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Sustainable Tourism

Régis Y. Chenavaz, Marta Leocata, Malgorzata Ogonowska, Dominique Torre

March 2022

Abstract

Current approach to the management of World Heritage sites and the associated tourism is not always effective. Indeed, operators of Chteau de Versailles, Alhambra, and Vatican Palace trade-off visitors' revenues against damage to these sites. We study the effective dynamic behavior of a firm managing a heritage site. In an optimal control model, the operator sets pricing and heritage conservation policies. Heritage sites benefit from spending on conservation but are deteriorated by large numbers of visitors. Visitor numbers increase with the attractiveness of the site but decrease with the entrance fee. Our results characterize optimal pricing and heritage conservation policies over time and provide a stronger foundation for sustainable tourism policies.

Keywords: sustainable tourism, World Heritage Sites, dynamic pricing, conservation policies, optimal control

1 Introduction

The media claim that mass tourism is causing irreversible damage to cultural heritage. For instance, the tomb of Tutankhamun in Egypt (Reuters-ABC, 2019), and Buddhist pagodas in Myanmar (Ban, 2016) have suffered severe damage due to excessive numbers of visitors to these sites, and following an initial period of intensive exploitation, the authorities have discontinued visitor access. In other, more numerous cases, the deterioration has been less spectacular, with the result that either access has continued or measures to reduce this heritage damage came too late to prevent permanent depreciation of the heritage. Take for instance the case of Venice over which the Editorial Board of *The Economist* (Board, 2019) recently highlighted concern. For the most part, myopic foresight is at the roots of poor management of heritage sites. Over time, it is possible to observe two periods when heritage is destroyed or severely damaged. In the first period, operators with a myopic long-term view exploit the resource intensively, as in the case of the city of Pompeii (Lyons, 2008). In the second period, sudden awareness of the depreciation of the heritage results in limitations to the number of tourists allowed to visit the site. The *Financial Times* highlights this limitation policy in relation to China's heritage sites, like the Forbidden City or the Great Wall (Chaguan, 2019). However, this sequence is ineffective: it is insufficiently proactive in the first period; in the second period, it deprives the public or private authorities responsible for the site of the funding necessary for its preservation and overly restricts visitors' access to the heritage. This situation requires a deeper consideration of the strategies needed to achieve a sustainable balance between economic exploitation and long-term preservation of heritage.

Sustainable management of heritage is of interest to practitioners and scholars. For instance, Hansen et al. (1998) and Kim et al. (2007) investigate the different methodologies to evaluate heritage projects; Garrod and Fyall (2000) and Choi et al. (2010) look at the

management of heritage tourism; Emerton et al. (2006) consider the sustainable financing of protected heritage; Voltaire (2017) analyses the pricing of natural heritage sites. The literature shows the interest in heritage site management. Despite its societal relevance, the pricing over time of sustainable heritage sites is still missing from the literature. Our paper fills the gap by discussing the importance of dynamic pricing solutions for heritage sustainability. Dynamic pricing concerns the price policy of a product over its lifecycle. The product price, which may change over the lifecycle, accounts for its effects on both current and future profits. Our research points to the important role of time awareness and the future effects of expensive policies aimed at heritage conservation. The use of a dynamic control setting seems appropriate for this study. It allows the simultaneous application of an adapted pricing policy control for the number of visitors and a preservation or protection policy funded in part by the revenues generated by visits. Both policies should be rationally generated using a single intertemporal objective function of the heritage operator, integrating the influence of expenses devoted to maintaining the heritage site with the quality of this heritage and the visitors' propensity to pay.

We propose a model in which heritage degrades with visits and may recover by appropriate expenditure. A private operator is the concessionaire of a World Heritage Site, seen as a profit center. Optimization of the operator's profit determines dynamically both the entrance fee, that is the price, paid by visitors and the amount of spending required to maintain the quality of the heritage. We consider a general form of demand for visits, but also introduce the usual additive and multiplicative specifications. We limit the study to the monopolistic case, which corresponds to many situations where a private operator is the sole manager of a unique heritage site. This case is relevant to the most outstanding World Heritage Sites such as the Eiffel Tower in Paris, the Forbidden City in Beijing, and the Vatican Palace in Rome. For simplicity, we omit other ways for the operator to extract revenues, like selling souvenirs or

filling hotels and restaurants.

We contribute to the literature on dynamic pricing and tourism management by examining dynamics policies for World Heritage Sites. The following findings emerge from our analysis. First, the price proposed by an operator aware of the need to preserve the future value of the heritage is higher than the price of a myopic operator (Proposition 1) and depends directly on demand elasticities (Proposition 2). The evolution of optimal heritage conservation expenses determines the evolution of the shadow price applied to the heritage over time (Proposition 3). If the operator is aware of the depreciation caused to the heritage by visitors, the price dynamics will depend on both the heritage dynamics and the shadow price of the heritage (Proposition 3). Over time, for more specific classes of the demand function, stronger pricing policies will emerge (Propositions 4-5).

Our article tries to provide a better understanding of the tension between short-term profit maximization and long-term conservation of World Heritage Sites. More broadly, it proposes dynamic pricing and heritage conservation rules for an operator managing a site, taking into account both profit and heritage preservation issues. These rules address some concerns of residents and administrative authorities whose interests are in economic development and site conservation. Thus, our results suggest more sustainable practices regarding tourism management.

The rest of the paper is as follows. Section 2 reviews the related literature. Section 3 formulates the model and section 4 describes the analysis. The results of the analysis appear in section 5. Section 7 is the discussion and section 8 concludes the paper.

2 Related Literature

This section surveys the existing literature on sustainable tourism and, more specifically, on heritage sites and on optimal control applications to dynamic pricing and tourism management. The section ends by stressing the essential ingredients of our approach, as compared to the literature.

2.1 Sustainable Tourism and Heritage

We first focus on sustainable tourism. The most useful definition of *cultural heritage site* comes from Chen and Lee (2017, p. 259):

The term “Cultural Heritage Site” encompasses cultural ruins, archeological artifacts, monuments, buildings, and other heritage sites (...) placed under protection.

Cultural heritage sites contribute to a society through aesthetic quality, spiritual meanings, historical and symbolic value; they represent for the community a shared experience and sense of continuity (Wright and Eppink, 2016). Thus, cultural heritage is at the heart of sustainable development due to its contribution to the richness of the living environment for present and future generations and is included in the United Nations Sustainable Development Goals (United Nations and Affairs, 2015) and the United Nations Educational, Scientific and Cultural Organization’s (UNESCO) Sustainable Development Agenda (Boccardi and Duvelle, 2013).

Since the 1990s, environmental protection and sustainability have become major issues. In this line tourists, and generally, citizens are more aware of environmental issues and conscious of damages caused by mass tourism and modify their perceptions and preferences, as surveyed

by Meleddu (2014). As a result, demand for sustainable products has emerged in this sector, and major tourist destinations are seeking to preserve high quality environmental, cultural, and heritage attributes, and are exploring sustainable development strategies and techniques in order to be still attractive in the future (Hassan, 2000). Following Wright and Eppink (2016, p. 278), maintenance of heritage include preservation, conservation, and renovation.

The fact that cultural heritage sites present the characteristics of a public good and contribute to social welfare justifies public subsidies to cover repair, maintenance, and enhancement costs (Choi et al., 2010, pp. 213-214). However, in most cases, the existing public subsidies are not sufficient to cover all of these costs (Voltaire, 2017). In this context, it has become especially important to identify additional sources of funding and to develop strategies to help to maintain the sites.

One solution to maintain heritage would be an adequate entrance price. This strategy is in line with the “user-pays principle” (Throsby, 2012). Site operators did not subscribe to this idea for many years, preferring to charge a “token” low entrance fee (Garrod and Fyall, 2000). Yet, a higher admission price helps site managers to control the number of visitors, matching site capacity constraints. A consequent entrance price represents a good solution to overcrowding issues and capacity management (Wu et al., 2017).

Price discrimination also represent a policy tool enabling underprivileged to visit heritage. People accept price discrimination, as it is based on an assessment of visitors’ willingness to pay or residents’ willingness to accept the costs of maintaining the site (Tuan and Navrud, 2008). Plus, research show that consumers’ willingness to pay is higher for sustainable and better preserved destinations (Dodds et al., 2010; Hedlund, 2011). By taking into account the preferences of the environmentally conscious population, the operator modernizes the heritage site, making it more sustainable (Brau, 2008; Minciu et al., 2010).

A dynamic pricing perspective is missing from the above literature specialized on sustain-

able heritage. We contribute to this literature by adding the temporal element.

2.2 Dynamic Pricing and Tourism

We review now prior literature on optimal control applications to dynamic pricing and tourism. We start with the literature on dynamic pricing. In a seminal article, Kalish (1983) examines the dynamic pricing of a new product. Price and quality are studied together by Vörös (2006) whereas, Chenavaz (2012) and Vörös (2013) examine both pricing and productivity policies. The impact of product quality on selling price is discussed by Chenavaz (2017) and is generalized by Ni and Li (2018) with goodwill and by Vörös (2019) with a salvage value. The above research uses structural (as opposed to parametric) demand and cost functions and provides general and robust results. Dye and Yang (2016) and Xue et al. (2016) characterize the dynamic optimal pricing of products that deteriorate over time. The literature above is essential to understand the price-quality relationship. Therefore, it represents a sound basis to investigate our price-heritage relationship.

Now, we turn our attention to the literature on dynamic tourism. The link between environment and tourism is investigated in Greiner et al. (2001) and Sirilersuang and Pongkijvorasin (2018). Heritage sites must be maintained over time (Hutter and Rizzo, 1997) and defended against threats related to natural degradation and also man-made actions such as wars, political and other instabilities, including terrorism, outright neglect, uncontrolled economic exploitation, and excessive tourism pressure (Feichtinger et al., 2001). Destinations's attributes attract tourists, which in turn damage those attributes. Tourism damage is modeled by Kort et al. (2002) in the case of a dynamic trade-off between environment and tourism.

More optimal control applications have considered tourism. Piga (2003) studies Pigouvian taxation in tourism. Candela and Cellini (2006) consider the tourism market with an oligopoly

setting. Claude and Zaccour (2009) look at reputation in the tourism market. Hernández and León (2007, 2013) examine tourism with a neoclassical demand function. Piazza and Roy (2015) consider deforestation, which is similar to site degradation.

Dynamic pricing and tourism literature considered neither the link between price and heritage quality nor the sustainability of heritage. Our work contributes to this literature by analyzing the price-heritage relationship and the sustainable heritage.

2.3 Our Approach

The above research on tourism management did not consider the price as a lever to control the visitor number, and maintain or improve heritage. Instead, the price only raises revenues, but without an explicit goal and with no systematic account for side effects. Despite its managerial relevance, no research examined the entrance fee over time of a World Heritage Site. The gap in the literature applying optimal control to tourism management is that of providing a policy that jointly maximizes profits and maintain heritage through a smart entrance fee mechanism. In other words, there is a need to examine sustainable joint pricing and heritage intertemporal policies. Such entrance free policy would be profitable in the long run only if the desirable condition of heritage preservation is met. Our research fills the remaining gap.

