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Efficient collective communication in optical networks, Lecture Notes in Computer Science, 1099

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► **To cite this version:**

Jean-Claude Bermond, Luisa Gargano, Stéphane Pérennes, Adele Rescigno, Ugo Vaccaro. Efficient collective communication in optical networks, Lecture Notes in Computer Science, 1099. ICALP 96, Jul 1996, Paderborn, Germany. pp.574-585, 10.1007/3-540-61440-0_160 . hal-03838865

HAL Id: hal-03838865

<https://hal.science/hal-03838865>

Submitted on 3 Nov 2022

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Efficient Collective Communication in Optical Networks*

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Abstract. This paper studies the problems of broadcasting and gossiping in optical networks. In such networks the vast bandwidth available is utilized through *wavelength division multiplexing*: a single physical optical link can carry several logical signals, provided that they are transmitted on different wavelengths. In this paper we consider both *single-hop* and *multihop* optical networks. In single-hop networks the information, once transmitted as light, reaches its destination without being converted to electronic form in between, thus reaching high speed communication. In multihop networks a packet may have to be routed through a few intermediate nodes before reaching its final destination. In both models we give efficient broadcasting and gossiping algorithms, in terms of time and number of wavelengths. We consider both networks with arbitrary topologies and particular networks of practical interest. Several of our algorithms exhibit optimal performances.

1 Introduction

Motivations. Optical networks offer the possibility of interconnecting hundreds to thousands of users, covering local to wide area and providing capacities exceeding those of traditional technologies by several orders of magnitude. Optical-fiber transmission systems also achieve very low bit error rate compared to their copper-wire predecessors, typically 10^{-9} compared to 10^{-5} . Optics is thus emerging as a key technology in state-of-the-art communication networks and is expected to dominate many applications. The most popular approach to realize these high-capacity networks is to divide the optical spectrum into many different channels, each channel corresponding to a different wavelength. This approach, called *wavelength-division multiplexing* (WDM) [11] allows multiple data streams to be transferred concurrently along the same fiber-optic, with different streams assigned separate wavelengths.

The major applications for such networks are video conferencing, scientific visualisation and real-time medical imaging, high-speed super-computing and

* The work of J-CB and SP was partially supported by the French GDR/PRC Project PRS and by Galileo Project, the work of LG, AAR, and UV was partially supported by the Italian Ministry of the University and Scientific Research, Project: "Algoritmi, Modelli di Calcolo e Strutture Informative", and by Galileo Project.

distributed computing [18, 39, 43]. We refer to the books of Green [18] and McAulay [29] for a presentation of the physical theory and applications of this emerging technology.

In order to state the new algorithmic issues and challenges concerning data communication in optical networks, we need first to describe the most accepted models of optical networks architectures.

The Optical Model. In WDM optical networks, the bandwidth available in optical fiber is utilised by partitioning it into several channels, each at a different wavelength. Each wavelength can carry a separate stream of data. In general, such a network consists of routing nodes interconnected by point-to-point fiber optic links. Each link can support a certain number of wavelengths. The routing nodes in the network are capable of routing a wavelength coming in on an input port to one or more output ports, independently of the other wavelengths. The same wavelength on two input ports *cannot* be routed to a same output port. WDM lightwave networks can be classified into two categories: *switchless* (also called *broadcast-and-select* or *non-reconfigurable*) and *switched* (also called *reconfigurable*). Each of these in turn can be classified as either *single-hop* (also called *all-optical*) or *multihop* [39]. In switchless networks, the transmission from each station is broadcast to all stations in the network. At the receiver, the desired signal is then extracted from all the signals. These networks are practically important since the whole network can be constructed out of passive optical components, hence it is reliable and easy to operate. However, switchless networks suffer of severe limitations that make problematic their extension to wide area networks. Indeed it has been proven in [1] that switchless networks require a large number of wavelengths to support even simple traffic patterns. Other drawbacks of switchless networks are discussed in [39]. Therefore, optical switches are required to build large networks.

A switched optical network consists of nodes interconnected by point-to-point optic communication lines. Each of the fiber-optic links supports a given number of wavelengths. The nodes can be terminals, switches, or both. Terminals send and receive signals. Switches direct their input signals to one or more of the output links. Each link is bidirectional and actually consists of a pair of unidirectional links [39].

