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# Virtual Path Layouts with Low Congestion or Low Diameter in ATM Networks\*

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## Abstract

Motivated by Asynchronous Transfer Mode (ATM) in telecommunication networks, we investigate the problem of designing a directed virtual topology on a directed physical topology, which consists in finding a set of *directed virtual paths (VPs)* satisfying some constraints in terms of *load* (the number of VPs sharing a physical link) and *hop count* (the number of VPs used to establish a connection). For both general and particular networks, such as paths, cycles, meshes, tori and trees, we derive tight bounds on the *virtual diameter* (the maximum hop count for a connection) as a function of the *network capacity* (the maximum load of a physical link). The dual issue is also addressed.

## 1 Introduction

The advent of fiber optic media has changed the classical views on the role and structure of digital communication networks. Specifically, the sharp distinction between telephone networks, cable television networks, and computer networks has been replaced by a unified approach. The most prevalent solution for this new network challenge is *Asynchronous Transfer Mode (ATM)*, which is thoroughly described in the literature [5, 12]. The transfer of data in ATM is based on packets of fixed length, termed *cells*. Each cell is routed independently, based on two routing fields at the cell header, called *virtual channel identifier (VCI)* and *virtual path identifier (VPI)*. This method effectively creates two types of predetermined simple routes in the network, namely, routes which are based on VPIs (called *virtual paths* or VPs) and routes based on VCIs and VPIs (called *virtual channels* or VCs). VCs are used for connecting network users (e.g., a telephone call); VPs are used for simplifying network management - routing of VCs in particular. Thus the route of a VC may be viewed as a concatenation of complete VPs. A major problem in this framework consists in defining the set of VPs in such a way that some good properties are achieved.

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1. A capacity (or bandwidth) is assigned to each VP. The sum of the capacities of the VPs that share a physical link constitutes the *load* of this link. Naturally, this load must not exceed the link capacity, i.e., the amount of data it can carry. The sum of the capacities of all the physical links is a major component in the cost of the network, and should be kept as low as possible.
2. The maximum number of VPs in a virtual channel, termed *hop count* in the literature, should also be kept as low as possible so as to guarantee low set up times for the virtual channels and high data transfer rates.

In its most general formulation, the *Virtual Path Layout (VPL)* problem is an optimization problem in which, given a certain communication demand between pairs of nodes and constraints on the maximum load and hop count, it is first required to design a system of VPs satisfying the constraints and then minimizing some given function of the load and hop count.

We employ a restricted model similar to the one presented by Cidon, Gerstel and Zaks in [10]. In particular, we assume that all VPs have equal capacities, normalized to 1. Hence the load of a physical link is simply the number of VPs sharing this link.

Although links based on optical fibers and cables are directed, traditional research uses an undirected model. Indeed, this model imposes the requirement that if there exists a VP from  $u$  to  $v$  then there exists also a VP from  $v$  to  $u$ . In fact, that is the way ATM networks are implemented at the present time. However, two symmetric VPs do not need to have the same capacity. Indeed, in many applications the flows on the VPs are not equal. Therefore, it seems more reasonable to use a directed model like the one introduced by Chanas and Goldschmidt in [4].

We focus on the *all-to-all problem* (all pairs of nodes are equally likely to communicate). Thus, the resulting maximum hop count can be viewed as the *diameter* of the graph induced by the set of VPs.

More formally, given a communication network, the VPs form a virtual directed graph (digraph) on the top of the physical one, with the same set of vertices but with a different set of arcs. (Specifically, a VP from  $u$  to  $v$  is represented by an arc from  $u$  to  $v$  in the virtual digraph.) This virtual digraph provides a *Directed Virtual Path Layout (DVPL)* for the physical graph. Each VC can be viewed as a simple dipath in the virtual digraph. Therefore, a central problem is to find a tradeoff between the maximum load and the virtual diameter.

