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► **To cite this version:**

Nabil Kahouadji. VARIATIONS ON TWO-PARAMETER FAMILIES OF ADVANCED TIME SERIES FORECASTING FUNCTIONS IV: SEASONAL MODELS AND APPLICATION TO UNIVERSITY ENROLLMENT. 2022. hal-03835677

**HAL Id: hal-03835677**

**<https://hal.science/hal-03835677>**

Preprint submitted on 1 Nov 2022

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# VARIATIONS ON TWO-PARAMETER FAMILIES OF ADVANCED TIME SERIES FORECASTING FUNCTIONS IV: SEASONAL MODELS AND APPLICATION TO UNIVERSITY ENROLLMENT

NABIL KAHOUADJI

ABSTRACT. Using the twenty four two-parameter families of advanced time series forecasting functions along with their associated seasonal models introduced in [6], we perform a series of seasonal forecasts (sixteen years times 3 trimesters) on eighteen university enrollment time series, and compare their performance to both Holt–Winters and ARIMA models. For each of these eighteen university enrollment time series, we show that our twenty four seasonal models outperform both the Holt–Winters and ARIMA models with respect to the sum of absolute errors, the number of closest forecasts, and the number of closest yearly forecasts.

Keywords: Time series forecasting, seasonal models, enrollment forecasting.  
MSC 2020: 90C10, 90C15, 62G99, 62P20, 62P99, 90C90, 60C99.

## 1. INTRODUCTION

This fourth paper is a continuation of a research program initiated in [6] in which we introduced twenty four two-parameter families of advanced time series forecasting functions, defined the concept of powering, derived nonseasonal and seasonal models with examples in education, sales, finance and economy, compared the performance of the latter models to both the Holt–Winters and ARIMA models, and applied and compared the performance of those twenty four models using five-year stock market data of 467 companies of the S&P500. We continue our research program with an application to university enrollment forecasting. Indeed, using eighteen university enrollment times series for the same university, from fall 2004 to summer 2022, we perform a series of seasonal forecasting using the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models [6], and compare their performance to both Holt–Winters and ARIMA models, using the sum of absolute error, the number of closest forecasts, and the number of closest yearly forecasts as metrics.

Time series forecasting is an important, required and a common task in business to help inform future decision making and planning. There is a wide range of time series forecasting methods, often developed within specific disciplines for specific purposes, each of which has its own level of performance and cost. Exponential smoothing [2, 4, 8] and ARIMA (Auto Regressive Integrated Moving Average) [1] methods are the two most widely used approaches to time series forecasting, and provide complementary approaches to the problem. While exponential smoothing models are based on a description of the trend and seasonality in the data, ARIMA models aim to describe the autocorrelations in the data [5]. For seasonal time series forecasting, both the exponential smoothing and ARIMA models require a decomposition of the seasonal time series into a trend, seasonal and random components. In [6], we introduced a new and different approach/method to advanced time series forecasting: given a time series  $X$  and a time series  $Y$  of the same size  $n$ , we defined twenty four two-parameter functions that output the forecasted/predicted  $(n + 1)$  data entry. These functions are then used as building blocs for various models for both nonseasonal and seasonal time series. Our approach is nonparametric, i.e., assumes no particular probability

distribution, and rely on the computations of estimates via three methods, and then choosing one of the estimate via a series of optimization processes using eight different optimization criteria. In section 2, we recall both the rate of interest and the rate of discount times series of a given time series  $Y$ , both of which are used to define  $\sharp$ - and  $\flat$ - mean and median forecast estimates of a given order for a given time series  $X$ . We then recall eight optimization criteria  $\kappa = 1, \dots, 8$ , which allows us to choose a preferred maximum order and a preferred optimization length, both of which are used to forecast the next (unknown) observation for the time series  $X$ . Combining both the mean and median forecasts enables us to define the  $\sharp$ - and  $\flat$ -forecasts, and then combining the  $\sharp$  and  $\flat$  estimates enables us to define the  $\natural$ -forecast, leading to twenty four new two-parameter families of forecasting functions. In Section 3, we recall the seasonal functionally balanced forecasting models, and recall the seasonal power time series mapping. We then recall the seasonal per-period sum of absolute error stochastic latest-optimized  $\alpha$ -power model. In Section 4, we apply the seasonal per-period sum of absolute error stochastic latest-optimized  $\alpha$ -power model to eighteen university enrollment (head counts and credit hours) time series (Examples 1–18) by computing the sequence of three-step-ahead forecasts for the last sixteen years (48 trimesters). We also compare our forecasts to both Holt–Winters and ARIMA Models. In particular, we compute the sum of absolute errors, and we count the number of times (out of 48 trimesterly forecasts) that each of the models lead to the best trimesterly and yearly forecasts. For all eighteen university enrollment time series, we find that each of our twenty four model variations outperform both Holt–Winters and ARIMA models.

## 2. TWO-PARAMETER FAMILIES OF FORECASTING FUNCTIONS

Let  $X = (x_1, x_2, \dots, x_n)$  and  $Y = (y_1, y_2, \dots, y_n)$  be two time series with  $n \geq 2$  observations each. We define the rate of interest time series  $R_Y$  associated to  $Y$  as the time series  $R_Y = (r_1, r_2, \dots, r_{n-1})$  of size  $n - 1$  defined by

$$r_i := \frac{y_{i+1} - y_i}{y_i} \quad \text{for } i = 1, \dots, n - 1 \quad (1)$$

We define also the rate of discount time series  $D_Y$  associated to  $Y$  as the time series  $D_Y = (d_1, d_2, \dots, d_{n-1})$  of size  $n - 1$  defined by

$$d_i := \frac{y_{i+1} - y_i}{y_{i+1}} \quad \text{for } i = 1, \dots, n - 1 \quad (2)$$

Given the rate of interest time series  $R_Y$ , we define the mean (resp., median)  $i$ th rate of interest  $\bar{r}_{i,\lambda}$  (resp.,  $\tilde{r}_{i,\lambda}$ ) of order  $\lambda$  as follows:

$$\bar{r}_{i,\lambda} = \text{mean}(r_{i-\lambda+1}, \dots, r_i) \quad \text{where } i = 1, \dots, n - 1 \text{ and } \lambda = 1, \dots, i - 1 \quad (3)$$

$$\tilde{r}_{i,\lambda} = \text{median}(r_{i-\lambda+1}, \dots, r_i) \quad \text{where } i = 1, \dots, n - 1 \text{ and } \lambda = 1, \dots, i - 1 \quad (4)$$

Similarly, given the rate of discount time series  $D_Y$ , we define the mean (resp., median)  $i$ th rate of discount  $\bar{d}_{i,\lambda}$  (resp.,  $\tilde{d}_{i,\lambda}$ ) of order  $\lambda$  as follows:

$$\bar{d}_{i,\lambda} = \text{mean}(d_{i-\lambda+1}, \dots, d_i) \quad \text{where } i = 1, \dots, n - 1 \text{ and } \lambda = 1, \dots, i - 1 \quad (5)$$

$$\tilde{d}_{i,\lambda} = \text{median}(d_{i-\lambda+1}, \dots, d_i) \quad \text{where } i = 1, \dots, n - 1 \text{ and } \lambda = 1, \dots, i - 1 \quad (6)$$

Given a time series  $X$  and a rate of interest times series  $R_Y$ , we define two types of forecast estimates of order  $\lambda$  for  $x_{i+1}$ . The  $\sharp$ -mean (resp.,  $\sharp$ -median) forecast estimate of  $x_{i+1}$  of order  $\lambda$ , denoted  $\sharp\widehat{x}_{i+1,\lambda}$  (resp.,  $\sharp\widetilde{x}_{i+1,\lambda}$ ), is defined as

$$\sharp\widehat{x}_{i+1,\lambda} = x_i \cdot (1 + \bar{r}_{i-1,\lambda}) \quad \text{where } i = 2, \dots, n \text{ and } \lambda = 1, \dots, i \quad (7)$$

$$\sharp\widetilde{x}_{i+1,\lambda} = x_i \cdot (1 + \tilde{r}_{i-1,\lambda}) \quad \text{where } i = 2, \dots, n \text{ and } \lambda = 1, \dots, i \quad (8)$$

Similarly, given a time series  $X$  and a rate of discount times series  $D_Y$ , we define another two types of forecast estimates of order  $\lambda$  for  $x_{i+1}$ . The  $\flat$ -mean (resp.,  $\flat$ -median) forecast estimate of  $x_{i+1}$  of order  $\lambda$ , denoted  ${}^{\flat}\widehat{x}_{i+1,\lambda}$  (resp.,  ${}^{\flat}\widetilde{x}_{i+1,\lambda}$ ), is defined as

$${}^{\flat}\widehat{x}_{i+1,\lambda} = x_i \div (1 - \bar{d}_{i-1,\lambda}) \quad \text{where } i = 2, \dots, n \text{ and } \lambda = 1, \dots, i \quad (9)$$

$${}^{\flat}\widetilde{x}_{i+1,\lambda} = x_i \div (1 - \widetilde{d}_{i-1,\lambda}) \quad \text{where } i = 2, \dots, n \text{ and } \lambda = 1, \dots, i \quad (10)$$

In what follows, the symbol  $\star$  stands for either  $\sharp$  or  $\flat$ . Given the  $\star$ -mean (resp.,  $\star$ -median) estimate forecasts  ${}^{\star}\widehat{x}_{i,\lambda}$  (resp.,  ${}^{\star}\widetilde{x}_{i,\lambda}$ ) of order  $\lambda$  for  $i = 3, \dots, n$  and  $\lambda = 1, \dots, i - 1$ , we define eight optimization criteria of sum of  $\star$ -mean (resp.,  $\star$ -median) estimate forecast errors to forecast  $x_i$  of length  $\nu$ , denoted  ${}^{\star}\Sigma_{i,\lambda,\nu}^{\kappa}$  (resp.,  ${}^{\star}\widetilde{\Sigma}_{i,\lambda,\nu}^{\kappa}$ ), where  $\nu = 1, \dots, n - 1$ ,  $\lambda + \nu < n$  and  $\kappa = 1, \dots, 8$ , by :

$${}^{\star}\Sigma_{i,\lambda,\nu}^1 := \sum_{j=i-\nu+1}^i \left( \frac{{}^{\star}\widehat{x}_{j,\lambda} - x_j}{{}^{\star}\widehat{x}_{j,\lambda}} \right)^2, \quad {}^{\star}\widetilde{\Sigma}_{i,\lambda,\nu}^1 := \sum_{j=i-\nu+1}^i \left( \frac{{}^{\star}\widetilde{x}_{j,\lambda} - x_j}{{}^{\star}\widetilde{x}_{j,\lambda}} \right)^2, \quad (11)$$

$${}^{\star}\Sigma_{i,\lambda,\nu}^2 := \sum_{j=i-\nu+1}^i \left| \frac{{}^{\star}\widehat{x}_{j,\lambda} - x_j}{{}^{\star}\widehat{x}_{j,\lambda}} \right|, \quad {}^{\star}\widetilde{\Sigma}_{i,\lambda,\nu}^2 := \sum_{j=i-\nu+1}^i \left| \frac{{}^{\star}\widetilde{x}_{j,\lambda} - x_j}{{}^{\star}\widetilde{x}_{j,\lambda}} \right|, \quad (12)$$

$${}^{\star}\Sigma_{i,\lambda,\nu}^3 := \sum_{j=i-\nu+1}^i \frac{({}^{\star}\widehat{x}_{j,\lambda} - x_j)^2}{|{}^{\star}\widehat{x}_{j,\lambda}|}, \quad {}^{\star}\widetilde{\Sigma}_{i,\lambda,\nu}^3 := \sum_{j=i-\nu+1}^i \frac{({}^{\star}\widetilde{x}_{j,\lambda} - x_j)^2}{|{}^{\star}\widetilde{x}_{j,\lambda}|}, \quad (13)$$

$${}^{\star}\Sigma_{i,\lambda,\nu}^4 := \sum_{j=i-\nu+1}^i \left( \frac{{}^{\star}\widehat{x}_{j,\lambda} - x_j}{x_j} \right)^2, \quad {}^{\star}\widetilde{\Sigma}_{i,\lambda,\nu}^4 := \sum_{j=i-\nu+1}^i \left( \frac{{}^{\star}\widetilde{x}_{j,\lambda} - x_j}{x_j} \right)^2, \quad (14)$$

$${}^{\star}\Sigma_{i,\lambda,\nu}^5 := \sum_{j=i-\nu+1}^i \left| \frac{{}^{\star}\widehat{x}_{j,\lambda} - x_j}{x_j} \right|, \quad {}^{\star}\widetilde{\Sigma}_{i,\lambda,\nu}^5 := \sum_{j=i-\nu+1}^i \left| \frac{{}^{\star}\widetilde{x}_{j,\lambda} - x_j}{x_j} \right|, \quad (15)$$

$${}^{\star}\Sigma_{i,\lambda,\nu}^6 := \sum_{j=i-\nu+1}^i \frac{({}^{\star}\widehat{x}_{j,\lambda} - x_j)^2}{|x_j|}, \quad {}^{\star}\widetilde{\Sigma}_{i,\lambda,\nu}^6 := \sum_{j=i-\nu+1}^i \frac{({}^{\star}\widetilde{x}_{j,\lambda} - x_j)^2}{|x_j|}, \quad (16)$$

$${}^{\star}\Sigma_{i,\lambda,\nu}^7 := \sum_{j=i-\nu+1}^i ({}^{\star}\widehat{x}_{j,\lambda} - x_j)^2, \quad {}^{\star}\widetilde{\Sigma}_{i,\lambda,\nu}^7 := \sum_{j=i-\nu+1}^i ({}^{\star}\widetilde{x}_{j,\lambda} - x_j)^2, \quad (17)$$

$${}^{\star}\Sigma_{i,\lambda,\nu}^8 := \sum_{j=i-\nu+1}^i |{}^{\star}\widehat{x}_{j,\lambda} - x_j|, \quad {}^{\star}\widetilde{\Sigma}_{i,\lambda,\nu}^8 := \sum_{j=i-\nu+1}^i |{}^{\star}\widetilde{x}_{j,\lambda} - x_j|. \quad (18)$$

For a given optimization criteria  $\kappa = 1, \dots, 8$ , a chosen maximum allowed order  $\lambda_{\max}$ , and a fixed length  $\nu$  such that  $\lambda_{\max} + \nu < n$ , the least sum of  $\star$ -mean (resp.,  $\star$ -median) estimate forecast errors  ${}^{\star}\Sigma_{\bar{n},\lambda_{\max},\nu}^{\kappa,*}$  (resp.,  ${}^{\star}\widetilde{\Sigma}_{\bar{n},\lambda_{\max},\nu}^{\kappa,*}$ ), to forecast  $x_n$ , of order  $\lambda_{\max}$  and length  $\nu$ , are defined as follows:

$${}^{\star}\Sigma_{\bar{n},\lambda_{\max},\nu}^{\kappa,*} = \min_{\lambda=1,\dots,\lambda_{\max}} {}^{\star}\Sigma_{\bar{n},\lambda,\nu}^{\kappa} \quad \text{and} \quad {}^{\star}\widetilde{\Sigma}_{\bar{n},\lambda_{\max},\nu}^{\kappa,*} = \min_{\lambda=1,\dots,\lambda_{\max}} {}^{\star}\widetilde{\Sigma}_{\bar{n},\lambda,\nu}^{\kappa} \quad (19)$$

From the latter, and for a given optimization criteria  $\kappa = 1, \dots, 8$ , a chosen maximum order  $\lambda_{\max} = 1, \dots, n - 1$  and fixed length  $\nu = 1, \dots, n - 1$  such that  $\lambda_{\max} + \nu < n$ , we extract the optimal  $\star$ -mean (resp.,  $\star$ -median) order  ${}^{\star}\lambda_{\lambda_{\max},\nu}^{k,*}$  (resp.,  ${}^{\star}\widetilde{\lambda}_{\lambda_{\max},\nu}^{k,*}$ ) from 1 to  $\lambda_{\max}$  to forecast  $x_n$  for a given length  $\nu$  as follows:

$${}^{\star}\lambda_{\lambda_{\max},\nu}^{k,*} = \arg \left( \min_{\lambda=1,\dots,\lambda_{\max}} {}^{\star}\Sigma_{\bar{n},\lambda_{\max},\nu}^{\kappa,*} \right) \quad \text{and} \quad {}^{\star}\widetilde{\lambda}_{\lambda_{\max},\nu}^{k,*} = \arg \left( \min_{\lambda=1,\dots,\lambda_{\max}} {}^{\star}\widetilde{\Sigma}_{\bar{n},\lambda_{\max},\nu}^{\kappa,*} \right) \quad (20)$$

These latter four optimal orders  $\star \lambda_{\lambda_{max}, \nu}^{k, \star}$  and  $\star \widetilde{\lambda}_{\lambda_{max}, \nu}^{k, \star}$ , where  $\star$  is either  $\sharp$  or  $\flat$ , are then used as preferred orders to forecast  $x_{n+1}$  from the  $n$  observations  $x_1, \dots, x_n$  and either rate of interest times series  $R(Y)$  or rate of discount time series  $D(Y)$ . Finally, we propose the following thirty two 2-parameter families of forecasting functions, that is for each of the four type of forecast estimate ( $\sharp$ -mean,  $\sharp$ -median,  $\flat$ -mean, and  $\flat$ -median) with each one of the eight optimization criteria  $\kappa = 1, \dots, 8$ .

**Definition 1.** Let  $X = (x_1, x_2, \dots, x_n)$  and  $Y = (y_1, y_2, \dots, y_n)$  be two time series with  $n \geq 2$  observations. Let  $R_Y$  be the rate of interest time series associated to  $Y$ . For each optimization criteria  $\kappa = 1, \dots, 8$ , a chosen maximum order  $\lambda_{max} = 1, \dots, n - 2$ , and a fixed length  $\nu$  in  $1, \dots, n - 2$  such that  $\lambda_{max} + \nu < n$ , the  $\sharp$ -mean (resp.,  $\sharp$ -median) forecasting function  $\sharp \Psi_{\lambda_{max}, \nu}^{\kappa}$  (resp.,  $\sharp \widetilde{\Psi}_{\lambda_{max}, \nu}^{\kappa}$ ) of order  $\lambda_{max}$  and length  $\nu$  maps the time series  $X$  and the rate of interest  $R_Y$  to a  $\sharp$ -mean (resp.,  $\sharp$ -median) forecast of  $x_{n+1}$ , denoted  $\sharp \widehat{x}_{n+1}^{\kappa}$  (resp.,  $\sharp \widetilde{\widehat{x}}_{n+1}^{\kappa}$ ), as follows:

$$\sharp \widehat{x}_{n+1}^{\kappa} = \sharp \Psi_{\lambda_{max}, \nu}^{\kappa}(X, R_Y) := x_n \cdot \left(1 + \bar{r}_{n-1, \sharp \lambda_{\lambda_{max}, \nu}^{\kappa, \star}}\right) \quad (21)$$

resp.,

$$\sharp \widetilde{\widehat{x}}_{n+1}^{\kappa} = \sharp \widetilde{\Psi}_{\lambda_{max}, \nu}^{\kappa}(X, R_Y) := x_n \cdot \left(1 + \widetilde{r}_{n-1, \sharp \widetilde{\lambda}_{\lambda_{max}, \nu}^{\kappa, \star}}\right) \quad (22)$$

where the  $\sharp$ -mean (resp.,  $\sharp$ -median) optimal order  $\sharp \lambda_{\lambda_{max}, \nu}^{k, \star}$  (resp.,  $\sharp \widetilde{\lambda}_{\lambda_{max}, \nu}^{k, \star}$ ) is as in (20).

**Definition 2.** Let  $X = (x_1, x_2, \dots, x_n)$  and  $Y = (y_1, y_2, \dots, y_n)$  be two time series with  $n \geq 2$  observations. Let  $D_Y$  be the rate of discount time series associated to  $Y$ . For each optimization criteria  $\kappa = 1, \dots, 8$ , a chosen maximum order  $\lambda_{max} = 1, \dots, n - 2$ , and a fixed length  $\nu$  in  $1, \dots, n - 2$  such that  $\lambda_{max} + \nu < n$ , the  $\flat$ -mean (resp.,  $\flat$ -median) forecasting function  $\flat \Psi_{\lambda_{max}, \nu}^{\kappa}$  (resp.,  $\flat \widetilde{\Psi}_{\lambda_{max}, \nu}^{\kappa}$ ) of order  $\lambda_{max}$  and length  $\nu$  maps the time series  $X$  and the rate of discount  $D_Y$  to a  $\flat$ -mean (resp.,  $\flat$ -median) forecast of  $x_{n+1}$ , denoted  $\flat \widehat{x}_{n+1}^{\kappa}$  (resp.,  $\flat \widetilde{\widehat{x}}_{n+1}^{\kappa}$ ), as follows:

$$\flat \widehat{x}_{n+1}^{\kappa} = \flat \Psi_{\lambda_{max}, \nu}^{\kappa}(X, D_Y) := x_n \div \left(1 - \bar{d}_{n-1, \flat \lambda_{\lambda_{max}, \nu}^{\kappa, \star}}\right) \quad (23)$$

resp.,

$$\flat \widetilde{\widehat{x}}_{n+1}^{\kappa} = \flat \widetilde{\Psi}_{\lambda_{max}, \nu}^{\kappa}(X, D_Y) := x_n \div \left(1 + \widetilde{d}_{n-1, \flat \widetilde{\lambda}_{\lambda_{max}, \nu}^{\kappa, \star}}\right) \quad (24)$$

where the  $\flat$ -mean (resp.,  $\flat$ -median) optimal order  $\flat \lambda_{\lambda_{max}, \nu}^{k, \star}$  (resp.,  $\flat \widetilde{\lambda}_{\lambda_{max}, \nu}^{k, \star}$ ) is as in (20).

One can optimize within a pair of forecast estimates, that is, choosing two forecast estimate types among the four. There are six possible pairs, and thus one can define another forty eight 2-parameter families of forecasting functions. We propose in the following only two pairs (among the six) leading to a  $\sharp$ - (resp.,  $\flat$ -) forecasting function, which combines the  $\sharp$ -mean (resp.,  $\flat$ -mean) and  $\sharp$ -median (resp.,  $\flat$ -median) forecasting functions.