Our paper supplements the above-cited studies by proposing an innovative approach based on optimal control theory which, to our knowledge, was unused to analyze the joint issue of pricing and cultural heritage site conservation. We analyze the explicit over time trade-off related to the number of visitors to a site since more visitors mean increased current revenues, but reduced future heritage quality. In our model, the site operator decides the level of the entrance fee and the investment in heritage quality. The entrance fee has a direct effect on the number of visitors, and in turn an indirect effect on the degradation of the site.

At the same time, heritage conservation attracts visitors, allowing the extraction of higher rents and thus higher revenues. For simplicity, we don't model other ways of generating profits, such as souvenir shops or hospitality facilities. Following a structural approach, we use general formulations for the demand and site maintenance functions to provide general results regarding the dynamic pricing and maintenance policies of a heritage site operator.

3 Model Formulation

In this section, we formulate our model. We propose an optimal control model of a monopolistic operator, that is a concession managed as a firm, in charge of a heritage site. The operator sets the price, that is the entrance fee, $p(t)$, and conservation expenditure, also called preservation investment, $u(t)$, over time, t . The conservation expenditure goes to maintaining or improving the attractiveness of the heritage site, $h(t)$. Visitors to the site, who prefer a lower price and better-preserved heritage, are aggregated in the demand $D(p(t), h(t))$. On the one hand, increasing the heritage attractiveness represents a current cost since it implies conservation expenditure; on the other hand, it generates future revenue by stimulating future demand. A higher number of visitors both increases the operator's current revenue and degrades future heritage quality. So, if no preservation action is undertaken, the site declines over time, implying the heritage becomes less attractive. The operator's profit results from the revenues minus the cost (investment in site conservation), that is $\pi(p(t), u(t), h(t)) = p(t)D(p(t), h(t)) - u(t)$. The operator trades off current profit, tied to higher demand, and future profit, linked to the preservation of heritage. This intertemporal focus requires a dynamic analysis. In the following sub-sections, we propose an optimal control framework to analyze the model. The operator solves an infinite horizon problem where the time variable t in $[0, +\infty)$ is continuous. For ease of reading, Table 1 defines the

main notations used in the model.

Table 1: Main Notations

t	= continuous time,
r	= discount factor, parameter,
$p(t)$	= price (or entrance fee) at time t , control variable,
$h(t)$	= heritage attractiveness at time t , state variable,
$D(p, h)$	= demand (or affluence) of visitors,
$u(t)$	= expenditure on heritage conservation at time t , control variable,
α	= impact of visitors on heritage degradation, parameter,
δ	= autonomous decay of heritage, parameter,
$\frac{dh}{dt}$	= $-\alpha D(p, h) - \delta h + K(u)$ = heritage attractiveness dynamics,
$\lambda(t)$	= shadow price of heritage at time t , co-state variable,
$\pi(p, u, h)$	= $pD(p, h) - u$ = current profit,
e_x	= $\left \frac{\partial D}{\partial x} \frac{x}{D} \right $ = elasticity of demand D with respect to x .

3.1 Demand

Demand aggregates the number of visitors to the site. Using a general functional form, demand (or visitor affluence) function $D : (\mathbb{R}^+)^2 \rightarrow \mathbb{R}^+$ is twice continuously differentiable. The demand for the site depends on the entrance fee and the site's attractiveness. Demand reads as, $p, h \geq 0$,

$$D = D(p(t), h(t)). \tag{1}$$

Hereafter for simplicity, unless there is a risk of confusion, we omit the dependence from certain variables, in particular the time variable. Demand decreases with price and increases with heritage quality (Kort et al., 2002; Claude and Zaccour, 2009; Sirilersuang and Pongkijvorasin, 2018). There is a diminishing impact of heritage quality on demand. Moreover, it is more difficult to increase demand by decreasing the price if heritage attractiveness is high rather than if it is low. In other words, visitors are less price-sensitive for a higher level of heritage attractiveness. (A similar assumption of *submodularity* of the demand function

concerning the arguments is made in Jørgensen and Zaccour 2012; Chenavaz 2017; Ni and Li 2018.) These four assumptions about the demand function translate formally into, $p, h \geq 0$,

$$\frac{\partial D}{\partial p} < 0, \quad \frac{\partial D}{\partial h} > 0, \quad \frac{\partial^2 D}{\partial h^2} < 0, \quad \frac{\partial^2 D}{\partial p \partial h} \leq 0. \quad (2)$$

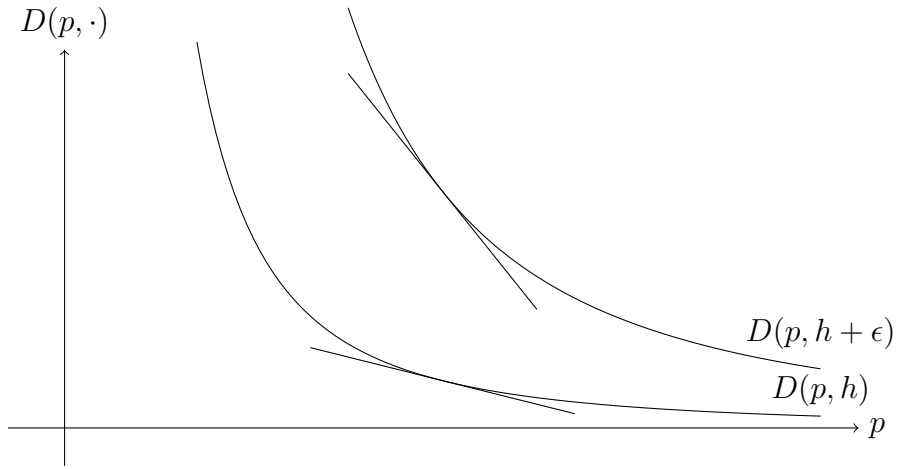


Figure 1: In this plot, we give a graphical interpretation of condition $\frac{\partial^2 D}{\partial p \partial h} \leq 0$. We plot the function D , just as a function of p and by fixing different values of heritage attractiveness, $h, h + \epsilon$. Since, we assume that visitors are less price sensitive if heritage attractiveness is high, we have that $D(p, h)$ tend to be close to zero, faster than $D(p, h + \epsilon)$. Indeed it is clear from the plot that $\frac{\partial D(p, h)}{\partial p} > \frac{\partial D(p, h + \epsilon)}{\partial p}$. Then, writing the different quotient with respect to the variable h , the previous implies that $\frac{\partial^2 D(p, h)}{\partial p \partial h} \leq 0$.

Assumptions (2) are confirmed using either a linear demand function $D = a_0 - a_1 p + a_2 h$ or a Cobb-Douglas demand function $D = a_0 p^{-a_1} h^{a_2}$ with $a_0, a_1, a_2 > 0$. Additionally, the demand function is assumed to be not “too” convex in price, $p, h \geq 0$,

$$2 - D \frac{\frac{\partial^2 D}{\partial p^2}}{\frac{\partial D}{\partial p}} > 0, \quad (3)$$

which is a condition required for profit optimization to ensure a unique profit maximizing point. This is a standard assumption in the dynamic pricing literature using structural demand functions. (See for instance Dockner et al. 2000; Vörös 2006, 2019; Jørgensen and Zaccour 2012; Chenavaz 2012, 2017; Ni and Li 2018.)

So that the sufficiency condition of Mangasarian holds¹, we also need to assume that

$$\left(-D \frac{\partial^2 D}{\partial h^2} \cdot \left(2 - D \frac{\frac{\partial^2 D}{\partial p^2}}{\left(\frac{\partial D}{\partial p} \right)^2} \right) - \left(\frac{\partial D}{\partial h} - D \frac{\frac{\partial^2 D}{\partial p \partial h}}{\frac{\partial D}{\partial p}} \right)^2 \right) > 0 \quad (4)$$

The Mangasarian condition (4) implies $\frac{\partial^2 D}{\partial h^2} < 0$ in (2). Although (4) appears difficult to check, it is satisfied in our cases of interest. Eventually, this condition (4) allow us to prove that the controls furnished by the maximum principle, are actually optimal point for the optimization problem.

3.2 Heritage

Next, we characterize heritage site attractiveness. The operator spends $u(t) \in \mathbb{R}^+$ to preserve the site heritage $h(t)$. Expenses for heritage conservation $u(t)$ and heritage attractiveness $h(t)$ are control and state variables respectively. The heritage conservation function $K : \mathbb{R}^+ \rightarrow \mathbb{R}$ is twice continuously differentiable. Heritage preservation depends on the associated expenditure (see for instance Throsby 1999; Creigh-Tyte 2000; Rizzo and Throsby 2006; Artal-Tur et al. 2018; Sirilersuang and Pongkijvorasin 2018) and reads $K = K(u)$.

Expenditure u increases heritage attractiveness h , but with diminishing returns (see McDonagh and Nahkies 2010; Rama 2012). It can be quite easy, and therefore cheap, to improve the quality of a low-quality heritage site but it can be difficult, and therefore expensive, to

¹The Hamiltonian (8) must be jointly concave in the control variable p and u and in the state variable h . See Mangasarian (1966) and Kamien and Schwartz (2012, Section 15).

improve an already high-quality heritage site. Thus, mathematically, we have, $u \geq 0$,

$$K' > 0, K'' < 0. \quad (5)$$

The dynamic of heritage attractiveness $h(t)$ can be written, for all $t > 0$,

$$\frac{dh(t)}{dt} = -\alpha D(p(t), h(t)) - \delta h(t) + K(u(t)), \text{ with } h(0) = h_0. \quad (6)$$

Formulation (6) accounts for several features of heritage dynamics. First, heritage decreases with the number of visitors, with $\alpha > 0$ the sensitivity of heritage dynamics to the visits (Kort et al., 2002; Claude and Zaccour, 2009; Sirilersuang and Pongkijvorasin, 2018).² Second, heritage attractiveness declines at a constant proportional rate $\delta > 0$.³ Heritage preservation depends on the associated expenditure. Indeed, if no investment maintains a site, then it deteriorates naturally over time (see for instance Patin 2010; Tan et al. 2018).⁴ Third, heritage can be improved thanks to the heritage conservation policy K as discussed above (Kort et al., 2002; Sirilersuang and Pongkijvorasin, 2018). Integrating (6) measures the stock (or cumulative level) of heritage attractiveness to the flow of demand, autonomous depreciation, and expenditure on heritage conservation $h(t) = h_0 + \int_0^t [-\alpha D(p(s), h(s)) - \delta h(s) + K(u(s))] ds$.

²Note that α cannot be “too” large, otherwise the number of visitors falls to zero, $D = 0$, which means that the site is closed to the public to be preserved. This corner solution has been implemented in some sites, such as Lascaux Cave, which was closed to the public in 1963 (see Bastian and Alabouvette 2009; Martin-Sanchez et al. 2015). Since then, only a precise replica of the cave can be visited. Interior solutions, $D > 0$, are the more common cases of sites open to the public. Because we are interested in accessible sites, we rule out $D = 0$ by imposing an upper bound on α .

