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Linear Controllers: PID controller design

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ABSTRACT

In this paper, the issue of designing and tuning a standard three-term PID controller is addressed. To achieve this goal, both the basics of linear controllers and the key principles of the PID controller are described. Subsequently, two popular design approaches are studied and then illustrated in a specific use case. The paper ends with a global conclusion including guidelines on extra material to address implementation issues.

KEYWORDS

Feedback control, time-domain specifications, linear controller, stability margins, PID control, zero-pole compensation, frequency-response design, performances robustness.

1. INTRODUCTION

Many plants require automatic control in order to enforce specific system variables to meet the related reference values in real time. To achieve this goal, engineers need to design feedback controls of dynamic systems. It contributes to stabilize an intrinsically unstable system and enables an effective reference tracking by rejecting, at any time, system and environment uncertainties [1].

Energetic Macroscopic Representation, which is a graphic formalism developed for multi physics system [2], is one very efficient tool to define a relevant control structure to any plant. Based on the plant model, it permits first to define the maximal control structure of the studied system and then, considering the sensors constraints, deduce the practical control structure [EEEPE 40011] [3, 4].

Designing each identified control loops comes into the realm of control systems design. This scientific field offers numerous feedback control laws. One of these solutions is the Proportional-Integral-Derivative (PID) controller which benefits from a high cost/performance ratio [1]. As this classic three-term controller proves to be used in most industrial applications, it is crucial for electrical engineers to master its basic principles.

The present article aims at providing the basics of PID control so that the reader could successfully tune the 3-term controller parameters in any specific use case. The suggested tuning methods are explained by way of a practical example.

The paper is organized as follows. Section 2 explains the frequency loop-shaping requirements that the selected analog controller has to meet so as to ensure high stability margin, high damping ratio (close to 1), low tracking errors and small sensitivity regarding noises and system model uncertainties. It then gives the PID controller structure and outlines the main role of each terms. Section 3 briefly describes the studied case and defines the related control design specifications.

Section 4 addresses the control tuning using the dominant poles cancellation method. Section 5 presents an alternative approach based on the frequency-response design method. Finally, section 6 draws conclusions and perspectives of the suggested approaches, mentioning practical implementation issues such as sensitivity assessment, computer aided design, integrative anti-windup strategy, feedforward terms and digital control law enabling using microcontrollers.

2. CONTROLLER DESIGN OUTLINES AND PID CONTROL BASICS

As shown in figure 1, an elementary feedback control is composed of 3 blocks:

1. A plant (also called system), consisting of an actuator, a process and a sensor.
2. A comparator computing the error $\varepsilon(t)$ between the reference signal $y_{ref}(t)$ and the controlled one $y(t)$.
3. A controller aiming at providing in real time the control variable $u(t)$ so that the measured variable $y(t)$ accurately follows its reference value $y_{ref}(t)$. Note that the variable being tracked is the sensor output and not the physical quantity at stake. In some cases, such as temperature or pressure controls, the slow dynamics of the sensor requires the use of a larger phase margin specification to prevent the real variable to face unexpected transient overshoot.

To analytically define the controller structure and adjust its settings, the designer needs a dynamic model of the plant. In the case where the plant mathematical description is a linear differential equation, Laplace transform is used and both the studied system and the controller are described by transfer functions: $H_P(s)$ and $C(s)$, respectively.

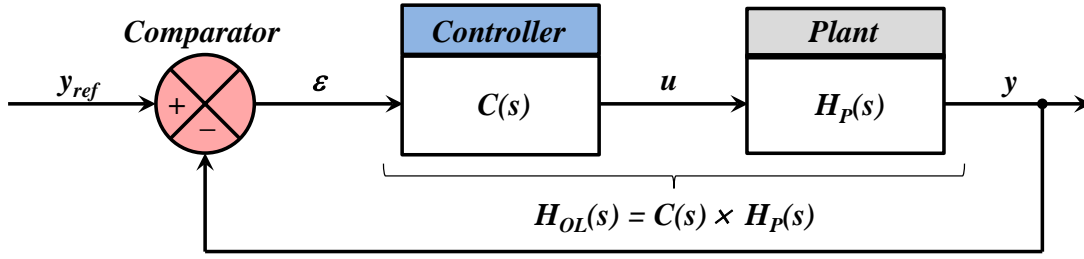


Fig. 1: block diagram of a feedback control.