**Definition 3.** Let  $X = (x_1, x_2, \dots, x_n)$  and  $Y = (y_1, y_2, \dots, y_n)$  be two time series with  $n \geq 2$  observations. Let  $R_Y$  (resp.,  $D_Y$ ) be the rate of interest (resp., discount) time series associated to  $Y$ . For each optimization criteria  $\kappa = 1, \dots, 8$ , a chosen maximum order  $\lambda_{max} = 1, \dots, n - 2$ , and a fixed length  $\nu$  in  $1, \dots, n - 2$  such that  $\lambda_{max} + \nu < n$ , the  $\sharp$ - (resp.,  $\flat$ -) forecasting function  $\sharp \Psi_{\lambda_{max}, \nu}^{\kappa}$  (resp.,  $\flat \Psi_{\lambda_{max}, \nu}^{\kappa}$ ) of order  $\lambda_{max}$  and length  $\nu$  maps the time series  $X$  and the rate of interest (resp., discount)  $R_Y$  (resp.,  $D_Y$ ) to a  $\sharp$ - (resp.,  $\flat$ -) forecast of  $x_{n+1}$ , denoted  $\sharp \widehat{x}_{n+1}^{\kappa}$  (resp.,  $\flat \widehat{x}_{n+1}^{\kappa}$ ), as follows:

$$\sharp \widehat{x}_{n+1}^{\kappa} = \sharp \Psi_{\lambda_{max}, \nu}^{\kappa}(X, R_Y) := \begin{cases} \sharp \widehat{x}_{n+1}^{\kappa} & \text{if } \sharp \sum_{\bar{n}, \lambda_{max}, \nu}^{\kappa, \star} \leq \sharp \sum_{\bar{n}, \lambda_{max}, \nu}^{\kappa, \star} \\ \flat \widehat{x}_{n+1}^{\kappa} & \text{if } \sharp \sum_{\bar{n}, \lambda_{max}, \nu}^{\kappa, \star} > \sharp \sum_{\bar{n}, \lambda_{max}, \nu}^{\kappa, \star} \end{cases} \quad (25)$$

resp.,

$$\widehat{x}_{n+1}^\kappa = {}^b\Psi_{\lambda_{max},\nu}^\kappa(X, D_Y) := \begin{cases} {}^b\widehat{x}_{n+1}^\kappa & \text{if } {}^b\Sigma_{\bar{n},\lambda_{max},\nu}^{\kappa,*} \leq {}^b\Sigma_{\bar{n},\lambda_{max},\nu}^{\kappa,*} \\ {}^b\widehat{x}_{n+1}^\kappa & \text{if } {}^b\Sigma_{\bar{n},\lambda_{max},\nu}^{\kappa,*} > {}^b\Sigma_{\bar{n},\lambda_{max},\nu}^{\kappa,*} \end{cases} \quad (26)$$

where the least sum of  $\#$ - (resp.,  $b$ -) mean estimate forecast error  $\# \Sigma_{\bar{n},\lambda_{max},\nu}^{\kappa,*}$  (resp.,  ${}^b \Sigma_{\bar{n},\lambda_{max},\nu}^{\kappa,*}$ ) and the least sum of  $\#$ - (resp.,  $b$ ) median estimate forecast error  $\# \Sigma_{\bar{n},\lambda_{max},\nu}^{\kappa,*}$  (resp.,  ${}^b \Sigma_{\bar{n},\lambda_{max},\nu}^{\kappa,*}$ ) are as in (19).

Similarly, one can optimize within a triple of forecast estimates. There are four possible triples, and thus one can add another thirty two 2-parameter families of forecasting functions. Lastly, one can optimize across the four forecast estimates, and thus add another eight 2-parameter families of forecasting functions, as shown in the following proposition.

**Definition 4.** Let  $X = (x_1, x_2, \dots, x_n)$  and  $Y = (y_1, y_2, \dots, y_n)$  be two time series with  $n \geq 2$  observations. Let  $R_Y$  (resp.,  $D_Y$ ) be the rate of interest (resp., discount) times series associated to  $Y$ . For each optimization criteria  $\kappa = 1, \dots, 8$ , a chosen maximum order  $\lambda_{max} = 1, \dots, n - 2$ , and fixed length  $\nu$  in  $1, \dots, n - 2$  such that  $\lambda_{max} + \nu < n$ , the forecasting function  ${}^b\Psi_{\lambda_{max},\nu}^\kappa$  of order  $\lambda_{max}$  and length  $\nu$  maps the time series  $X$ , the rate of interest  $R_Y$ , and the rate of discount  $D_Y$  to a forecast of  $x_{n+1}$ , denoted  ${}^b\widehat{x}_{n+1}^\kappa$ , as follows:

$${}^b\widehat{x}_{n+1}^\kappa = {}^b\Psi_{\lambda_{max},\nu}^\kappa(X, R_Y, D_Y) := \begin{cases} {}^b\widehat{x}_{n+1}^\kappa & \text{if } \# \Sigma_{n,\lambda_{max},\nu}^{\kappa,*} \leq {}^b \Sigma_{n,\lambda_{max},\nu}^{\kappa,*} \\ {}^b\widehat{x}_{n+1}^\kappa & \text{if } \# \Sigma_{n,\lambda_{max},\nu}^{\kappa,*} > {}^b \Sigma_{n,\lambda_{max},\nu}^{\kappa,*} \end{cases} \quad (27)$$

where the least  $\#$ -estimate forecast error  $\# \Sigma_{n,\lambda_{max},\nu}^{\kappa,*}$  is

$$\# \Sigma_{n,\lambda_{max},\nu}^{\kappa,*} := \min(\# \Sigma_{\bar{n},\lambda_{max},\nu}^{\kappa,*}, \# \Sigma_{\bar{n},\lambda_{max},\nu}^{\kappa,*}), \quad (28)$$

and the least  $b$ -estimate forecast error  ${}^b \Sigma_{n,\lambda_{max},\nu}^{\kappa,*}$  is

$${}^b \Sigma_{n,\lambda_{max},\nu}^{\kappa,*} := \min({}^b \Sigma_{\bar{n},\lambda_{max},\nu}^{\kappa,*}, {}^b \Sigma_{\bar{n},\lambda_{max},\nu}^{\kappa,*}) \quad (29)$$

Note that given a times series  $X$ , one can use its associated rate of interest  $R_X$  and/or rate of discount  $D_X$ , i.e., set  $Y = X$  in for forecasting functions. Note also that one can define a total of one hundred twenty forecasting functions. However, we focus our attention on only twenty four of them, i.e.,  $\# \Psi^\kappa$ ,  ${}^b \Psi^\kappa$  and  ${}^b \Psi^\kappa$  for  $\kappa = 1, \dots, 8$ .

### 3. SEASONAL FUNCTIONALLY BALANCED FORECASTING MODELS

Let  $X$  be a time series with a frequency  $f$  and size  $nf$ , i.e.,

$$X = (x_1, \dots, x_f, x_{f+1}, \dots, x_{2f}, \dots, x_{(n-1)f+1}, \dots, x_{nf}) \quad (30)$$

We denote by  $X_{s/f}$  the sub time series of size  $n$  defined as follows:

$$X_{s/f} = (x_s, x_{f+s}, x_{2f+s}, \dots, x_{(n-1)f+s}), \quad \text{where } s = 1, \dots, f \quad (31)$$

For instance, if  $X$  is a monthly time series starting in January, then the frequency is  $f = 12$  and  $X_{1/12}$  is the January sub time series,  $X_{7/12}$  is the July sub time series, and  $X_{12/12}$  is the December sub time series. Similarly, if  $X$  is a quarterly time series starting in the first quarter of the year (from January 1<sup>st</sup> to March 31<sup>st</sup>), then the frequency is  $f = 4$ ,  $X_{1/4}$  is the first quarter sub time series,  $X_{2/4}$  is the second quarter sub time series,  $X_{3/4}$  is the third quarter sub time series and  $X_{4/4}$  is the last quarter sub time series.

**Proposition 1** (Seasonal functionally balanced forecasting models). *Let  $X = (x_1, \dots, x_{nf})$  and  $Y = (y_1, \dots, y_{nf})$  be two time series with frequency  $f$  and with  $nf$  observations, where  $n \geq 2$ . For each optimization criteria  $\kappa = 1, \dots, 8$ , the  $\sharp$ - (resp.,  $\flat$ -,  $\natural$ -) functionally balanced forecasting function  $\sharp\Psi_f^\kappa$  (resp.,  $\flat\Psi_f^\kappa$ ,  $\natural\Psi_f^\kappa$ ) maps the seasonal time series  $X$  and the rate of interest (resp., discount) time series  $R_{\Phi(Y)}$  (resp.,  $D_{\Phi(Y)}$ ) to a  $\sharp$ - (resp.,  $\flat$ -,  $\natural$ -) balanced forecast of  $x_{nf+s}$ , denoted  $\sharp\hat{x}_{nf+s}^\kappa$  (resp.,  $\flat\hat{x}_{nf+s}^\kappa$ ,  $\natural\hat{x}_{nf+s}^\kappa$ ), where  $s = 1, \dots, f$ , and  $\Phi$  is a function that maps  $Y$  to a time series of the same size, as follows:*

$$\sharp\hat{x}_{nf+s}^\kappa = \sharp\Psi_f^\kappa(X, R_{\Phi(Y)}) := \sharp\Psi_{\lceil \frac{n-1}{2} \rceil, \lfloor \frac{n-1}{2} \rfloor}^\kappa(X_{s/f}, R_{\Phi(Y)_{s/f}}) \quad (32)$$

$$\flat\hat{x}_{nf+s}^\kappa = \flat\Psi_f^\kappa(X, D_{\Phi(Y)}) := \flat\Psi_{\lceil \frac{n-1}{2} \rceil, \lfloor \frac{n-1}{2} \rfloor}^\kappa(X_{s/f}, D_{\Phi(Y)_{s/f}}) \quad (33)$$

$$\natural\hat{x}_{nf+s}^\kappa = \natural\Psi_f^\kappa(X, R_{\Phi(Y)}, D_{\Phi(Y)}) := \natural\Psi_{\lceil \frac{n-1}{2} \rceil, \lfloor \frac{n-1}{2} \rfloor}^\kappa(X_{s/f}, R_{\Phi(Y)_{s/f}}, D_{\Phi(Y)_{s/f}}) \quad (34)$$

Therefore, given a seasonal time series of size  $nf$ , one can forecast the next  $f$  values of the time series, i.e.,  $\bullet\hat{x}_{nf+1}^\kappa, \bullet\hat{x}_{nf+2}^\kappa, \dots, \bullet\hat{x}_{(n+1)f}^\kappa$ , where  $\bullet$  is either  $\sharp$ ,  $\flat$  or  $\natural$ . In order to compare these seasonal functionally balanced forecasting functions for a given seasonal time series  $X = (x_1, \dots, x_{nf})$  with frequency  $f$ , where  $n \geq 2$ , let's compute the sequence of forecasts of  $x_{2f+1}$ ,  $x_{2f+2}$ ,  $\dots$ ,  $x_{nf}$ . For  $s = 1, \dots, f$ , and  $i = 2, \dots, n-1$ , the sequence of  $f$ -step-ahead forecasts for  $X$  are then

$$\left(\sharp\hat{x}_{if+s}^\kappa\right)_{i,s} := \left(\sharp\Psi_{\lceil \frac{i-1}{2} \rceil, \lfloor \frac{i-1}{2} \rfloor}^\kappa(X_{[1:i]_{s/f}}, R_{\Phi(Y_{[1:i]_{s/f}})})\right)_{i,s} \quad (35)$$

$$\left(\flat\hat{x}_{if+s}^\kappa\right)_{i,s} := \left(\flat\Psi_{\lceil \frac{i-1}{2} \rceil, \lfloor \frac{i-1}{2} \rfloor}^\kappa(X_{[1:i]_{s/f}}, D_{\Phi(Y_{[1:i]_{s/f}})})\right)_{i,s} \quad (36)$$

$$\left(\natural\hat{x}_{if+s}^\kappa\right)_{i,s} := \left(\natural\Psi_{\lceil \frac{i-1}{2} \rceil, \lfloor \frac{i-1}{2} \rfloor}^\kappa(X_{[1:i]_{s/f}}, R_{\Phi(Y_{[1:i]_{s/f}})}, D_{\Phi(Y_{[1:i]_{s/f}})})\right)_{i,s} \quad (37)$$

To compare these balanced forecasting functions, one may use two types of sum of absolute errors, or the sum of square errors: per period and global. Indeed, for each optimization criteria  $\kappa = 1, \dots, 8$ , the  $\sharp$ - (resp.,  $\flat$ -,  $\natural$ -) sum of absolute errors for the  $i^{\text{th}}$  period and the  $\sharp$ - (resp.,  $\flat$ -,  $\natural$ -) sum of square errors for the  $i^{\text{th}}$  period, where  $i = 3, \dots, n$ , are defined as follows:

$$\sharp\text{SAE}_i^\kappa := \sum_{s=1}^f \left| \sharp\hat{x}_{(i-1)f+s}^\kappa - x_{(i-1)f+s} \right| \quad \text{and} \quad \sharp\text{SSE}_i^\kappa := \sum_{s=3}^f \left( \sharp\hat{x}_{(i-1)f+j}^\kappa - x_{(i-1)f+s} \right)^2 \quad (38)$$

$$\flat\text{SAE}_i^\kappa := \sum_{s=1}^f \left| \flat\hat{x}_{(i-1)f+s}^\kappa - x_{(i-1)f+s} \right| \quad \text{and} \quad \flat\text{SSE}_i^\kappa := \sum_{s=3}^f \left( \flat\hat{x}_{(i-1)f+j}^\kappa - x_{(i-1)f+s} \right)^2 \quad (39)$$

$$\natural\text{SAE}_i^\kappa := \sum_{s=1}^f \left| \natural\hat{x}_{(i-1)f+s}^\kappa - x_{(i-1)f+s} \right| \quad \text{and} \quad \natural\text{SSE}_i^\kappa := \sum_{s=3}^f \left( \natural\hat{x}_{(i-1)f+j}^\kappa - x_{(i-1)f+s} \right)^2 \quad (40)$$

and the  $\sharp$ - (resp.,  $\flat$ -,  $\natural$ -) sum of absolute errors and the  $\sharp$ - (resp.,  $\flat$ -,  $\natural$ -) sum of square errors are defined as follows:

$$\sharp\text{SAE}^\kappa := \sum_{i=3}^n \sharp\text{SAE}_i^\kappa \quad \text{and} \quad \sharp\text{SSE}^\kappa := \sum_{i=3}^n \sharp\text{SSE}_i^\kappa \quad (41)$$

$$\flat\text{SAE}^\kappa := \sum_{i=3}^n \flat\text{SAE}_i^\kappa \quad \text{and} \quad \flat\text{SSE}^\kappa := \sum_{i=3}^n \flat\text{SSE}_i^\kappa \quad (42)$$

$$\natural\text{SAE}^\kappa := \sum_{i=3}^n \natural\text{SAE}_i^\kappa \quad \text{and} \quad \natural\text{SSE}^\kappa := \sum_{i=3}^n \natural\text{SSE}_i^\kappa \quad (43)$$

The model that minimizes either the sum of absolute errors or the sum of square errors (depending on the forecaster's interests) is said to be optimal for  $X$ . One can also count the number of optimal per-period sum of absolute or square errors. We illustrate the latter in the following two examples using one seasonal monthly time series and one seasonal quarterly time series using the following function  $\Phi$ .

**Definition 5** (Seasonal power time series mapping). *Let  $Y = (y_1, \dots, y_{nf})$  be a time series with frequency  $f$  and with  $nf$  observations, where  $n \geq 2$ . Let  $\alpha = (\alpha_1, \dots, \alpha_f)$  be an  $f$ -tuple where  $\alpha_s \in [0, 1]$  for  $s = 1, \dots, f$ . The seasonal power times series mapping  $\Phi$  maps  $Y$  and  $\alpha$  to a seasonal time series  $\Phi(Y)$  of the same size and frequency such that for each  $s = 1, \dots, f$ ,*

$$\Phi(Y)_{s/f} := Y_{s/f}^{\alpha_1} + Y_{(s+1)/f}^{\alpha_2} + \dots + Y_{f/f}^{\alpha_{f-s+1}} + Y_{1/f}^{\alpha_{f-s+2}} + Y_{2/f}^{\alpha_{f-s+3}} + \dots + Y_{(s-1)/f}^{\alpha_f}. \quad (44)$$

In Example 6 in [6], we used three fixed  $\alpha$  power to compute the seasonally balanced forecasts. While the sum of absolute errors of our  $\sharp$ -,  $\flat$ - and  $\natural$ -forecasts were consistently lower than both Holt–Winters and ARIMA models, the number of best monthly and/or yearly forecasts were fairly higher. Therefore, we introduced in [6] an optimized and dynamic method to select the  $\alpha$ -powers for the seasonally balanced forecasts. The latter leads to a substantially improved forecasts for seasonal time series.

**Definition 6** (Seasonal per-period SAE latest-optimized  $\alpha$ -power). *Let  $X = (x_1, \dots, x_{nf})$  and  $Y = (y_1, \dots, y_{nf})$  be two time series with frequency  $f$  and with  $nf$  observations, where  $n \geq 2$ . For each optimization criteria  $\kappa = 1, \dots, 8$ , the seasonal  $\sharp$  (resp.,  $\flat$ ,  $\natural$ ) SAE latest-optimized  $\alpha$ -power for the period  $i = 3, \dots, n$ , denoted  $\sharp\alpha_i^\kappa$  (resp.,  $\flat\alpha_i^\kappa$ ,  $\natural\alpha_i^\kappa$ ), are defined as the  $f$ -tuples*

$$\sharp\alpha_3^\kappa = \flat\alpha_3^\kappa = \natural\alpha_3^\kappa = (1, 1, \dots, 1) \quad (45)$$

and

$$\sharp\alpha_i^\kappa = (\sharp\alpha_{1,i}^\kappa, \dots, \sharp\alpha_{f,i}^\kappa) := \arg \left( \min_{\beta_1, \dots, \beta_f \in [0,1]} \sharp SAE_{i-1}^\kappa \right) \quad \text{for } i = 4, \dots, n \quad (46)$$

$$\flat\alpha_i^\kappa = (\flat\alpha_{1,i}^\kappa, \dots, \flat\alpha_{f,i}^\kappa) := \arg \left( \min_{\beta_1, \dots, \beta_f \in [0,1]} \flat SAE_{i-1}^\kappa \right) \quad \text{for } i = 4, \dots, n \quad (47)$$

$$\natural\alpha_i^\kappa = (\natural\alpha_{1,i}^\kappa, \dots, \natural\alpha_{f,i}^\kappa) := \arg \left( \min_{\beta_1, \dots, \beta_f \in [0,1]} \natural SAE_{i-1}^\kappa \right) \quad \text{for } i = 4, \dots, n \quad (48)$$

where  $\sharp SAE_i^\kappa$ ,  $\flat SAE_i^\kappa$  and  $\natural SAE_i^\kappa$  are as in (38), (39) and (40), and  $\Phi(Y)$  as in (44).

We define the sequence of seasonal per-period SSE latest-optimized  $\alpha$ -power in a similar way. While an analytic derivation of the sequence of these seasonal per-period SAE latest-optimized  $\alpha$ -powers can be (in practice) tedious, we introduce in the following stochastic (and more pragmatic) method to obtain an approximate of the sequence of these seasonal per-period SAE latest-optimized  $\alpha$ -powers, which we use for the forecasting.

**Definition 7** (Seasonal per-period SAE stochastic latest-optimized  $\alpha$ -power). *Let  $X = (x_1, \dots, x_{nf})$  and  $Y = (y_1, \dots, y_{nf})$  be two time series with frequency  $f$  and with  $nf$  observations, where  $n \geq 2$ . For each optimization criteria  $\kappa = 1, \dots, 8$ , the random  $\sharp$  (resp.,  $\flat$ ,  $\natural$ ) SAE  $\beta$ -power for the period  $i = 3, \dots, n$ , denoted  $\sharp_{\beta} \widetilde{SAE}_i^\kappa$  is the sum of absolute error for the  $i$ th period where the power  $\beta = (\beta_1, \dots, \beta_f)$  in  $\Phi(Y)$  are such that each of the  $\beta_i$  are drawn from a uniform distribution on the interval  $[0, 1]$ . Thus, for each optimization criteria  $\kappa = 1, \dots, 8$ , the seasonal  $\sharp$  (resp.,  $\flat$ ,  $\natural$ ) SAE stochastic latest-optimized  $\alpha$ -power for the period  $i = 3, \dots, n$ , denoted  $\sharp\widetilde{\alpha}_i^\kappa$  (resp.,  $\flat\widetilde{\alpha}_i^\kappa$ ,  $\natural\widetilde{\alpha}_i^\kappa$ ), are defined as the  $f$ -tuples*

$$\sharp\widetilde{\alpha}_3^\kappa = \flat\widetilde{\alpha}_3^\kappa = \natural\widetilde{\alpha}_3^\kappa = (1, 1, \dots, 1) \quad (49)$$



and

$$\# \tilde{\alpha}_i^\kappa = (\# \tilde{\alpha}_{1,i}^\kappa, \dots, \# \tilde{\alpha}_{f,i}^\kappa) := \arg_{\beta_1, \dots, \beta_f} \left( \min_{j=1, \dots, N} \# \beta \widetilde{SAE}_{i-1}^\kappa \right) \quad \text{for } i = 4, \dots, n \quad (50)$$

$$b \tilde{\alpha}_i^\kappa = (b \tilde{\alpha}_{1,i}^\kappa, \dots, b \tilde{\alpha}_{f,i}^\kappa) := \arg_{\beta_1, \dots, \beta_f} \left( \min_{j=1, \dots, N} b \beta \widetilde{SAE}_{i-1}^\kappa \right) \quad \text{for } i = 4, \dots, n \quad (51)$$

$$\natural \tilde{\alpha}_i^\kappa = (\natural \tilde{\alpha}_{1,i}^\kappa, \dots, \natural \tilde{\alpha}_{f,i}^\kappa) := \arg_{\beta_1, \dots, \beta_f} \left( \min_{j=1, \dots, N} \natural \beta \widetilde{SAE}_{i-1}^\kappa \right) \quad \text{for } i = 4, \dots, n \quad (52)$$

where  $N$  is a large number (for instance,  $N = 1000$ ).