³The case $\delta < 0$ studied in Hernández and León (2007, 2013); Sirilersuang and Pongkijvorasin (2018) corresponds to a natural recovery process. This case makes sense in their context of the natural site, but not in our context of World Heritage Site.

⁴The surveys by Jørgensen et al. (2010) and by Bencheikroun and Long (2012) provide more information on the proportional depreciation of the state variable in sustainable economies.

3.3 Profit

The current profit function $\pi : (\mathbb{R}^+)^3 \rightarrow \mathbb{R}$ is twice continuously differentiable. Profit corresponds to current revenue minus conservation expenses. The heritage expense u , is decided independently of demand. In this economic sense it represents a fixed cost. The cost of a visit c , that is the unit production cost, is constant. Without loss of generality, for simplicity, c is normalized to zero. Based on the revenues and costs, the profit function can be written, $p, u, h, D \geq 0$,

$$\pi(p, u, h) = pD(p, h) - u.$$

Here, conservation expenses maintain or increase heritage, modeled as the capital stock. In turn, increased heritage attractiveness stimulates visitor interest, allowing the operator to extract higher rents. Formally, $\frac{\partial \pi}{\partial h} > 0$ follows from $\frac{\partial D}{\partial h} > 0$ assumed in (3).

3.4 Operator's Dynamic Optimization Problem

The operator maximizes its intertemporal profit (or present value of the profit stream) over the planning horizon by choosing conservation and pricing policies simultaneously, accounting for the dynamics of heritage. For simplicity, we let the salvage value of heritage be null. Let the discount rate be r in \mathbb{R}^+ and the objective function of the firm be

$$\begin{aligned} & \max_{u, p \geq 0} \int_0^{\infty} \exp(-rt) \pi(p(t), u(t), h(t)) dt, & (7) \\ & \text{subject to } \frac{dh(t)}{dt} = -\alpha D(p(t), h(t)) - \delta h(t) + K(u(t)), \text{ for all } t > 0, \text{ with } h(0) = h_0. \end{aligned}$$

3.5 Model Intuitions

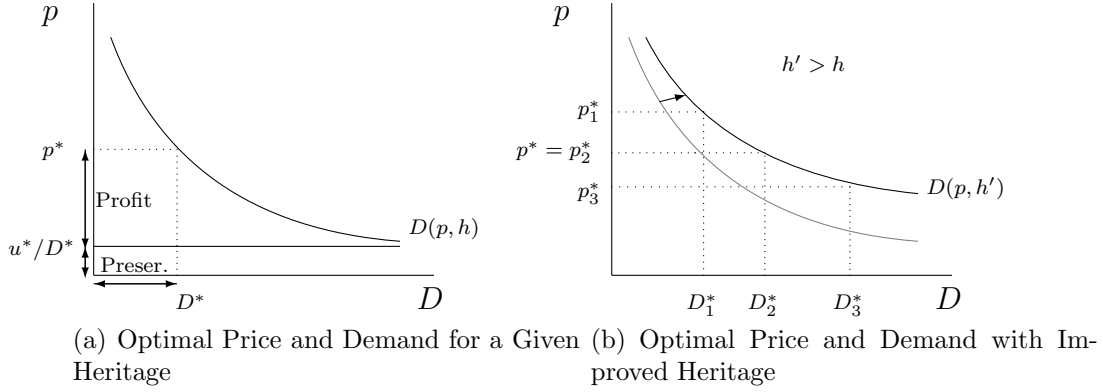


Figure 2: The Effect of Heritage on Price and Demand

Before presenting our analytical results, we discuss the intuitions behind the effects at play in our model. Figure 2 includes two panels, namely Figure 2(a) which depicts the situation of an operator for a given heritage, h , and Figure 2(b) which describes the new situation after an improvement to the heritage such that $h' > h$. Together they provide intuitions about the link between heritage and the optimal price. The star notation $*$ refers to the optimal value, which maximizes the intertemporal profit. More specifically, Figure 2(a) determines the optimal price p^* for heritage h and demand $D(p, h)$. Following the pricing rule (19), the optimal price p^* sums the markup $-\frac{D}{D_p}$ and the impact of future heritage $\alpha\lambda$. Substitution of $p = p^*$ in $D(p, h)$ gives the optimal sales $D^* = D(p^*, h)$. Profit is represented by a rectangle in which the length is the markup and the width is sales. The profit rectangle increases in size with both markup and sales. Figure 2(b) depicts the impact of an improvement to the heritage $h' > h$ on price. Heritage stimulates visitor interest. Therefore demand shifts both upward (greater markup for the same level of sales) and to the right (more sales for the same price), and $D(p, h') > D(p, h)$. A more attractive heritage exerts an ambiguous effect on price. Indeed there are three optimal price candidates for profit optimality: $p_1^* > p^*$, $p_2^* = p^*$,

and $p_3^* < p^*$. The operator trades off markup and sales which work in opposite directions along with the demand function. Profit may be maximized either by a higher markup and lower sales, as with p_1^* , or by a lower markup and higher sales, as with p_3^* . In the situation p_2^* , the markup and sales effects cancel each other out.

Section 4 quantifies the effects on the price-heritage policies which are depicted qualitatively in Figure 2. It is necessary to derive precise analytical results to ensure that the operator maximizes profit by applying the appropriate pricing and conservation policies.

4 Model Analysis

Here we analyze the model. The shadow price (or current-value adjoint variable) $\lambda(t)$ represents the marginal value of the heritage at time t . The current-value Hamiltonian H with the current-value adjoint variable (or shadow price) λ for heritage dynamics reads,

$$u, p, h \geq 0, \lambda \text{ in } \mathbb{R},$$

$$H(u, p, h, \lambda) = pD(p, h) - u + \lambda [-\alpha D(p, h) - \delta h + K(u)]. \quad (8)$$

The Hamiltonian H measures the intertemporal profit from summing the current profit $pD - u$ and the future profit $\lambda[-\alpha D - \delta h + K]$. The maximum principle for our optimization

problem writes, for all $t \geq 0$. Recall that the $*$ notation refers to the optimal solution.

$$\left\{ \begin{array}{l} \frac{dh^*}{dt} = -\alpha D(p^*(t), h^*(t)) - \delta h^*(t) + K(u^*(t)), \\ \frac{d\lambda^*}{dt} = -p^* \frac{\partial D(p^*(t), u^*(t))}{\partial h} + \left(r + \alpha \frac{\partial D(p^*(t), u^*(t))}{\partial h} + \delta \right) \lambda^*(t), \\ h^*(0) = h_0, \\ \lim_{t \rightarrow +\infty} e^{-rt} \lambda^*(t) = 0 \\ H(u^*(t), p^*(t), h^*(t), \lambda^*(t)) = \max_{u, p \geq 0} H(u, p, h^*(t), \lambda^*(t)), \text{ for each } t \geq 0. \end{array} \right. \quad (9)$$

The dynamic of the shadow price is combined with the transversality condition for a free terminal state and infinite terminal time. See Aseev and Kryazhimskiy (2004) for a theoretical explanation of the transversality condition and Chenavaz et al. (2020) for a recent application on dynamic pricing.

Lemma 1. *The quadruplets $(u^*(t), p^*(t), h^*(t), \lambda^*(t))$, solution of the system (9) return a local maximum for the optimization problem (7). If (9) admits an inner solution, then, it is given by the resolution of the following system,*

$$\left\{ \begin{array}{l} \lambda^* \cdot K'(u^*) = 1, \\ p^* = -\frac{D(p^*, h^*)}{\frac{\partial D(p^*, h^*)}{\partial p}} + \alpha \lambda^*. \end{array} \right.$$

Moreover, $\lambda^*(t) \geq 0$ and $u^*(t), p^*(t) \geq 0$ for all $t \geq 0$.

Proof. The proof is in Appendix A.2. □

In the remainder of the article, we eliminate the $*$ notation; all equations refer to the optimal solution, unless explicitly written. The shadow price of heritage, λ , represents the

impact on future profits of a one-unit increase in current heritage attractiveness. As such, λ exerts an influence on the intertemporal trade-off between current and future profits and is worthy of further investigation. Note the heritage elasticity of demand is $e_h = \frac{\partial D}{\partial h} \frac{h}{D}$. Substituting e_h and (19) in (9) implies for all $t > 0$

$$\frac{d\lambda}{dt} - \left(r + \delta + \alpha \frac{\partial D}{\partial h} \right) \lambda = -e_h \frac{pD}{h}, \text{ with } \lim_{t \rightarrow +\infty} e^{-rt} \lambda(t) = 0.$$

Lemma 2. *The value of λ over time can be written for all $t \geq 0$*

$$\lambda(t) = \left(\int_t^\infty e_h \frac{pD}{h} \exp \left(- \int_t^\ell \left(r + \delta + \alpha \frac{\partial D}{\partial h} \right) ds \right) d\ell \right).$$

Proof. The proof is in Appendix A.1. □

The shadow price of heritage, λ , increases with the heritage elasticity of demand, e_h , and the heritage deflated revenue, $\frac{pD}{h}$. Also, λ increases with the discount rate, r , and increases with autonomous heritage decay, δ . Eventually, λ decreases with the degrading impact of visitors, α , because $\frac{\partial D}{\partial h} < 0$.

5 Results

We present our analysis results below. The main objective of the model is to determine the optimal dynamic pricing and conservation policies of a heritage site operator. Remember that for simplicity, when presenting the results we may omit some function variables, especially the time variable. We determine the pricing rules through Propositions 1-2. We consider variations in conservation and pricing levels in Propositions 3 for a general demand function formulation. We specify the demand function and obtain stronger results for the price

variations in Propositions 4-5. All proofs are in Appendix A.

5.1 Pricing Rules

The optimal price p , comes from the first-order optimality condition,

$$p = -\frac{D}{\frac{\partial D}{\partial p}} + \alpha\lambda.$$

If the firm ignore the long run effect of the shadow price of heritage ($\lambda = 0$), then the static (myopic) price is considered,

$$p_{static} = -\frac{D}{\frac{\partial D}{\partial p}}.$$

See Fruchter et al. (2006); Fruchter (2009) for a similar distinction. Here, we derive the two main pricing rules, namely Propositions 1 and 2.

Proposition 1. *For all $t \geq 0$, the difference between the optimal dynamic (foresighted) and static (myopic) price reads*

$$p - p_{static} = \frac{\alpha}{K'} > 0.$$

Proof. The proof is in Appendix A.3 □

Note then that the difference between optimal dynamic and static prices does not depend on the discount factor, r , or on heritage deterioration, δ , while the shadow price of heritage depends on both of these parameters. The rationale for this result is that while the infinite horizon of the forecast tends to decrease with the shadow price of the heritage, the current optimal decision continues to be to set rather high fees if the objective is intertemporal. The dynamic pattern of λ is only technical. When, in each period, the operator of the heritage site also considers future profits, it always sets a higher price dynamic (foresighted) than the

static (myopic) price. The difference between the optimal dynamic and the static price is larger with greater visitor damage α and larger with the difficulty involved in repairing the site K' .