In this paper we consider switched networks with *generalised switches*, as done in [1, 3, 10, 38]. In this kind of networks, signals for different requests may travel on a same communication link into a node v (on different wavelengths) and then exit v along different links. Thus the photonic switch can differentiate between several wavelengths coming along a communication link and direct each of them to a different output of the switch. The only constraint is that no two paths in the network sharing a same optical link have the same wavelength assignment. In switched networks it is possible to "reuse wavelengths" [39], thus obtaining a drastic reduction on the number of required wavelengths with respect to switchless networks [1]. We remark that optical switches do not modulate the wavelengths of the signals passing through them; rather, they direct the incoming waves to one or more of their outputs.

Single-hop networks (or all-optical networks) are networks where the information, once transmitted as light, reaches its final destination directly without being converted to electronic form in between. Maintaining the signal in optic form allows to reach high speed in these networks since there is no overhead due to conversions to and from the electronic form. However, engineering reasons [39] suggest that in some situations the multihop approach can be preferable. In these networks, a packet from a terminal node may have to be routed through a few terminal nodes before reaching its final destination. At each terminal node, the packet is converted from light to electronic form and retransmitted on another wavelength. See [32, 33] for more on these questions. In the present paper we consider both switched single-hop and switched multihop networks.

Our Results. In this paper we initiate the study of the problem of designing efficient algorithms for collective communication in switched optical networks.

Collective communication among the processors is one of the most important issues in multi-processor systems. The need for collective communication arises in many problems of parallel and distributed computing including many scientific computations [9, 12, 15] and database management [17, 44]. Due to the considerable practical relevance in parallel and distributed computation and the related interesting theoretical issues, collective communication problems have been extensively studied in the literature (see the surveys [20, 25, 16]). In this paper we will consider the design of efficient algorithms for two widely used collective communication operations: *Broadcasting* and *Gossiping* (also called all-to-all broadcasting). Formally, the broadcasting and gossiping processes can be described as follows.

Broadcasting: One terminal node v , called the source, has a block of data $B(v)$. The goal is to disseminate this block so that each other terminal node in the network gets $B(v)$.

Gossiping: Each terminal node v in the network has a block of data $B(v)$. The goal is to disseminate these blocks so that each terminal node gets all the blocks $B(u)$, for each terminal u in the network.

Although our work seems to be the first to address the problem of collective communication in switched optical networks, there is a substantial body of literature that has considered related problems. Optical routing in arbitrary networks has been recently considered in [1, 3, 30, 38]. Above papers contain also efficient algorithms for routing in networks of practical interest. Routing in hypercube based networks has been considered by [3, 34, 38]. Lower bounds on the number of wavelengths necessary for routing permutations have been given in [34, 4, 37]. Gossiping in broadcast-and-select optical networks has been considered in [1]. Other work related to ours is contained in [13, 23, 14, 24, 25]. In these papers the problem of designing efficient broadcasting and gossiping algorithms in traditional networks has been considered under the assumption that data exchange can take place through edge-disjoint paths in the network.

In this paper we consider both single-hop and multihop networks. In case of single-hop networks we design broadcasting and gossiping algorithms that do not need buffering at intermediate nodes. The algorithms have to guarantee

that there is a path between each pair of nodes requiring communication and no link will carry two different signals on the same wavelength. For our purposes, a wavelength will be an integer in the interval $[1, W]$. Generally, we wish to minimise the quantity W , since the cost of switching and amplification devices depends on the number of wavelengths they handle. For single-hop networks we obtain:

- *Optimal* broadcasting algorithms for *all* maximally edge-connected graphs;
- *optimal* gossiping algorithm for rings and hypercubes, *quasi-optimal* algorithms for toruses;
- upper and lower bounds on the number of wavelengths necessary to gossip in arbitrary graphs in terms of the edge-expansion factor.

For multihop networks we derive non-trivial tradeoffs between the number of wavelengths and the number of hops (rounds) necessary to complete the process. We obtain, among several results:

- Asymptotically tight bounds for bounded degree networks;
- Tight bounds for hypercubes, meshes, and toruses.

Some of our results generalise previously known ones; indeed the results of [13] and [23] can be seen as particular cases of our results, when only *one* wavelength is available.

Due to the space limits, all proofs are omitted. We refer to the full version [8] for all omitted proofs.