**Related Work** The problem has been considered in the undirected case, for example, in [10, 9, 15, 8, 13, 6]. The problem of minimizing the maximum load over all VPL with bounded hop-count is studied in [7, 1], and minimizing also the average load is considered in [9]. The one-to-many problem is handled in [7, 9], where the focus is on minimizing the eccentricity of the virtual graph from a special point called the root (this problem is the *rooted virtual path layout problem*) rather than minimizing the diameter of the virtual graph. A duality in the chain network between the problem of minimizing the hop-count knowing the maximum load, and the one of minimizing the load, knowing the maximum hop-count, is established in [7]. The reader can find an excellent survey of the results in the undirected model in [16].

The techniques involved in our constructions bear a certain resemblance to various embedding techniques used previously in the context of parallel computing, in order to implement a useful virtual

architecture on a given practical machine topology (cf. [14, 11]). The parameters of interest in such embeddings are the number of virtual processors mapped onto any physical processor, the load on the physical links, and the dilation of the embedding, namely, the maximum length of the physical path corresponding to a given virtual link. The relevant concerns in our context are somewhat different, as dilation is of no consequence, and on the other hand, we have the freedom of designing the virtual topology as desired, in order to optimize its diameter.

## 2 Model

A physical network is represented by a *capacitated digraph*  $G = (V, E, c)$ , that is a directed graph with vertex set  $V$  and arc set  $E$ , together with a positive integral capacity function  $c$  on the set of arcs. We always denote by  $n$  the number of vertices and in this paper we consider only constant capacity functions.

The network formed by a set of VPs is represented by a digraph  $H = (V, E')$  together with a layout  $P$  assigning to each arc  $e' = (x, y) \in E'$  a simple directed path (dipath)  $P(e')$  connecting  $x$  to  $y$  in  $G$ . In our terminology, the pair  $(H, P)$  is a *virtual digraph on  $G$* , an arc of  $H$  is a *virtual arc*, and the dipath  $P(e')$  in  $G$  associated with a virtual arc  $e'$  is a *virtual dipath (VP)*.

The *load* of an arc  $e$  of  $G$  is the number of virtual dipaths containing the arc  $e$ , that is,  $l(e) = |\{e' \in E', e \in P(e')\}|$ . A virtual digraph  $(H, P)$  satisfying the requirement  $\forall e \in E, l(e) \leq c(e)$  is referred to as a  *$c$ -admissible Directed Virtual Paths Layout* of  $G$ , shortly denoted  *$c$ -DVPL* of  $G$ . The aim is to design  $c$ -DVPL of  $G$  with minimum hop-count, i.e, to find a virtual digraph with minimum diameter.

For any digraph  $F$ ,  $d_F(x, y)$  denotes the distance from  $x$  to  $y$  in  $F$ , and  $D_F$  denotes diameter of  $F$ . The *virtual diameter*,  $\tilde{D}(G, c)$ , of the digraph  $G$  with respect to the capacity  $c$ , is the minimum of  $D_H$  over all the  $c$ -DVPL  $H$  of  $G$ .

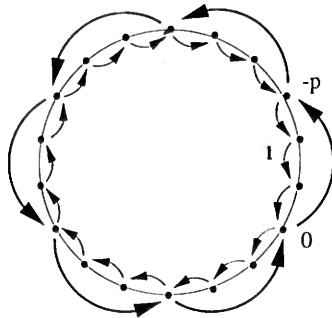


Figure 1: Example of VPL: the cycle, capacity 1

In Figure 1,  $G$  consists of the symmetric directed cycle  $C_n$ . The virtual graph  $H$  consists of arcs  $(i, i + 1)$  in the clockwise direction and arcs  $(ip, (i - 1)p)$  in the opposite direction (assuming that  $p$  divides  $n$ ). The load of every arc of  $C_n$  is one. Choosing  $p = \sqrt{\frac{n}{2}}$  gives good DVPL with diameter at most  $2\sqrt{2n} + 1$ .