Proposition 1 compares with the reformulation of the price optimality condition of Kalish (1983, Equation (2c), p. 138)

$$p - p_{static} = -\lambda < 0,$$

In our setting, the substitution of (19) in (10) allows rewriting Proposition 1 as

$$p - p_{static} = \alpha\lambda > 0.$$

The difference is that in Kalish (1983), which considers the sales of a new product, the dynamic price is lower than the static price to stimulate current sales. Greater current sales, in turn, boosts future goodwill and profits. In contrast to our model of tourism management, the dynamic price is larger than the static price to reduce current visits. Eventually, fewer current visits maintain or improve future heritage and profits.

Consider now the variation in the optimal price related to variation in demand. Thus, note the price elasticity of demand, $e_p = -\frac{\partial D}{\partial p} \frac{p}{D}$.

Proposition 2. *For $t \geq 0$, the optimal relationship between price and heritage expenses implies*

$$p = \frac{e_p}{e_p - 1} \frac{\alpha}{K'}. \quad (10)$$

Proof. The proof is in Appendix A.4. □

Proposition 2 originates from the joint optimal price and conservation policies if the heritage site operator is maximizing profit. Proposition 2 builds on the classic static part $\frac{e_p}{e_p - 1}$ and on our model's specific dynamic part $\frac{\alpha}{K'}$ which integrates future heritage value. When

visitors' willingness to pay the fee has a strong negative effect on the heritage, the operator will charge a higher price to reduce the number of visitors, and preserve the heritage; p increases with λ . When spending on conservation allows easy recovery of the heritage, then the operator charges less to increase demand and accepts more visitors; p decreases with K' .

Proposition 2 is best understood compared with the classical dynamic pricing rule of Kalish (1983, Equation (3), p. 138)

$$p = \frac{e_p}{e_p - 1}(c - \lambda),$$

in which c is the marginal cost.

In our framework, the substitution of (19) in (10) rewrites Proposition 2 as a similar result to Kalish (1983) with

$$p = \frac{e_p}{e_p - 1}\alpha\lambda.$$

The difference is that we don't consider a marginal cost, which is low for a visitor and approximated by zero, but we integrate the impact of visitors on heritage degradation, α .

Proposition 2 is the consequence of the equalization of marginal revenues and marginal costs over the planning period. For maximal profit, and given the optimal control variable values, changes to marginal revenue must balance changes to marginal costs. In turn, changes to the heritage drive changes to pricing. The causality in the changes results in a relationship between price and heritage.

Proposition 2 highlights the optimal relationship between price and preservation expenses. However, it does not specify the conditions under which the optimal price increases or decreases after a heritage improvement. Proposition 2 equates marginal revenues to marginal costs at all times. It must hold throughout the planning period. To achieve the maximal profit, changes in marginal revenue must offset changes in marginal costs. In turn, changes

to the heritage drive price changes. The causality among these changes is linked to the relationship between price and heritage.

To the best of our knowledge, former optimal control models investigating tourism management do not provide any dynamic pricing rule (Feichtinger et al., 2001; Greiner et al., 2001; Kort et al., 2002; Piga, 2003; Candela and Cellini, 2006; Claude and Zaccour, 2009; Hernández and León, 2007, 2013; Sirilersuang and Pongkijvorasin, 2018). Consequently, Propositions 1 and 2 represent the first dynamic pricing rules applied to tourism management.

5.2 Variations of $p(t)$ and $u(t)$

Next, we study the evolution of heritage preservation expenses and of the price over time, t .

Proposition 3. *For all $t > 0$, the dynamics of price p reads*

$$\frac{dp}{dt} \left[\underbrace{2 - D \frac{\partial^2 D}{\partial p^2}}_{+} \right] = \frac{dh}{dt} \left[\underbrace{\frac{e_h p}{e_p h}}_{+} + \underbrace{D \frac{\partial^2 D}{\partial p \partial h}}_{-} \right] + \frac{d\lambda}{dt} \underbrace{\alpha}_{+}. \quad (11)$$

For all $t > 0$, the dynamics of u imitate the dynamics of λ . Formally,

$$\text{sgn} \frac{du}{dt} = \text{sgn} \frac{d\lambda}{dt}. \quad (12)$$

Proof. The proof is in Appendix A.5. □

The operator preserves the heritage in accordance with the shadow price of the heritage. In other words, the operator spends more for a higher shadow price and less for a lower shadow price. This simple rule of thumb is easy to apply.

Corollary 1. *The optimal conservation expense u is dependent on the sole dynamics of λ .*

Proof. The proof is in Appendix A.6. □

Corollary 1 simplifies the marketing-mix of the operator. Of course, p and u are tied through λ as attested in the first order conditions (19) and (20). However, when determining the conservation policy, the operator can focus on λ only, and disregard p .

Proposition 3, more specifically (11), relates the price dynamics $\frac{dp}{dt}$ to the heritage dynamics $\frac{dh}{dt}$ and the shadow price $\frac{d\lambda}{dt}$. On the left-hand side of (11), the second factor $\left(2 - D \frac{\frac{\partial^2 D}{\partial p^2}}{\left(\frac{\partial D}{\partial p}\right)^2}\right)$ is strictly positive because of the second-order condition (3). On the right-hand side, for a given λ , the effect of heritage on price originates from two opposing effects: the markup effect $\frac{e_h p}{e_p h}$ and the sales effect $D \frac{\frac{\partial^2 D}{\partial p \partial h}}{\left(\frac{\partial D}{\partial p}\right)^2}$:

- The markup effect $\frac{e_h p}{e_p h}$ measures the rise in consumers' willingness to pay for a better heritage. The markup effect is positive. It is more pronounced for a superior product with a low level of competition (a unique monument) such as the Eiffel Tower in Paris or the Alhambra in Granada; it is less pronounced for an inferior product with strong competition such as the many historical villages and city centers in Europe.
- The sales effect $D \frac{\frac{\partial^2 D}{\partial p \partial h}}{\left(\frac{\partial D}{\partial p}\right)^2}$ quantifies the increase in sales after a decrease in price combined with a heritage improvement. The sales effect is negative. It is larger for a mass consumption product such as the Colosseum in Rome or the Chateau de Versailles; it is lower for a niche product such as the military heritage in Northern France, which includes the Omaha Beach Memorial in Normandy.

Proposition 3 states also that with a fixed heritage ($\frac{dh}{dt} = 0$), the dynamics of p follows the dynamics of λ . In other words, the price increases, if the future heritage is more valuable; a higher price reduces the number of visitors, which preserves the heritage. Conversely, the

price decreases, if the future heritage is less valuable; a lower price increases revenues by attracting a higher number of visitors, with the consequence that heritage might decline.

In brief, Proposition 3 shows how the price dynamics account for the current markup and sales effects on the demand side, and the future heritage effect on the supply side.

6 Special Cases

The general demand function provides an information rule regarding the price-heritage relationship in Proposition 3. The caveat is that it can sometimes be difficult to apply. Specifying the demand function can be a way of providing clearer price-heritage relationship rules. The gain in applicability can be set off against the loss of generality.

Additive Separable Demand Function A demand function additively separable in price and heritage is the simplest and most natural modeling approach. With the additive separable case, demand (1) can be written, $p, h \geq 0$,

$$D = m(p) + n(h), \tag{13}$$

where $m' < 0$, $n' > 0$ and

$$2 - (m(p) + n(h)) \frac{m''(p)}{(m'(p))^2} > 0,$$

$$-n''(h) \left(2 - (m(p) + n(h)) \frac{m''(p)}{(m'(p))^2} \right) (m(p) + n(h)) > (n'(h))^2,$$

According to this assumptions, conditions (2)-(3)-(4) are automatically satisfied.

Proposition 4. *For $t > 0$, with an additively separable demand function, the dynamics of price p is written*

$$\frac{dp}{dt} \underbrace{\left[2 - (m(p) + n(h)) \frac{m''(p)}{(m'(p))^2} \right]}_{+} = \frac{dh}{dt} \underbrace{\left[-\frac{n'(h)}{m'(p)} \right]}_{+} + \frac{d\lambda}{dt} \underbrace{\alpha}_{+}. \quad (14)$$

Proof. The proof is in Appendix A.7. □

With an additive separable demand function, the markup effect (positive) holds, whereas the sales effect (negative) disappears. Therefore, price is more likely to increase with greater heritage because of the sole role of the markup effect. Yet, because of the dynamics of the shadow price, a positive price-heritage relationship is not sure.

Example 1. *Linear price demand function.*

Assume the demand function $D = a_0 - a_1p + a_2h^\alpha$, with $a_0, a_1, a_2 > 0$ and $0 < \alpha < 1$. The derivatives are written $\frac{\partial D}{\partial p} = -a_1$, $\frac{\partial^2 D}{\partial p^2} = \frac{\partial^2 D}{\partial p \partial h} = 0$. Substituting the derivatives in (14) yields

$$2 \frac{dp}{dt} = \frac{dh}{dt} \underbrace{\frac{a_2}{a_1} \alpha h^{\alpha-1}}_{+} + \frac{d\lambda}{dt} \underbrace{\alpha}_{+},$$

confirming that price increases with heritage, provided that λ is not “overly” decreasing.

The property of additive separability is observed for instance when demand aggregates two different types of individual populations, the first mainly motivated by the quality of the heritage and the second by the price of the visits. This is a case when the heritage attracts both an expert and an intrinsically motivated population, and families or average revenue standard visitors. Those two populations react differently to the dynamics of price and heritage. In the myopic case, as the level of preservation of the heritage is given in the short term, the demand of the first population is inelastic and the price depends on the price elasticity of the second population. During the time the same price is then maintained, but given the heritage depletion, the profit decreases due to the collapse of the experts’ demand, until the

destruction of the heritage. When the price is determined dynamically, it is higher during all periods, with the effect of restoring heritage but also the demand of the first population and the profit of the operator. In definitive, the dynamic approach is in line with experts' interest while the static method would be, in the short run, in line with the wishes of low revenue tourists.

Multiplicative Separable Demand Function A demand function multiplicatively separable in price and heritage remains relatively general with few constraints. Multiplicative separability is both simple and natural and ensures analytical tractability and fits well with the data (Chenavaz, 2012; Helmes and Schlosser, 2013). With the multiplicative separable case, demand (1) is written as, $p, h \geq 0$,

$$D = m(p)n(h), \tag{15}$$

with $m' > 0, n' < 0, n'' < 0$ and

$$2 - m \frac{m''}{(m')^2} > 0.$$

Then $\frac{\partial D}{\partial p} = nm' < 0$, $\frac{\partial D}{\partial h} = mn' > 0$, $\frac{\partial^2 D}{\partial p \partial h} = m'n' < 0$, $\frac{\partial^2 D}{\partial p^2} = m''n$. According to this assumptions, conditions (2)-(3)-(4) are automatically satisfied.