2 Notations and Definitions

We represent the network as a graph $G = (V(G), E(G))$. For physical reasons, each edge in G is to be considered bidirectional and consisting of a pair of unidirectional optical links [39, 30]. In graph-theoretic language, this is equivalent to say that the network should be represented by a *directed symmetric* graph. For sake of simplicity, we prefer to consider G as an *undirected* graph. However, we will be always careful to count the number of signals crossing an edge taking into account their directions, that is, our algorithms will always assign *different* wavelengths to signals crossing an edge in the *same* direction. We will use the term graph and network interchangeably. The number of vertices of G will be always denoted by n . Given $v \in V(G)$, we denote with $d(v)$ the *degree* of v , with d_{\max} and d_{\min} we denote the maximum and minimum degree of G , respectively.

Processes are accomplished by a set of calls; a call consists of the transmission of a message from some node x to some destination node y along a path from x to y in G . Each call requires one round and is assigned a fixed wavelength. A node can be involved in an arbitrary number of calls during each round, but we require that if two calls share an edge in the same direction during the same round then they must be assigned different wavelengths.

Given a network G , a node $x \in V(G)$, and an integer t , we denote by $wb(G, x, t)$ the minimum possible number of *wavelengths* necessary to complete the broadcasting in G in at most t rounds, when x is the source of the broadcast;

we set $\text{wb}(G, t) = \max_{x \in V(G)} \text{wb}(G, x, t)$. Analogously, with $\text{wg}(G, t)$ we shall denote the minimum possible number of *wavelengths* necessary to complete the gossiping process in G in at most t rounds.

Given G , a node $x \in V(G)$, and an integer w , we denote by $\text{tb}(G, x, w)$ the minimum possible number of *rounds* necessary to complete the broadcasting process in G using up to w wavelengths per round, when x is the source of the broadcast; we set $\text{tb}(G, w) = \max_{x \in V(G)} \text{tb}(G, x, w)$. We denote by $\text{tg}(G, w)$ the minimum possible number of *rounds* necessary to complete the gossiping process using up to w wavelengths per round.

The *edge-expansion* $\beta(G)$ of G [26], (also called *isoperimetric number* in [31, 42] and *conductance* in [27]) is the minimum over all subsets of nodes $S \subset V(G)$ of size $|S| \leq n/2$, of the ratio of the number of edges having exactly one endpoint in S to the size of S .

A graph G is k -edge-connected if k is the minimum number of edges to be removed in order to disconnect G , G is *maximally edge-connected* if its edge-connectivity equals its minimum degree.

A *routing* for a graph G is a set of $n(n-1)$ paths $R = \{R_{x,y} \mid x, y \in V(G), x \neq y\}$, where $R_{x,y}$ is a path in G from x to y . Given a routing R for the graph G , the load of an edge $e \in E(G)$, denoted by $\text{load}(R, e)$, is the number of paths of R going through e in either directions. The *edge-forwarding index* of G [21], denoted by $\pi(G)$, is the minimum over all routings R for G of the maximum over all the edges of G of the load posed by the routing R on the edge, that is, $\pi(G) = \min_R \max_{e \in E(G)} \text{load}(R, e)$. It is known that [42]

$$\pi(G) \geq \frac{n}{\beta(G)}. \quad (1)$$

Unless otherwise specified, all logarithms in this paper are in base 2.

3 Single-Hop Networks

In this section we consider the number of wavelengths necessary to realize the broadcasting and gossiping processes in single-hop (all-optical) networks.

In the single-hop model it is sufficient to study the number of wavelengths necessary when only *one* communication round is used. Indeed, any one-round algorithm that uses w wavelengths can also be executed in t rounds using $\lceil w/t \rceil$ wavelengths per round, that is,

$$\text{wb}(G, t) \leq \left\lceil \frac{\text{wb}(G, 1)}{t} \right\rceil, \quad \text{wg}(G, t) \leq \left\lceil \frac{\text{wg}(G, 1)}{t} \right\rceil. \quad (2)$$

On the other hand, the assumption of a single-hop system implies that if we have a realization of a process in t rounds using up to w wavelengths per round, we can easily obtain a new realization using wt wavelengths and one round. Therefore, in the sequel of this section we will focus on one-round algorithms; we will write $\text{wb}(G)$ and $\text{wg}(G)$ to denote $\text{wb}(G, 1)$ and $\text{wg}(G, 1)$, respectively.