In the dual problem, given a physical graph  $G = (V, E)$  and an integer  $h$ , we seek for a virtual graph  $H = (V, E')$  and a layout  $P$  with minimum load, such that  $D_H \leq h$ . More formally, if  $\forall e \in E$ ,  $\pi(G, H, P, e) = |\{e' \in E', e \in P(e')\}|$ , we give bounds on  $\pi(G, h) = \min_{D_H \leq h} (\max_{e \in E} \pi(G, H, P, e))$ . Note that  $\pi(G, 1)$  is called the arc forwarding index of  $G$ . More generally, one can define the minimum load  $\pi(G, R, h)$  for any subset of requests  $R \subset V \times V$ . Given  $G, R$  and  $h$ , we seek for a virtual graph  $H = (V, E')$  and a layout  $P$ , such that  $\forall (x, y) \in R, d_H(x, y) \leq h$  and  $\max_{e \in E} \{\pi(G, H, P, e)\}$  is minimum. This minimum is denoted by  $\pi(G, R, h)$ . Note that  $\pi(G, AA, h) = \pi(G, h)$ , where  $AA$  denotes the all-to-all set of requests  $V \times V$ .

### 3 Results

The following table summarizes our results on the first problem, giving lower and upper bounds on the virtual diameter (the minimum diameter of an admissible virtual digraph) as a function of the number of vertices  $n$  in the physical graph, its diameter  $D_G$ , its maximum in- and out-degree  $d$ , and the constant capacity  $c$ . The results mentioned for the path in the special case  $c = 1$  are due to [2, 3].

Graph $G$	Capacity	Lower Bound	Upper Bound
General Graph	$c = o(n)$	$\frac{\log n}{\log(cd)} - 1$	$D_G$ $\mathcal{O}(c \cdot n^{\frac{1}{2c-1}})$ $\mathcal{O}(D_G^{1/(2c-1)} \cdot \log n)$
Path $P_n$	$c = 1$	$\frac{n}{2} + \log n - \mathcal{O}(1)$	$\frac{n}{2} + \log n$
	$c = o(n)$	$n^{\frac{1}{2c-1}}/2$	$\mathcal{O}(c \cdot n^{\frac{1}{2c-1}})$
Cycle $C_n$	$c = 1$	$2\sqrt{2n} + \mathcal{O}(1)$	$2\sqrt{2n} + 1$
	$c = o(n)$	$n^{\frac{1}{2c}}/2$	$4c \left(\frac{n}{2}\right)^{\frac{1}{2c}} + 1$
Torus $TM(a, b), a \leq b$	$c = o(n)$	$\Omega((a \cdot b)^{1/2ac})$	$\mathcal{O}(a \cdot b^{1/2ac})$
Mesh $M(a, b), \log b \leq a \leq b$	$c = o(n)$	$\Omega(\log n)$	$\mathcal{O}(\log n)$
Arbitrary Tree $T$	$c = o(n)$	$D_T^{1/(2c-1)}/2$	$10c \cdot n^{1/(2c-1)}$ $\mathcal{O}(c \cdot D_T^{1/(2c-1)} \cdot \log n)$
Complete $k$ -ary Tree $T$ $h = \text{depth}(T)$	$c = 2, k = 2$	$h$ ( $h$ even) $h - 1$ ( $h$ odd)	$h + 1$
	$c = o(n)$	$2 \left\lfloor \frac{h-1}{\lfloor \log_k c \rfloor + 1} \right\rfloor + 1$	$2 \left\lfloor \frac{h}{\lfloor \log_k c \rfloor + 1} \right\rfloor + 2$

The following theorems concern the dual problem. They give relations between  $\pi(G, R, 2)$  and  $\pi(G, R, 1)$ , for  $R = V \times V$  (AA all-to-all) and  $R = \{s\} \times V$  (OA one-to-all).

**Theorem 1**  $\forall G, \forall \alpha \geq 14$ , if  $\pi(G, AA, 1) \geq \frac{3}{\alpha} \sqrt{n \ln n}$  then  $\pi(G, AA, 2) \leq \alpha \pi(G, AA, 1) \sqrt{\frac{\ln n}{n}}$ .

**Corollary 2**  $\forall G$ , if  $\pi(G, AA, 1) \geq \frac{\sqrt{n \log n}}{5}$  then  $\pi(G, AA, 2) \leq 15 \cdot \pi(G, AA, 1) \sqrt{\frac{\log n}{n}}$  and if  $\pi(G, AA, 1) \leq \frac{\sqrt{n \log n}}{5}$  then  $\pi(G, AA, 2) \leq 15 \log n$ .

