0}{\bar{v}_0^2}(\bar{v}_L/2 + \bar{v}_R) \\ 3m_0\bar{v}_T/h_0^2 \end{pmatrix}$

Model (M3): linear acoustics

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Small fluctuations

2) $H_5(\mathbf{x}_5) = \frac{m_0}{2\ell_0^2}(\bar{v}_L^2 + \bar{v}_R^2 + \bar{v}_L\bar{v}_R) + \frac{3m_0}{2h_0^2}\bar{v}_T^2 + \frac{P_0V_0\gamma}{2}\left(\frac{\bar{m}}{m_0}\right)^2$

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\bar{q} Acoustical mass rate
 $\bar{\psi}$ Acoustical enthalpy

Model (M3): linear acoustics

Approximation 2

The fluctuations of the kinetic variables are small:

(App 2) : $\tilde{v}_i = \bar{v}_i + O(\tilde{v}_i^2) \ll 1 \quad \forall i \in \{L, R, T\}.$

1) $\mathbf{x}_5 = [\bar{v}_L, \bar{v}_R, \bar{v}_T, \bar{m}]^T$

Small fluctuations

2) $H_5(\mathbf{x}_5) = \frac{m_0}{2\ell_0^2}(\bar{v}_L^2 + \bar{v}_R^2 + \bar{v}_L\bar{v}_R) + \frac{3m_0}{2h_0^2}\bar{v}_T^2 + \frac{P_0V_0\gamma}{2} \left(\frac{\bar{m}}{m_0}\right)^2$

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3) Quadratic form $\rightarrow \mathbf{e}_5 = \mathbf{Q}\mathbf{x}_5$

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\bar{q} Acoustical mass rate
 $\bar{\psi}$ Acoustical enthalpy



Model (M3): linear acoustics

Approximation 2

The fluctuations of the kinetic variables are small:

(App 2) : $\tilde{v}_i = \bar{v}_i + O(\tilde{v}_i^2) \ll 1 \quad \forall i \in \{L, R, T\}.$

1) $\mathbf{x}_5 = [\bar{v}_L, \bar{v}_R, \bar{v}_T, \bar{m}]^\top$

Small fluctuations

2) $H_5(\mathbf{x}_5) = \frac{m_0}{2\ell_0^2}(\bar{v}_L^2 + \bar{v}_R^2 + \bar{v}_L\bar{v}_R) + \frac{3m_0}{2h_0^2}\bar{v}_T^2 + \frac{P_0V_0\gamma}{2}\left(\frac{\bar{m}}{m_0}\right)^2$

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\bar{q} Acoustical mass rate
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Links with linear acoustics
 Acoustical pressure:

Model (M3): linear acoustics

Approximation 2

The fluctuations of the kinetic variables are small:

(App 2) : $\tilde{v}_i = \bar{v}_i + O(\tilde{v}_i^2) \ll 1 \quad \forall i \in \{L, R, T\}.$

1) $\mathbf{x}_5 = [\bar{v}_L, \bar{v}_R, \bar{v}_T, \bar{m}]^\top$

Small fluctuations

2) $H_5(\mathbf{x}_5) = \frac{m_0}{2\ell_0^2}(\bar{v}_L^2 + \bar{v}_R^2 + \bar{v}_L\bar{v}_R) + \frac{3m_0}{2h_0^2}\bar{v}_T^2 + \frac{P_0V_0\gamma}{2} \left(\frac{\bar{m}}{m_0}\right)^2$

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\bar{q} Acoustical mass rate
 $\bar{\psi}$ Acoustical enthalpy

Links with linear acoustics

Acoustical pressure:

$$\bar{m}P_0V_0/\rho_0^2V_0 = c_0^2\bar{\rho}$$

Model (M3): linear acoustics

Approximation 2

The fluctuations of the kinetic variables are small:

(App 2) : $\tilde{v}_i = \bar{v}_i + O(\tilde{v}_i^2) \ll 1 \quad \forall i \in \{L, R, T\}.$

1) $\mathbf{x}_5 = [\bar{v}_L, \bar{v}_R, \bar{v}_T, \bar{m}]^T$

Small fluctuations

2) $H_5(\mathbf{x}_5) = \frac{m_0}{2\ell_0^2}(\bar{v}_L^2 + \bar{v}_R^2 + \bar{v}_L\bar{v}_R) + \frac{3m_0}{2h_0^2}\bar{v}_T^2 + \frac{P_0V_0\gamma}{2} \left(\frac{\bar{m}}{m_0}\right)^2$

$$= \frac{1}{2} \mathbf{x}_5^T \underbrace{\begin{bmatrix} m_0/\ell_0^2 & m_0/2\ell_0^2 & 0 & 0 \\ m_0/2\ell_0^2 & m_0/\ell_0^2 & 0 & 0 \\ 0 & 0 & 3m_0/h_0^2 & 0 \\ 0 & 0 & 0 & P_0V_0\gamma/m_0^2 \end{bmatrix}}_{\mathbf{Q}} \mathbf{x}_5$$