2.1. Loop-shaping

The controller is designed to meet the 3 main feedback control requirements, namely: stability, a damping ratio close to 1, accuracy (at least in steady states) and settling time regarding step response (with respect to a 5% error band) [IEEEPE 40002]. To achieve these 3 goals, the controller $C(s)$ modifies the global open-loop transfer function $H_{OL}(s) = C(s)H_P(s)$ such a way that the frequency response $\underline{H_{OL}}(j\omega)$ complies with the frequency templates depicted in figure 2. Indeed, the closed-loop transfer function $H_{CL}(s)$ is given by:

$$H_{CL}(s) = \frac{Y(s)}{Y_{ref}(s)} = \frac{H_{OL}(s)}{1 + H_{OL}(s)} \quad (1)$$

Hence, it derives that:

$$\begin{aligned} |H_{OL}(j\omega)| \gg 1 & \rightarrow H_{CL}(j\omega) \cong \frac{H_{OL}(j\omega)}{H_{OL}(j\omega)} = 1 \\ |H_{OL}(j\omega)| \ll 1 & \rightarrow H_{CL}(j\omega) \cong \frac{H_{OL}(j\omega)}{1} = H_{OL}(j\omega) \end{aligned} \quad (2)$$

Consequently, to comply with the feedback specification, the key ideas of the loop-shaping are:

1. At Low Frequency (LF), a high open-loop gain $|H_{OL}(j\omega)|$, above LF boundary, implies a closed-loop gain value close to unity and hence a high steady state accuracy and an efficient tracking of LF reference signal even considering model uncertainties.

2. At medium frequency (MF), i.e. in the transition band $[\omega_X/3, 3\omega_X]$ around the cross-frequency ω_X where $|H_{OL}(j\omega_X)| = 1$, it is crucial that the MF slope is as close to -20 dB/decade as possible to ensure large stability margins.

- In practice, a phase margin value more than 60° combined with a gain margin value above a 10 to 15 dB threshold usually enables to limit step overshoot below 10% and ensure attractive robustness with respect to plant parameter uncertainties.
- Moreover, based on a pure integrator behavior (ω_0/s) , a rough estimate of the 5% settling time $t_{ST,5\%}$ versus the cross-frequency is provided by:

$$t_{ST,5\%} \cong \frac{3}{\omega_X} \quad (3)$$

3. At High Frequency (HF), a low open-loop gain $|H_{OL}(j\omega)|$, below HF boundary, implies a low closed-loop gain value and hence a proper filtering of sensor noise as well as a suitable mitigation of the unmodelled fast dynamics of the plant.

Next sub-section shows that the PID controller is a simple and usually efficient option to address these 3 feedback control challenges.