More generally, we have

**Theorem 3**  $\forall G$ , if  $\pi(G, AA, 1) \geq n^{\frac{h-1}{h}} \log n$  then  $\pi(G, AA, h) = \mathcal{O}\left(\pi(G, AA, 1) \frac{\sqrt{\log n}}{n^{\frac{h-1}{h}}}\right)$ .

**Theorem 4**  $\forall T$  tree,  $\pi(T, AA, 2) \leq \sqrt{\pi(T, AA, 1)}$

**Theorem 5**  $\forall G$ ,  $\pi(G, OA, 2) \leq 2 \cdot \sqrt{\pi(G, OA, 1)}$

More generally, we have

**Theorem 6**  $\forall G$ ,  $\pi(G, OA, h) = \mathcal{O}\left(\pi(G, OA, 1)^{\frac{1}{h}}\right)$

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## References

- [1] L. Becchetti, P. Bertolazzi, C. Gaibisso, and G. Gambosi. On the design of efficient ATM routing schemes. manuscript, 1997.
- [2] M. Burlet, P. Chanas, and O. Goldschmidt. Optimization of VP layout in ATM networks. In preparation, 1998.
- [3] P. Chanas. *Dimensionnement de réseaux ATM*. PhD thesis, CNET Sophia, Sept. 1998. In preparation.
- [4] P. Chanas and O. Goldschmidt. Conception de réseau de VP de diamètre minimum pour les réseaux ATM. In *Road-f'98*, pages 38–40, 1998.
- [5] M. De Pricker. *Asynchronous Transfer Mode, Solution for Broadband ISDN*. Prentice Hall, August 1995. 3rd edition 332p.
- [6] T. Eilam, M. Flammini, and S. Zaks. A complete characterization of the path layout construction problem for ATM networks with given hop count and load. In *24th International Colloquium on Automata, Languages and Programming (ICALP)*, volume 1256 of *Lecture Notes in Computer Science*, pages 527–537. Springer-Verlag, 1997.
- [7] M. Feiglstein and S. Zaks. Duality in chain ATM virtual path layouts. In *4th International Colloquium on Structural Information and Communication Complexity (SIROCCO)*, Monte Verita, Ascona, Switzerland, July 1997.
- [8] O. Gerstel, I. Cidon, and S. Zaks. The layout of virtual paths in ATM networks. *IEEE/ACM Transactions on Networking*, 4(6):873–884, 1996.
- [9] O. Gerstel, A. Wool, and S. Zaks. Optimal layouts on a chain ATM network. In *3rd Annual European Symposium on Algorithms*, volume LNCS 979, pages 508–522. Springer Verlag, 1995.

- [10] O. Gerstel and S. Zaks. The virtual path layout problem in fast networks. In *Symposium on Principles of Distributed Computing (PODC '94)*, pages 235–243, New York, USA, Aug. 1994. ACM Press.
- [11] J.-W. Hong, K. Mehlhorn, and A. Rosenberg. Cost trade-offs in graph embedding with applications. *J. ACM*, 30, 1983.
- [12] D. Kofman and M. Gagnaire. *Réseaux Haut Débit, réseaux ATM, réseaux locaux et réseaux tout-optiques*. InterEditions-Masson, 1998. 2eme édition.
- [13] E. Kranakis, D. Krizanc, and A. Pelc. Hop-congestion trade-offs for high-speed networks. *International Journal of Foundations of Computer Science*, 8:117–126, 1997.
- [14] A. L. Rosenberg. Issues in the study of graph embeddings. In H. Noltemeier, editor, *Proceedings of the International Workshop on Graphtheoretic Concepts in Computer Science*, volume 100 of *LNCS*, pages 150–176, Bad Honnef, FRG, June 1980. Springer.
- [15] L. Stacho and I. Vrt'o. Virtual path layouts for some bounded degree networks. In *Structure, Information and Communication Complexity, 3rd Colloquium, SIROCCO*, pages 269–278. Carleton University Press, 1996.
- [16] S. Zaks. Path layout in ATM networks - a survey. In *The DIMACS Workshop on Networks in Distributed Computing, DIMACS Center, Rutgers University*, Oct. 1997. manuscript.