3.1 Broadcasting

Given a graph G and a node $v \in V(G)$, when v is the source of the broadcasting process there must exist at least $(n-1)/d(v)$ calls of the $n-1$ originated at v that share a same edge incident on v . Therefore,

Lemma 3.1 For each graph G on n nodes $\text{wb}(G) \geq \left\lceil \frac{n-1}{d_{\min}(G)} \right\rceil$.

We give now an upper bound that allows to determine the exact value of $\text{wb}(G)$ for all maximally edge-connected graphs and, therefore, for most of the used interconnection networks.

Theorem 3.1 For each k -edge-connected graph G on n nodes $\text{wb}(G) \leq \left\lceil \frac{n-1}{k} \right\rceil$.

From Lemma 3.1 and Theorem 3.1 we get

Corollary 3.1 If G is maximally edge-connected then $\text{wb}(G) = \left\lceil \frac{n-1}{d_{\min}(G)} \right\rceil$.

The above corollary gives the exact value of the number of wavelengths necessary to broadcast in one round in various classes of important networks. By Mader's theorem [28], Corollary 3.1 gives the exact value of $\text{wb}(G)$ for the wide class of vertex-transitive graphs. In particular, we have

- for the d -dimensional hypercube H_d $\text{wb}(H_d) = \lceil (2^d - 1)/d \rceil$;
- for the $r \times s$ mesh $M_{r,s}$ $\text{wb}(M_{r,s}) = \lceil (rs - 1)/2 \rceil$;
- for the d dimensional torus C_m^d $\text{wb}(C_m^d) = \lceil (m^d - 1)/(2d) \rceil$;
- for any Cayley graph G of degree d $\text{wb}(G) = \lceil (n - 1)/d \rceil$.

For other classes of graphs G for which the edge connectivity is equal to d_{\min} and, therefore, for which $\text{wb}(G) = \left\lceil \frac{n-1}{d_{\min}(G)} \right\rceil$ by Corollary 3.1, see the survey paper [7].

3.2 Gossiping

In this section we study the minimum possible number of wavelengths necessary to perform gossiping in single-hop networks in exactly one round. In the following lemma we put in relation $\text{wg}(G)$ with $\pi(G)$.

Lemma 3.2 For each graph G it holds that $\text{wg}(G) \geq \pi(G)/2$.

Minimising the number of wavelengths is in general not the same problem as that of realizing a routing that minimises the number of paths sharing a same edge. Indeed, our problem is made much harder due to the further requirement of wavelengths assignment on the paths. In order to get equality in Lemma 3.2 one should find a routing R achieving the bound $\pi(G)/2$ for which the associated *conflict graph*, that is, the graph with a node for each path in R and an edge between any two paths sharing an edge in the same direction, is $\pi(G)/2$ -vertex

colorable. We also notice that the problem of determining the edge-forwarding index of a graph is NP-complete [41].

In the rest of this section we will put in relation the minimum possible number of wavelengths necessary to perform gossiping in G in one round with the edge-expansion of G . From Lemma 3.2 and (1) we get the lower bound $\mathbf{wg}(G) = \Omega(n/\beta(G))$. We now show that gossiping can be efficiently realized in any bounded degree graph with a number of wavelengths within a $(\log^2 n)/\beta(G)$ factor from the optimal. In order to gossip in one round one has to choose a path for each pair of nodes and use these paths concurrently, this is equivalent to the problem of embedding the nodes of the complete graph K_n in G and route the edges of K_n as paths in G . For a bounded degree graph G , Leighton and Rao [26] showed that this problem can be efficiently solved with congestion $O(\frac{n \log n}{\beta(G)})$ and dilation $O(\frac{\log n}{\beta(G)})$. Since each vertex in the conflict graph of the resulting routing has degree upper bounded by (congestion \times dilation) $= O(\frac{n \log^2 n}{\beta^2(G)})$, the greedy colouring algorithm can be used to colour the vertices of the conflict graph with $O(\frac{n \log^2 n}{\beta^2(G)})$ colours, that is, it can be used to assign $O(\frac{n \log^2 n}{\beta^2(G)})$ wavelengths to the paths of the routing so that no two paths sharing an edge have the same wavelength assignment. Summarising,