**Proposition 2** (Seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models). *Let  $X = (x_1, \dots, x_{nf})$  and  $Y = (y_1, \dots, y_{nf})$  be two time series with frequency  $f$  and with  $nf$  observations, where  $n \geq 2$ . For each optimization criteria  $\kappa = 1, \dots, 8$ , the  $\#$ - (resp.,  $b$ -,  $\natural$ -) seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting function  $\# \Psi_f^\kappa$  (resp.,  $b \Psi_f^\kappa$ ,  $\natural \Psi_f^\kappa$ ) maps the seasonal time series  $X$  and the rate of interest (resp., discount) time series  $R_{\tilde{\Phi}(Y)}$  (resp.,  $D_{\tilde{\Phi}(Y)}$ ) to a  $\#$ - (resp.,  $b$ -,  $\natural$ -) balanced forecast of  $x_{nf+s}$ , denoted  $\# \hat{x}_{nf+s}^\kappa$  (resp.,  $b \hat{x}_{nf+s}^\kappa$ ,  $\natural \hat{x}_{nf+s}^\kappa$ ), where  $s = 1, \dots, f$ , and  $\tilde{\Phi}$  is as in (44) with the  $\#$ -power  $\# \tilde{\alpha}^\kappa$  (resp.,  $b \tilde{\alpha}^\kappa$ ,  $\natural \tilde{\alpha}^\kappa$ ) is as in (50) (resp., (51), (52)), as follows:*

$$\# \hat{x}_{nf+s}^\kappa = \# \Psi_f^\kappa(X, R_{\tilde{\Phi}(Y)}) := \# \Psi_{\lceil \frac{n-1}{2} \rceil, \lfloor \frac{n-1}{2} \rfloor}^\kappa(X_{s/f}, R_{\tilde{\Phi}(Y)_{s/f}}) \quad (53)$$

$$b \hat{x}_{nf+s}^\kappa = b \Psi_f^\kappa(X, D_{\tilde{\Phi}(Y)}) := b \Psi_{\lceil \frac{n-1}{2} \rceil, \lfloor \frac{n-1}{2} \rfloor}^\kappa(X_{s/f}, D_{\tilde{\Phi}(Y)_{s/f}}) \quad (54)$$

$$\natural \hat{x}_{nf+s}^\kappa = \natural \Psi_f^\kappa(X, R_{\tilde{\Phi}(Y)}, D_{\tilde{\Phi}(Y)}) := \natural \Psi_{\lceil \frac{n-1}{2} \rceil, \lfloor \frac{n-1}{2} \rfloor}^\kappa(X_{s/f}, R_{\tilde{\Phi}(Y)_{s/f}}, D_{\tilde{\Phi}(Y)_{s/f}}) \quad (55)$$

Thus, for  $s = 1, \dots, f$ , and  $i = 2, \dots, n-1$ , the sequence of  $f$ -step-ahead forecasts for  $X$  are then

$$\left( \# \hat{x}_{if+s}^\kappa \right)_{i,s} := \left( \# \Psi_{\lceil \frac{i-1}{2} \rceil, \lfloor \frac{i-1}{2} \rfloor}^\kappa(X_{[1:if]_{s/f}}, R_{\tilde{\Phi}(Y_{[1:if]_{s/f}})}) \right)_{i,s} \quad (56)$$

$$\left( b \hat{x}_{if+s}^\kappa \right)_{i,s} := \left( b \Psi_{\lceil \frac{i-1}{2} \rceil, \lfloor \frac{i-1}{2} \rfloor}^\kappa(X_{[1:if]_{s/f}}, D_{\tilde{\Phi}(Y_{[1:if]_{s/f}})}) \right)_{i,s} \quad (57)$$

$$\left( \natural \hat{x}_{if+s}^\kappa \right)_{i,s} := \left( \natural \Psi_{\lceil \frac{i-1}{2} \rceil, \lfloor \frac{i-1}{2} \rfloor}^\kappa(X_{[1:if]_{s/f}}, R_{\tilde{\Phi}(Y_{[1:if]_{s/f}})}, D_{\tilde{\Phi}(Y_{[1:if]_{s/f}})}) \right)_{i,s} \quad (58)$$

#### 4. APPLICATION TO UNIVERSITY ENROLLMENT

Let's consider eighteen Northeastern Illinois University time series of enrollment and credit hours from fall 2004 to summer 2022. Table 1 contains eight of these times series. The remaining ten time series are derived by addition of such time series.

- $X_1$  (resp.,  $X_{10}$ ) represent the time series of total undergraduate enrollment (resp., credit hours), and is defined as  $X_1 = X_2 + X_3$  (resp.,  $X_{10} = X_{11} + X_{12}$ ).
- $X_2$  (resp.,  $X_{11}$ ) represent the time series of continuing undergraduate enrollment (resp., credit hours).
- $X_3$  (resp.,  $X_{12}$ ) represent the time series of new undergraduate enrollment (resp., credit hours).
- $X_4$  (resp.,  $X_{13}$ ) represent the time series of total graduate enrollment (resp., credit hours), and is defined as  $X_4 = X_5 + X_6$  (resp.,  $X_{13} = X_{14} + X_{15}$ ).
- $X_5$  (resp.,  $X_{14}$ ) represent the time series of continuing graduate enrollment (resp., credit hours).
- $X_6$  (resp.,  $X_{15}$ ) represent the time series of new graduate enrollment (resp., credit hours).
- $X_7$  (resp.,  $X_{16}$ ) represent the time series of total undergraduate + graduate enrollment (resp., credit hours), and is defined as  $X_7 = X_1 + X_4$  (resp.,  $X_{16} = X_{10} + X_{13}$ ).

- $X_8$  (resp.,  $X_{17}$ ) represent the time series of continuing undergraduate + graduate enrollment (resp., credit hours), and is defined as  $X_8 = X_2 + X_5$  (resp.,  $X_{17} = X_{11} + X_{14}$ ).
- $X_9$  (resp.,  $X_{18}$ ) represent the time series of new undergraduate + graduate enrollment (resp., credit hours), and is defined as  $X_9 = X_3 + X_6$  (resp.,  $X_{18} = X_{12} + X_{15}$ ).

Term	$X_2$	$X_3$	$X_5$	$X_6$	$X_{11}$	$X_{12}$	$X_{14}$	$X_{15}$
Fall 2004	6723	2582	2174	685	66952	27802	10970	3665
Spring 2005	7791	853	2392	368	79796	8080.5	12345	1847.5
Summer 2005	3742	601	1889	412	21773	3207	8812	1813
Fall 2005	6812	2606	2065	744	68614.5	27934.5	10485.5	4242
Spring 2006	7890	801	2402	435	80561	7499	12785.5	2262.5
Summer 2006	3757	568	1839	303	21708	2994	8956	1303
Fall 2006	6710	2547	2148	651	66656.5	27819	11121	3710.5
Spring 2007	7787	831	2353	296	78701	7840.5	12420	1472
Summer 2007	3682	527	1716	230	21528.5	2820	8002	1014.5
Fall 2007	6635	2480	2015	514	66213.5	26682	10221	2767
Spring 2008	7658	748	2133	267	77788	7103	11022	1402
Summer 2008	3667	444	1501	261	22097	2474	7020.5	1106.5
Fall 2008	6537	2450	1652	554	65679.5	26822	8649	2933.5
Spring 2009	7654	682	1810	441	79042.5	6536	9464	2367
Summer 2009	3549	573	1411	403	21155	3218	6970	1920
Fall 2009	6422	2769	1816	624	65681	30651	9445	3609
Spring 2010	7852	908	1830	476	82308	8950.5	9845.5	2687
Summer 2010	3580	581	1342	350	21866	3292	6483	1664
Fall 2010	6757	2741	1533	715	70106	30859	7964	4358
Spring 2011	8038	902	1767	459	84263.5	8671	9401.5	2753
Summer 2011	3607	458	1281	292	22395	2717	6274.5	1421
Fall 2011	6819	2602	1501	658	70515	29115	7876	4158
Spring 2012	7964	792	1630	415	82889.5	7566	9004	2463.5
Summer 2012	3147	594	1087	244	18303	3241	5123.5	1257
Fall 2012	6516	2624	1342	667	67272	29258.5	7196.5	4007
Spring 2013	7648	790	1465	373	79634	7758.5	7942	2182
Summer 2013	3036	553	1009	173	17427	2949	4694	813
Fall 2013	6478	2472	1304	567	65500	26139.5	6940	3298
Spring 2014	7235	722	1478	268	74053.5	6517	8058	1558
Summer 2014	2888	548	988	165	16219	2938	4750	800
Fall 2014	6086	2326	1299	564	61502.5	24647	6870.5	3447
Spring 2015	6847	680	1497	219	69983	6360	8360	1305
Summer 2015	2830	526	1023	197	15695	2902	4954	1008
Fall 2015	5776	2319	1191	605	58270	24562	6518	3607
Spring 2016	6520	751	1494	325	66691	7061	8389	1624
Summer 2016	2637	511	1116	132	14943	2742	5717	606
Fall 2016	5345	2331	1266	596	53850.5	24524.5	6750	3761
Spring 2017	6139	717	1567	273	62371.5	6595	8600	1435
Summer 2017	2395	426	1149	205	13661	2824	5664	1195
Fall 2017	4842	2276	1263	603	49268	24287	6820	3658
Spring 2018	5697	595	1517	242	59103.5	5599	8662	1278.5
Summer 2018	2297	350	1102	176	13304	2250	5551	925
Fall 2018	4646	1754	1168	535	47271	18472	6340	3474
Spring 2019	5115	490	1373	248	52945.5	4643	8208	1428
Summer 2019	2189	315	977	174	12621	1839	5006	880
Fall 2019	4037	1664	1113	609	41859.5	17758	6360	4049
Spring 2020	4556	482	1333	392	48376	4507	8184	2362
Summer 2020	2224	275	1014	263	14185	1626	5170	1463
Fall 2020	3745	1475	1287	612	39523.5	15432	8023.5	4071
Spring 2021	4251	468	1579	319	44765	4421	10096	1815
Summer 2021	1901	265	1178	212	11978	1512	5859	1075
Fall 2021	3311	1297	1303	529	34305.5	13687	7787	3681
Spring 2022	3637	413	1467	245	38621	3765	9147.5	1573.5
Summer 2022	1511	220	962	151	8734	1081	4577	776

TABLE 1. Northeastern Illinois University Enrollment and Credit Hours

*Example 1.* Let's consider undergraduate total enrollment (fall, spring and summer) over 18 years (fall 2004 to summer 2022). Thus,  $f = 3$  and  $X = (x_1, \dots, x_{54})$ . For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead forecasts for  $X$  using both Holt–Winters and ARIMA models. The sum of absolute errors for each model are

$$SAE^{HW} = \sum_{j=7}^{54} |\hat{x}_j^{HW} - x_j| = 6,329 \text{ and } SAE^{ARIMA} = \sum_{j=7}^{54} |\hat{x}_j^{ARIMA} - x_j| = 12,556 \quad (59)$$

Comparing these 16 years times 3 trimesters forecasts, i.e., 48 trimester forecasts, the Holt–winters model lead to better trimesterly forecast (smallest absolute residual) 28 times as opposed

to 20 times for the ARIMA model, i.e.,  $HW/AR = 28/20$ . Moreover, if one compares the sum of absolute error per year, the Holt–Winters model lead to a better yearly forecast nine times as opposed to seven times for ARIMA, i.e.,  $HW/AR = 9/7$ .

For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead  $\sharp, b$  and  $\natural$  forecasts for  $X$  as in (56), (57) and (58) for each optimization criteria  $\kappa = 1, \dots, 8$  using the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models (with  $N = 1000$  to find the optimal latest-optimized alpha power). Moreover, we compute the (global) sum of absolute error, and we compare our forecasts to both Holt–Winters and ARIMA models by counting the number of best monthly and yearly forecasts. The results are summarized in Table 2.

$\bullet \kappa$	SAE	HW / AR / K.	HW / K.	AR / K.	HW / AR / K.	HW / K.	AR / K.
#1	4,936	12 / 9 / 27	17 / 31	13 / 35	4 / 2 / 10	5 / 11	4 / 12
#2	4,952	13 / 12 / 23	18 / 30	16 / 32	4 / 2 / 10	5 / 11	4 / 12
#3	4,921	13 / 9 / 26	18 / 30	13 / 35	5 / 2 / 9	6 / 10	4 / 12
#4	4,868	13 / 9 / 26	18 / 30	13 / 35	4 / 2 / 10	5 / 11	4 / 12
#5	4,882	13 / 10 / 25	18 / 30	14 / 34	4 / 2 / 10	5 / 11	4 / 12
#6	4,915	13 / 10 / 25	18 / 30	14 / 34	4 / 2 / 10	5 / 11	4 / 12
#7	4,900	12 / 9 / 27	17 / 31	13 / 35	5 / 2 / 9	6 / 10	4 / 12
#8	4,917	11 / 11 / 26	16 / 32	15 / 33	4 / 2 / 10	5 / 11	4 / 12
b1	4,984	12 / 10 / 26	17 / 31	14 / 34	4 / 2 / 10	5 / 11	4 / 12
b2	4,926	12 / 10 / 26	17 / 31	14 / 34	4 / 2 / 10	5 / 11	4 / 12
b3	4,960	11 / 11 / 26	16 / 32	15 / 33	4 / 2 / 10	5 / 11	4 / 12
b4	4,961	13 / 11 / 24	18 / 30	16 / 32	4 / 2 / 10	5 / 11	4 / 12
b5	4,886	13 / 12 / 23	18 / 30	16 / 32	4 / 2 / 10	5 / 11	4 / 12
b6	4,944	12 / 11 / 25	17 / 31	15 / 33	4 / 2 / 10	5 / 11	4 / 12
b7	4,945	11 / 10 / 27	16 / 32	15 / 33	4 / 2 / 10	5 / 11	4 / 12
b8	4,964	12 / 9 / 27	17 / 31	12 / 36	4 / 2 / 10	5 / 11	4 / 12
‡1	4,891	13 / 10 / 25	18 / 30	15 / 33	4 / 2 / 10	5 / 11	4 / 12
‡2	4,932	12 / 10 / 26	17 / 31	14 / 34	4 / 2 / 10	5 / 11	4 / 12
‡3	4,979	10 / 11 / 27	16 / 32	14 / 34	4 / 2 / 10	5 / 11	4 / 12
‡4	4,905	12 / 12 / 24	17 / 31	16 / 32	4 / 2 / 10	5 / 11	4 / 12
‡5	4,849	12 / 10 / 26	17 / 31	14 / 34	4 / 2 / 10	5 / 11	4 / 12
‡6	4,902	12 / 11 / 25	18 / 30	15 / 33	4 / 2 / 10	5 / 11	4 / 12
‡7	4,944	13 / 10 / 25	18 / 30	14 / 34	4 / 2 / 10	5 / 11	4 / 12
‡8	4,945	12 / 10 / 26	17 / 31	14 / 34	4 / 2 / 10	5 / 11	4 / 12

TABLE 2. Undergraduate Total Enrollment: Comparison of the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models to the seasonal Holt–Winters and ARIMA models

*Example 2.* Let’s consider undergraduate continuing enrollment (fall, spring and summer) over 18 years (fall 2004 to summer 2022). Thus,  $f = 3$  and  $X = (x_1, \dots, x_{54})$ . For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead forecasts for  $X$  using both Holt–Winters and ARIMA models. The sum of absolute errors for each model are

$$\text{SAE}^{\text{HW}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{HW}} - x_j \right| = 6,115 \quad \text{and} \quad \text{SAE}^{\text{ARIMA}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{ARIMA}} - x_j \right| = 11,884 \quad (60)$$

Comparing these 16 years times 3 trimesters forecasts, i.e., 48 trimester forecasts, the Holt–winters model lead to better trimesterly forecast (smallest absolute residual) 30 times as opposed to 18 times for the ARIMA model, i.e.,  $HW/AR = 30/18$ . Moreover, if one compares the sum of absolute error per year, the Holt–Winters model lead to a better yearly forecast twelve times as opposed to four times for ARIMA, i.e.,  $HW/AR = 12/4$ .

For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead  $\sharp, b$  and  $\natural$  forecasts for  $X$  as in (56), (57) and (58) for each optimization criteria  $\kappa = 1, \dots, 8$  using the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models (with  $N = 1000$  to find the optimal latest-optimized alpha power). Moreover,

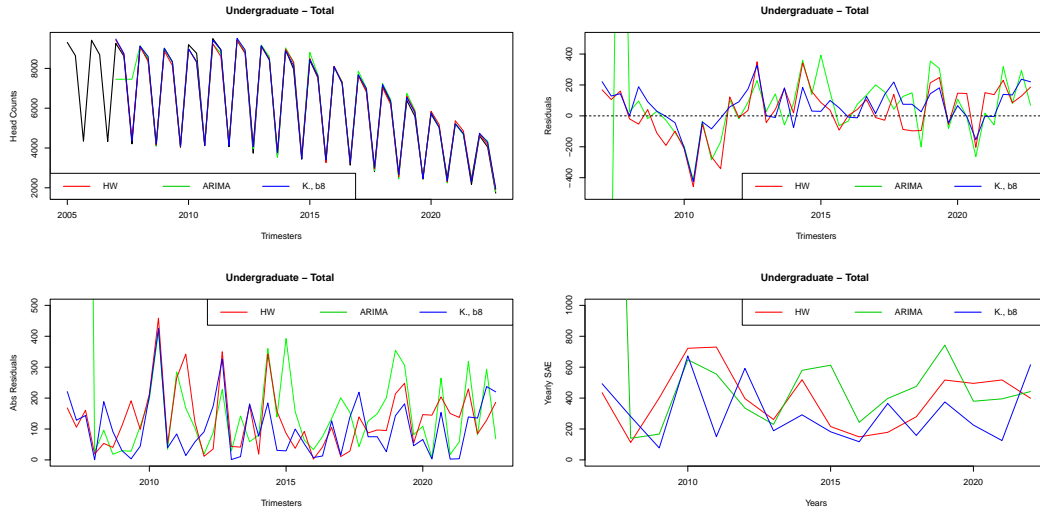


FIGURE 1. Undergraduate Total Enrollment: Forecasts, residuals, absolute residuals and yearly sum of absolute errors

we compute the (global) sum of absolute error, and we compare our forecasts to both Holt–Winters and ARIMA models by counting the number of best monthly and yearly forecasts. The results are summarized in Table 3.

• $f_c$	SAE	HW / AR / K.	HW / K.	AR / K.	HW / AR / K.	HW / K.	AR / K.
#1	4,408	13 / 6 / 29	16 / 32	8 / 40	6 / 0 / 10	6 / 10	2 / 14
#2	4,476	13 / 8 / 27	16 / 32	10 / 38	5 / 0 / 11	5 / 11	2 / 14
#3	4,380	11 / 6 / 31	15 / 33	8 / 40	6 / 0 / 10	6 / 10	2 / 14
#4	4,425	12 / 7 / 29	16 / 32	9 / 39	6 / 0 / 10	6 / 10	2 / 14
#5	4,475	13 / 8 / 27	18 / 30	11 / 37	5 / 0 / 11	5 / 11	2 / 14
#6	4,416	12 / 9 / 27	17 / 31	11 / 37	6 / 0 / 10	6 / 10	2 / 14
#7	4,362	11 / 7 / 30	15 / 33	9 / 39	6 / 0 / 10	6 / 10	1 / 15
#8	4,521	12 / 9 / 27	16 / 32	12 / 36	5 / 0 / 11	5 / 11	2 / 14
b1	4,325	11 / 6 / 31	15 / 33	8 / 40	6 / 0 / 10	6 / 10	2 / 14
b2	4,461	12 / 8 / 28	16 / 32	11 / 37	5 / 0 / 11	5 / 11	2 / 14
b3	4,381	11 / 7 / 30	15 / 33	9 / 39	6 / 0 / 10	6 / 10	2 / 14
b4	4,382	11 / 9 / 26	16 / 32	11 / 37	6 / 0 / 10	6 / 10	2 / 14
b5	4,425	12 / 8 / 28	17 / 31	10 / 38	5 / 0 / 11	5 / 11	2 / 14
b6	4,417	12 / 8 / 28	17 / 31	10 / 38	6 / 0 / 10	6 / 10	1 / 15
b7	4,420	11 / 7 / 30	15 / 33	9 / 39	6 / 0 / 10	6 / 10	2 / 14
b8	4,468	12 / 8 / 28	16 / 32	10 / 38	5 / 0 / 11	5 / 11	2 / 14
h1	4,331	12 / 8 / 28	16 / 32	10 / 38	6 / 0 / 10	6 / 10	2 / 14
h2	4,505	13 / 8 / 27	18 / 30	10 / 38	5 / 0 / 11	5 / 11	2 / 14
h3	4,438	11 / 7 / 30	17 / 31	10 / 38	6 / 0 / 10	6 / 10	2 / 14
h4	4,445	12 / 7 / 29	16 / 32	9 / 39	6 / 0 / 10	6 / 10	2 / 14
h5	4,412	13 / 8 / 27	17 / 31	10 / 38	5 / 0 / 10	5 / 11	2 / 14
h6	4,426	12 / 9 / 27	17 / 31	11 / 37	6 / 0 / 10	6 / 10	2 / 14
h7	4,499	12 / 8 / 28	16 / 32	11 / 37	6 / 0 / 10	6 / 10	2 / 14
h8	4,515	12 / 8 / 28	17 / 31	11 / 37	5 / 0 / 11	5 / 11	2 / 14

TABLE 3. Undergraduate Continuing Enrollment: Comparison of the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models to the seasonal Holt–Winters and ARIMA models

*Example 3.* Let’s consider undergraduate new enrollment (fall, spring and summer) over 18 years (fall 2004 to summer 2022). Thus,  $f = 3$  and  $X = (x_1, \dots, x_{54})$ . For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead forecasts for  $X$  using both Holt–Winters and ARIMA models. The sum of absolute errors for each model are

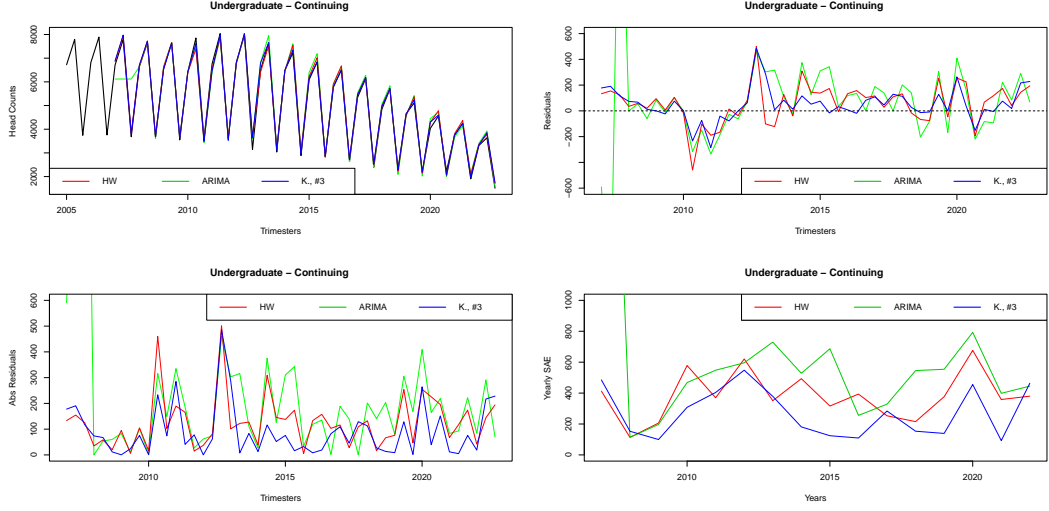


FIGURE 2. Undergraduate Continuing Enrollment: Forecasts, residuals, absolute residuals and yearly sum of absolute errors

$$\text{SAE}^{\text{HW}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{HW}} - x_j \right| = 3,719 \text{ and } \text{SAE}^{\text{ARIMA}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{ARIMA}} - x_j \right| = 6,289 \quad (61)$$

Comparing these 16 years times 3 trimesters forecasts, i.e., 48 trimester forecasts, the Holt–Winters model lead to better trimesterly forecast (smallest absolute residual) 32 times as opposed to 16 times for the ARIMA model, i.e.,  $HW/AR = 32/16$ . Moreover, if one compares the sum of absolute error per year, the Holt–Winters model lead to a better yearly forecast nine times as opposed to seven times for ARIMA, i.e.,  $HW/AR = 9/7$ .

