



**HAL**  
open science

## Transfer between waves and eddies in stratified or rotating turbulence

Henri Lam, Alexandre Delache, Fabien S. Godeferd

► **To cite this version:**

Henri Lam, Alexandre Delache, Fabien S. Godeferd. Transfer between waves and eddies in stratified or rotating turbulence. International Congress on Theoretical and Applied Mechanics 2021, Aug 2021, Milan (online), Italy. hal-03834810

**HAL Id: hal-03834810**

**<https://hal.science/hal-03834810>**

Submitted on 12 Oct 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

## Transfer between waves and eddies in stratified or rotating turbulence

Henri Lam <sup>\*1</sup>, Alexandre Delache<sup>1,2</sup>, and Fabien S Godefert<sup>2</sup><sup>1</sup>Laboratoire de Mécanique des Fluides et d'Acoustique, Univ Lyon, École centrale de Lyon, INSA Lyon, Université Claude Bernard Lyon I, CNRS, France<sup>1</sup>Laboratoire de Mécanique des Fluides et d'Acoustique, Univ Lyon, site de Saint-Étienne, Université Jean Monnet de Saint-Étienne, CNRS, France

**Summary** We propose to separate the waves and eddies in stratified and/or rotating turbulence by using an extended method exposed in Lam et al. (2020) [1]. This separation takes into account the spatial and temporal distortion due to the advecting flow, which modifies the dispersion relation of the waves through the sweeping effect. From this separation, we analyse the transfers and interactions that occur between eddies and waves. Furthermore, this permits the analysis of the detailed energy cascade and fluxes from the input scale to the dissipative ones.

## INTRODUCTION

Internal gravity waves (IGW) in the case of stratified turbulence and inertial waves (IW) in the case of rotating turbulence are waves sharing some remarkable properties. They both satisfy a dispersion relation linking their angular frequency  $\omega$  and the direction of the wavevector  $\mathbf{k}$ . For IGW  $\omega_r = \pm N \cos \theta$  and for IW  $\omega_r = \pm 2\Omega \sin \theta$ , where  $N$  is the Brunt-Väisälä frequency,  $\Omega$  is the rotation rate and  $\theta$  is the angle made by the wavevector  $\mathbf{k}$  against the horizontal plane. This was experimentally shown in experiments and in simulations of IW propagation [3, 4], likewise for IGW in stratified flows [2]. However, the frequency of the waves can be modified by the sweeping effect (similar to the Doppler shift). This corresponds to the advection of the waves by a large scale, slowly varying in time background flow. In stratified turbulence, this corresponds to the vertically shear horizontal flow (VSHF) composed of large velocity band invariant in the horizontal direction and in rotating turbulence to the geostrophic mode (GM) composed of two opposite vortices invariant in the  $z$  direction. In a rotating experiment [4], the geostrophic mode is shown to be the main responsible of the modification of the frequency of the waves through the sweeping effect. A new 4D decomposition has been proposed [1] that takes into account the frequency of the waves  $\omega$  and its wave vector  $\mathbf{k}$  to make an estimation of the sweeping effect to separate the waves from the eddies. It requires the estimation of a uniform and constant in time advecting velocity  $\mathbf{c}$  responsible of the sweeping effect. Using this estimation the waves are separated from the eddies because the waves are expected to satisfy the inequality  $\omega_r - |\mathbf{c} \cdot \mathbf{k}| \leq \omega \leq \omega_r + |\mathbf{c} \cdot \mathbf{k}|$ .

## SEPARATION TECHNIQUE OF WAVES AND EDDIES

We propose a new separation technique for waves and eddies where the advecting velocity  $\mathbf{c}(\mathbf{x}, t)$  varies in time and space. This allows us to take into account the non-uniform and the slow variation in time of the expected advecting velocity. This algorithm requires two stages:

1. The advecting flow of the DNS is extracted to convect the response of a linear Navier-Stokes equation forced by a series of Dirac forces in time and space. The dispersion relation of the waves is modified by the advecting flow  $\mathbf{c}(\mathbf{x}, t)$ . A filter  $\zeta(\omega, \mathbf{k})$  is created which considers waves to be in areas of the  $(\omega, \mathbf{k})$  space with a high level of energy. On the contrary eddies are expected to be in areas of the  $(\omega, \mathbf{k})$  space with a low level of energy.
2. The filter  $\zeta(\omega, \mathbf{k})$  is applied to the original DNS to separate the waves from the eddies.