Theorem 3.2 *In any bounded degree graph G on n nodes $\mathbf{wg}(G) = O\left(\frac{n \log^2 n}{\beta^2(G)}\right)$.*

Computing $\beta(G)$ seems an hard computational problem (see [31]), therefore it can be useful also to relate $\mathbf{wg}(G)$ with easily computable parameters of G . In particular, we can obtain bounds on $\mathbf{wg}(G)$ in terms of the spectrum of matrices associated to G . Recalling that the Laplacian of a graph with adjacency matrix A and degree function $d(\cdot)$ is the $n \times n$ matrix with entries $d(x)\delta_{x,y} - A_{x,y}$, where $\delta_{x,y}$ is the Kronecker symbol, from Lemma 2.1 of [2], Theorem 4.2 of [31], Lemma 3.2, Theorem 3.2, and formulæ (1) of the present paper we get:

Theorem 3.3 *Let λ be the second smallest eigenvalue of the Laplacian associated to G . We have $\mathbf{wg}(G) = \Omega\left(\frac{n}{\sqrt{\lambda(2d_{\max} - \lambda)}}\right)$ and $\mathbf{wg}(G) = O\left(\frac{n \log^2 n}{\lambda}\right)$.*

We show now that for some classes of important networks the lower bound on $\mathbf{wg}(G)$ given in Lemma 3.2 can be efficiently reached.

In case of the path P_n on n nodes it is not hard to prove that the shortest path routing gives a set of paths that can be coloured with an optimal number of colours $\pi(P_n)/2 = \frac{1}{2} \left\lceil \frac{n^2}{2} \right\rceil$, so that all paths sharing an edge in the same direction have different colours. In the next theorems we determine $\mathbf{wg}(\cdot)$ for the ring, the torus, and the hypercube.

Theorem 3.4 *Let C_n be the ring on n nodes. Then*

$$\mathbf{wg}(C_n) = \left\lceil \frac{1}{2} \left\lceil \frac{n^2}{4} \right\rceil \right\rceil.$$

Theorem 3.5 *Let C_k^2 be the $k \times k$ torus. If k is odd then*

$$\mathbf{wg}(C_k^2) = k \lfloor k^2/4 \rfloor / 2,$$

if k is even then

$$k^3/8 \leq \mathbf{wg}(C_k^2) \leq (k+1)(k^2/8 + k/2).$$

Theorem 3.6 *Let H_d be the d -dimensional hypercube. We have*

$$\mathbf{wg}(H_d) = 2^{d-1}.$$

4 Multihop Networks

In this section we show that by exploiting the capabilities of the multihop optical model, a drastic reduction on the number of wavelengths can be obtained with respect to (2).

In the following, we will be mostly interested in investigating broadcasting algorithms. Indeed, as it is well known, the gossiping process can be accomplished by first accumulating all blocks at one node and then broadcasting the resulting message from this node. Since accumulation corresponds to the inverse process of broadcasting we get the obvious result

Lemma 4.1 *For each graph G and number of wavelengths w*

$$\mathbf{tb}(G, w) \leq \mathbf{tg}(G, w) \leq 2 \mathbf{tb}(G, w).$$

4.1 Lower Bounds

Lemma 4.2 *For each graph G on n nodes of minimum degree d_{\min} and maximum degree d_{\max}*

$$\mathbf{tb}(G, w) \geq \left\lceil \frac{\log(1 + (n-1)d_{\max}/d_{\min})}{\log(wd_{\max} + 1)} \right\rceil. \quad (3)$$

Lemma 4.3 *Given a graph G on n nodes of maximum degree d , let $\tau_0 = \mathbf{tb}(G, w)$. It is possible to perform gossiping on G in t rounds using w wavelengths only if*

$$2(n-1) \frac{(wd+1)^{t-\tau_0} - 1}{wd} + (2\tau_0 - t)(wd+1)^{t-1} \geq \pi(G)/(2w).$$

4.2 Upper Bounds

In order to obtain our general upper bound on the number of rounds to broadcast in G with a fixed number of wavelengths, we need the following covering property.