For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead  $\hat{x}_j^{\text{HW}}$ ,  $\hat{x}_j^{\text{ARIMA}}$  and  $\hat{x}_j^{\text{K. \#3}}$  forecasts for  $X$  as in (56), (57) and (58) for each optimization criteria  $\kappa = 1, \dots, 8$  using the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models (with  $N = 1000$  to find the optimal latest-optimized alpha power). Moreover, we compute the (global) sum of absolute error, and we compare our forecasts to both Holt–Winters and ARIMA models by counting the number of best monthly and yearly forecasts. The results are summarized in Table 4..

*Example 4.* Let's consider graduate total enrollment (fall, spring and summer) over 18 years (fall 2004 to summer 2022). Thus,  $f = 3$  and  $X = (x_1, \dots, x_{54})$ . For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead forecasts for  $X$  using both Holt–Winters and ARIMA models. The sum of absolute errors for each model are

$$\text{SAE}^{\text{HW}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{HW}} - x_j \right| = 5,013 \text{ and } \text{SAE}^{\text{ARIMA}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{ARIMA}} - x_j \right| = 6,572 \quad (62)$$

Comparing these 16 years times 3 trimesters forecasts, i.e., 48 trimester forecasts, the Holt–Winters model lead to better trimesterly forecast (smallest absolute residual) 32 times as opposed to 16 times for the ARIMA model, i.e.,  $HW/AR = 32/16$ . Moreover, if one compares the sum of

$\bullet\kappa$	SAE	HW / AR / K.	HW / K.	AR / K.	HW / AR / K.	HW / K.	AR / K.
#1	2,276	10 / 8 / 30	10 / 38	10 / 38	2 / 0 / 14	2 / 14	0 / 16
#2	2,274	10 / 6 / 32	10 / 38	8 / 40	3 / 0 / 13	3 / 13	0 / 16
#3	2,207	11 / 9 / 28	13 / 35	11 / 37	2 / 0 / 14	2 / 14	0 / 16
#4	2,186	8 / 9 / 31	9 / 39	10 / 38	1 / 0 / 15	1 / 15	0 / 16
#5	2,213	9 / 7 / 32	9 / 39	8 / 40	2 / 0 / 14	2 / 14	0 / 16
#6	2,162	8 / 8 / 32	9 / 39	9 / 39	1 / 0 / 15	1 / 15	0 / 16
#7	2,146	9 / 7 / 32	12 / 36	8 / 40	2 / 0 / 14	2 / 14	0 / 16
#8	2,205	9 / 6 / 33	9 / 39	6 / 42	3 / 0 / 13	3 / 13	0 / 16
b1	2,219	11 / 5 / 32	11 / 37	8 / 40	1 / 0 / 15	1 / 15	0 / 16
b2	2,260	11 / 7 / 30	12 / 36	10 / 38	0 / 0 / 16	0 / 16	0 / 16
b3	2,213	10 / 8 / 30	11 / 37	9 / 39	1 / 0 / 15	1 / 15	0 / 16
b4	2,175	9 / 8 / 31	10 / 38	9 / 39	1 / 0 / 15	1 / 15	0 / 16
b5	2,212	11 / 7 / 30	13 / 35	8 / 40	1 / 0 / 15	1 / 15	0 / 16
b6	2,158	10 / 9 / 29	11 / 37	10 / 38	1 / 0 / 15	1 / 15	0 / 16
b7	2,176	8 / 8 / 32	8 / 40	9 / 39	0 / 0 / 16	0 / 16	0 / 16
b8	2,171	8 / 6 / 34	9 / 39	6 / 42	1 / 0 / 15	1 / 15	0 / 16
‡1	2,282	13 / 9 / 26	13 / 35	11 / 37	1 / 0 / 15	1 / 15	0 / 16
‡2	2,305	11 / 7 / 30	12 / 36	10 / 38	2 / 0 / 14	2 / 14	0 / 16
‡3	2,211	11 / 7 / 30	12 / 36	9 / 39	1 / 0 / 15	1 / 15	0 / 16
‡4	2,197	10 / 8 / 30	10 / 38	9 / 39	1 / 0 / 15	1 / 15	0 / 16
‡5	2,245	11 / 6 / 31	11 / 37	7 / 41	2 / 0 / 14	2 / 14	0 / 16
‡6	2,296	10 / 8 / 30	10 / 38	9 / 39	2 / 0 / 14	2 / 14	0 / 16
‡7	2,201	9 / 8 / 31	9 / 39	9 / 39	1 / 0 / 15	1 / 15	0 / 16
‡8	2,190	9 / 5 / 34	10 / 38	10 / 38	2 / 0 / 14	2 / 14	0 / 16

TABLE 4. Undergraduate New Enrollment: Comparison of the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models to the seasonal Holt–Winters and ARIMA models

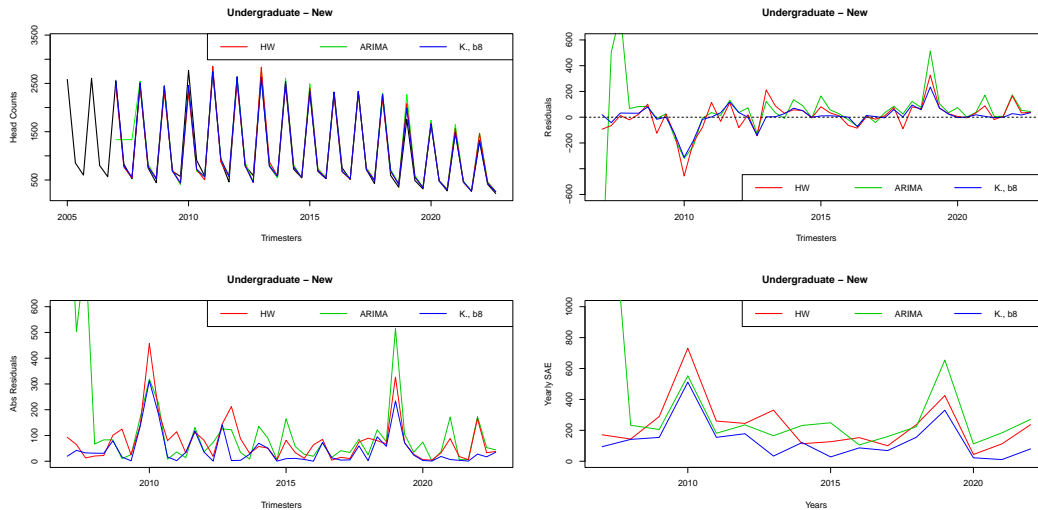


FIGURE 3. Undergraduate New Enrollment: Forecasts, residuals, absolute residuals and yearly sum of absolute errors

absolute error per year, the Holt–Winters model lead to a better yearly forecast thirteen times as opposed to three times for ARIMA, i.e.,  $HW/AR = 13/3$ .

For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead  $\ddagger, b$  and  $\ddagger$  forecasts for  $X$  as in (56), (57) and (58) for each optimization criteria  $\kappa = 1, \dots, 8$  using the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models (with  $N = 1000$  to find the optimal latest-optimized alpha power). Moreover,

we compute the (global) sum of absolute error, and we compare our forecasts to both Holt–Winters and ARIMA models by counting the number of best monthly and yearly forecasts. The results are summarized in Table 5..

● <sub>FC</sub>	SAE	HW / AR / K.	HW / K.	AR / K.	HW / AR / K.	HW / K.	AR / K.
#1	3,602	11 / 3 / 34	14 / 34	4 / 44	1 / 1 / 14	1 / 15	1 / 15
#2	3,525	10 / 5 / 33	13 / 35	6 / 42	1 / 1 / 14	1 / 15	1 / 15
#3	3,619	11 / 3 / 34	14 / 34	4 / 44	1 / 1 / 14	1 / 15	1 / 15
#4	3,657	12 / 4 / 32	14 / 34	5 / 43	2 / 1 / 13	2 / 14	2 / 14
#5	3,545	11 / 5 / 32	13 / 35	6 / 42	1 / 1 / 14	1 / 15	1 / 15
#6	3,640	11 / 5 / 32	13 / 35	6 / 42	2 / 1 / 13	2 / 14	2 / 14
#7	3,637	13 / 4 / 31	16 / 32	5 / 43	2 / 1 / 13	2 / 14	1 / 15
#8	3,559	11 / 5 / 32	14 / 34	6 / 42	1 / 1 / 14	1 / 15	1 / 15
b1	3,618	12 / 4 / 32	15 / 33	5 / 43	1 / 0 / 15	1 / 15	0 / 16
b2	3,537	9 / 7 / 32	11 / 37	8 / 40	1 / 1 / 14	1 / 15	1 / 15
b3	3,622	11 / 5 / 32	14 / 34	6 / 42	1 / 1 / 14	1 / 15	1 / 15
b4	3,680	11 / 6 / 31	14 / 34	7 / 41	2 / 1 / 13	2 / 14	1 / 15
b5	3,563	12 / 4 / 32	14 / 34	5 / 43	1 / 1 / 14	1 / 15	1 / 15
b6	3,707	12 / 5 / 31	15 / 33	7 / 41	2 / 1 / 13	2 / 14	2 / 14
b7	3,655	14 / 5 / 29	17 / 31	6 / 42	2 / 1 / 13	2 / 14	1 / 15
b8	3,567	12 / 4 / 32	14 / 34	5 / 43	1 / 1 / 14	1 / 15	1 / 15
t1	3,605	12 / 4 / 32	15 / 33	5 / 43	1 / 0 / 15	1 / 15	0 / 16
t2	3,512	11 / 5 / 32	14 / 34	6 / 42	1 / 1 / 14	1 / 15	1 / 15
t3	3,809	11 / 5 / 32	14 / 34	7 / 41	1 / 1 / 14	1 / 15	1 / 15
t4	3,636	12 / 4 / 32	13 / 35	6 / 42	2 / 0 / 14	2 / 14	0 / 16
t5	3,523	12 / 5 / 31	14 / 34	6 / 42	1 / 1 / 14	1 / 15	1 / 15
t6	3,654	14 / 5 / 29	16 / 32	7 / 41	2 / 1 / 13	2 / 14	2 / 14
t7	3,687	13 / 4 / 31	15 / 33	5 / 43	2 / 1 / 13	2 / 14	1 / 15
t8	3,552	12 / 5 / 31	14 / 34	6 / 42	1 / 1 / 14	1 / 15	1 / 15

TABLE 5. Graduate Total Enrollment: Comparison of the seasonal per-period SAE stochastic latest-optimized  $\hat{\alpha}$ -power balanced forecasting models to the seasonal Holt–Winters and ARIMA models

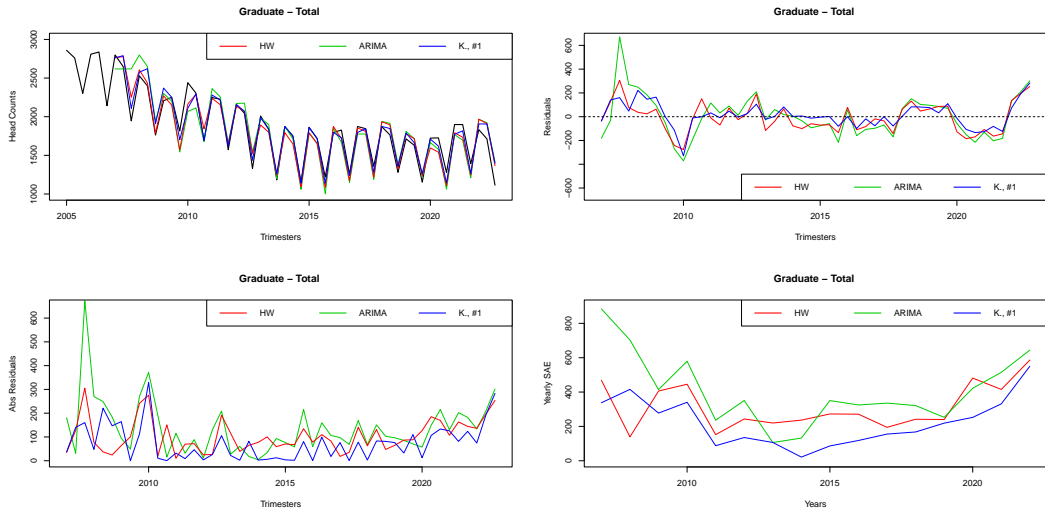


FIGURE 4. Graduate Total Enrollment: Forecasts, residuals, absolute residuals and yearly sum of absolute errors

*Example 5.* Let’s consider graduate continuing enrollment (fall, spring and summer) over 18 years (fall 2004 to summer 2022). Thus,  $f = 3$  and  $X = (x_1, \dots, x_{54})$ . For  $s = 1, \dots, 3$  (trimesters),

and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead forecasts for  $X$  using both Holt–Winters and ARIMA models. The sum of absolute errors for each model are

$$\text{SAE}^{\text{HW}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{HW}} - x_j \right| = 4,288 \text{ and } \text{SAE}^{\text{ARIMA}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{ARIMA}} - x_j \right| = 5,781 \quad (63)$$

Comparing these 16 years times 3 trimesters forecasts, i.e., 48 trimester forecasts, the Holt–winters model lead to better trimesterly forecast (smallest absolute residual) 34 times as opposed to 14 times for the ARIMA model, i.e.,  $HW/AR = 34/14$ . Moreover, if one compares the sum of absolute error per year, the Holt–Winters model lead to a better yearly forecast fourthree times as opposed to two times for ARIMA, i.e.,  $HW/AR = 14/2$ .

For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead  $\sharp, \flat$  and  $\natural$  forecasts for  $X$  as in (56), (57) and (58) for each optimization criteria  $\kappa = 1, \dots, 8$  using the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models (with  $N = 1000$  to find the optimal latest-optimized alpha power). Moreover, we compute the (global) sum of absolute error, and we compare our forecasts to both Holt–Winters and ARIMA models by counting the number of best monthly and yearly forecasts. The results are summarized in Table 6..

$\bullet \kappa$	SAE	HW / AR / K.	HW / K.	AR / K.	HW / AR / K.	HW / K.	AR / K.
$\sharp 1$	3,327	13 / 4 / 31	15 / 33	8 / 40	4 / 1 / 11	5 / 11	1 / 15
$\sharp 2$	3,286	14 / 4 / 30	17 / 31	7 / 41	4 / 1 / 11	5 / 11	1 / 15
$\sharp 3$	3,317	13 / 5 / 30	15 / 33	9 / 39	4 / 1 / 11	5 / 11	1 / 15
$\sharp 4$	3,310	14 / 5 / 29	17 / 31	10 / 38	5 / 1 / 10	6 / 10	1 / 15
$\sharp 5$	3,317	14 / 4 / 30	16 / 32	8 / 40	5 / 1 / 10	6 / 10	1 / 15
$\sharp 6$	3,344	12 / 5 / 31	15 / 33	9 / 39	5 / 1 / 10	6 / 10	1 / 15
$\sharp 7$	3,319	16 / 5 / 27	18 / 30	10 / 38	5 / 1 / 10	6 / 10	1 / 15
$\sharp 8$	3,308	14 / 3 / 31	16 / 32	7 / 41	4 / 1 / 11	5 / 11	1 / 15
$\flat 1$	3,327	13 / 4 / 31	15 / 33	8 / 40	5 / 1 / 10	6 / 10	1 / 15
$\flat 2$	3,299	12 / 5 / 31	15 / 33	8 / 40	3 / 1 / 12	4 / 12	1 / 15
$\flat 3$	3,287	14 / 3 / 31	16 / 32	7 / 41	4 / 1 / 11	5 / 11	1 / 15
$\flat 4$	3,254	14 / 4 / 30	16 / 32	9 / 39	4 / 1 / 11	5 / 11	1 / 15
$\flat 5$	3,300	13 / 3 / 32	15 / 33	7 / 41	4 / 1 / 11	5 / 11	1 / 15
$\flat 6$	3,307	14 / 6 / 28	17 / 31	9 / 39	4 / 1 / 11	5 / 11	1 / 15
$\flat 7$	3,310	14 / 6 / 28	15 / 33	11 / 37	3 / 1 / 12	4 / 12	1 / 15
$\flat 8$	3,307	13 / 4 / 31	14 / 34	8 / 40	4 / 1 / 11	5 / 11	1 / 15
$\natural 1$	3,309	14 / 4 / 30	17 / 31	7 / 41	4 / 1 / 11	5 / 11	1 / 15
$\natural 2$	3,310	15 / 3 / 30	17 / 31	7 / 41	5 / 1 / 10	6 / 10	1 / 15
$\natural 3$	3,443	15 / 6 / 27	18 / 30	10 / 38	5 / 1 / 10	6 / 10	2 / 14
$\natural 4$	3,322	13 / 5 / 30	15 / 33	10 / 38	5 / 1 / 10	6 / 10	1 / 15
$\natural 5$	3,323	14 / 4 / 30	16 / 32	8 / 40	4 / 1 / 11	5 / 11	1 / 15
$\natural 6$	3,341	12 / 6 / 30	15 / 33	9 / 39	5 / 1 / 10	6 / 10	1 / 15
$\natural 7$	3,318	13 / 4 / 31	14 / 34	9 / 39	5 / 1 / 10	6 / 10	1 / 15
$\natural 8$	3,309	15 / 4 / 29	17 / 31	8 / 40	5 / 1 / 10	6 / 10	1 / 15

TABLE 6. Graduate Continuing Enrollment: Comparison of the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models to the seasonal Holt–Winters and ARIMA models

*Example 6.* Let’s consider graduate new enrollment (fall, spring and summer) over 18 years (fall 2004 to summer 2022). Thus,  $f = 3$  and  $X = (x_1, \dots, x_{54})$ . For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead forecasts for  $X$  using both Holt–Winters and ARIMA models. The sum of absolute errors for each model are

$$\text{SAE}^{\text{HW}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{HW}} - x_j \right| = 3,219 \text{ and } \text{SAE}^{\text{ARIMA}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{ARIMA}} - x_j \right| = 3,274 \quad (64)$$

Comparing these 16 years times 3 trimesters forecasts, i.e., 48 trimester forecasts, the Holt–winters model lead to slightly worse trimesterly forecast (smallest absolute residual) 23 times as opposed to 25 times for the ARIMA model, i.e.,  $HW/AR = 23/25$ . Moreover, if one compares



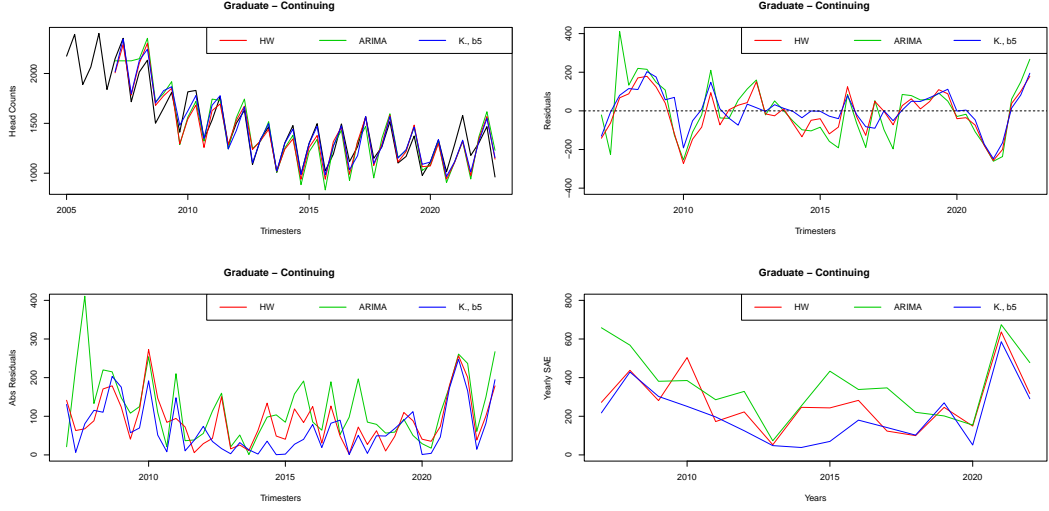


FIGURE 5. Graduate Continuing Enrollment: Forecasts, residuals, absolute residuals and yearly sum of absolute errors

the sum of absolute error per year, the Holt–Winters model lead to the same number of best yearly forecast as ARIMA, i.e.,  $HW/AR = 8/8$ .