The Navier-Stokes equation with the Boussinesq approximation are solved on a pseudo-spectral code with a three-dimensional  $2\pi$  periodic domain size. The forcing is done on the kinetic part of the Navier-Stokes equation, it consists of a constant influx of energy  $P$ . From this algorithm the velocity vector  $\mathbf{u}$  and buoyancy field  $b$  are separated in a wave (superscript  $w$ ), an eddy (superscript  $e$ ) part and a large scale flow like the VSHF or GM (superscript  $L$ ). We obtain  $\mathbf{u} = \mathbf{u}^w + \mathbf{u}^e + \mathbf{u}^L$  and  $b = b^w + b^e + b^L$ .

## ENERGY BUDGET AND TRANSFER IN STRATIFIED TURBULENCE

We illustrate here the results of our wave-eddy decomposition only for IGW in stratified turbulence, but similar results are obtained for IW in rotating flows. In a statistically stationary flow, with constant injected power  $P$ , we have an equation of energy budget for waves and eddies on a large period  $T_0$ . In stratified turbulence without considering the VSHF flow, it is:

---

\*Corresponding author. E-mail: henri.lam@ec-lyon.fr

$$\begin{aligned} 0 &= \partial_t E^w = T_{ee}^w + T_{we}^w + \epsilon^w + P^w \\ 0 &= \partial_t E^e = T_{ww}^e + T_{ew}^e + \epsilon^e + P^e \end{aligned} \quad (1)$$

where the total dissipation is  $\epsilon^l = \frac{1}{T_0} \int_{T_0} \sum_k \frac{\nu k^2}{2} (|\mathbf{u}^l|^2 + \frac{|b^l|^2}{N^2})$ , the total energy is  $E^l = \frac{1}{T_0} \int_{T_0} \sum_k \frac{1}{2} (|\mathbf{u}^l|^2 + \frac{|b^l|^2}{N^2})$  and the total transfer is  $T_{ij}^l = \frac{1}{T_0} \int_{T_0} \sum_k (\mathbf{u}^i \times \omega^j) \mathbf{u}^l - \frac{(\mathbf{u}^i \cdot \nabla b^j) b^l}{N^2}$ .  $T_{ij}^l$  means that  $i$  interacts with  $j$  to give or take energy to  $l$  (the subscripts or superscripts  $i, j, l$  stand for  $w$  or  $e$ ). The input of energy  $P$  due to the forcing is separated in an eddy  $P^e$  and a wave  $P^w$  input of energy.

The energy budget can be represented by a Sankey diagram (see Figure 1) where the width of the bar is proportional to the flow rate. The injected power  $P$  on the left is dissipated by  $\epsilon^w$  and  $\epsilon^e$  on the right and subject to different interactions  $T_{ij}^l$ , where the color indicates the origin of the flow (red= $w$ , blue= $e$ ). The overall energy transfer is from the waves with the transfers  $T_{ee}^w$  and  $T_{we}^w$ , and to the eddies with the transfers  $T_{ew}^e$  and  $T_{ww}^e$ . By exploring different values of Froude and buoyancy Reynolds numbers, we show that the maximum of energy transfer from waves to eddies occurs when both are of similar magnitudes (at  $Re_b \sim 1$ ). Similar analyses will be presented for rotating flows.