Proposition 5. *For $t > 0$, the dynamics of price p is written*

$$\frac{dp}{dt} \underbrace{\left[2 - m \frac{m''}{(m')^2} \right]}_{+} = \frac{d\lambda}{dt} \underbrace{\alpha}_{+}. \tag{16}$$

Proof. The proof is in Appendix A.8. □

With a multiplicative separable demand function, the markup effect and the sales effect

outweigh each other exactly. Therefore, the dynamics of price are independent of the dynamics of heritage. In other words, the price-heritage relationship is null.

Corollary 2. *For a multiplicatively separable demand function, the dynamics of both p and u imitate the dynamics of λ . Formally,*

$$\text{sgn} \frac{dp}{dt} = \text{sgn} \frac{du}{dt} = \text{sgn} \frac{d\lambda}{dt}.$$

Proof. The proof is in Appendix A.9. □

The marketing-mix implications of Proposition 5 and Corollary 2 are straightforward. The dynamics of price and heritage preservation expenses emulate the dynamics of the shadow price. To decide on price and preservation expenses, the operator only needs to know the value of the shadow price.

Example 2. *Cobb-Douglas price-heritage demand function.*

Suppose the demand function $D = a_0 p^{-a_1} h^{a_2}$, with $a_0, a_1 > 1$, and $0 < a_2 < 1$. The derivatives write $\frac{\partial D}{\partial p} = -a_1 \frac{D}{p}$, $\frac{\partial D}{\partial h} = a_2 \frac{D}{h}$, $\frac{\partial^2 D}{\partial p^2} = a_1(1 + a_1) \frac{D}{p^2}$, $\frac{\partial^2 D}{\partial h^2} = a_2(a_2 - 1) \frac{D}{h^2}$, $\frac{\partial^2 D}{\partial p \partial h} = -a_1 a_2 \frac{D}{ph}$. Substituting the derivatives in (16) yields

$$\frac{dp}{dt} \underbrace{\left[1 - \frac{1}{a_1} \right]}_{+} = \frac{d\lambda}{dt} \underbrace{\alpha}_{+}, \tag{17}$$

verifying that the dynamics of p simulates the dynamics of λ .

The case of multiplicative separability is a sort of “textbook case.” It expresses the imperfect substitutability between price and heritage in the arguments of indirect utility and consumer demand. This situation may correspond to a demand expressed by visitors both motivated by the price of the visit and the quality of heritage, and able to confirm or not

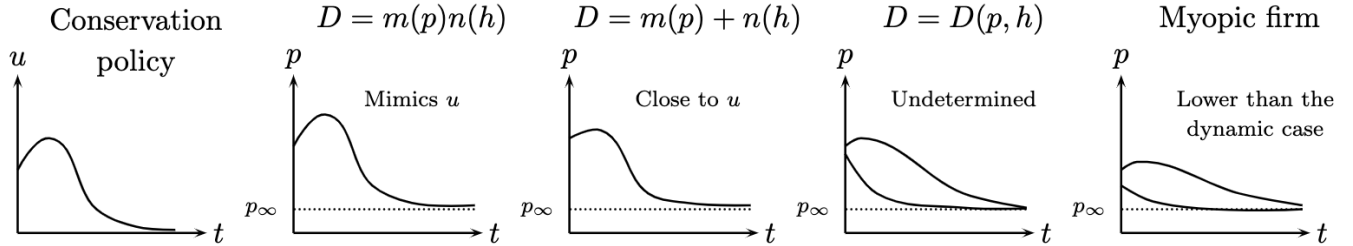


Figure 3: Conservation and Pricing Policies over Time for Different Classes of Demand Functions.

their demand according to the nature of the trade-off between the two determinants of their choice. The dynamic profile of prices exhibited by Proposition 5 could then be considered as a normative result presenting a sufficient degree of generality when the heritage belongs to standard leisure consumption which corresponds to these characteristics.

7 Discussion

In this section, we discuss the theoretical and managerial implications of our results.

7.1 Theoretical Implications

For illustrative reasons, we focus on the case where $\lim_{t \rightarrow \infty} \lambda(t) = 0$. Figure 3 summarizes the main insights of our research regarding the conservation and pricing policies over time. The first subfigure depicts the conservation policy, u , which mimics the evolution of the shadow price, λ (Proposition 3 and Corollary 1). The implication of the pricing condition (19) guarantees that λ is always positive as a greater heritage invariably increases future profits. Moreover, since $\lim_{t \rightarrow \infty} \lambda(t) = 0$, the operator has an incentive to invest in heritage preservation. Further, conservation expenses may increase in the first phase but always de-

crease after some time. As heritage does not generate any additional rent at the end of the horizon, the operator does not maintain the heritage anymore, that is $\lim_{t \rightarrow \infty} u(t) = 0$.

The following subfigures depict the pricing policy under different classes of demand functions. The price at the end of the horizon is the same for each class, with $p_\infty = \lim_{t \rightarrow \infty} p(t) = \lim_{t \rightarrow \infty} -\frac{D(p(t), h(t))}{\frac{\partial D}{\partial p}(p(t), h(t))}$. In effect, the vanishing shadow price of heritage at the end of the horizon, $\lim_t e^{-\rho t} \lambda(t) = 0$, dictates a single first-order condition on price (20) at infinite. The class of demand functions generates different temporal paths for the pricing policies. Indeed with multiplicative separability, $D = m(p)n(h)$, the pricing policy exactly mimics the conservation policy (Proposition 5 and Corollary 2). With additive separability, $D = m(p) + n(h)$, the pricing policy is similar, but does not exactly match the conservation policy (Proposition 4). With a general demand function, $D = D(p, h)$, the shape of the pricing policy may not be inferred from the conservation policy. Indeed Proposition 3 points out different elements exerting influence in opposing directions for the dynamics of price. As such, the pricing policy is undetermined, and distinct pricing schemes may appear.

We considered above a dynamic operator in the sense that it accounts for the impact of its pricing and conservation policies on future heritage attractiveness and thus on future profits. In such a situation the shadow price, $\lambda(t)$, is given at time t by Lemma 2. Alternatively, in the final subfigure, we consider a myopic (or static) operator, which disregards future heritage. In this situation the discount factor is infinite, $r = \infty$. Consequently the shadow price, given by Lemma 2, is null over the whole planning horizon, that is $\lambda(t) = 0$ for all $t \geq 0$. The implication is that the operator considers the current profit alone. Proposition 1 reveals that the myopic price is always lower than the dynamic price. The reason is that the myopic operator charges less to increase current sales, as it disregards future site degradation and the consequences on future profits. The final subfigure depicts that, even if the operator is myopic and behaves statically by maximizing current profit at each time t , price is not constant over

time. Indeed the optimal price is tied to demand and in turn to heritage attractiveness. Because attractiveness decreases over time, if not maintained by conservation investment, then demand and price change over the planning horizon.

7.2 Managerial Implications

The main motivation of this paper is that World Heritage Sites depreciate rapidly when widely accessible to visitors. Consequently, after the first period of intense exploitation, private or public organizations operating the asset may severely restrict or even prohibit any access and visits. Some sites or parts of them may be closed temporarily to the public, like for instance Tutankhamun’s Tomb, which fully reopened in 2019 after 10 years of restoration. Others, because of irreversible damage caused by visiting, like Lascaux cave, will never be accessible to the public again.⁵

We questioned this policy with a dynamic optimization setting. It proved unsustainable because the first period accelerates the depreciation of assets and the second deprives managers of financial resources necessary for their restoration. This sequence rapidly destroys the heritage and makes its restoration difficult when the exploitation of heritage becomes unprofitable. Our model supports the thesis that visiting, considered as the main cause of the problem, could also be its remedy if controlled by a suitable pricing policy. The number of visitors could be reduced using monitored access prices.

The preservation of World Heritage Sites requires a sound pricing policy. Operators aim at resource preservation and at the same time making the site accessible to visitors in the long run. Therefore, they must charge higher access prices than those which maximize short-term

⁵It is interesting to notice that in the text of the decisions adopted during the 43rd session of World Heritage Committee (UNESCO, Baku, 2019) the word “access” is used 44 times, the words “restrict,” “restriction,” “restrictive,” 10 times, while the words “fees” and “price,” are used respectively 1 and 0 times (UNESCO, 2019).

profits (Proposition 1). The revenues from the access prices finance heritage preservation, with a balance between access price and preservation expense (Proposition 2). Higher prices trade off a lower heritage degradation with higher exploitation revenues (Maximization problem (7)). Proposition 3 examines the link between the dynamics of price and heritage attractiveness in the general case (general demand function). Counterintuitively, it shows that price does not necessarily increase with heritage. In other words, it may be in the interest of the operator to charge less a better heritage site. Propositions 4 and 5 consider specific cases of demand behavior, obtaining stronger results, at the cost of a generality loss.

Premium access prices are necessary to reduce the pressure on the resource, especially when numerous potential visitors are highly price-sensitive and ask for discounts or when some shareholders promote other policies. The same people, depending on their current situation, may have contradictory and incoherent claims. As a citizen, intellectual, or activist one may promote heritage preservation and argue for higher entrance prices, but as a tourist one would prefer no or low entrance fees to cultural heritage.

Propositions 1 and 2 guide the calibration of the spread between dynamic and static prices, preserving a sustainable heritage. This spread increases with the damage generated by tourists and with the difficulty to repair the site. It is important to disentangle these two effects: some sites could be both resilient and easy to repair, like castles, fortifications, or military installations. Others integrate fragile and not reproducible components, such as cave painting or old stained-glass windows. In the first case, the dynamic component of prices could be neglected, while they should impose a very high access price in the second case. In many cases, the situation is between those two extreme cases: cathedrals, pagodas, palaces, can be restored quite in their original form and the damage resulting from visits is generally neither negligible nor dramatic.

8 Conclusion

To conclude, we summarize our approach, list our contributions, and show future research areas.

8.1 Summary

This article has examined the joint dynamic policies of heritage conservation and pricing for World Heritage Sites. Working on the structural properties of the general (non-parametric) class of demand functions, we have highlighted different elements influencing the interplay between price and heritage. We extend the literature on heritage sites by proposing analytic (as opposed to quantitative) pricing and heritage conservation rules (Propositions 1, 2, and 3). Such optimality rules balance the total effect on profit of current demand-side effects linked to markup and sales, and future supply-side effects linked to heritage (Propositions 3).

More precisely, Proposition 1 indicates that World Heritage Sites' operators, who ignore the long-term consequences of their pricing strategies (in terms of heritage degradation) will consistently price too low, thereby systematically increasing current profit at the cost of future profit and heritage damage. Our approach offers a new lens on the demand and supply sides of World Heritage Sites managed by profit-maximizing operators. Proposition 2 provides a deeper understanding of the linkage between pricing and investment strategies related to World Heritage Sites. Proposition 3 informs the operator about the heritage investment over time. Eventually, Proposition 3 offers clear-cut theoretical implications regarding the critical challenge of achieving a sustainable balance between the conflicting goals of World Heritage Site economic exploitation and conservation.