For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead  $\hat{x}_{t,b}$  and  $\hat{x}_{t,b}$  forecasts for  $X$  as in (56), (57) and (58) for each optimization criteria  $\kappa = 1, \dots, 8$  using the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models (with  $N = 1000$  to find the optimal latest-optimized alpha power). Moreover, we compute the (global) sum of absolute error, and we compare our forecasts to both Holt–Winters and ARIMA models by counting the number of best monthly and yearly forecasts. The results are summarized in Table 7. .

*Example 7.* Let’s consider student (undergraduate + graduate) total enrollment (fall, spring and summer) over 18 years (fall 2004 to summer 2022). Thus,  $f = 3$  and  $X = (x_1, \dots, x_{54})$ . For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead forecasts for  $X$  using both Holt–Winters and ARIMA models. The sum of absolute errors for each model are

$$SAE^{HW} = \sum_{j=7}^{54} |\hat{x}_j^{HW} - x_j| = 9,653 \text{ and } SAE^{ARIMA} = \sum_{j=7}^{54} |\hat{x}_j^{ARIMA} - x_j| = 17,491 \quad (65)$$

Comparing these 16 years times 3 trimesters forecasts, i.e., 48 trimester forecasts, the Holt–winters model lead to better trimesterly forecast (smallest absolute residual) 31 times as opposed to 17 times for the ARIMA model, i.e.,  $HW/AR = 31/17$ . Moreover, if one compares the sum of absolute error per year, the Holt–Winters model lead to a better yearly forecast eleven times as opposed to five times for ARIMA, i.e.,  $HW/AR = 11/5$ .

For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead  $\hat{x}_{t,b}$  and  $\hat{x}_{t,b}$  forecasts for  $X$  as in (56), (57) and (58) for each optimization criteria  $\kappa = 1, \dots, 8$  using the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced

• $\alpha$	SAE	HW / AR / K.	HW / K.	AR / K.	HW / AR / K.	HW / K.	AR / K.
#1	2,215	11 / 15 / 22	16 / 32	21 / 27	2 / 3 / 11	4 / 12	4 / 12
#2	2,161	10 / 13 / 25	13 / 35	18 / 30	2 / 2 / 12	4 / 12	3 / 13
#3	2,177	11 / 13 / 24	14 / 34	20 / 28	2 / 3 / 11	4 / 12	4 / 12
#4	2,122	10 / 12 / 26	15 / 33	17 / 31	3 / 2 / 11	4 / 12	3 / 13
#5	2,190	11 / 13 / 24	15 / 33	18 / 30	2 / 2 / 12	4 / 12	3 / 13
#6	2089	10 / 12 / 26	15 / 33	18 / 30	2 / 2 / 12	3 / 13	3 / 13
#7	2,149	11 / 13 / 24	15 / 33	17 / 31	3 / 2 / 11	5 / 11	3 / 13
#8	2,139	11 / 13 / 24	15 / 33	19 / 29	2 / 2 / 12	4 / 12	3 / 13
b1	2,174	11 / 14 / 23	16 / 32	17 / 31	3 / 2 / 11	4 / 12	3 / 13
b2	2,174	10 / 13 / 25	15 / 33	16 / 32	2 / 2 / 12	4 / 12	3 / 13
b3	2,187	11 / 11 / 26	16 / 32	16 / 32	2 / 2 / 12	3 / 13	3 / 13
b4	2,172	10 / 14 / 24	14 / 34	19 / 29	3 / 2 / 11	4 / 12	4 / 12
b5	2,152	11 / 14 / 23	16 / 32	19 / 29	2 / 2 / 12	4 / 12	3 / 13
b6	2,134	10 / 14 / 24	16 / 32	19 / 29	1 / 2 / 13	2 / 14	3 / 13
b7	2,156	10 / 12 / 26	14 / 34	17 / 31	2 / 2 / 12	3 / 13	3 / 13
b8	2,176	12 / 14 / 22	17 / 31	19 / 29	2 / 2 / 12	4 / 12	3 / 13
h1	2,190	10 / 14 / 24	15 / 33	19 / 29	2 / 2 / 12	4 / 12	3 / 13
h2	2,161	9 / 14 / 25	14 / 34	18 / 30	2 / 2 / 12	4 / 12	3 / 13
h3	2,156	11 / 12 / 25	15 / 33	18 / 30	3 / 3 / 10	4 / 12	5 / 11
h4	2,156	9 / 13 / 26	14 / 34	16 / 32	2 / 2 / 12	3 / 13	3 / 13
h5	2,176	9 / 14 / 25	15 / 33	18 / 30	2 / 2 / 12	4 / 12	3 / 13
h6	2,131	9 / 12 / 27	14 / 34	17 / 31	2 / 2 / 12	3 / 13	3 / 13
h7	2,157	10 / 12 / 26	14 / 34	16 / 32	2 / 2 / 12	4 / 12	3 / 13
h8	2,159	10 / 14 / 24	15 / 33	18 / 30	2 / 2 / 12	4 / 12	3 / 13

TABLE 7. Graduate New Enrollment: Comparison of the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models to the seasonal Holt–Winters and ARIMA models

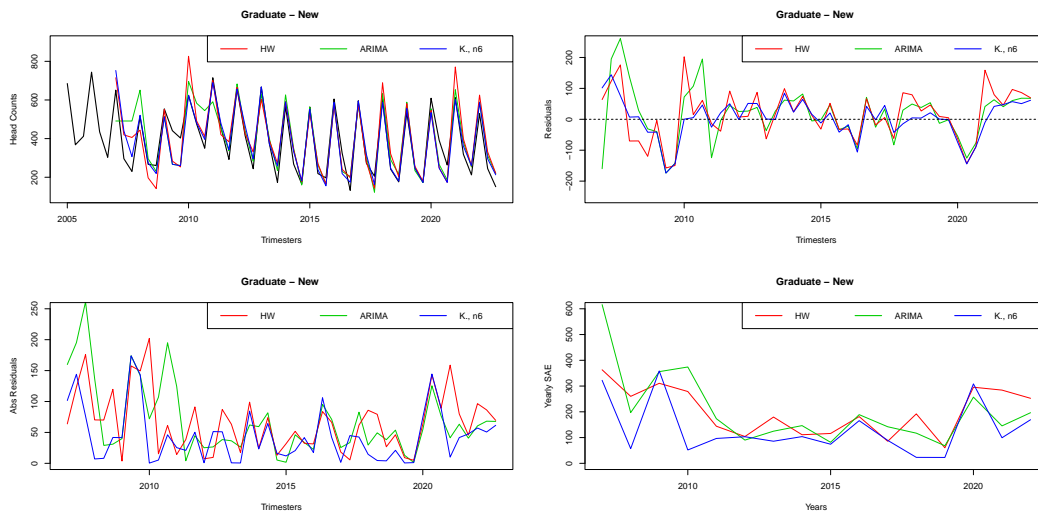


FIGURE 6. Graduate New Enrollment: Forecasts, residuals, absolute residuals and yearly sum of absolute errors

forecasting models (with  $N = 1000$  to find the optimal latest-optimized alpha power). Moreover, we compute the (global) sum of absolute error, and we compare our forecasts to both Holt–Winters and ARIMA models by counting the number of best monthly and yearly forecasts. The results are summarized in Table 8..

*Example 8.* Let’s consider student (undergraduate + graduate) continuing enrollment (fall, spring and summer) over 18 years (fall 2004 to summer 2022). Thus,  $f = 3$  and  $X = (x_1, \dots, x_{54})$ . For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead forecasts for  $X$  using both Holt–Winters and ARIMA models. The sum of absolute errors for

● <sub>FC</sub>	SAE	HW / AR / K.	HW / K.	AR / K.	HW / AR / K.	HW / K.	AR / K.
#1	7,487	14 / 9 / 25	18 / 30	13 / 35	3 / 2 / 11	3 / 13	3 / 13
#2	7,387	13 / 8 / 27	15 / 33	12 / 36	4 / 1 / 11	5 / 11	2 / 14
#3	7,469	13 / 7 / 28	15 / 33	11 / 37	3 / 2 / 11	4 / 12	3 / 13
#4	7,410	14 / 7 / 27	16 / 32	11 / 37	3 / 2 / 11	3 / 13	3 / 13
#5	7,540	13 / 8 / 27	17 / 31	12 / 36	4 / 2 / 10	4 / 12	4 / 12
#6	7,500	14 / 9 / 25	16 / 32	13 / 35	3 / 2 / 11	4 / 12	3 / 13
#7	7,623	15 / 9 / 24	18 / 30	15 / 33	3 / 2 / 11	5 / 11	3 / 13
#8	7,336	13 / 7 / 28	15 / 33	11 / 37	4 / 2 / 10	4 / 12	3 / 13
b1	7,567	16 / 9 / 23	18 / 30	14 / 34	3 / 2 / 11	3 / 13	3 / 13
b2	7,396	13 / 8 / 27	15 / 33	12 / 36	4 / 1 / 11	4 / 12	2 / 14
b3	7,562	15 / 9 / 24	18 / 30	15 / 33	3 / 3 / 10	4 / 12	4 / 12
b4	7,513	13 / 10 / 25	16 / 32	15 / 33	3 / 2 / 11	4 / 12	3 / 13
b5	7,546	12 / 10 / 26	16 / 32	15 / 33	4 / 2 / 10	5 / 11	4 / 12
b6	7,523	14 / 9 / 25	16 / 32	14 / 34	3 / 3 / 10	5 / 11	4 / 12
b7	7,579	16 / 9 / 23	18 / 30	15 / 33	3 / 3 / 10	4 / 12	4 / 12
b8	7,268	14 / 8 / 26	17 / 31	12 / 36	4 / 2 / 10	5 / 11	3 / 13
t1	7,540	13 / 8 / 27	17 / 31	12 / 36	3 / 3 / 10	3 / 13	4 / 12
t2	7,434	12 / 7 / 29	15 / 33	11 / 37	4 / 2 / 10	4 / 12	4 / 12
t3	7,742	15 / 9 / 24	20 / 28	14 / 34	4 / 3 / 9	5 / 11	4 / 12
t4	7,432	15 / 8 / 25	18 / 30	12 / 36	3 / 2 / 11	3 / 13	3 / 13
t5	7,631	13 / 9 / 26	18 / 30	13 / 35	4 / 2 / 10	4 / 12	4 / 12
t6	7,519	14 / 9 / 25	16 / 32	13 / 35	3 / 3 / 10	5 / 11	4 / 12
t7	7,640	16 / 8 / 24	20 / 28	14 / 34	3 / 2 / 11	5 / 11	3 / 13
t8	7,381	13 / 7 / 28	16 / 32	11 / 37	4 / 2 / 10	4 / 12	3 / 13

TABLE 8. Undergraduate + Graduate Total Enrollment: Comparison of the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models to the seasonal Holt–Winters and ARIMA models

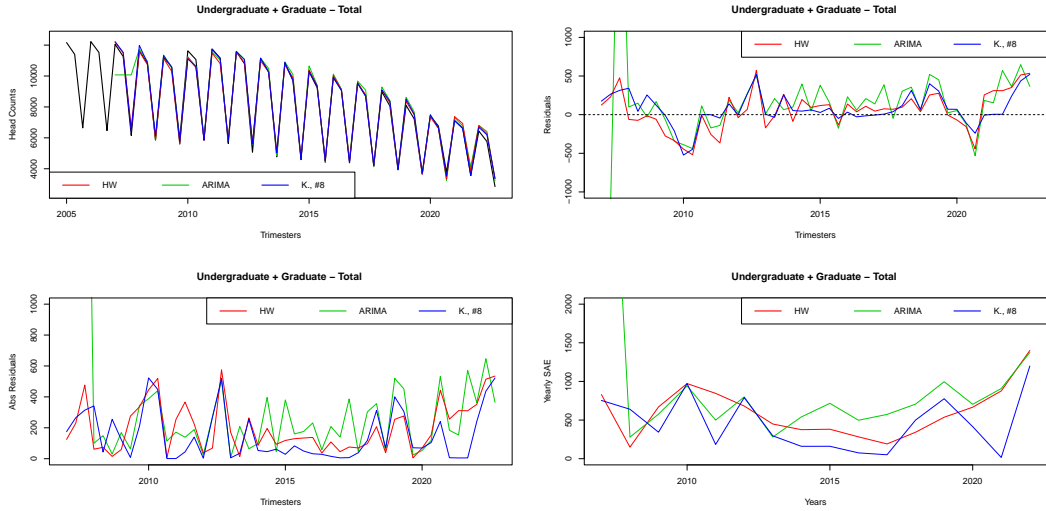


FIGURE 7. Undergraduate + Graduate Total Enrollment: Forecasts, residuals, absolute residuals and yearly sum of absolute errors

each model are

$$SAE^{HW} = \sum_{j=7}^{54} |\hat{x}_j^{HW} - x_j| = 7,843 \text{ and } SAE^{ARIMA} = \sum_{j=7}^{54} |\hat{x}_j^{ARIMA} - x_j| = 14,796 \quad (66)$$

Comparing these 16 years times 3 trimesters forecasts, i.e., 48 trimester forecasts, the Holt–Winters model lead to better trimesterly forecast (smallest absolute residual) 32 times as opposed to 16 times for the ARIMA model, i.e.,  $HW/AR = 32/16$ . Moreover, if one compares the sum of absolute error per year, the Holt–Winters model lead to a better yearly forecast twelve times

as opposed to four times for ARIMA, i.e.,  $HW/AR = 12/4$ .

For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead  $\sharp, \flat$  and  $\natural$  forecasts for  $X$  as in (56), (57) and (58) for each optimization criteria  $\kappa = 1, \dots, 8$  using the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models (with  $N = 1000$  to find the optimal latest-optimized alpha power). Moreover, we compute the (global) sum of absolute error, and we compare our forecasts to both Holt–Winters and ARIMA models by counting the number of best monthly and yearly forecasts. The results are summarized in Table 9..

$\bullet\kappa$	SAE	HW / AR / K.	HW / K.	AR / K.	HW / AR / K.	HW / K.	AR / K.
$\sharp 1$	5,484	13 / 7 / 28	18 / 30	9 / 39	2 / 1 / 13	2 / 14	1 / 15
$\sharp 2$	5,418	11 / 6 / 31	15 / 33	8 / 40	2 / 1 / 13	2 / 14	1 / 15
$\sharp 3$	5,483	12 / 7 / 29	16 / 32	9 / 39	2 / 1 / 13	2 / 14	1 / 15
$\sharp 4$	5,472	13 / 6 / 29	16 / 32	7 / 41	2 / 1 / 13	2 / 14	1 / 15
$\sharp 5$	5,356	14 / 7 / 27	18 / 30	8 / 40	3 / 1 / 12	3 / 13	1 / 15
$\sharp 6$	5,430	12 / 7 / 29	16 / 32	8 / 40	2 / 1 / 13	2 / 14	1 / 15
$\sharp 7$	5,504	13 / 7 / 28	17 / 31	9 / 39	2 / 1 / 13	2 / 14	1 / 15
$\sharp 8$	5,384	12 / 7 / 29	16 / 32	8 / 40	3 / 1 / 12	3 / 13	1 / 15
$\flat 1$	5,487	13 / 7 / 28	17 / 31	9 / 39	2 / 1 / 13	2 / 14	1 / 15
$\flat 2$	5,487	13 / 7 / 28	17 / 31	9 / 39	2 / 1 / 13	2 / 14	1 / 15
$\flat 3$	5,478	13 / 7 / 28	18 / 30	9 / 39	2 / 1 / 13	2 / 14	1 / 15
$\flat 4$	5,452	12 / 6 / 30	16 / 32	8 / 40	2 / 1 / 13	2 / 14	1 / 15
$\flat 5$	5,422	13 / 7 / 28	18 / 30	8 / 40	3 / 1 / 12	3 / 13	1 / 15
$\flat 6$	5,435	13 / 5 / 30	16 / 32	6 / 42	2 / 1 / 13	2 / 14	1 / 15
$\flat 7$	5,513	12 / 7 / 29	17 / 31	8 / 40	2 / 1 / 13	2 / 14	1 / 15
$\flat 8$	5,391	14 / 6 / 28	18 / 30	7 / 41	3 / 1 / 12	3 / 13	1 / 15
$\natural 1$	5,532	17 / 7 / 28	18 / 30	8 / 40	2 / 1 / 13	3 / 13	1 / 15
$\natural 2$	5,487	14 / 7 / 27	19 / 29	9 / 39	2 / 1 / 13	2 / 14	1 / 15
$\natural 3$	5,462	13 / 6 / 29	18 / 30	7 / 41	3 / 1 / 12	3 / 13	1 / 15
$\natural 4$	5,508	13 / 6 / 29	17 / 31	8 / 40	2 / 1 / 13	2 / 14	1 / 15
$\natural 5$	5,452	15 / 7 / 26	19 / 29	8 / 40	3 / 1 / 12	3 / 13	1 / 15
$\natural 6$	5,478	13 / 7 / 28	16 / 32	8 / 40	3 / 1 / 12	3 / 13	1 / 15
$\natural 7$	5,444	12 / 7 / 29	16 / 32	9 / 39	2 / 1 / 13	2 / 14	1 / 15
$\natural 8$	5,449	15 / 7 / 26	19 / 29	8 / 40	3 / 1 / 12	3 / 13	1 / 15

TABLE 9. Undergraduate + Graduate Continuing Enrollment: Comparison of the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models to the seasonal Holt–Winters and ARIMA models

*Example 9.* Let’s consider student (undergraduate + graduate) new enrollment (fall, spring and summer) over 18 years (fall 2004 to summer 2022). Thus,  $f = 3$  and  $X = (x_1, \dots, x_{54})$ . For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead forecasts for  $X$  using both Holt–Winters and ARIMA models. The sum of absolute errors for each model are

$$SAE^{HW} = \sum_{j=7}^{54} \left| \hat{x}_j^{HW} - x_j \right| = 4,784 \text{ and } SAE^{ARIMA} = \sum_{j=7}^{54} \left| \hat{x}_j^{ARIMA} - x_j \right| = 8,938 \quad (67)$$

Comparing these 16 years times 3 trimesters forecasts, i.e., 48 trimester forecasts, the Holt–winters model lead to better trimesterly forecast (smallest absolute residual) 34 times as opposed to 14 times for the ARIMA model, i.e.,  $HW/AR = 34/14$ . Moreover, if one compares the sum of absolute error per year, the Holt–Winters model lead to a better yearly forecast twelve times as opposed to four times for ARIMA, i.e.,  $HW/AR = 12/4$ .

For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead  $\sharp, \flat$  and  $\natural$  forecasts for  $X$  as in (56), (57) and (58) for each optimization criteria  $\kappa = 1, \dots, 8$  using the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models (with  $N = 1000$  to find the optimal latest-optimized alpha power). Moreover,

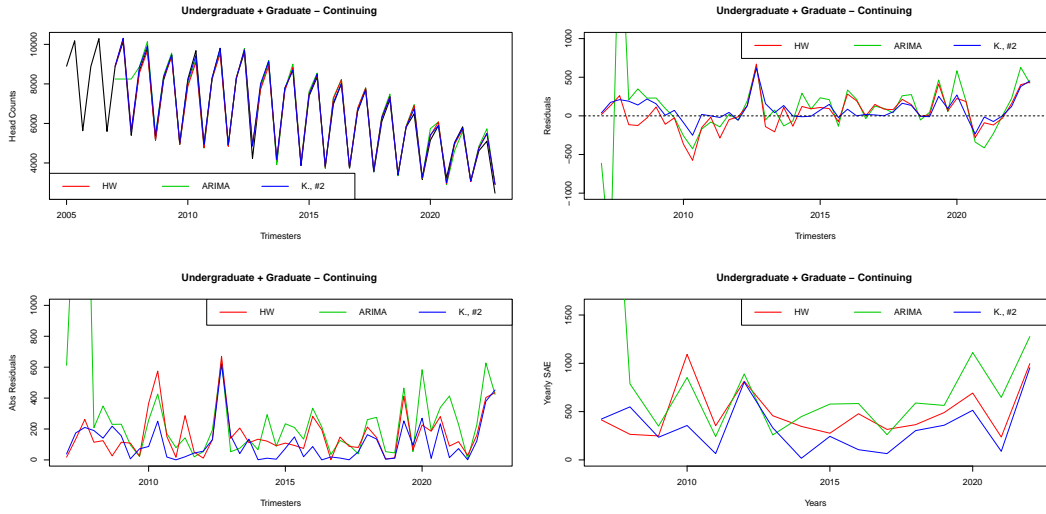


FIGURE 8. Undergraduate + Graduate Continuing Enrollment: Forecasts, residuals, absolute residuals and yearly sum of absolute errors

we compute the (global) sum of absolute error, and we compare our forecasts to both Holt–Winters and ARIMA models by counting the number of best monthly and yearly forecasts. The results are summarized in Table 10..