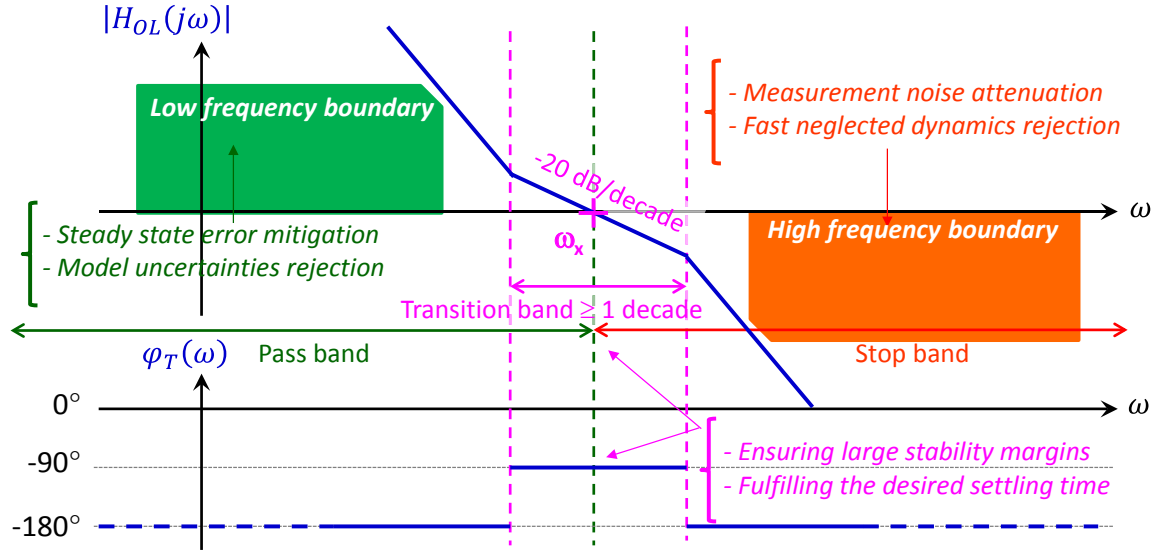


Fig. 2: Open-loop frequency requirements for efficient feedback control.

2.2. PID controller basics

The PID controller is defined by the time-domain equation:

$$u(t) = K_P \left(\varepsilon(t) + \frac{1}{T_I} \int_0^t \varepsilon(\tau) \cdot d\tau + T_D \frac{d\varepsilon}{dt}(t) \right) \quad (4)$$

which, in Laplace transform, is:

$$C_{PID}(s) = \frac{U(s)}{\varepsilon(s)} = K_P \left(1 + \frac{1}{T_I s} + T_D s \right) = K_P \left(\frac{1 + T_I s + T_I T_D s^2}{T_I s} \right) \quad (5)$$

The PID transfer function combines 3 independent additional terms:

1. The proportional term (K_P) which acts at any frequency and hence permits to adjust the cross frequency ω_x and consequently the settling time $t_{ST,5\%}$,
2. The integral term ($K_P/T_I s$) which mainly acts at low frequency ($\omega < 1/T_I$) providing a high gain and hence a high LF accuracy combined with an attractive rejection of model uncertainties. This term also introduces a possibly destabilizing 90° phase delay.

3. The derivative term ($K_P T_D s$) which mainly acts at high frequency $\omega > 1/T_D$ providing a stabilizing 90° phase lead which enables to adjust a cross frequency combined with large phase and gain margins.

Note that the derivative term is a non-causal element and most importantly provides high gain at HF which may amplify sensor noise. Therefore, the derivative term may be filtered in the high frequency domain as shown by the following adapted transfer function:

$$C_{PID,f}(s) = \frac{U(s)}{\varepsilon(s)} = K_P \left(1 + \frac{1}{T_I s} + \frac{T_D s}{1 + (T_D/R)s} \right) \quad (6)$$

where R is usually selected in the $[5 ; 20]$ range. Nevertheless, in discrete implementation, the mandatory approximation of the derivative term already provides the required noise filtering effect.

Finally, in simple cases where stability margins are intrinsically ensured, the use of the D term would be useless. It leads to ($T_D = 0$ s) and the PID controller simplifies into a PI controller. In [EEEEPE 40005], Bao-Huy Nguyen illustrates this aspect through the implementation of a current regulation in a power converter.

Based on a unity proportional gain, Figure 3 illustrates the PID frequency response in the standard case where the transfer function has two repeated real zeros ($T_I = 4T_D$). The figure also shows the response both in the case where $T_I < 4T_D$, which enables a notch response around the center frequency $\omega_0 = \sqrt{1/(T_I T_D)}$ and conversely in the case of $T_I > 4T_D$, which permits a smooth phase response.