8.2 Contributions

The challenge of sustainable policies for World Heritage Sites resides in two opposing forces. On the one hand, the heritage deteriorates with the number of visitors; on the other, the heritage quality may increase thanks to the revenue generated by entrance fees. Therefore, the question is no longer whether to allow or to prohibit visits to the site, but how to regulate this access. We contribute to extant literature by focusing on pricing policies rather than quantitative restrictions to optimize the trade-off between current profit and preservation of the heritage, i.e. the source of future profits.

Further, our contributions show that dynamic prices targeting intertemporal profit maximization are always higher than static prices targeting short-term objectives. More fundamentally, we were able, with general demand functions, to distinguish the components of the spread between both dynamic and static prices. Stronger is the depreciation of the heritage, and the more efficient are the investments able to reconstitute it, the greater is the difference between dynamic and myopic prices. When the time pattern of prices is considered, the elasticities of demand on prices and quality are fundamental determinants of these dynamics.

Eventually, our contributions emphasize that there is not one single recommendation for all cases, independently of the type of visitors, and of the existence of more or less close substitutes to the site or monument under consideration. Managerial policies should be adapted to the nature of the heritage, to the more or less heterogeneity of demand, and the competitive environment. But there are no cases such that dynamic optimization would impose lower access fees than a static one.

8.3 Research Limitations and Future Research

Our model may be extended in several ways to address several policy issues that we discuss hereafter. A public policy concern may be the access to all people, that is, access should be possible not only to the ‘rich,’ but also to the ‘underprivileged.’ Such concern may be achieved by a discriminating pricing policy, with a low or free entrance fee for underprivileged, as for children, seniors, large families, unemployed, or local people of poor countries. Apollo (2014) reviews such policies in poor countries: to access the TajMahal, foreign visitors pay 3,500% higher fees than local visitors. The model generalization may be done in future work.

Another policy issue is that of the founding of a high conservation cost. Our model models an entrance fee; it could be extended to restrict also visitor access by a quota. For instance, Barcelona’s Park Güell manages jointly its access pricing policy with the limitation of the number of visitors admitted per hour (Goodwin, 2016; Crespi-Vallbona and Smith, 2019). Plus, all conservation costs are paid by the entrance fee. In the case of quota, a high conservation cost may also be born by government subsidies. The adequate private/public partnership remains an open question, calling for other model extensions.

Our model framework could also be useful to protect natural heritage sites. Indeed, some natural sites set an entrance fee to both raise revenues and limit visits. That’s the case of the neolithic cave of Galdar in Gran Canaria Isle. However, for natural heritage sites, the model could be adapted or extended to allow flow regulation by restricting access, with or without an entrance fee. This model would correspond to the practice of restricting the access of Lascaut’s cave (with entrance fee) and of the Callanques of Marseille (without entrance fee). Such model extension to natural heritage conservation would complement our model framework.

A Proofs

A.1 Proof of Lemma 2

We found the explicit solution for λ , by using integrating factor method, (for further details on the method see Boyce et al. 2021),

$$\frac{d\lambda}{dt} - \left(r + \delta + \alpha \frac{\partial D}{\partial h} \right) \lambda = -e_h \frac{pD}{h}, \text{ with } \lim_{t \rightarrow +\infty} e^{-rt} \lambda(t) = 0.$$

We look for a function $u(t)$ such that

$$\frac{d}{dt} (u\lambda) = u \frac{d\lambda}{dt} - u \left(r + \delta + \alpha \frac{\partial D}{\partial h} \right) \lambda. \quad (18)$$

$$\begin{aligned} \frac{d}{dt} (u\lambda) = u \frac{d\lambda}{dt} - u \left(r + \delta + \alpha \frac{\partial D}{\partial h} \right) \lambda &\Rightarrow \frac{du}{dt} \lambda + \frac{d\lambda}{dt} u = u \frac{d\lambda}{dt} - u \left(r + \delta + \alpha \frac{\partial D}{\partial h} \right) \lambda \\ &\Rightarrow \frac{du}{dt} \lambda = -u \left(r + \delta + \alpha \frac{\partial D}{\partial h} \right) \lambda \Rightarrow \frac{du}{dt} = -u \left(r + \delta + \alpha \frac{\partial D}{\partial h} \right) \\ &\Rightarrow u(t) = \exp \left(- \int_0^t \left(r + \delta + \alpha \frac{\partial D}{\partial h} \right) ds + C \right). \end{aligned}$$

Combining equation (18) and the equation satisfied by λ , we get

$$\begin{aligned} \frac{d}{dt} (u\lambda) = -e_h \frac{pD}{h} u &\Rightarrow \lambda(t) = \frac{1}{u(t)} \left(- \int_0^t e_h \frac{pD}{h} u(\ell) d\ell + C_1 \right) \\ \Rightarrow \lambda(t) = \exp \left(\int_0^t \left(r + \delta + \alpha \frac{\partial D}{\partial h} \right) ds \right) &\left(- \int_0^t e_h \frac{pD}{h} \exp \left(- \int_0^\ell \left(r + \delta + \alpha \frac{\partial D}{\partial h} \right) ds \right) d\ell + C_2 \right). \end{aligned}$$

In order to have the transversality condition to be verified, we must impose that

$$\lim_{t \rightarrow \infty} \left(- \int_0^t e_h \frac{pD}{h} \exp \left(- \int_0^\ell \left(r + \delta + \alpha \frac{\partial D}{\partial h} \right) ds \right) d\ell + C_2 \right) = 0$$

which means that

By plugging the value of C_2 in the expression for λ , we conclude that

$$\lambda(t) = \left(\int_t^\infty e_h \frac{pD}{h} \exp \left(- \int_t^\ell \left(r + \delta + \alpha \frac{\partial D}{\partial h} \right) ds \right) d\ell \right).$$

A.2 Results about Optimization

Lemma 3. *The shadow price is non negative, $\lambda^* \geq 0$. In particular, two possible scenarios can occur,*

1. $\lambda^*(t) > 0$ for $t \geq 0$;
2. $\lambda^*(t) > 0$ for $t \in [0, T^*)$ and $\lambda^*(t) = 0$ for $t \in [T^*, \infty)$.

Proof. From Lemma 2, we have that $\lambda^*(t)$ can be written as $e^{\int_0^t g(s)ds} \int_t^\infty f(\ell) d\ell$ with some $f(\ell) \geq 0$ for each $\ell \in [0, T]$. If there exists $T > 0$ such that $f(\ell) > 0$ for a.e. $\ell \in [T, \infty]$, then for each $t \geq T$, $\lambda(t) \geq 0$ and moreover for $t < T$

$$\lambda(t) \gtrsim \int_T^\infty f(\ell) d\ell > 0.$$

Now, we assume that the condition above is not verified. This means that for each $T > 0$, there exists a non negligible set $\mathcal{A}(T) \subseteq [T, +\infty)$ such that $f(\ell) = 0$, for each $\ell \in \mathcal{A}(T)$. If

for each $T > 0$, $\mathcal{A}(T) \subset [T, \infty)$, then $\lambda(t) > 0$ for each $t \geq 0$. Otherwise, there exists T^* such that $\mathcal{A}(T) = [T, \infty)$. This means that for $t < T^*$

$$\lambda(t) = e^{\int_0^t g(s) ds} \int_{[t, \infty) \cap \mathcal{A}(t)} f(\ell) d\ell > 0,$$

and for $t \geq T^*$, $\lambda(t) = 0$. □

Proof of Lemma 1. The solution of the Pontryagin system is a candidate optimal solution. Its shape depends strongly on $\lambda^*(t)$. Indeed, in order to solve the Pontryagin system, we need to find

$$(u^*(t), p^*(t)) \in \operatorname{argmax}_{(u,p) \in (\mathbb{R}^+)^2} H(u, p, h^*(t), \lambda^*(t)), \text{ for each } t \geq 0.$$

In the next lines, we will find out that if $\lambda^* \neq 0$ there exists a solution in the inner, namely $u^*, p^* > 0$. Otherwise, we can only find a solution on the border. Since, a priori we do not know the sign of $\lambda^*(t)$, we study the different cases presented in the Lemma 3.

If the first case occurs, we study

$$(u^*(t), p^*(t)) \in \operatorname{argmax}_{(u,p) \in (\mathbb{R}^+)^2} H(u, p, h^*(t), \underbrace{\lambda^*(t)}_{\neq 0}), \text{ for each } t \geq 0.$$

In this case the critical point satisfies the following equations,

$$\frac{\partial H(u^*, p^*, h^*, \lambda^*)}{\partial u} = 0 \implies \lambda^* \cdot K'(u^*) = 1, \tag{19}$$

$$\frac{\partial H(u^*, p^*, h^*, \lambda^*)}{\partial p} = 0 \implies p^* = -\frac{D(p^*, h^*)}{\frac{\partial D(p^*, h^*)}{\partial p}} + \alpha \lambda^*. \tag{20}$$

Since u^* is obtained by reversing the function K' , then $u^* > 0$. Assume that the function $F(p) = -\frac{D(p, h^*)}{\frac{\partial D(p, h^*)}{\partial p}} + \alpha \lambda^*$ admits a fixed point different from zero, p^* . Then the positivity

of λ^* , together with the $\frac{\partial D}{\partial p} < 0$ implies that $p^* > 0$. Moreover, we can say that the critical point furnishes a candidate maximum for (7). Indeed, since $\lambda^* > 0$ and $K'' < 0$

$$\frac{\partial^2 H(u^*, p^*, h^*, \lambda^*)}{\partial u^2} = \lambda K''(u^*) \implies \frac{\partial^2 H(u^*, p^*, h^*, \lambda)}{\partial u^2} < 0. \quad (21)$$

Given (2) and (3),

$$\frac{\partial^2 H(u^*, p^*, h^*, \lambda)}{\partial p^2} = \frac{\partial D(p^*, h^*)}{\partial p} \left(2 - D(p^*, h^*) \frac{\frac{\partial^2 D(p^*, h^*)}{\partial p^2}}{\frac{\partial D(p^*, h^*)}{\partial p}} \right) \implies \frac{\partial^2 H}{\partial p^2} < 0. \quad (22)$$

Given (21) and (22),

$$\frac{\partial^2 H}{\partial u^2} \frac{\partial^2 H}{\partial p^2} - \left(\frac{\partial^2 H}{\partial u \partial p} \right)^2 = \frac{\partial^2 H}{\partial u^2} \frac{\partial^2 H}{\partial p^2} \implies \frac{\partial^2 H}{\partial u^2} \frac{\partial^2 H}{\partial p^2} - \left(\frac{\partial^2 H}{\partial u \partial p} \right)^2 > 0,$$