● <sub>k</sub>	SAE	HW / AR / K.	HW / K.	AR / K.	HW / AR / K.	HW / K.	AR / K.
#1	3,146	7 / 8 / 33	12 / 36	10 / 38	1 / 1 / 14	1 / 15	1 / 15
#2	3,062	9 / 6 / 33	13 / 35	7 / 41	2 / 1 / 13	2 / 14	1 / 15
#3	3,114	11 / 8 / 29	17 / 31	10 / 38	2 / 1 / 13	2 / 14	1 / 15
#4	3,112	8 / 10 / 30	14 / 34	12 / 36	2 / 2 / 12	2 / 14	2 / 14
#5	3,096	10 / 7 / 31	15 / 33	9 / 39	1 / 1 / 14	1 / 15	1 / 15
#6	3,090	8 / 9 / 31	13 / 35	11 / 37	2 / 2 / 12	2 / 14	2 / 14
#7	3,044	8 / 8 / 32	13 / 35	10 / 38	1 / 0 / 15	1 / 15	0 / 16
#8	3,106	11 / 8 / 29	15 / 33	10 / 38	1 / 1 / 14	1 / 15	1 / 15
b1	3,179	9 / 8 / 31	14 / 34	9 / 39	3 / 1 / 12	3 / 13	1 / 15
b2	3,069	10 / 8 / 30	16 / 32	10 / 38	2 / 1 / 13	2 / 14	1 / 15
b3	3,128	9 / 7 / 32	15 / 33	8 / 40	2 / 1 / 13	2 / 14	1 / 15
b4	3,054	9 / 8 / 31	15 / 33	10 / 38	3 / 1 / 12	3 / 13	1 / 15
b5	3,065	11 / 7 / 30	15 / 33	9 / 39	1 / 1 / 14	1 / 15	1 / 15
b6	3,107	9 / 8 / 31	15 / 33	10 / 38	2 / 1 / 13	2 / 14	1 / 15
b7	3,039	9 / 10 / 29	14 / 34	12 / 36	2 / 0 / 14	2 / 14	0 / 16
b8	3,04	10 / 6 / 32	14 / 34	8 / 40	1 / 1 / 14	1 / 15	1 / 15
†1	3,130	9 / 9 / 30	15 / 33	10 / 38	2 / 1 / 13	2 / 14	1 / 15
†2	3,103	9 / 10 / 29	13 / 35	12 / 36	1 / 2 / 13	1 / 15	2 / 14
†3	3,342	9 / 7 / 32	15 / 33	8 / 40	3 / 1 / 12	3 / 13	1 / 15
†4	3,043	9 / 9 / 30	15 / 33	10 / 38	1 / 1 / 14	1 / 15	1 / 15
†5	3,073	11 / 7 / 30	15 / 33	9 / 39	1 / 1 / 14	1 / 15	1 / 15
†6	3,087	9 / 9 / 30	14 / 34	11 / 37	2 / 0 / 14	2 / 14	0 / 16
†7	3,052	11 / 7 / 30	15 / 33	9 / 39	1 / 1 / 14	1 / 15	1 / 15
†8	3,052	11 / 7 / 30	15 / 33	9 / 39	1 / 1 / 14	1 / 15	1 / 15

TABLE 10. Undergraduate + Graduate New Enrollment: Comparison of the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models to the seasonal Holt–Winters and ARIMA models

*Example 10.* Let’s consider undergraduate total credit hours (fall, spring and summer) over 18 years (fall 2004 to summer 2022). Thus,  $f = 3$  and  $X = (x_1, \dots, x_{54})$ . For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead forecasts for  $X$  using both

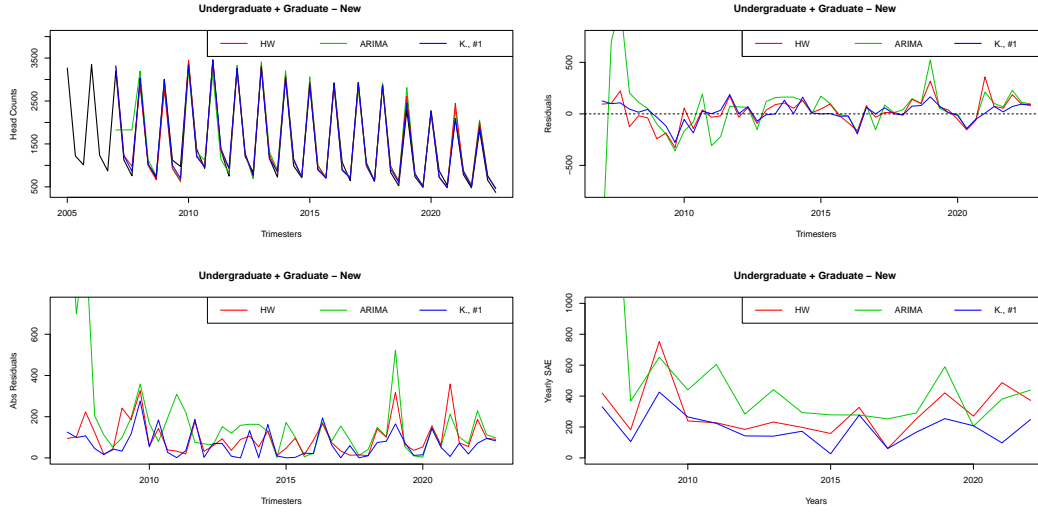


FIGURE 9. Undergraduate + Graduate New Enrollment: Forecasts, residuals, absolute residuals and yearly sum of absolute errors

Holt–Winters and ARIMA models. The sum of absolute errors for each model are

$$\text{SAE}^{\text{HW}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{HW}} - x_j \right| = 93,088 \quad \text{and} \quad \text{SAE}^{\text{ARIMA}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{ARIMA}} - x_j \right| = 171,033 \quad (68)$$

Comparing these 16 years times 3 trimesters forecasts, i.e., 48 trimester forecasts, the Holt–Winters model lead to better trimesterly forecast (smallest absolute residual) 32 times as opposed to 16 times for the ARIMA model, i.e.,  $HW/AR = 32/16$ . Moreover, if one compares the sum of absolute error per year, the Holt–Winters model lead to a better yearly forecast nine times as opposed to seven times for ARIMA, i.e.,  $HW/AR = 9/7$ .

For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead  $\sharp, b$  and  $\natural$  forecasts for  $X$  as in (56), (57) and (58) for each optimization criteria  $\kappa = 1, \dots, 8$  using the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models (with  $N = 1000$  to find the optimal latest-optimized alpha power). Moreover, we compute the (global) sum of absolute error, and we compare our forecasts to both Holt–Winters and ARIMA models by counting the number of best monthly and yearly forecasts. The results are summarized in Table 11..

*Example 11.* Let’s consider undergraduate continuing credit hours (fall, spring and summer) over 18 years (fall 2004 to summer 2022). Thus,  $f = 3$  and  $X = (x_1, \dots, x_{54})$ . For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead forecasts for  $X$  using both Holt–Winters and ARIMA models. The sum of absolute errors for each model are

$$\text{SAE}^{\text{HW}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{HW}} - x_j \right| = 89,007 \quad \text{and} \quad \text{SAE}^{\text{ARIMA}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{ARIMA}} - x_j \right| = 147,859 \quad (69)$$

Comparing these 16 years times 3 trimesters forecasts, i.e., 48 trimester forecasts, the Holt–Winters model lead to better trimesterly forecast (smallest absolute residual) 28 times as opposed to 20 times for the ARIMA model, i.e.,  $HW/AR = 28/20$ . Moreover, if one compares the sum

•/κ	SAE	HW / AR / K.	HW / K.	AR / K.	HW / AR / K.	HW / K.	AR / K.
#1	55,490	16 / 7 / 25	19 / 29	10 / 38	4 / 2 / 10	6 / 10	2 / 14
#2	54,052	14 / 6 / 28	16 / 32	9 / 39	4 / 2 / 10	6 / 10	2 / 14
#3	55,601	18 / 7 / 23	10 / 29	9 / 39	3 / 2 / 11	5 / 11	2 / 14
#4	55,949	16 / 7 / 25	18 / 30	10 / 38	4 / 2 / 10	6 / 10	2 / 14
#5	54,621	14 / 7 / 27	17 / 31	10 / 38	4 / 2 / 10	6 / 10	2 / 14
#6	56,656	17 / 7 / 24	19 / 29	10 / 38	4 / 2 / 10	6 / 10	2 / 14
#7	56,264	14 / 7 / 27	15 / 33	9 / 39	5 / 2 / 9	7 / 9	2 / 14
#8	53,830	15 / 7 / 26	16 / 32	11 / 37	4 / 2 / 10	6 / 10	2 / 14
‡1	55,714	16 / 7 / 25	17 / 31	11 / 37	4 / 2 / 10	6 / 10	2 / 14
‡2	54,597	15 / 7 / 26	17 / 31	11 / 37	4 / 2 / 10	6 / 10	2 / 14
‡3	56,149	17 / 6 / 25	19 / 29	9 / 39	4 / 2 / 10	6 / 10	2 / 14
‡4	55,733	14 / 7 / 27	16 / 32	9 / 39	4 / 2 / 10	6 / 10	2 / 14
‡5	54,500	14 / 6 / 28	16 / 32	10 / 38	4 / 2 / 10	6 / 10	2 / 14
‡6	56,058	16 / 7 / 25	18 / 30	9 / 39	3 / 2 / 11	5 / 11	2 / 14
‡7	55,918	15 / 7 / 26	17 / 31	9 / 39	5 / 2 / 9	7 / 9	2 / 14
‡8	54,360	14 / 7 / 27	17 / 31	10 / 38	4 / 2 / 10	6 / 10	2 / 14
‡1	56,484	15 / 6 / 27	17 / 31	9 / 39	4 / 2 / 10	6 / 10	2 / 14
‡2	53,906	14 / 6 / 28	17 / 31	9 / 39	4 / 2 / 10	6 / 10	2 / 14
‡3	55,252	16 / 6 / 26	18 / 30	9 / 39	4 / 2 / 10	6 / 10	2 / 14
‡4	56,108	16 / 7 / 25	19 / 29	10 / 38	4 / 2 / 10	6 / 10	2 / 14
‡5	54,387	13 / 6 / 29	15 / 33	10 / 38	4 / 2 / 10	6 / 10	2 / 14
‡6	56,531	15 / 7 / 29	16 / 32	9 / 39	4 / 2 / 10	6 / 10	2 / 14
‡7	56,770	15 / 7 / 26	17 / 31	9 / 39	5 / 2 / 9	7 / 9	2 / 14
‡8	54,332	14 / 6 / 28	15 / 33	10 / 38	4 / 2 / 10	6 / 10	2 / 14

TABLE 11. Undergraduate Total Credit Hours: Comparison of the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models to the seasonal Holt–Winters and ARIMA models

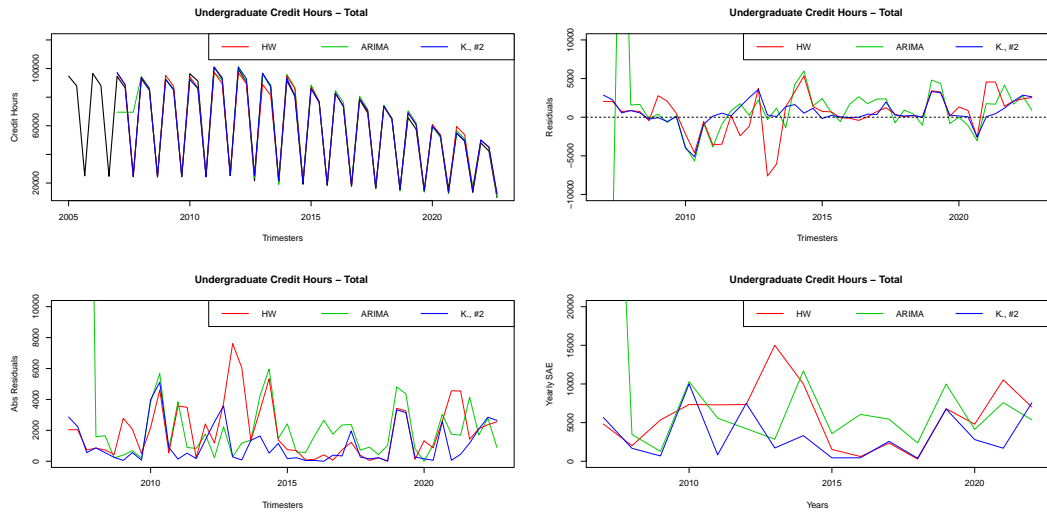


FIGURE 10. Undergraduate Total Credit Hours: Forecasts, residuals, absolute residuals and yearly sum of absolute errors

of absolute error per year, the Holt–Winters model lead to a better yearly forecast nine times as opposed to seven times for ARIMA, i.e.,  $HW/AR = 9/7$ .

For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead  $\ddagger, \ddagger$  and  $\ddagger$  forecasts for  $X$  as in (56), (57) and (58) for each optimization criteria  $\kappa = 1, \dots, 8$  using the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models (with  $N = 1000$  to find the optimal latest-optimized alpha power). Moreover,

we compute the (global) sum of absolute error, and we compare our forecasts to both Holt–Winters and ARIMA models by counting the number of best monthly and yearly forecasts. The results are summarized in Table 12..

•#	SAE	HW / AR / K.	HW / K.	AR / K.	HW / AR / K.	HW / K.	AR / K.
#1	46,307	9 / 6 / 33	10 / 38	9 / 39	2 / 1 / 13	2 / 14	1 / 15
#2	45,926	9 / 8 / 31	12 / 36	11 / 37	1 / 1 / 14	1 / 15	1 / 15
#3	46,317	8 / 9 / 31	9 / 39	12 / 36	2 / 1 / 13	2 / 14	1 / 15
#4	45,977	8 / 7 / 33	10 / 38	9 / 39	2 / 1 / 13	2 / 14	1 / 15
#5	45,678	8 / 8 / 32	11 / 37	10 / 38	1 / 1 / 14	1 / 15	1 / 15
#6	46,389	7 / 7 / 34	9 / 39	8 / 40	2 / 1 / 13	2 / 14	1 / 15
#7	46,323	10 / 6 / 32	11 / 37	7 / 41	2 / 1 / 13	2 / 14	1 / 15
#8	46,229	8 / 8 / 32	11 / 37	11 / 37	1 / 1 / 14	1 / 15	1 / 15
b1	46,694	9 / 8 / 31	12 / 36	10 / 38	2 / 1 / 13	2 / 14	1 / 15
b2	45,117	6 / 8 / 34	9 / 39	9 / 39	1 / 1 / 14	1 / 15	1 / 15
b3	45,663	9 / 8 / 31	11 / 37	10 / 38	2 / 1 / 13	2 / 14	1 / 15
b4	45,736	9 / 8 / 31	10 / 38	10 / 38	2 / 1 / 13	2 / 14	1 / 15
b5	45,927	7 / 9 / 32	11 / 37	11 / 37	1 / 1 / 14	1 / 15	1 / 15
b6	46,173	7 / 8 / 33	10 / 38	9 / 39	2 / 1 / 13	2 / 14	1 / 15
b7	46,161	7 / 9 / 32	8 / 40	11 / 37	2 / 1 / 13	2 / 14	1 / 15
b8	45,662	7 / 9 / 32	11 / 37	11 / 37	1 / 1 / 14	1 / 15	1 / 15
†1	46,610	8 / 10 / 30	10 / 38	13 / 35	2 / 1 / 13	2 / 14	1 / 15
†2	45,659	8 / 7 / 33	12 / 36	8 / 40	1 / 1 / 14	1 / 15	1 / 15
†3	46,403	9 / 8 / 31	12 / 36	10 / 38	2 / 1 / 13	2 / 14	1 / 15
†4	45,971	8 / 9 / 31	11 / 37	11 / 37	2 / 1 / 13	2 / 14	1 / 15
†5	45,730	8 / 8 / 32	12 / 36	11 / 37	1 / 1 / 14	1 / 15	1 / 15
†6	45,865	8 / 9 / 31	10 / 38	10 / 38	2 / 1 / 13	2 / 14	1 / 15
†7	45,799	6 / 8 / 34	8 / 40	10 / 38	2 / 1 / 13	2 / 14	1 / 15
†8	45,506	8 / 8 / 32	12 / 36	9 / 39	1 / 1 / 14	1 / 15	1 / 15

TABLE 12. Undergraduate Continuing Credit Hours: Comparison of the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models to the seasonal Holt–Winters and ARIMA models

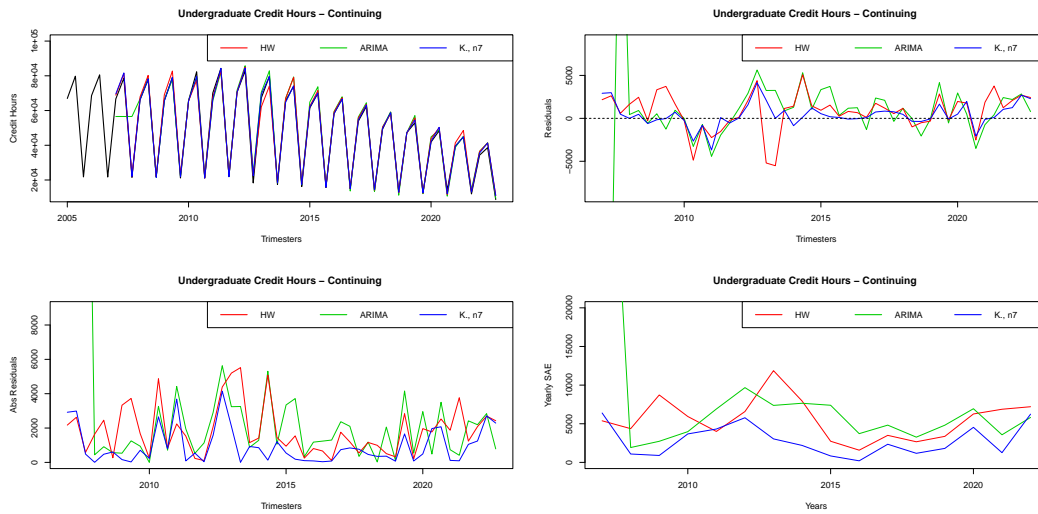


FIGURE 11. Undergraduate Continuing Credit Hours: Forecasts, residuals, absolute residuals and yearly sum of absolute errors

*Example 12.* Let’s consider undergraduate new credit hours (fall, spring and summer) over 18 years (fall 2004 to summer 2022). Thus,  $f = 3$  and  $X = (x_1, \dots, x_{54})$ . For  $s = 1, \dots, 3$  (trimesters),



and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead forecasts for  $X$  using both Holt–Winters and ARIMA models. The sum of absolute errors for each model are

$$\text{SAE}^{\text{HW}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{HW}} - x_j \right| = 34,265 \text{ and } \text{SAE}^{\text{ARIMA}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{ARIMA}} - x_j \right| = 67,620 \quad (70)$$

Comparing these 16 years times 3 trimesters forecasts, i.e., 48 trimester forecasts, the Holt–Winters model lead to better trimesterly forecast (smallest absolute residual) 32 times as opposed to 16 times for the ARIMA model, i.e.,  $HW/AR = 32/16$ . Moreover, if one compares the sum of absolute error per year, the Holt–Winters model lead to a better yearly forecast ten times as opposed to six times for ARIMA, i.e.,  $HW/AR = 10/6$ .

For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead  $\sharp, \flat$  and  $\natural$  forecasts for  $X$  as in (56), (57) and (58) for each optimization criteria  $\kappa = 1, \dots, 8$  using the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models (with  $N = 1000$  to find the optimal latest-optimized alpha power). Moreover, we compute the (global) sum of absolute error, and we compare our forecasts to both Holt–Winters and ARIMA models by counting the number of best monthly and yearly forecasts. The results are summarized in Table 13..

$\bullet \kappa$	SAE	HW / AR / K.	HW / K.	AR / K.	HW / AR / K.	HW / K.	AR / K.
#1	22,391	13 / 6 / 29	14 / 34	11 / 37	1 / 0 / 15	1 / 15	0 / 16
#2	22,958	14 / 6 / 28	16 / 32	10 / 38	2 / 0 / 14	2 / 14	0 / 16
#3	22,093	13 / 6 / 29	16 / 32	9 / 39	1 / 0 / 15	1 / 15	0 / 16
#4	21,986	13 / 6 / 29	15 / 33	9 / 39	1 / 0 / 15	1 / 15	0 / 16
#5	22,551	14 / 7 / 27	17 / 31	11 / 37	3 / 0 / 13	3 / 13	0 / 16
#6	22,319	13 / 5 / 30	14 / 34	9 / 39	2 / 0 / 14	2 / 14	0 / 16
#7	22,507	12 / 8 / 28	13 / 35	12 / 36	1 / 0 / 15	1 / 15	0 / 16
#8	22,294	12 / 9 / 27	13 / 35	12 / 36	2 / 0 / 14	2 / 14	0 / 16
♯1	21,708	14 / 4 / 30	15 / 33	8 / 40	0 / 0 / 16	0 / 16	0 / 16
♯2	23,246	14 / 6 / 28	16 / 32	11 / 37	2 / 0 / 14	2 / 14	0 / 16
♯3	22,004	12 / 7 / 29	13 / 35	11 / 37	1 / 0 / 15	1 / 15	0 / 16
♯4	22,265	12 / 8 / 28	13 / 35	11 / 37	2 / 0 / 14	2 / 14	0 / 16
♯5	22,945	12 / 7 / 29	14 / 34	10 / 38	2 / 0 / 14	2 / 14	0 / 16
♯6	22,206	14 / 7 / 27	15 / 33	11 / 37	2 / 0 / 14	2 / 14	0 / 16
♯7	22,145	14 / 5 / 29	16 / 32	8 / 40	2 / 0 / 14	2 / 14	0 / 16
♯8	22,537	11 / 7 / 30	13 / 35	11 / 37	3 / 0 / 13	3 / 13	0 / 16
♭1	21,412	13 / 7 / 28	15 / 33	11 / 37	0 / 0 / 16	0 / 16	0 / 16
♭2	23,530	14 / 7 / 27	16 / 32	12 / 36	2 / 0 / 14	2 / 14	0 / 16
♭3	21,654	11 / 7 / 30	13 / 35	10 / 38	0 / 0 / 16	0 / 16	0 / 16
♭4	22,104	14 / 5 / 29	14 / 34	9 / 39	2 / 0 / 14	2 / 14	0 / 16
♭5	22,657	11 / 8 / 29	12 / 36	11 / 37	3 / 0 / 13	3 / 13	0 / 16
♭6	22,311	13 / 7 / 28	14 / 34	10 / 38	2 / 0 / 14	2 / 14	0 / 16
♭7	22,058	15 / 7 / 26	17 / 31	9 / 39	2 / 0 / 14	2 / 14	0 / 16
♭8	22,373	12 / 8 / 28	14 / 34	12 / 36	3 / 0 / 13	3 / 13	0 / 16

TABLE 13. Undergraduate New Credit Hours: Comparison of the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models to the seasonal Holt–Winters and ARIMA models

*Example 13.* Let’s consider graduate total credit hours (fall, spring and summer) over 18 years (fall 2004 to summer 2022). Thus,  $f = 3$  and  $X = (x_1, \dots, x_{54})$ . For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead forecasts for  $X$  using both Holt–Winters and ARIMA models. The sum of absolute errors for each model are

$$\text{SAE}^{\text{HW}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{HW}} - x_j \right| = 33,070 \text{ and } \text{SAE}^{\text{ARIMA}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{ARIMA}} - x_j \right| = 37,629 \quad (71)$$

Comparing these 16 years times 3 trimesters forecasts, i.e., 48 trimester forecasts, the Holt–Winters model lead to better trimesterly forecast (smallest absolute residual) 26 times as opposed to 22 times for the ARIMA model, i.e.,  $HW/AR = 26/22$ . Moreover, if one compares the sum of

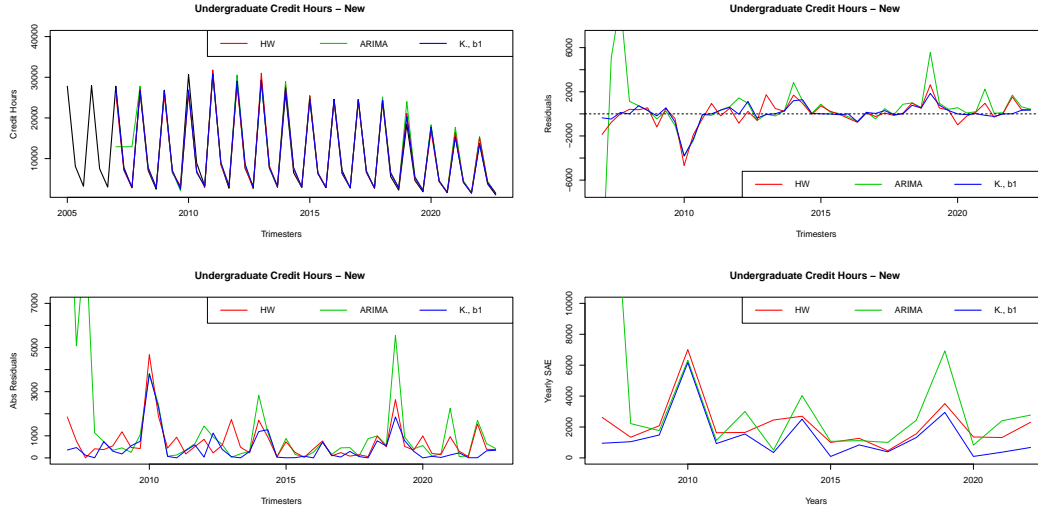


FIGURE 12. Undergraduate New Credit Hours: Forecasts, residuals, absolute residuals and yearly sum of absolute errors

absolute error per year, the Holt–Winters model lead to the same number of best yearly forecast as ARIMA, i.e.,  $HW/AR = 8/8$ .