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Links with linear acoustics

Acoustical pressure:

$$\bar{m}P_0V_0/\rho_0^2V_0 = c_0^2\bar{\rho}$$

For a more classical representation

Model (M3): linear acoustics

Approximation 2

The fluctuations of the kinetic variables are small:

(App 2) : $\tilde{v}_i = \bar{v}_i + O(\tilde{v}_i^2) \ll 1 \quad \forall i \in \{L, R, T\}.$

1) $\mathbf{x}_5 = [\bar{v}_L, \bar{v}_R, \bar{v}_T, \bar{m}]^\top$

Small fluctuations

2) $H_5(\mathbf{x}_5) = \frac{m_0}{2\ell_0^2}(\bar{v}_L^2 + \bar{v}_R^2 + \bar{v}_L\bar{v}_R) + \frac{3m_0}{2h_0^2}\bar{v}_T^2 + \frac{P_0V_0\gamma}{2}\left(\frac{\bar{m}}{m_0}\right)^2$

$$= \frac{1}{2} \mathbf{x}_5^\top \underbrace{\begin{bmatrix} m_0/\ell_0^2 & m_0/2\ell_0^2 & 0 & 0 \\ m_0/2\ell_0^2 & m_0/\ell_0^2 & 0 & 0 \\ 0 & 0 & 3m_0/h_0^2 & 0 \\ 0 & 0 & 0 & P_0V_0\gamma/m_0^2 \end{bmatrix}}_{\mathbf{Q}} \mathbf{x}_5$$

3) Quadratic form $\rightarrow \mathbf{e}_5 = \mathbf{Q}\mathbf{x}_5$

4) $\left(\frac{d\mathbf{x}_5}{dt}\right) = \mathbf{S}_5 \left(\frac{\mathbf{e}_5}{\mathbf{u}_5}\right)$ where $\mathbf{u}_5 = \begin{pmatrix} \bar{\psi}_L \\ \bar{\psi}_R \\ \bar{\psi}_T \end{pmatrix} = \begin{pmatrix} \bar{m}P_0\gamma V_0/m_0^2 \\ \bar{m}P_0\gamma V_0/m_0^2 \\ \bar{m}P_0\gamma V_0/m_0^2 \end{pmatrix}$

and $\mathbf{y}_5 = \begin{pmatrix} \bar{q}_L \\ \bar{q}_R \\ \bar{q}_T \end{pmatrix} = \begin{pmatrix} \frac{m_0}{\ell_0^2}(\bar{v}_L + \bar{v}_R/2) \\ \frac{m_0}{\ell_0^2}(\bar{v}_L/2 + \bar{v}_R) \\ 3m_0\bar{v}_T/h_0^2 \end{pmatrix}$

\bar{q} Acoustical mass rate
 $\bar{\psi}$ Acoustical enthalpy

Links with linear acoustics

Acoustical pressure:

$$\bar{m}P_0V_0/\rho_0^2V_0 = c_0^2\bar{\rho}$$

For a more classical representation

$$\begin{pmatrix} p_i \\ U_i \end{pmatrix} = \begin{bmatrix} \rho_0 & 0 \\ 0 & 1/\rho_0 \end{bmatrix} \begin{pmatrix} \bar{\psi}_i \\ \bar{q}_i \end{pmatrix} \quad \forall i \in \{L, R, T\}.$$

Model (M3): linear acoustics

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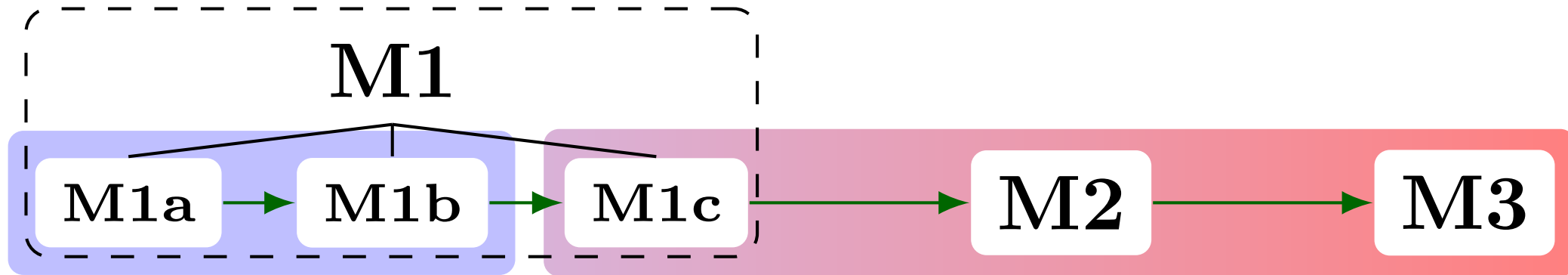
$$\bar{m}P_0V_0/\rho_0^2V_0 = c_0^2\bar{\rho}$$

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Classical state-space representation also available !

Quick Recap



Fluid mechanics

Nonlinear acoustics

Linear acoustics

Conclusion



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Thank you for your attention !

Victor.wetzel@pm.me