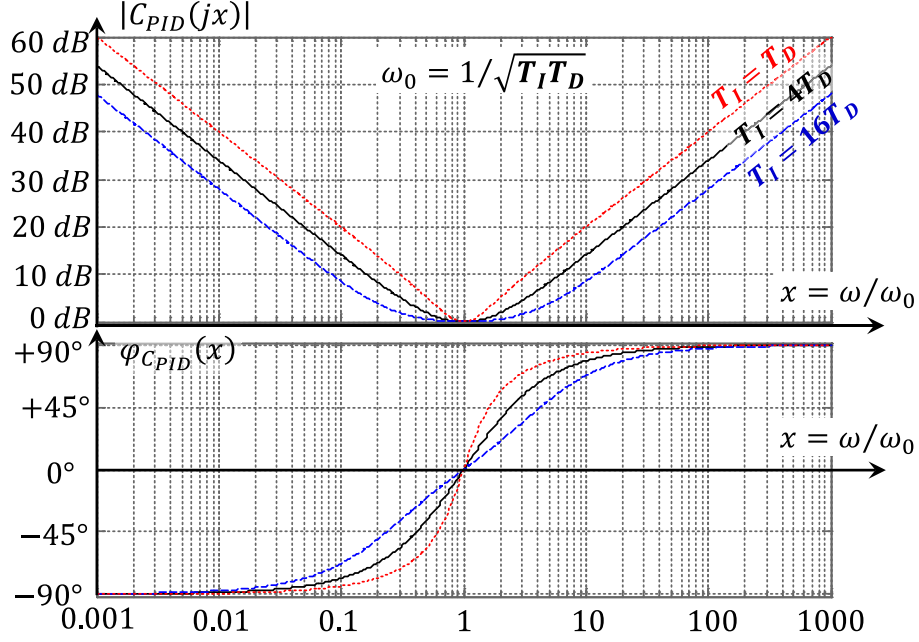


Fig. 3: PID controller frequency response $\underline{C}(j\omega)$ based on $K_P = 1$

where the frequency ω is normalized with respect to $\omega_0 = \sqrt{1/(T_I T_D)}$.

(a) solid line: $T_I = 4T_D$ (b) dashed line: $T_I = 16T_D$ (c) dotted line: $T_I = T_D$

3. USE CASE

Many approaches permit tuning the 3 parameters of the PID controller to fulfill the feedback control requirements [5]. The present article focuses on two main ways of addressing this issue and implements them on a practical use case to validate the suggested strategies.

3.1. Adopted use case

The mathematical description of the process at stake is a third order linear differential equation. Its related transfer function can be described by:

$$H_P(s) = \frac{H_{P,0}}{(1 + 2m_a\tau_a s + \tau_a^2 s^2) \cdot (1 + \tau_b s)} \quad (7)$$

Where the dynamic model settings are given in Tab. 1.

Tab.1: Dynamic model settings

$H_{P,0}$	τ_a	m_a	τ_b
0.70	159.2 ms	0.55	15.9 ms

Tab. 1: dynamic model settings.

3.2. Feedback control specification

The feedback control goal is threefold (Tab 2).

1. First, obtaining a zero steady state error ε_∞ ,
2. Second, achieving a settling time $t_{ST,5\%}$ of the step response on 100 ms time scale,
3. Third, having an overshoot (OS%) in step response lower than 10%.

Tab.2: Feedback control specification

ε_∞	$t_{ST,5\%}$	OS%
0.0	100.0 ms	10.0 %

Tab. 2: the desired feedback specification.

4. 1ST APPROACH: ZERO-POLE CANCELLATION COMBINED WITH KP TUNING

4.1. 1st Strategy outlines

The first adopted strategy is to simplify the open-loop transfer function. For this purpose, T_I and T_D are adjusted so that the 2 polynomial roots of the PID transfer function could cancel the plant dominant poles (i.e. LF poles). Then, the third parameter K_P is tuned to fulfill the feedback specification as best as possible.

4.2. 1st strategy implementation

In the present case, the plant function has 3 poles divided into a complex conjugate pair such as $p_{1,2} = -1/\tau_a (m_a \pm j\sqrt{1 - m_a^2})$ and a distinct HF real pole, namely $p_3 = -1/\tau_b$. Hence, setting $T_I = 2m_a\tau_a$ and $T_D = \tau_a/2m_a$ permits to achieve the zero-pole cancellation approach. Therefore, it derives the following open-loop transfer function:

$$H_{OL,1}(s) = \frac{K_P \cdot H_{P,0}}{T_I s \cdot (1 + \tau_b s)} \quad (8)$$

Where the only tuning parameter is the proportional gain K_P .