For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead  $\sharp, \flat$  and  $\natural$  forecasts for  $X$  as in (56), (57) and (58) for each optimization criteria  $\kappa = 1, \dots, 8$  using the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models (with  $N = 1000$  to find the optimal latest-optimized alpha power). Moreover, we compute the (global) sum of absolute error, and we compare our forecasts to both Holt–Winters and ARIMA models by counting the number of best monthly and yearly forecasts. The results are summarized in Table 14.

*Example 14.* Let's consider graduate continuing credit hours (fall, spring and summer) over 18 years (fall 2004 to summer 2022). Thus,  $f = 3$  and  $X = (x_1, \dots, x_{54})$ . For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead forecasts for  $X$  using both Holt–Winters and ARIMA models. The sum of absolute errors for each model are

$$\text{SAE}^{\text{HW}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{HW}} - x_j \right| = 26,863 \quad \text{and} \quad \text{SAE}^{\text{ARIMA}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{ARIMA}} - x_j \right| = 31,672 \quad (72)$$

Comparing these 16 years times 3 trimesters forecasts, i.e., 48 trimester forecasts, the Holt–winters model lead to better trimesterly forecast (smallest absolute residual) 26 times as opposed to 22 times for the ARIMA model, i.e.,  $HW/AR = 26/22$ . Moreover, if one compares the sum of absolute error per year, the Holt–Winters model lead to a better yearly forecast twelve times as opposed to four times for ARIMA, i.e.,  $HW/AR = 12/4$ .

For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead  $\sharp, \flat$  and  $\natural$  forecasts for  $X$  as in (56), (57) and (58) for each optimization criteria  $\kappa = 1, \dots, 8$  using the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models (with  $N = 1000$  to find the optimal latest-optimized alpha power). Moreover,

•/s	SAE	HW / AR / K.	HW / K.	AR / K.	HW / AR / K.	HW / K.	AR / K.
#1	22,292	7 / 8 / 33	10 / 38	11 / 37	2 / 1 / 13	2 / 14	1 / 15
#2	22,853	8 / 10 / 30	10 / 38	14 / 34	2 / 1 / 13	2 / 14	1 / 15
#3	22,591	9 / 8 / 31	12 / 36	11 / 37	2 / 1 / 13	2 / 14	1 / 15
#4	22,617	8 / 7 / 33	10 / 38	10 / 38	2 / 1 / 13	2 / 14	1 / 15
#5	22,498	8 / 8 / 32	11 / 37	12 / 36	2 / 1 / 13	2 / 14	1 / 15
#6	22,957	9 / 9 / 30	12 / 36	12 / 36	2 / 1 / 13	2 / 14	1 / 15
#7	22,613	9 / 8 / 31	12 / 36	11 / 37	2 / 1 / 13	2 / 14	1 / 15
#8	22,577	9 / 9 / 30	11 / 37	13 / 35	2 / 1 / 13	2 / 14	1 / 15
b1	22,603	9 / 9 / 30	12 / 36	13 / 35	2 / 1 / 13	2 / 14	1 / 15
b2	22,739	9 / 10 / 29	11 / 37	14 / 34	2 / 1 / 13	2 / 14	1 / 15
b3	22,433	8 / 9 / 31	10 / 38	13 / 35	2 / 1 / 13	2 / 14	1 / 15
b4	22,750	8 / 11 / 29	10 / 38	15 / 22	2 / 1 / 13	2 / 14	1 / 15
b5	22,441	7 / 8 / 33	9 / 39	11 / 37	2 / 1 / 13	2 / 14	1 / 15
b6	22,999	9 / 10 / 29	12 / 36	14 / 34	2 / 1 / 13	2 / 14	1 / 15
b7	22,714	8 / 8 / 32	11 / 37	11 / 37	2 / 1 / 13	2 / 14	1 / 15
b8	22,594	7 / 10 / 31	9 / 39	12 / 36	2 / 1 / 13	2 / 14	1 / 15
‡1	22,435	7 / 9 / 32	10 / 38	12 / 36	2 / 1 / 13	2 / 14	1 / 15
‡2	22,801	7 / 10 / 31	9 / 39	14 / 34	2 / 1 / 13	2 / 14	1 / 15
‡3	23,498	9 / 9 / 30	11 / 37	13 / 35	2 / 1 / 13	2 / 14	1 / 15
‡4	22,758	8 / 9 / 31	10 / 38	12 / 36	2 / 1 / 13	2 / 14	1 / 15
‡5	22,631	8 / 8 / 32	10 / 38	12 / 36	2 / 1 / 13	2 / 14	1 / 15
‡6	22,710	9 / 10 / 29	12 / 36	13 / 35	2 / 1 / 13	2 / 14	1 / 15
‡7	22,796	8 / 8 / 32	11 / 37	11 / 37	2 / 1 / 13	2 / 14	1 / 15
‡8	22,913	8 / 9 / 31	11 / 37	13 / 35	2 / 1 / 13	2 / 14	1 / 15

TABLE 14. Graduate Total Credit Hours: Comparison of the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models to the seasonal Holt–Winters and ARIMA models

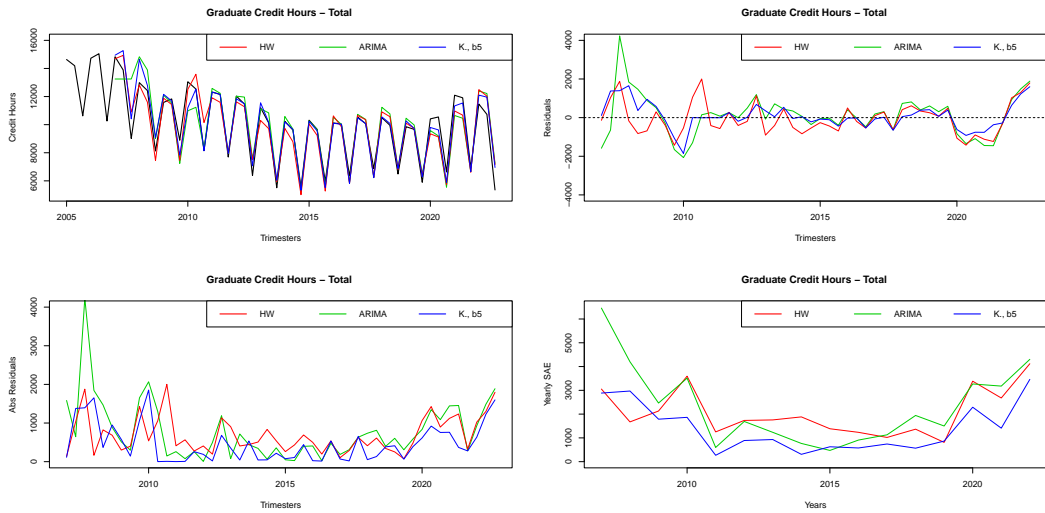


FIGURE 13. Graduate Total Credit Hours: Forecasts, residuals, absolute residuals and yearly sum of absolute errors

we compute the (global) sum of absolute error, and we compare our forecasts to both Holt–Winters and ARIMA models by counting the number of best monthly and yearly forecasts. The results are summarized in Table 15..

*Example 15.* Let’s consider graduate new credit hours (fall, spring and summer) over 18 years (fall 2004 to summer 2022). Thus,  $f = 3$  and  $X = (x_1, \dots, x_{54})$ . For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead forecasts for  $X$  using both

#/s	SAE	HW / AR / K.	HW / K.	AR / K.	HW / AR / K.	HW / K.	AR / K.
#1	19,752	9 / 10 / 29	15 / 33	11 / 37	3 / 1 / 12	4 / 12	2 / 14
#2	19,770	11 / 11 / 26	18 / 30	12 / 36	3 / 1 / 12	4 / 12	2 / 14
#3	19,759	10 / 10 / 28	16 / 32	11 / 37	3 / 1 / 12	4 / 12	2 / 14
#4	20,002	9 / 10 / 29	15 / 33	11 / 37	4 / 1 / 11	5 / 11	2 / 14
#5	19,732	12 / 11 / 25	19 / 29	12 / 36	4 / 1 / 11	5 / 11	2 / 14
#6	20,173	11 / 10 / 27	18 / 30	11 / 37	5 / 2 / 9	6 / 10	3 / 13
#7	19,942	12 / 9 / 27	19 / 29	10 / 38	4 / 1 / 11	5 / 11	2 / 14
#8	19,702	11 / 11 / 26	18 / 30	12 / 36	4 / 1 / 11	5 / 11	2 / 14
b1	19,679	10 / 9 / 29	16 / 32	10 / 38	3 / 1 / 12	4 / 12	2 / 14
b2	19,744	10 / 10 / 28	17 / 31	11 / 37	4 / 1 / 11	5 / 11	2 / 14
b3	19,571	10 / 8 / 30	16 / 32	9 / 39	3 / 1 / 12	4 / 12	2 / 14
b4	19,991	10 / 9 / 29	17 / 31	10 / 38	4 / 1 / 11	5 / 11	2 / 14
b5	19,790	11 / 10 / 27	18 / 30	11 / 37	4 / 1 / 11	5 / 11	2 / 14
b6	20,094	10 / 11 / 27	17 / 31	12 / 36	4 / 1 / 11	5 / 11	2 / 14
b7	19,753	10 / 10 / 28	17 / 31	11 / 37	3 / 1 / 12	4 / 12	2 / 14
b8	19,762	11 / 10 / 27	18 / 30	11 / 37	4 / 1 / 11	5 / 11	2 / 14
♯1	19,829	9 / 10 / 29	15 / 33	11 / 37	3 / 1 / 12	4 / 12	2 / 14
♯2	19,699	11 / 11 / 26	18 / 30	12 / 36	3 / 1 / 12	4 / 12	2 / 14
♯3	20,202	10 / 10 / 28	18 / 30	12 / 36	3 / 1 / 12	4 / 12	2 / 14
♯4	20,146	12 / 11 / 25	18 / 30	13 / 35	4 / 2 / 10	5 / 11	3 / 13
♯5	19,758	10 / 11 / 27	17 / 31	12 / 36	4 / 1 / 11	5 / 11	2 / 14
♯6	20,083	12 / 9 / 27	19 / 29	10 / 38	4 / 1 / 11	5 / 11	2 / 14
♯7	19,776	10 / 10 / 28	17 / 31	11 / 37	3 / 1 / 12	4 / 12	2 / 14
♯8	19,819	9 / 11 / 28	16 / 32	12 / 36	4 / 1 / 11	5 / 11	2 / 14

TABLE 15. Graduate Continuing Credit Hours: Comparison of the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models to the seasonal Holt–Winters and ARIMA models

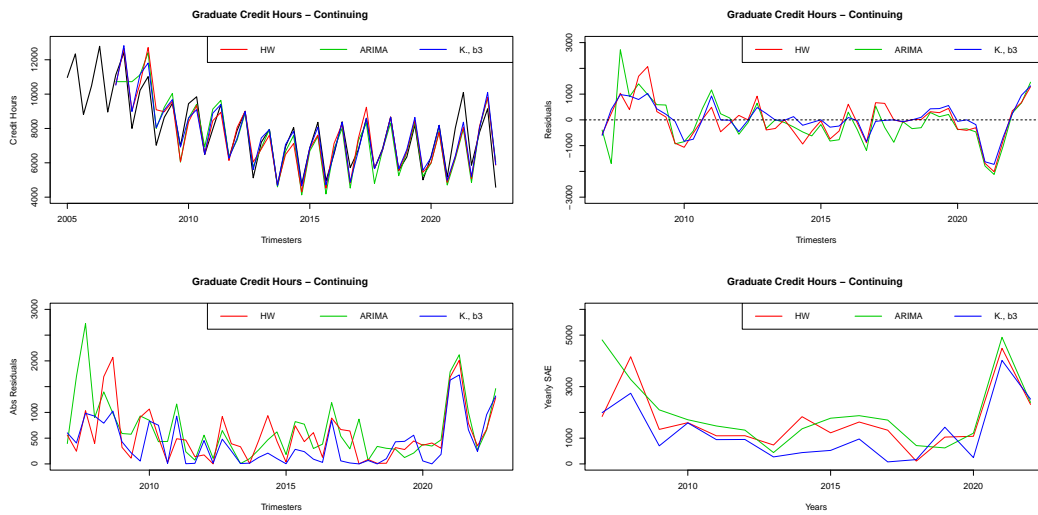


FIGURE 14. Graduate Continuing Credit Hours: Forecasts, residuals, absolute residuals and yearly sum of absolute errors

Holt–Winters and ARIMA models. The sum of absolute errors for each model are

$$SAE^{HW} = \sum_{j=7}^{54} \left| \hat{x}_j^{HW} - x_j \right| = 20,138 \text{ and } SAE^{ARIMA} = \sum_{j=7}^{54} \left| \hat{x}_j^{ARIMA} - x_j \right| = 19,258 \quad (73)$$

Comparing these 16 years times 3 trimesters forecasts, i.e., 48 trimester forecasts, the Holt–winters model lead to slightly worse trimesterly forecast (smallest absolute residual) 23 times as opposed to 25 times for the ARIMA model, i.e.,  $HW/AR = 23/25$ . Moreover, if one compares the sum of absolute error per year, the Holt–Winters model lead to a better yearly forecast nine

times as opposed to seven times for ARIMA, i.e.,  $HW/AR = 9/7$ .

For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead  $\sharp, \flat$  and  $\natural$  forecasts for  $X$  as in (56), (57) and (58) for each optimization criteria  $\kappa = 1, \dots, 8$  using the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models (with  $N = 1000$  to find the optimal latest-optimized alpha power). Moreover, we compute the (global) sum of absolute error, and we compare our forecasts to both Holt–Winters and ARIMA models by counting the number of best monthly and yearly forecasts. The results are summarized in Table 16..

$\bullet\kappa$	SAE	HW / AR / K.	HW / K.	AR / K.	HW / AR / K.	HW / K.	AR / K.
#1	12,829	12 / 12 / 24	13 / 35	16 / 32	4 / 2 / 10	4 / 12	3 / 13
#2	12,950	12 / 12 / 24	13 / 35	16 / 32	4 / 1 / 11	4 / 12	2 / 14
#3	12,820	11 / 13 / 24	12 / 36	16 / 32	4 / 2 / 10	4 / 12	3 / 13
#4	12,316	9 / 12 / 27	9 / 39	15 / 33	4 / 1 / 11	4 / 12	2 / 14
#5	12,606	9 / 13 / 26	10 / 38	15 / 33	4 / 1 / 11	4 / 12	2 / 14
#6	12,639	12 / 13 / 23	13 / 35	18 / 30	4 / 2 / 10	4 / 12	3 / 13
#7	12,779	12 / 14 / 22	13 / 35	17 / 31	4 / 2 / 10	4 / 12	3 / 13
#8	12,779	10 / 14 / 24	11 / 37	18 / 30	4 / 1 / 11	4 / 12	2 / 14
b1	12,550	13 / 11 / 24	14 / 34	15 / 33	2 / 1 / 13	2 / 14	2 / 14
b2	12,605	12 / 11 / 25	12 / 36	17 / 31	3 / 1 / 12	3 / 13	2 / 14
b3	12,834	13 / 12 / 23	14 / 34	15 / 33	4 / 1 / 11	4 / 12	2 / 14
b4	12,560	12 / 11 / 25	12 / 36	15 / 33	3 / 1 / 12	3 / 13	2 / 14
b5	12,346	11 / 12 / 25	12 / 36	17 / 31	3 / 1 / 12	3 / 13	2 / 14
b6	12,733	12 / 12 / 24	12 / 36	15 / 33	4 / 1 / 11	4 / 12	2 / 14
b7	12,883	14 / 13 / 21	15 / 33	18 / 30	4 / 2 / 10	4 / 12	3 / 13
b8	12,607	13 / 13 / 22	14 / 34	17 / 31	3 / 1 / 12	3 / 13	2 / 14
‡1	12,751	12 / 12 / 24	13 / 35	16 / 32	4 / 2 / 10	4 / 12	3 / 13
‡2	12,895	11 / 12 / 25	12 / 36	16 / 32	4 / 1 / 11	4 / 12	2 / 14
‡3	12,492	13 / 12 / 23	13 / 35	15 / 33	3 / 2 / 11	3 / 13	3 / 13
‡4	12,350	10 / 13 / 25	10 / 38	17 / 31	4 / 1 / 11	4 / 12	2 / 14
‡5	12,565	11 / 13 / 24	12 / 36	17 / 31	4 / 1 / 11	4 / 12	2 / 14
‡6	12,719	12 / 11 / 25	12 / 36	14 / 34	4 / 1 / 11	4 / 12	2 / 14
‡7	12,817	12 / 12 / 24	13 / 35	16 / 32	4 / 2 / 10	4 / 12	3 / 13
‡8	12,834	11 / 13 / 24	12 / 36	16 / 32	3 / 1 / 12	3 / 13	2 / 14

TABLE 16. Graduate New Credit Hours: Comparison of the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models to the seasonal Holt–Winters and ARIMA models

*Example 16.* Let’s consider student (undergraduate + graduate) total credit hours (fall, spring and summer) over 18 years (fall 2004 to summer 2022). Thus,  $f = 3$  and  $X = (x_1, \dots, x_{54})$ . For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead forecasts for  $X$  using both Holt–Winters and ARIMA models. The sum of absolute errors for each model are

$$\text{SAE}^{\text{HW}} = \sum_{j=7}^{54} |\hat{x}_j^{\text{HW}} - x_j| = 105,985 \text{ and } \text{SAE}^{\text{ARIMA}} = \sum_{j=7}^{54} |\hat{x}_j^{\text{ARIMA}} - x_j| = 199,672 \quad (74)$$

Comparing these 16 years times 3 trimesters forecasts, i.e., 48 trimester forecasts, the Holt–winters model lead to better trimesterly forecast (smallest absolute residual) 29 times as opposed to 19 times for the ARIMA model, i.e.,  $HW/AR = 29/19$ . Moreover, if one compares the sum of absolute error per year, the Holt–Winters model lead to a better yearly forecast nine times as opposed to seven times for ARIMA, i.e.,  $HW/AR = 9/7$ .

For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead  $\sharp, \flat$  and  $\natural$  forecasts for  $X$  as in (56), (57) and (58) for each optimization criteria  $\kappa = 1, \dots, 8$  using the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models (with  $N = 1000$  to find the optimal latest-optimized alpha power). Moreover,

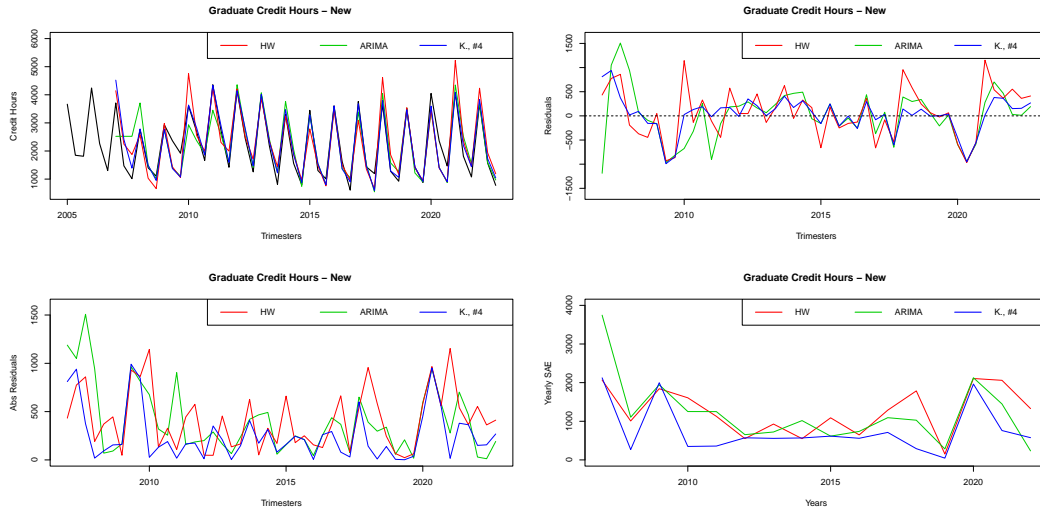


FIGURE 15. Graduate New Credit Hours: Forecasts, residuals, absolute residuals and yearly sum of absolute errors

we compute the (global) sum of absolute error, and we compare our forecasts to both Holt–Winters and ARIMA models by counting the number of best monthly and yearly forecasts. The results are summarized in Table 17..