Now, we observe that the Hamiltonian H , evaluated in the critical point, is negative definite. Indeed, let us consider the Hessian matrix associated to the hamiltonian H , \mathcal{H}_H , evaluated in the critical point obtained in (19)-(20) ,

$$\mathcal{H}_H(u^*, p^*, h^*, \lambda^*) = \begin{pmatrix} \lambda K''(u^*) & 0 & 0 \\ 0 & \frac{\partial D(p^*, h^*)}{\partial p} \left(2 - D(p^*, h^*) \frac{\frac{\partial^2 D(p^*, h^*)}{\partial p^2}}{\left(\frac{\partial D(p^*, h^*)}{\partial p} \right)^2} \right) & \frac{\partial D(p^*, h^*)}{\partial h} - D(p^*, h^*) \frac{\frac{\partial^2 D(p^*, h^*)}{\partial p \partial h}}{\frac{\partial D(p^*, h^*)}{\partial p}} \\ 0 & \frac{\partial D(p^*, h^*)}{\partial h} - D(p^*, h^*) \frac{\frac{\partial^2 D(p^*, h^*)}{\partial p \partial h}}{\frac{\partial D(p^*, h^*)}{\partial p}} & -D(p^*, h^*) \frac{\frac{\partial^2 D(p^*, h^*)}{\partial h^2}}{\frac{\partial D}{\partial p}} \end{pmatrix}.$$

We call D_k the leading minor of order k . Thus, from the hypothesis on K and the sign on λ^*

$$D_1 = \lambda K''(u^*) < 0, \quad D_2 = \underbrace{\lambda K''(u^*)}_{<0} \underbrace{\frac{\partial D(p^*, h^*)}{\partial p}}_{<0} \underbrace{\left(2 - D(p^*, h^*) \frac{\frac{\partial^2 D(p^*, h^*)}{\partial p^2}}{\left(\frac{\partial D(p^*, h^*)}{\partial p} \right)^2} \right)}_{>0} > 0$$

$$D_3 = \underbrace{\lambda K''(u^*)}_{<0} \times \underbrace{\left(-D(p^*, h^*) \frac{\partial^2 D(p^*, h^*)}{\partial h^2} \left(2 - D(p^*, h^*) \frac{\frac{\partial^2 D(p^*, h^*)}{\partial p^2}}{\left(\frac{\partial D(p^*, h^*)}{\partial p} \right)^2} \right) - \left(\frac{\partial D(p^*, h^*)}{\partial h} - D(p^*, h^*) \frac{\frac{\partial^2 D(p^*, h^*)}{\partial p \partial h}}{\frac{\partial D(p^*, h^*)}{\partial p}} \right)^2 \right)}_{>0}$$

Now, we study the case when λ^* satisfies the second case of Lemma 3. Then, we need to find

$$(u^*(t), p^*(t)) \in \operatorname{argmax}_{(u,p) \in (\mathbb{R}^+)^2} H(u, p, h^*(t), 0), \quad t \in [T^*, \infty).$$

Notice that $H(u, p, h^*(t), 0) = pD(p, h^*(t)) - u$. It can be easily proven that the point $(0, p^*)$ with $p^* = -\frac{D}{\partial D \partial p}$ is a local maximum for $H(u, p, h^*(t), 0)$. Moreover, the hessian of the function $pD(p, h)$ is negative definite in the point (p^*, h^*) , which implies that the point (p^*, h^*) is a local maximum for $pD(p, h)$. Since $u > 0$ we conclude that $(0, p^*, h^*)$ is a local maximum for $H(u, p, h, 0)$,

$$H(u, p, h, 0) = pD(p, h) - u \leq pD(p, h) \leq p^* D(p^*, h^*) = H(0, p^*, h^*, 0).$$

To conclude, we prove that for all h, u that stay close enough to h^*, u^*

$$\int_0^\infty \exp(-rt) (\pi(t, h(t), u(t)) - \pi(p^*(t), h^*(t), u^*(t))) dt \leq 0 \quad (23)$$

where we indicate with p^*, h^*, u^* the solution of the maximum principle. If case 1. of Lemma 3 is verified, then since H is concave in $p^*(t), h^*(t), u^*(t)$ for $t \in [0, T)$, to prove (23) we just adapt the proof of Theorem 2.5 in Yong and Zhou (1999). Now consider we focus on the case

2. of Lemma 3. We call

$$b = -\alpha D(p(t), h(t)) - \delta h(t) + K(u(t)),$$

$$b^* = -\alpha D(p^*(t), h^*(t)) - \delta h^*(t) + K(u^*(t)),$$

and since (u^*, p^*, h^*) is a local maximum for H , we get

$$\begin{aligned} & \int_0^\infty \exp(-rt) (\pi(p(t), h(t), u(t)) - \pi(p^*(t), h^*(t), u^*(t))) dt = \\ &= \int_0^\infty \exp(-rt) (H(u(t), p(t), h(t), \lambda(t)) - H(u^*(t), p^*(t), h^*(t), \lambda(t))) dt + \int_0^\infty \exp(-rt) \lambda(t) (b^* - b) dt \\ &= \int_0^{T^*} \exp(-rt) (H(u(t), p(t), h(t), \lambda(t)) - H(u^*(t), p^*(t), h^*(t), \lambda(t))) dt + \int_0^{T^*} \exp(-rt) \lambda(t) (b^* - b) dt + \\ &+ \underbrace{\int_{T^*}^\infty \exp(-rt) (H(u(t), p(t), h(t), 0) - H(u^*(t), p^*(t), h^*(t), 0)) dt}_{\leq 0} + \underbrace{\int_{T^*}^\infty \exp(-rt) 0 \cdot (b^* - b) dt}_{=0} \\ &\leq \int_0^{T^*} \exp(-rt) (H(u(t), p(t), h(t), \lambda(t)) - H(u^*(t), p^*(t), h^*(t), \lambda(t))) dt + \int_0^{T^*} \exp(-rt) \lambda(t) (b^* - b) dt. \end{aligned}$$

Then, we conclude that the last line is negative, by repeating the same argument proposed in the proof of Theorem 2.5 in Yong and Zhou (1999).

□

A.3 Proof of Proposition 1

Following (20), the dynamic price is $p = -\frac{D(p(t), h(t))}{\frac{\partial D(p(t), h(t))}{\partial p}} + \alpha \lambda$, with λ given by Lemma 2. For the static price, the shadow price of heritage is null over the whole planning period, that is $\lambda = 0$ for all $t \geq 0$, and $p_{static} = -\frac{D}{\frac{\partial D}{\partial p}}$. Note with (19) that $\lambda = \frac{1}{K'}$ and substituting it in (20) offers $p = -\frac{D(p(t), h(t))}{\frac{\partial D(p(t), h(t))}{\partial p}} + \frac{\alpha}{K'}$. The difference between the two prices is straightforward.

A.4 Proof of Proposition 2

Isolating λ from (19), and substituting λ in (20) yields $p = -\frac{D}{\frac{\partial D}{\partial p}} + \frac{\alpha}{K'}$. Then we rearrange this result with the price elasticity notation.

A.5 Proof of Proposition 3

In order to study the evolution of the price variable, we differentiate the first-order condition on price $D + p\frac{\partial D}{\partial p} - \alpha\lambda\frac{\partial D}{\partial p} = 0$, rearranged from (20), with respect to time t . Mathematically, for all $t > 0$, we have:

$$\frac{\partial D}{\partial p} \frac{dp}{dt} + \frac{\partial D}{\partial h} \frac{dh}{dt} + \left[\frac{\partial^2 D}{\partial p^2} \frac{dp}{dt} + \frac{\partial^2 D}{\partial p \partial h} \frac{dh}{dt} \right] \left[p - \frac{\alpha}{K'} \right] + \frac{\partial D}{\partial p} \left[\frac{dp}{dt} + \frac{\alpha K''}{(K')^2} \frac{du}{dt} \right] = 0. \quad (24)$$

Substituting (19) in (20) yields

$$p - \frac{\alpha}{K'} = -\frac{D}{\frac{\partial D}{\partial p}} > 0. \quad (25)$$

Note that

$$-\frac{\frac{\partial D}{\partial h}}{\frac{\partial D}{\partial p}} = \frac{e_h}{e_p} \frac{p}{h}. \quad (26)$$

Substituting (27), (25), and (26) in (24) and rearranging provides the result (11).

Now we focus on the evolution of u . Note that the first order condition on conservation expenses (19) must hold for all optimal values and at any time. We differentiate both sides of (19) with respect to time, noting that both sides must remain equal for optimal values. Thus we obtain for all $t > 0$:

$$\frac{d\lambda}{dt} K' + \lambda K'' \frac{du}{dt} = 0.$$

Substituting in the previous result $\lambda = \frac{1}{K'}$ from (19), dividing by K' , and rearranging yields

$$\frac{K''}{(K')^2} \frac{du}{dt} = -\frac{d\lambda}{dt}. \quad (27)$$

Recalling $K'' < 0$ from assumption (5) gives for all $t > 0$,

$$\text{sgn} \frac{du}{dt} = \text{sgn} \frac{d\lambda}{dt},$$

which proves the result.

A.6 Proof of Corollary 1

Immediate with Proposition 3.

A.7 Proof of Proposition 4

Substitute (13) and its derivatives in Proposition 3. With an additively separable demand function, the sales effect is null. Only the markup effect remains.

A.8 Proof of Proposition 5

Substitute (15) and its derivatives in Proposition 3. With a multiplicatively separable demand function, the markup and sales effects are of same magnitude, but have opposite signs. They cancel each other out.

A.9 Proof of Corollary 2

Immediate with Propositions 3 and 5.

References

- Apollo, M. (2014). Dual pricing—two points of view (citizen and non-citizen). Case of entrance fees in tourist facilities in Nepal. *Procedia-Social and Behavioral Sciences* 120, 414–422.
- Artal-Tur, A., M. Villena-Navarro, and L. Alamá-Sabater (2018). The relationship between cultural tourist behaviour and destination sustainability. *Anatolia* 29(2), 237–251.
- Aseev, S. M. and A. V. Kryazhimskiy (2004). The pontryagin maximum principle and transversality conditions for a class of optimal control problems with infinite time horizons. *SIAM Journal on Control and Optimization* 43(3), 1094–1119.
- Ban, B. (2016, February 23). Myanmar stops tourists scaling prized pagodas. *Agence France Presse*. Retrieved on July 12, 2019, <https://gulfnews.com/world/asia/myanmar-stops-tourists-scaling-prized-pagodas-1.1678033>.
- Bastian, F. and C. Alabouvette (2009). Lights and shadows on the conservation of a rock art cave: the case of Lascaux Cave. *International Journal of Speleology* 38(1), 6.
- Benckekroun, H. and N. V. Long (2012). Collaborative environmental management: A review of the literature. *International Game Theory Review* 14(04), 1240002.
- Board, E. (2019, August 9). Venice cruise crash is a sign of the risks of tourism. *Financial Times*. Retrieved on August 10, 2019, <https://www.ft.com/content/44e9e118-b9ce-11e9-96bd-8e884d3ea203>.
- Boccardi, G. and C. Duvelle (2013). Introducing cultural heritage into the sustainable development agenda. In *Hangzhou International Congress*, pp. 15–17.
- Boyce, W. E., R. C. Di Prima, and D. B. Meade (2021). *Elementary Differential Equations and Boundary Value Problems*. John Wiley & Sons.