Subsequently, the closed-loop transfer function is expressed as a unit static gain second-order system.

$$H_{CL,1}(s) = \frac{1}{1 + \frac{T_I}{K_P H_{P,0}} s + \frac{T_I \tau_b}{K_P H_{P,0}} s^2} \quad (9)$$

Its natural frequency and damping ratio are both defined using the K_P gain. Based on second-order characteristics or using a software tool, an adequate value is computed enabling to fulfil the settling time and overshoot requirements.

In sum, Tab 3 shows the settings defined by this first approach while figure 4 illustrates the closed-loop step response compared with the plant step one.

Tab.3: PID settings based on the zero-pole cancellation approach

K_P	T_I	T_D
5.5	175.1 ms	144.7 ms

Tab. 3: PID settings based on the zero-pole cancellation approach.

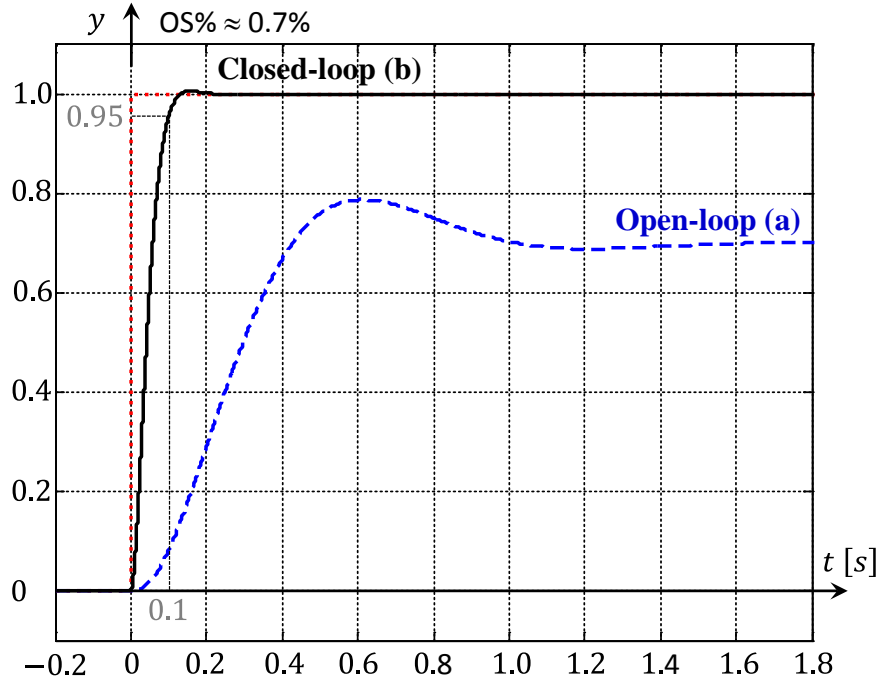


Fig. 4: Step response of the plant in (a) open-loop (dashed line) and (b) in closed-loop tuned using the zero-pole cancellation approach (solid line).

5. 2ND APPROACH: FREQUENCY COMPENSATION

5.1. Essential features of the 2nd strategy

The second suggested strategy consists in compensating the plant frequency response to achieve a global open-loop transfer function suitable with the feedback specification. For this purpose, T_D is adjusted to enhance the phase margin while K_P setting enables to get the open-loop cross frequency ω_X consistent with the settling time demand. T_I parameter is set to achieve the specification without impacting the stability margins.

5.2. 2nd strategy implementation

Referring to eq. 3, a rough estimate of the desired cross frequency is: $\omega_X \cong \frac{3}{t_{ST,5\%}} = 30 \text{ rad.s}^{-1}$.