● FC	SAE	HW / AR / K.	HW / K.	AR / K.	HW / AR / K.	HW / K.	AR / K.
#1	67,807	8 / 8 / 32	10 / 38	11 / 37	3 / 1 / 12	4 / 12	2 / 14
#2	68,195	9 / 8 / 31	11 / 37	11 / 37	3 / 1 / 12	4 / 12	2 / 14
#3	68,629	10 / 7 / 31	12 / 36	11 / 37	3 / 1 / 12	4 / 12	2 / 14
#4	68,035	8 / 6 / 34	10 / 38	9 / 39	3 / 1 / 12	4 / 12	2 / 14
#5	68,152	9 / 7 / 32	11 / 37	10 / 38	3 / 1 / 12	4 / 12	2 / 14
#6	69,908	10 / 7 / 31	12 / 36	11 / 37	3 / 1 / 12	4 / 12	2 / 14
#7	69,003	10 / 6 / 32	12 / 36	10 / 38	3 / 1 / 12	4 / 12	2 / 14
#8	67,842	8 / 7 / 33	10 / 38	11 / 37	3 / 0 / 13	3 / 13	1 / 15
b1	66,854	9 / 7 / 32	11 / 37	10 / 38	3 / 1 / 12	4 / 12	2 / 14
b2	67,568	10 / 8 / 30	12 / 36	11 / 37	3 / 1 / 12	4 / 12	2 / 14
b3	67,605	9 / 7 / 32	11 / 37	11 / 37	3 / 1 / 12	4 / 12	2 / 14
b4	66,966	9 / 7 / 32	11 / 37	10 / 38	3 / 1 / 12	4 / 12	2 / 14
b5	67,480	11 / 8 / 29	13 / 35	11 / 37	3 / 1 / 12	4 / 12	2 / 14
b6	68,033	10 / 7 / 31	12 / 36	11 / 37	3 / 1 / 12	4 / 12	2 / 14
b7	68,211	9 / 7 / 32	11 / 37	11 / 37	3 / 1 / 12	4 / 12	2 / 14
b8	67,202	10 / 8 / 30	12 / 36	11 / 37	3 / 1 / 12	4 / 12	2 / 14
̄1	68,272	9 / 7 / 32	11 / 37	10 / 38	3 / 1 / 12	4 / 12	2 / 14
̄2	67,838	9 / 8 / 31	11 / 37	12 / 36	3 / 1 / 12	4 / 12	2 / 14
̄3	69,186	10 / 6 / 32	12 / 36	11 / 37	3 / 1 / 12	4 / 12	2 / 14
̄4	67,559	9 / 6 / 33	11 / 37	10 / 38	3 / 1 / 12	4 / 12	2 / 14
̄5	67,925	8 / 8 / 32	10 / 38	11 / 37	3 / 1 / 12	4 / 12	2 / 14
̄6	68,603	10 / 6 / 32	12 / 36	10 / 38	3 / 1 / 12	4 / 12	2 / 14
̄7	68,570	10 / 7 / 31	12 / 36	11 / 37	3 / 1 / 12	4 / 12	2 / 14
̄8	67,446	9 / 8 / 31	11 / 37	11 / 37	3 / 1 / 12	3 / 13	2 / 14

TABLE 17. Undergraduate + Graduate Total Credit Hours: Comparison of the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models to the seasonal Holt–Winters and ARIMA models

*Example 17.* Let’s consider student (undergraduate + graduate) continuing credit hours (fall, spring and summer) over 18 years (fall 2004 to summer 2022). Thus,  $f = 3$  and  $X = (x_1, \dots, x_{54})$ . For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead forecasts for  $X$  using both Holt–Winters and ARIMA models. The sum of absolute errors

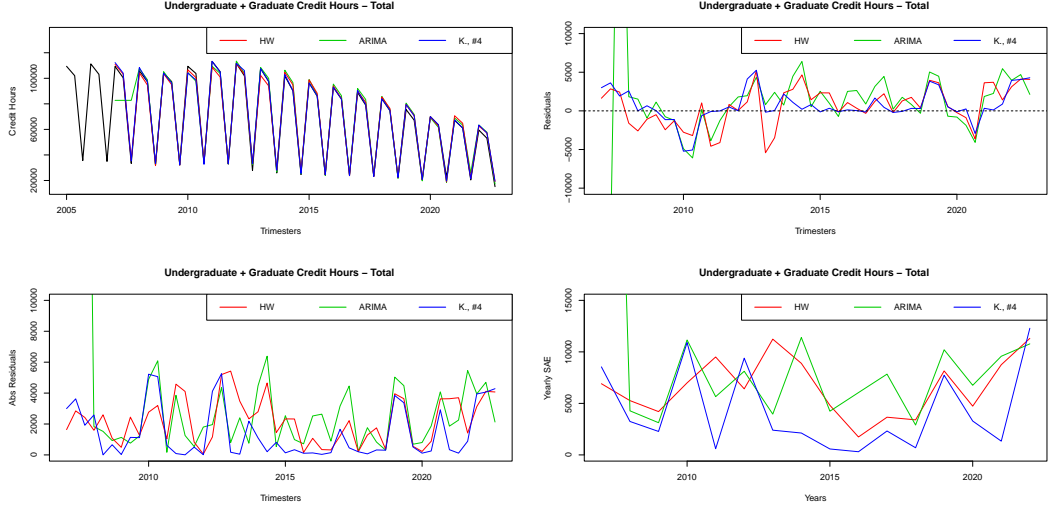


FIGURE 16. Undergraduate + Graduate Total Credit Hours: Forecasts, residuals, absolute residuals and yearly sum of absolute errors

for each model are

$$\text{SAE}^{\text{HW}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{HW}} - x_j \right| = 92,711 \quad \text{and} \quad \text{SAE}^{\text{ARIMA}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{ARIMA}} - x_j \right| = 156,508 \quad (75)$$

Comparing these 16 years times 3 trimesters forecasts, i.e., 48 trimester forecasts, the Holt–Winters model lead to better trimesterly forecast (smallest absolute residual) 28 times as opposed to 20 times for the ARIMA model, i.e.,  $\text{HW}/\text{AR} = 28/20$ . Moreover, if one compares the sum of absolute error per year, the Holt–Winters model lead to a better yearly forecast eleven times as opposed to five times for ARIMA, i.e.,  $\text{HW}/\text{AR} = 11/5$ .

For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead  $\hat{x}_t^{\text{HW}}$ ,  $\hat{x}_t^{\text{ARIMA}}$  and  $\hat{x}_t^{\text{K, \#4}}$  forecasts for  $X$  as in (56), (57) and (58) for each optimization criteria  $\kappa = 1, \dots, 8$  using the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models (with  $N = 1000$  to find the optimal latest-optimized alpha power). Moreover, we compute the (global) sum of absolute error, and we compare our forecasts to both Holt–Winters and ARIMA models by counting the number of best monthly and yearly forecasts. The results are summarized in Table 18.

*Example 18.* Let's consider student (undergraduate + graduate) new credit hours (fall, spring and summer) over 18 years (fall 2004 to summer 2022). Thus,  $f = 3$  and  $X = (x_1, \dots, x_{54})$ . For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead forecasts for  $X$  using both Holt–Winters and ARIMA models. The sum of absolute errors for each model are

$$\text{SAE}^{\text{HW}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{HW}} - x_j \right| = 56,764 \quad \text{and} \quad \text{SAE}^{\text{ARIMA}} = \sum_{j=7}^{54} \left| \hat{x}_j^{\text{ARIMA}} - x_j \right| = 84,466 \quad (76)$$

Comparing these 16 years times 3 trimesters forecasts, i.e., 48 trimester forecasts, the Holt–Winters model lead to better trimesterly forecast (smallest absolute residual) 29 times as opposed

$\bullet/\#s$	SAE	HW / AR / K.	HW / K.	AR / K.	HW / AR / K.	HW / K.	AR / K.
#1	51,659	9 / 10 / 29	11 / 37	11 / 37	2 / 1 / 13	2 / 14	1 / 15
#2	51,644	7 / 10 / 31	12 / 36	11 / 37	1 / 1 / 14	1 / 15	1 / 15
#3	52,501	10 / 9 / 29	12 / 36	10 / 38	2 / 1 / 13	2 / 14	1 / 15
#4	51,888	10 / 10 / 28	13 / 35	11 / 37	2 / 1 / 13	2 / 14	1 / 15
#5	50,919	7 / 10 / 31	11 / 37	11 / 37	1 / 1 / 14	1 / 15	1 / 15
#6	52,325	10 / 9 / 29	12 / 36	11 / 37	2 / 1 / 13	2 / 14	1 / 15
#7	53,028	10 / 10 / 28	12 / 36	12 / 36	2 / 1 / 13	2 / 14	1 / 15
#8	51,262	9 / 8 / 31	11 / 37	0 / 39	1 / 1 / 14	1 / 15	1 / 15
b1	52,007	9 / 10 / 29	12 / 36	12 / 36	2 / 0 / 14	2 / 14	0 / 16
b2	51,214	6 / 10 / 32	10 / 38	11 / 37	1 / 1 / 14	1 / 15	1 / 15
b3	52,543	10 / 9 / 29	12 / 36	11 / 37	2 / 1 / 13	2 / 14	1 / 15
b4	52,097	10 / 10 / 28	13 / 35	12 / 36	2 / 1 / 13	2 / 14	1 / 15
b5	50,952	6 / 10 / 32	11 / 37	11 / 37	1 / 1 / 14	1 / 15	1 / 15
b6	52,142	10 / 10 / 28	12 / 36	12 / 36	2 / 1 / 13	2 / 14	1 / 15
b7	52,575	9 / 9 / 30	11 / 37	10 / 38	2 / 1 / 13	2 / 14	1 / 15
b8	50,263	6 / 10 / 32	10 / 38	11 / 37	1 / 0 / 15	1 / 15	0 / 16
‡1	52,284	11 / 10 / 27	13 / 35	12 / 36	2 / 1 / 13	2 / 14	1 / 15
‡2	51,293	7 / 10 / 31	11 / 37	11 / 37	1 / 1 / 14	1 / 15	1 / 15
‡3	51,545	8 / 10 / 30	9 / 39	11 / 37	2 / 0 / 14	2 / 14	0 / 16
‡4	52,308	11 / 9 / 28	13 / 35	11 / 37	2 / 1 / 13	2 / 14	1 / 15
‡5	51,587	8 / 10 / 30	13 / 35	11 / 37	1 / 1 / 14	1 / 15	1 / 15
‡6	53,126	11 / 10 / 27	14 / 34	12 / 36	2 / 1 / 13	2 / 14	1 / 15
‡7	53,635	10 / 8 / 30	12 / 36	9 / 39	2 / 1 / 13	2 / 14	1 / 15
‡8	50,883	6 / 10 / 32	11 / 37	11 / 37	1 / 1 / 14	1 / 15	1 / 15

TABLE 18. Undergraduate + Graduate Continuing Credit Hours: Comparison of the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models to the seasonal Holt–Winters and ARIMA models

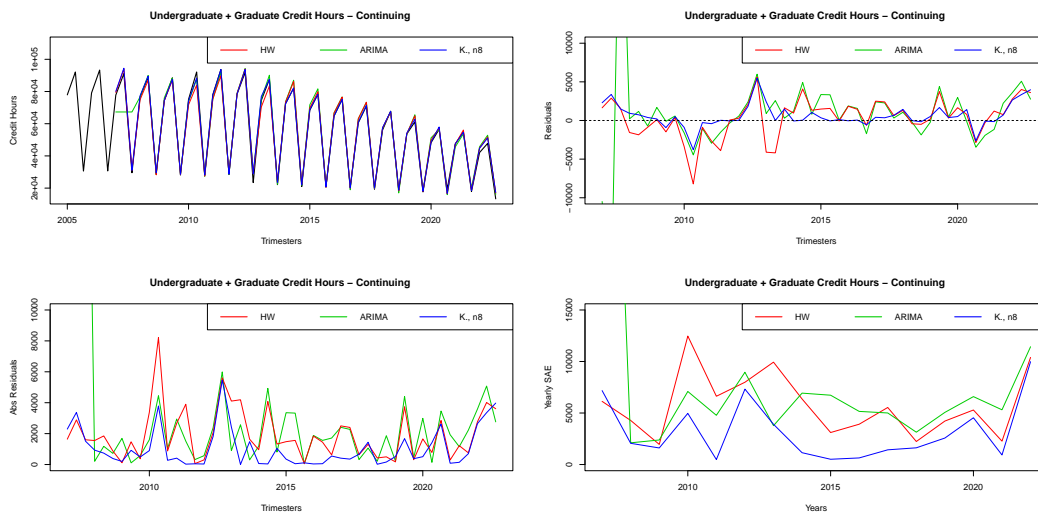


FIGURE 17. Undergraduate + Graduate Continuing Credit Hours: Forecasts, residuals, absolute residuals and yearly sum of absolute errors

to 19 times for the ARIMA model, i.e.,  $HW/AR = 29/19$ . Moreover, if one compares the sum of absolute error per year, the Holt–Winters model lead to a better yearly forecast ten times as opposed to six times for ARIMA, i.e.,  $HW/AR = 10/6$ .

For  $s = 1, \dots, 3$  (trimesters), and  $i = 2, \dots, 17$  (years), we compute the sequence of three-step-ahead  $\ddagger, b$  and  $\ddagger$  forecasts for  $X$  as in (56), (57) and (58) for each optimization criteria  $\kappa = 1, \dots, 8$  using the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models (with  $N = 1000$  to find the optimal latest-optimized alpha power). Moreover,



we compute the (global) sum of absolute error, and we compare our forecasts to both Holt–Winters and ARIMA models by counting the number of best monthly and yearly forecasts. The results are summarized in Table 19..

● <sub>FC</sub>	SAE	HW / AR / K.	HW / K.	AR / K.	HW / AR / K.	HW / K.	AR / K.
#1	24,360	11 / 3 / 34	13 / 35	7 / 41	3 / 0 / 13	3 / 13	0 / 16
#2	23,922	11 / 5 / 32	14 / 34	10 / 38	2 / 1 / 13	2 / 14	1 / 15
#3	24,359	13 / 4 / 31	16 / 32	9 / 39	2 / 0 / 14	2 / 14	0 / 16
#4	24,585	14 / 4 / 30	16 / 32	11 / 37	2 / 1 / 13	2 / 14	1 / 15
#5	24,777	12 / 4 / 32	15 / 33	11 / 37	3 / 0 / 13	3 / 13	0 / 16
#6	24,034	14 / 3 / 31	17 / 31	10 / 38	3 / 0 / 13	3 / 13	0 / 16
#7	24,076	12 / 4 / 32	14 / 34	9 / 39	2 / 1 / 13	2 / 14	1 / 15
#8	24,289	11 / 4 / 33	14 / 34	11 / 37	2 / 1 / 13	2 / 14	1 / 15
b1	24,066	13 / 4 / 31	16 / 32	11 / 37	2 / 0 / 14	2 / 14	0 / 16
b2	24,146	12 / 4 / 32	16 / 32	9 / 39	2 / 0 / 14	2 / 14	0 / 16
b3	24,245	14 / 4 / 30	16 / 32	11 / 37	2 / 0 / 14	2 / 14	0 / 16
b4	24,198	13 / 4 / 31	15 / 33	11 / 37	3 / 0 / 13	3 / 13	0 / 16
b5	24,423	12 / 4 / 32	15 / 33	11 / 37	3 / 0 / 13	3 / 13	0 / 16
b6	24,341	15 / 3 / 30	17 / 31	10 / 38	3 / 0 / 13	3 / 13	0 / 16
b7	24,239	13 / 5 / 30	15 / 33	12 / 36	3 / 1 / 12	3 / 13	1 / 15
b8	24,335	12 / 4 / 32	15 / 33	11 / 37	2 / 1 / 13	2 / 14	1 / 15
t1	24,286	12 / 4 / 32	14 / 34	9 / 39	2 / 1 / 13	2 / 14	1 / 15
t2	24,371	12 / 5 / 31	16 / 32	10 / 38	3 / 0 / 13	3 / 13	0 / 16
t3	24,580	12 / 5 / 31	16 / 32	10 / 38	2 / 0 / 14	2 / 14	0 / 16
t4	24,938	14 / 4 / 30	16 / 32	11 / 37	3 / 1 / 12	3 / 13	1 / 15
t5	24,882	12 / 4 / 32	15 / 33	11 / 37	2 / 0 / 14	2 / 14	0 / 16
t6	24,472	b13 / 4 / 31	16 / 32	11 / 37	3 / 0 / 13	3 / 13	0 / 16
t7	23,965	12 / 5 / 31	15 / 33	11 / 37	2 / 1 / 13	2 / 14	1 / 15
t8	24,651	12 / 3 / 33	15 / 33	10 / 38	2 / 0 / 14	2 / 14	0 / 16

TABLE 19. Undergraduate + Graduate New Credit Hours: Comparison of the seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced forecasting models to the seasonal Holt–Winters and ARIMA models

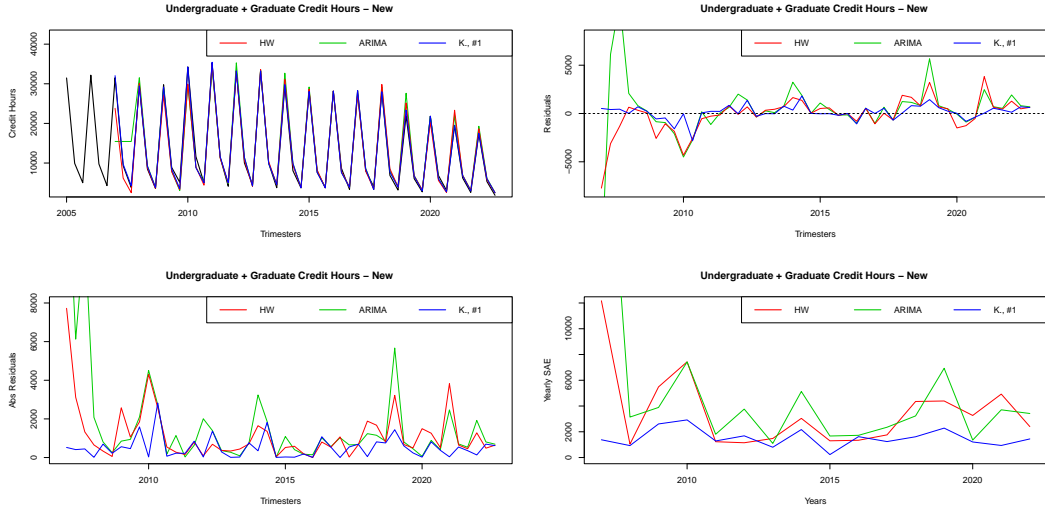


FIGURE 18. Undergraduate + Graduate New Credit Hours: Forecasts, residuals, absolute residuals and yearly sum of absolute errors

### 5. DISCUSSION

In light of Examples 2–19, several conclusions can be formed. First, for all eighteen times series, each of our twenty four seasonal per-period SAE stochastic latest-optimized  $\tilde{\alpha}$ -power balanced

forecasting models outperformed both Holt–Winters and ARIMA models with respect to the sum of absolute error, the number of closest trimesterly forecasts, and the number of closest yearly forecasts. Second, Holt–Winters model performed better than the ARIMA model for sixteen out of the eighteen time series. Indeed, for the new graduate enrollment time series, ARIMA had two more closest trimesterly forecast, i.e.,  $HW/AR = 23/25$ , but had the same number of closest yearly forecasts, i.e.,  $HW/AR = 8/8$ . For the new graduate credit hours, ARIMA also had two more closest trimesterly forecast, i.e.,  $HW/AR = 23/25$ , but the Holt–Winters model had two more closest yearly forecasts, i.e.,  $HW/AR = 9/7$ . Third, for almost all considered seasonal time series in [6] and in Example 2–19, ARIMA models leads to horrible first year forecast, that is, given the first two periods,  $(x_1, \dots, x_f, x_{f+1}, \dots, x_{2f})$ , the forecasted values  $\hat{x}_{2f+1}, \dots, \hat{x}_{3f}$  using the ARIMA models lead to a substantially larger absolute residual than Holt–Winters and our models. Thus, using ARIMA to predict the third sound of period using the the first two periods is not reliable.

## REFERENCES

- [1] G. E. P. Box, G. M. Jenkins and G. C. Reinsel, *Time Series Analysis*, Forecasting and Control, Third Edition. Holden-Day. Series G (1976).
- [2] R. G. Brown, *Statistical forecasting for inventory control*, McGraw/Hill (1959).
- [3] P. S.P. Cowpertwait and A. V. Metcalfe, *Introductory Time Series with R*, Springer (2009).
- [4] C. E. Holt, *Forecasting seasonal and trends by exponentially weighted averages*, Carnegie Institute of Technology, Pittsburgh USA (1957).
- [5] R. J. Hyndman and G. Athanasopoulos, *Forecasting: Principles and Practice*, 2nd edition, OTexts: Melbourne, Australia. OTexts.com/fpp2 '(2018).
- [6] N. Kahouadji, *Variations on two-parameter families of forecasting functions: seasonal/nonseasonal Models, comparison to the exponential smoothing and ARIMA models, and applications to stock market data*, arXiv:2207.04882, July 2022.
- [7] Makridakis, Wheelwright and Hyndman, *Forecasting: methods and applications*, John Wiley & Sons: New York. Chapter 2 (1998).
- [8] P. R. Winters, *Forecasting sales by exponentially weighted moving averages*, Management Science, 6(3), (1960), 324?342.