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# Inertial waves focusing by vertically oscillating torus in rotating fluid

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## Abstract :

In geophysics or astrophysics, flows are often submitted to stable-stratification-induced buoyancy force or to rotation-induced Coriolis force, that lead to the appearance of internal gravity waves (IGW) or inertial waves (IW). In the ocean, energy injected by tidal forces generates internal gravity waves that participate a lot to energy transport and mixing. Waves can interact with the topography (ocean floor) but also interact with one another [1], leading to a complex dynamics of the resulting turbulent flow, especially in a neighborhood of the focal zone between two waves.

This situation can be geometrically idealized in a flow containing an oscillating solid torus-shaped ring, hence reproducing the focalization of waves at the apex of the propagation cone with an angle  $\theta$  (fig. 1). Several studies for IGW in stratified fluid, but also for IW in a rotating fluid, have analyzed this torus configuration by using numerical simulations of IW [2], experiments of IGW [3], or analytical theory of IGW [4]. Note that IGW and IW are both transverse waves with similar dispersion relation, so that their dynamics can be expected to be comparable, at least qualitatively.

Based on the theoretical work on an oscillating sphere [4], a preferential angle  $\theta_p$  has been obtained, at which the growth of the fluid velocity amplitude in the focal region is maximal for IGW [3]. For IW in rotating fluid, this preferential angle was found to be  $\theta_p \simeq 57^\circ$  [5]. At this angle, the maximum energy of waves is transferred from torus oscillation to the focus zone.

In the present study, a vertically oscillating torus in a rotating fluid is used to reproduce the wave-focusing phenomenon at the apex of the waves propagation cone. In order to explore this energy conversion in the focus zone, direct numerical simulations (DNS) have been performed using the standard pseudo-spectral method in a periodic domain with spatial resolution of  $512^3$  points and by using the penalization method to model the oscillation of a rigid torus. We vary the propagation angle of the IW by adjusting the ratio of oscillation frequency and rotating frequency, in order to study the focus region and the local turbulence generated due to this energy focusing. We perform two kinds of DNS : linear simulations that permit to identify the mere kinematic motion, and non-linear DNS in order to analyze the complete dynamics.

First, the linear and nonlinear DNS results show that a stationary geostrophic mode develops in the flow, elongated in the vertical direction and with a radius of the order of the small radius of the torus. **Secondly, our linear DNS confirm the theoretical linear result of Liu et al. (2022) [5]** and the preferential angle  $\theta_p \simeq 57^\circ$ . This remains true for non-linear DNS. Moreover, the nonlinear DNS at large oscillation amplitude and high frequency exhibit a transfer from large eddies (from the inertial waves) to small

eddies (local turbulence) in the focus zone. In order to study the transition from laminar to turbulent state, the spectrogram of the vertical velocity signal on the focus zone in nonlinear DNS shows that the principal forcing frequency coming from the oscillating torus transfer energy to higher frequency (superharmonics). Surprisingly, due to high nonlinear interactions, the lower (subharmonics) frequencies are also excited in the form of a stationary mode ( $\omega \simeq 0$ ) which is a vertically stretched central vortex and which appears in addition to the geostrophic mode (fig. 2). A series of statistics are used to characterize the flow dynamics and topology. Among them, bispectra and bicoherences will also be presented in order to propose a transition scenario in terms of triadic wave resonance [6].

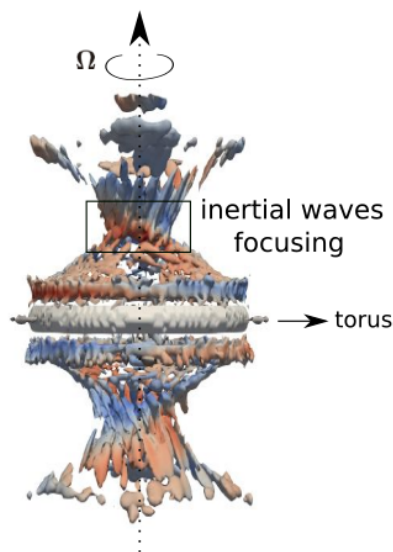


FIGURE 1 – Isosurfaces of the vertical velocity  $u_3$  from numerical simulation

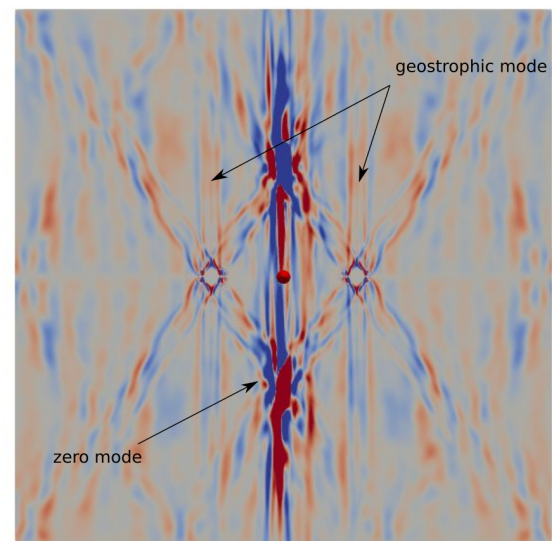


FIGURE 2 – The normalized amplitude of the vertical velocity component  $u_3$  in the focus zone as a function in the waves propagation angle  $\theta$

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