Definition 4.1 An s -tree cover for a graph $G = (V, E)$ is a family \mathcal{F} of subtrees of G such that:

1. $\cup_{F \in \mathcal{F}} V(F) = V$;
2. For each $F, F' \in \mathcal{F}$ it holds $|V(F) \cap V(F')| \leq 1$;
3. For each $F \in \mathcal{F}$ it holds $|V(F)| \leq s$.

The s -tree cover number of G is the minimum size of an s -tree cover for G .

The following result upper bounds the s -tree cover number of any graph; its proof also furnishes an efficient way to determine an s -tree cover which attains the bound.

Lemma 4.4 For each graph G on n nodes and bound s , the s -tree cover number of G is upper bounded by $2n/s$.

Before giving the upper bound on the broadcasting time in general graphs, we notice the following application of Lemma 4.4 to the function $\text{wb}(\cdot)$.

Theorem 4.1 For each k -edge connected graph G on n nodes

$$\left\lceil \frac{\sqrt{1 + (n-1)d_{\max}/d_{\min}} - 1}{d_{\max}} \right\rceil \leq \text{wb}(G, 2) \leq \left\lceil \sqrt{\frac{2n}{k}} \right\rceil.$$

By using Lemma 4.4 we can prove a general upper bound on $\text{tb}(G, w)$ for any $w \geq 2$; in the case $w = 1$ the bound $\text{tb}(G, 1) \leq \lceil \log n \rceil$ has been given in [13].

Theorem 4.2 For each graph G on n nodes and number of wavelengths $w \geq 2$

$$\text{tb}(G, w) \leq \lceil \log n / (\log(w+1) - 1) \rceil.$$

By Lemma 4.2 and Theorem 4.2 we get

Corollary 4.1 For each bounded degree graph G on n nodes

$$\text{tb}(G, w) = \Theta(\log_{w+1} n).$$

We give now a sharper bound on the broadcasting time in the d -dimensional hypercube in terms of the maximum number of wavelengths. In the special case $w = 1$ it is proved in [23] that $\text{tb}(H_d, 1) = \Theta(d/\log d)$.

Theorem 4.3 For each d and number of wavelengths w

$$\left\lceil \frac{d}{\log(wd+1)} \right\rceil \leq \text{tb}(H_d, w) \leq c(d, w) \frac{d}{\log(wd+1)} + 2$$

with $c(d, w) \leq 4$ and $\lim_{d \rightarrow \infty} c(d, w) \leq \begin{cases} 1 & \text{if } \log w = o(2^d), \\ 1 + \frac{\log e}{e} & \text{otherwise.} \end{cases}$

For meshes and toruses we have the following result

Theorem 4.4 Let M_{k_1, k_2} and C_{k_1, k_2} be the $k_1 \times k_2$ mesh and torus, respectively, on the $n = k_1 k_2$ nodes in the set $\{(x_1, x_2) : 0 \leq x_i < k_i, i = 1, 2\}$. For each w , k and $k_1, k_2 \leq k$

$$\left\lceil \frac{\log(2n-1)}{\log(4w+1)} \right\rceil \leq \text{tb}(M_{k_1, k_2}, w) \leq \left\lceil \frac{\log k}{\log \lfloor \sqrt{4w+1} \rfloor} \right\rceil + 1,$$

$$\left\lceil \frac{\log n}{\log(4w+1)} \right\rceil \leq \text{tb}(C_{k_1, k_2}, w) \leq \left\lceil \frac{\log k}{\log \lfloor \sqrt{4w+1} \rfloor} \right\rceil.$$

5 Conclusions and Open Problems

In this paper we have initiated the study of efficient collective communication in switched optical networks. Although we have obtained a number of results, several open problems can be investigated for future lines of research. We list the most important of them here.

- The computation complexity of the quantities $\text{wb}(G, t)$, $\text{wg}(G, t)$, $\text{tb}(G, w)$, $\text{tg}(G, w)$ deserves to be investigated. It is likely that for some of them it is NP-hard. In this view, approximation algorithms in the sense of [40] and [19] could be interesting to design.
- Our algorithm require a centralised control. This seems not to be a severe limitation in that the major applications for optical networks require connections that last for long periods once set up; therefore, the initial overhead is acceptable as long as sustained throughput at high data rates is subsequently available [38]. Still distributed algorithms are worth investigating.
- We did not consider fault tolerant issues here. See the recent survey [35] for an account of the vast literature on fault-tolerance in traditional networks.

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