At this specific frequency, the plant phase is only: $\varphi_P(\omega_X) = -187^\circ$. With a view to attaining the

stability and overshoot objectives, a phase margin of no less than 60° is required. Also, a 67° phase compensation is needed. Additionally, the global gain should be 0 dB at the selected cross frequency. Adopting the standard ratio $T_I/T_D = 4$ (suggested by Ziegler and Nichols [6] or Åström et Hägglund [7] in their heuristic methods) and referring to the PID controller frequency response plotted in Figure 3, lead to adopt the parameter pair (K_P, T_D) as: $(11.2, 0.1)$. The choice of $T_I = 4T_D = 0.4 \text{ s}$ enables keeping attractive transient behavior while achieving zero steady state error. Tab 4 sums up the settings defined by this second method while figure 5 shows the feedback time response regarding a step reference. Note that the actual settling time is around the desired value but slightly higher than it. This is caused by the approximate relationship between the desired settling time and the tuned cross frequency (Eq. 3). If needed, and based on this first settings, the PID parameters can be adjusted.

Tab.4: PID settings based on the frequency compensation approach

K_P	T_I	T_D
11.2	400.0 ms	100.0 ms

Tab. 4: PID settings based on the frequency compensation approach.

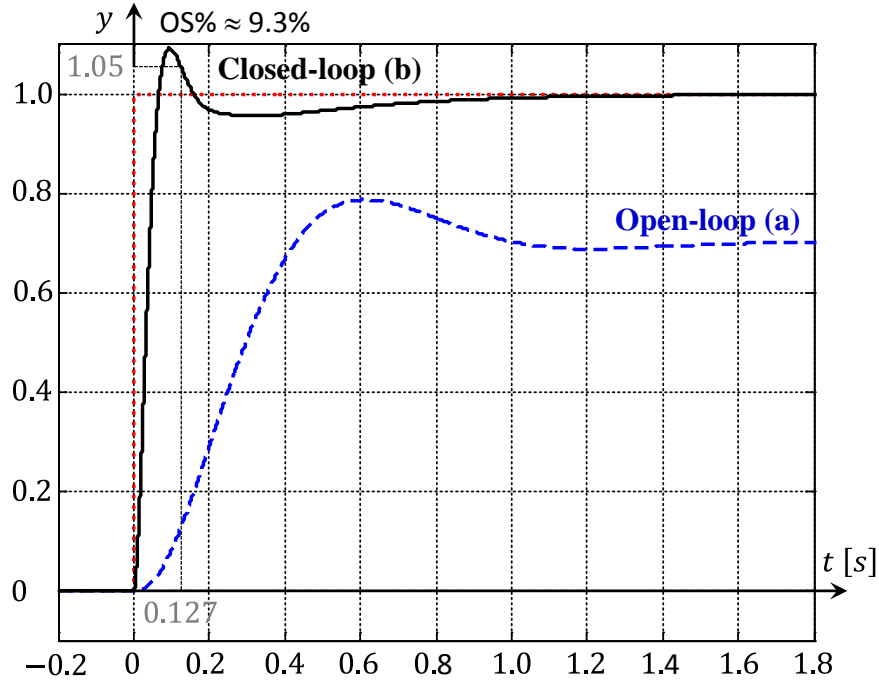


Fig. 5: Step response of the plant in

(a) open-loop (dashed line)

(b) closed-loop tuned using the frequency compensation approach (solid line).

6. CONCLUSION AND PERSPECTIVES

This paper aims at providing the key elements to design and tune the PID controller which remains the most commonly used control technique. Firstly, it explains the crucial concepts enabling to develop this controller in any use case. Second, it describes two of the standard approaches used to tune the 3-term controller. To make the document easier to understand and reuse, each approach is developed using the same specific example.

This basic material should facilitate the extensive understanding of PID control, which is a broad scientific and technologic area addressed in numerous scientific books. In addition, on certain matters progress is still being made by research labs over the world.