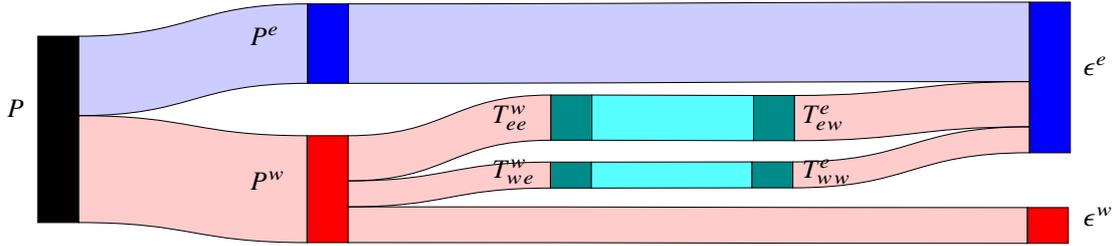


Figure 1: Energy budget for a DNS with buoyancy Reynolds number  $Re_b = 0.9$  and horizontal Froude number  $Fr = 0.015$ .

It is hard to draw a clear pattern from the direct study of the fluxes of waves and eddies because they fluctuate a lot. Therefore, we decide to study these statistics through four values (see Figure 2):

- the total positive  $T^+$  (resp. negative  $T^-$ ) transfer, which correspond to the amount of energy given (resp. taken) from waves or eddies.
- the average scale of positive  $k^+$  (resp. negative  $k^-$ ) transfer  $T^+$  (resp.  $T^-$ ), which correspond to the scale at which on average the positive (resp. negative) transfer occurs.

For example, in Figure 2a, the strength of positive and negative transfer  $T_{ee}^e$  increases when the Froude number increases. It means that eddies interact more with each other when the flow is more turbulent. In Figure 2b, the transfer  $T_{ee}^e$  participates in a forward cascade (as  $k^+ > k^-$ ), and we show that the forward cascade is stronger at high Froude number (when the flow is more turbulent). The same analysis can be done for the wave turbulence transfer  $T_{ww}^w$  and for all remaining transfers. Surprisingly, we observe that the wave-eddy interaction results in an inverse cascade for some Froude numbers.

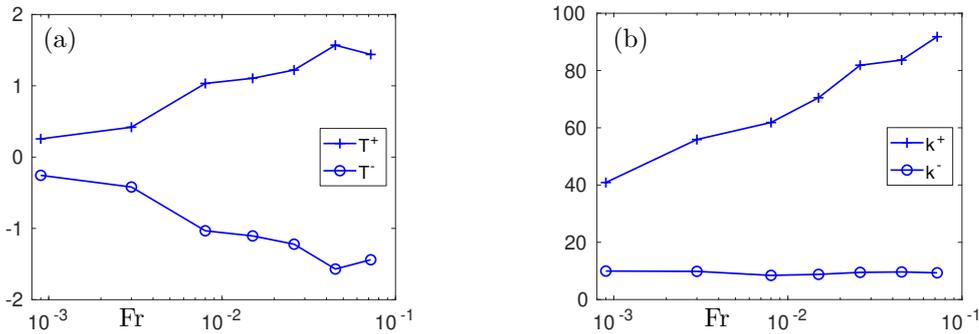


Figure 2: Statistics of the transfer  $T_{ee}^e$  for different Froude number. (a) Total positive  $T^+$  and negative transfer  $T^-$ . (b) Average scale of positive  $k^+$  and negative  $k^-$  transfer.

## References

- [1] Lam H., Delache A., Godefert F. S. Partitioning Waves and Eddies in Stably Stratified Turbulence. Atmosphere 11: 4, 2020
- [2] Yarom E., Sharon E. Experimental observation of steady inertial wave turbulence in deep rotating flows Nature, 10: 7, 2014
- [3] Le Reun T., Favier B., Barker A. J., Le Bars M. Inertial Wave Turbulence Driven by Elliptical Instability Phys. Rev. Lett., 119: 3, 2017
- [4] Campagne A., Gallet B., Moisy F., Cortet P. Disentangling inertial waves from eddy turbulence in a forced rotating-turbulence experiment Phy. Rev. E, 91: 4, 2015