- Brau, R. (2008). Demand-driven sustainable tourism? a choice modelling analysis. *Tourism Economics* 14(4), 691–708.
- Candela, G. and R. Cellini (2006). Investment in tourism market: a dynamic model of differentiated oligopoly. *Environmental and Resource Economics* 35(1), 41.
- Chaguan (2019, January 26). China must embrace global norms if it wants to defend the environment. *The Economist*. Retrieved on July 12, 2019, <https://www.economist.com/china/2019/01/26/china-must-embrace-global-norms-if-it-wants-to-defend-the-environment>.
- Chen, C.-C. and C.-H. Lee (2017). Economic benefits of improving the quality of cultural heritage sites. *Journal of Economics and Management* 13(2), 241–264.
- Chenavaz, R. (2012). Dynamic pricing, product and process innovation. *European Journal of Operational Research* 222(3), 553–557.
- Chenavaz, R. (2017). Better product quality may lead to lower product price. *The BE Journal of Theoretical Economics* 17(1), 1–22.
- Chenavaz, R., G. Feichtinger, R. F. Hartl, and P. M. Kort (2020). Modeling the impact of product quality on dynamic pricing and advertising policies. *European Journal of Operational Research* 284(3), 990–1001.
- Choi, A. S., B. W. Ritchie, F. Papandrea, and J. Bennett (2010). Economic valuation of cultural heritage sites: A choice modeling approach. *Tourism Management* 31(2), 213–220.
- Claude, D. and G. Zaccour (2009). Investment in tourism market and reputation. *Journal of Public Economic Theory* 11(5), 797–817.

- Creigh-Tyte, S. W. (2000). The built heritage: Some British experiences. *Recherches Économiques de Louvain/Louvain Economic Review* 66(2), 213–229.
- Crespi-Vallbona, M. and S. G. Smith (2019). Managing sociocultural sustainability in public heritage spaces. *Tourism Planning & Development* 0(0), 1–19.
- Dockner, E., S. Jorgensen, V. N. Long, and S. Gerhard (2000). *Differential Games in Economics and Management Science*. Cambridge University Press.
- Dodds, R., S. R. Graci, and M. Holmes (2010). Does the tourist care? a comparison of tourists in koh phi phi, thailand and gili trawangan, indonesia. *Journal of Sustainable Tourism* 18(2), 207–222.
- Dye, C.-Y. and C.-T. Yang (2016). Optimal dynamic pricing and preservation technology investment for deteriorating products with reference price effects. *Omega* 62, 52–67.
- Emerton, L., J. Bishop, and L. Thomas (2006). *Sustainable Financing of Protected Areas: A Global Review of Challenges and Options*. IUCN.
- Feichtinger, G., R. Hartl, P. Kort, and A. Novak (2001). Terrorism control in the tourism industry. *Journal of Optimization Theory and Applications* 108(2), 283–296.
- Fruchter, G. (2009). Signaling quality: Dynamic price-advertising model. *Journal of Optimization Theory and Applications* 143(3), 479–496.
- Fruchter, G. E., E. D. Jaffe, and I. D. Nebenzahl (2006). Dynamic brand-image-based production location decisions. *Automatica* 42(8), 1371–1380.
- Garrod, B. and A. Fyall (2000). Managing heritage tourism. *Annals of Tourism Research* 27(3), 682–708.

- Goodwin, H. (2016). Managing tourism in Barcelona. Technical report, Responsible Tourism Partnership Working Paper. Retrieved on July 12, 2019, <https://responsibletourismpartnership.org/wp-content/uploads/2019/11/Managing-tourism-in-Barcelona.pdf>.
- Greiner, A., G. Feichtinger, J. L. Haunschmied, P. M. Kort, and R. F. Hartl (2001). Optimal periodic development of a pollution generating tourism industry. *European Journal of Operational Research* 134(3), 582–591.
- Hansen, T. B., H. Christoffersen, and S. Wanhill (1998). The economic evaluation of cultural and heritage projects: Conflicting methodologies. *Tourism Culture & Communication* 1(1), 27–48.
- Hassan, S. S. (2000). Determinants of market competitiveness in an environmentally sustainable tourism industry. *Journal of Travel Research* 38(3), 239–245.
- Hedlund, T. (2011). The impact of values, environmental concern, and willingness to accept economic sacrifices to protect the environment on tourists intentions to buy ecologically sustainable tourism alternatives. *Tourism and Hospitality Research* 11(4), 278–288.
- Helmes, K. L. and R. Schlosser (2013). Dynamic advertising and pricing with constant demand elasticities. *Journal of Economic Dynamics and Control* 37(12), 2814–2832.
- Hernández, J. M. and C. J. León (2007). The interactions between natural and physical capitals in the tourist lifecycle model. *Ecological Economics* 62(1), 184–193.
- Hernández, J. M. and C. J. León (2013). Welfare and environmental degradation in a tourism-based economy. *Tourism Economics* 19(1), 5–35.
- Hutter, M. and I. Rizzo (1997). *Economic Perspectives on Cultural Heritage*. Springer.

- Jørgensen, S., G. Martín-Herrán, and G. Zaccour (2010). Dynamic games in the economics and management of pollution. *Environmental Modeling & Assessment* 15(6), 433–467.
- Jørgensen, S. and G. Zaccour (2012). *Differential Games in Marketing*, Volume 15. Springer Science & Business Media.
- Kalish, S. (1983). Monopolist pricing with dynamic demand and production cost. *Marketing Science* 2(2), 135–159.
- Kamien, M. I. and N. L. Schwartz (2012). *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management*. Courier Corporation, London, United Kingdom.
- Kim, S. S., K. K. Wong, and M. Cho (2007). Assessing the economic value of a world heritage site and willingness-to-pay determinants: A case of Changdeok Palace. *Tourism Management* 28(1), 317–322.
- Kort, P. M., A. Greiner, G. Feichtinger, J. L. Haunschmied, A. Novak, and R. F. Hartl (2002). Environmental effects of tourism industry investments: An inter-temporal trade-off. *Optimal Control Applications and Methods* 23(1), 1–19.
- Lyons, C. (2008, August 23). The paradox of Pompeii. *Financial Times*. Retrieved on July 12, 2019, <https://www.ft.com/content/44e47ed0-6fde-11dd-986f-0000779fd18c>.
- Mangasarian, O. L. (1966). Sufficient conditions for the optimal control of nonlinear systems. *SIAM Journal on control* 4(1), 139–152.
- Martin-Sanchez, P. M., A. Z. Miller, and C. Saiz-Jimenez (2015). Lascaux cave: an example of fragile ecological balance in subterranean environments. *Microbial Life in Cave Systems*, ed. by Engel, AS, De Gruyter, Berlin/Boston, 279–301.

- McDonagh, J. and P. B. Nahkies (2010). Heritage building preservation—the ultimate in green building? In *Proceedings of the 16th annual conference of the Pacific Rim Real Estate Society, Wellington, New Zealand*, pp. 24–27.
- Meleddu, M. (2014). Tourism, residents welfare and economic choice: A literature review. *Journal of Economic Surveys* 28(2), 376–399.
- Minciu, R., D. Popescu, M. Padurean, R. Hornoiu, and A. Baltaretu (2010). Commercialization of holidays in the protected natural areas-form of the sustainable development in tourism. *Amfiteatru Economic* 12(27), 83–98.
- Ni, J. and S. Li (2018). When better quality or higher goodwill can result in lower product price: A dynamic analysis. *Journal of the Operational Research Society* 70(5), 726–736.
- Patin, V. (2010). The economy of cultural heritage, tourism and conservation. *International Preservation News* (52), 6.
- Piazza, A. and S. Roy (2015). Deforestation and optimal management. *Journal of Economic Dynamics and Control* 53, 15–27.
- Piga, C. A. (2003). Pigouvian taxation in tourism. *Environmental and Resource Economics* 26(3), 343–359.
- Rama, M. (2012). Investing in the sense of place: The economics of urban upgrading projects with a cultural dimension. *The Economics of Uniqueness*, 15.
- Reuters-ABC (2019, February 1). Tutankhamun’s tomb restored after years of damage by tourists. *Reuters-ABC*. Retrieved on July 12, 2019, <https://www.abc.net.au/news/2019-02-01/tutankhamans-tomb-to-restored-after-years-of-damage/10770272>.

- Rizzo, I. and D. Throsby (2006). Cultural heritage: economic analysis and public policy. *Handbook of the Economics of Art and Culture 1*, 983–1016.
- Sirilersuang, P. and S. Pongkijvorasin (2018). Dynamic optimal joint policies for tourism promotion and environmental restoration. *Environmental Economics and Policy Studies 20*(2), 261–286.
- Tan, S.-K., S.-H. Tan, Y.-S. Kok, and S.-W. Choon (2018). Sense of place and sustainability of intangible cultural heritage—The case of George Town and Melaka. *Tourism Management 67*, 376–387.
- Throsby, D. (1999). Cultural capital. *Journal of Cultural Economics 23*(1–2), 3–12.
- Throsby, D. (2012). *The economics of uniqueness. Investing in Historic City Cores and Cultural Heritage Assets for Sustainable Development?*, Chapter Heritage Economics: A Conceptual Framework, pp. 45–74. The World Bank.
- Tuan, T. H. and S. Navrud (2008). Capturing the benefits of preserving cultural heritage. *Journal of Cultural Heritage 9*(3), 326–337.
- UNESCO (2019). Decisions adopted during the 43rd session of the world heritage committee. Technical report, World Heritage Committee, Baku.
- United Nations, D. o. E. and S. Affairs (2015). Transforming our world: The 2030 agenda for sustainable development. Technical report, United Nations.
- Voltaire, L. (2017). Pricing future nature reserves through contingent valuation data. *Ecological Economics 135*(C), 66–75.
- Vörös, J. (2006). The dynamics of price, quality and productivity improvement decisions. *European Journal of Operational Research 170*(3), 809–823.

- Vörös, J. (2013). Multi-period models for analyzing the dynamics of process improvement activities. *European Journal of Operational Research* 230(3), 615–623.
- Vörös, J. (2019). An analysis of the dynamic price-quality relationship. *European Journal of Operational Research* 277(3), 1037–1045.
- Wright, W. C. and F. V. Eppink (2016). Drivers of heritage value: A meta-analysis of monetary valuation studies of cultural heritage. *Ecological Economics* 130, 277–284.
- Wu, W., L. Zhang, and F. Qiu (2017). Determinants of tourism ticket pricing for ancient villages and towns: Case studies from Jiangsu, Zhejiang, Shanghai and Anhui provinces. *Tourism Management* 58, 270–275.
- Xue, M., W. Tang, and J. Zhang (2016). Optimal dynamic pricing for deteriorating items with reference-price effects. *International Journal of Systems Science* 47(9), 2022–2031.
- Yong, J. and X. Y. Zhou (1999). *Stochastic controls: Hamiltonian systems and HJB equations*, Volume 43. Springer Science & Business Media.