To go further, partial remaining issues are:

1. The robustness of the feedback control performance. For instance, in case where the effective natural frequency is 20% lower than expected, the first approach is more impacted than the second since the optimal zero-pole cancellation is not effective anymore. Figure 6 illustrates the current step response of each method in this peculiar case.
2. The extensive use of computer-aided control systems design software tools. Once the PID control guidelines are fully understood and assimilated, these tools can provide optimized settings [EEEPE 40006]. For instance, Linear Quadratic Regulator (LQR) theory can be successfully used to design optimal PID controllers which is an attractive way to address the feedback control sensitivity with respect to noise and parameter uncertainties [8,9].
3. The practical saturation of the control variable due to actuator limitation. Whenever the control value reaches its threshold, the plant is in open-loop while the integrator keeps integrating, gradually providing a huge value. It results in large overshoots and possibly unstable response. This phenomenon is known as integrator windup. Many strategies, called anti-windup techniques, can mitigate this problem [10].
4. When the model characteristics are well known, it is effective to add feedforward terms which would limit the action of the PID controller and hence reduce its deviation. For instance, the steady state control value deduced both from the plant model and the reference value is an appropriate feed-forward term [11].
5. Keeping the closed-loop characteristic equation (and hence the same closed-loop poles), alternative PID structures may be introduced to modify the zeros of the closed loop transfer function. Consequently, it impacts the transient response. The key idea is to avoid the initial pulse in the control signal u induced by the step change in the reference variable y_{ref} .

- Indeed, it may induce a larger overshoot than expected using the selected phase margin and damping ratio. Hence, in alternative PID structures, the integrator term still acts on the error variable ε while the proportional and derivative terms only acts on the output variable y .
6. Discrete PID. Modern PID controller are usually implemented in a digital system such as a microcontroller or a FPGA device [EEEPE 40029]. Digital control requires some specific techniques to at least adapt the continuous control law. In case of specific requirements, defining the correct algebraic equation is an extensive issue based on a deep theory [12, 13].

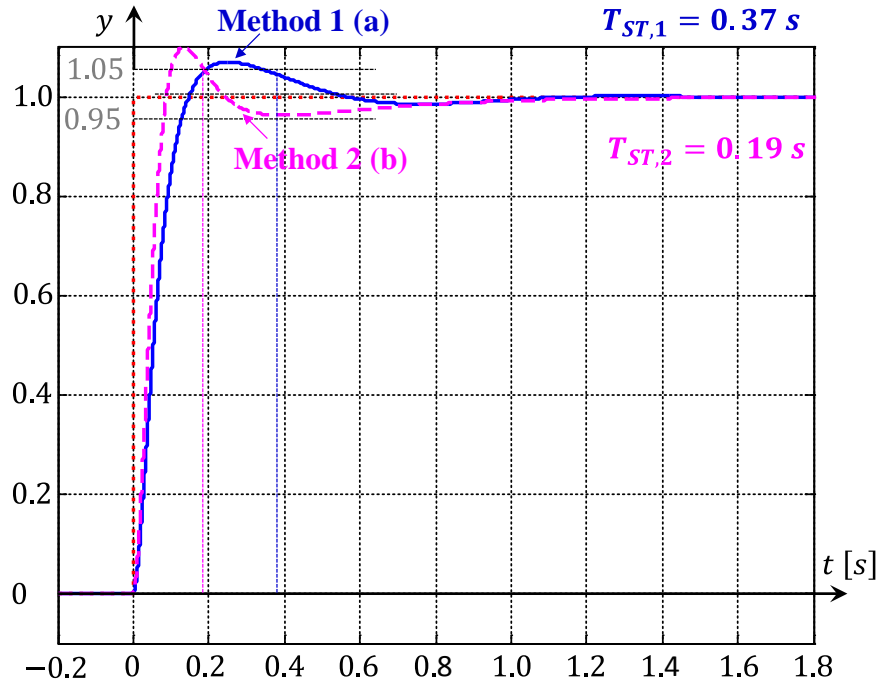


Fig. 6: Sensitivity of the closed-loop step response
with respect to plant parameters uncertainties ($\tau_a = 191 \text{ ms} \neq 159.2 \text{ ms}$)
(a) solid line: method 1 and (b) dashed line: method 2.

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