# A Rolling-Horizon Approach for a Surgery Case Scheduling Problem with Sterilizing Constraints <br> H. Al Hasan, Christelle Guéret, David Lemoine, David Rivreau 

## To cite this version:

H. Al Hasan, Christelle Guéret, David Lemoine, David Rivreau. A Rolling-Horizon Approach for a Surgery Case Scheduling Problem with Sterilizing Constraints. MIM 2022: 10th IFAC Conference on Manufacturing Modelling, Management and Control, Jun 2022, Nantes, France. pp.1625-1630, 10.1016/j.ifacol.2022.09.623 . hal-03832852

## HAL Id: hal-03832852

## https://hal.science/hal-03832852

Submitted on 4 Nov 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. publics ou privés.

Distributed under a Creative Commons Attribution - NonCommercial - NoDerivatives 4.0
International License

# A Rolling-Horizon Approach for a Surgery Case Scheduling Problem with Sterilizing Constraints 

H. Al Hasan * C. Guéret ${ }^{* *}$ D. Lemoine ${ }^{* * *}$ D. Rivreau ${ }^{* * * *}$<br>* UCO, LARIS, 44 Rue Rabelais, Angers 49000, France (e-mail: alhasan@uco.fr).<br>** Univ Angers, LARIS, SFR MATHSTIC, Angers 49000, France<br>(e-mail: c.jussien@univ-angers.fr)<br>*** IMT Atlantique, LS2N, 4 Rue Alfred Kastler, Nantes (e-mail: david.lemoine@imt-atlantique.fr) **** UCO, LARIS, 44 Rue Rabelais, Angers 49000, France (e-mail: david.rivreau@uco.fr).


#### Abstract

In this paper, we study a real scheduling problem which consists in scheduling a set of elective surgical cases that arrive over the time taking into account the uncertainties on their duration, and the delay to sterilize surgical instruments. The objectives are to schedule as many surgeries as possible, and to minimize the overtime of the surgical block staff and the number of instruments processed in emergency in the sterilizing unit. This research was performed in collaboration with the University Hospital of Angers in France, which has also provided historical data for the experiments. We propose a rolling horizon approach based on the iterative resolution of a robust mixed integer linear programming model, in which the objective functions are taken into account in a lexicographic order. The experiments on real data show a reduction of more than $50 \%$ of overtime, and around $87 \%$ less stress situations in the sterilizing unit.


Copyright © 2022 The Authors. This is an open access article under the CC BY-NC-ND license
(https://creativecommons.org/licenses/by-nc-nd/4.0/)
Keywords: Operations Research; Elective surgeries scheduling; Mixed-integer linear programming; Rolling horizon approach; Robust scheduling

## 1. INTRODUCTION

Operating rooms are recognized as the main source of income for hospitals: they generate around two thirds of hospital revenues (Jackson (2002)), and count for around $40 \%$ of hospital costs (Macario et al. (1995)) throughout the use of facilities (operating rooms, intensive care beds, etc.) and the personnel costs (surgeons, nurses, anaesthetists, etc.). For these reasons, a lot of researchers studied the operating room (OR) planning and scheduling problems in both elective (non urgent) and non-elective (urgent) contexts (Zhu et al. (2018)).

In this paper we consider the case study of the Centre Hospitalier Universitaire d'Angers (CHU), a French public hospital. This hospital serves multiple cities in the area and as such, it is very important that the surgical department runs efficiently. Due to the high number of limiting resources (surgeons, medical instruments,...etc.), and the increasing demand on surgery due to population ageing, the current manual scheduling process implemented at the CHU is showing its limits. This is clearly visible when we examine the poor performance measures such as the high numbers of staffs' overtime hours ( $\approx 28$ hours per month on average) and the medical instruments that are sterilized urgently in order to respect the proposed planning.

The main factor behind this problem, is the big uncertainty found when estimating surgeries durations. Despite using somewhat accurate but limited historic data in their calculations, they still end up with a big difference between the estimated and actual duration of surgeries. For this, our goal is to propose an operational optimization approach to ensure feasible schedules and generate real performance gains (in terms of overtime and medical equipment availability) while ensuring that the occupancy rates of the surgery block doesn't get affected negatively by such gains.
To the best of our knowledge there are very few literature and research on such particular problem. For instance in Beroule et al. (2016) the authors study an operating room scheduling problem where the set of surgeries to schedule is known in advance. The authors included medical devices sterilization but with the objective of reducing the number of medical devices needed at a time to respect a planning. A branch-and-price technique is applied in Cardoen et al. (2009) to solve a sequencing problem in a day care center, where the dates are fixed and they try to find the best order for surgeries. Their objective is to minimize the peak use of recovery beds, the occurrence of recovery overtime and the violation of various patient and surgeon preferences while including the medical devices sterilizing constraints.

In this research, we propose a robust optimization model to solve the static version of a surgical case scheduling problem that is not yet treated in the literature. We then integrate our model in a rolling horizon approach to solve the dynamic version of the problem. We show that the obtained solutions provide competitive results in term of the number of scheduled surgeries, and significantly improve those operationally implemented in terms of overtime and stress situations at the sterilizing unit.

## 2. PROBLEM DESCRIPTION

The Centre Hospitalier Universitaire d'Angers (CHU) currently employs approximately 6300 hospital staff. It includes several operating theatres, and a sterilizing unit which is in charge of the sterilization of all the medical instruments used in the hospital. In this study, we focus on the Orthopaedic Surgical Block (OSB) and on the Sterilizing Unit (SU). The OSB is composed of 3 operating rooms, with different opening hours : room 1 and 2 are open 5 days a week from 8:15 to 17:00, and room 3 is open only 4 days a week from $8: 15$ to $14: 30$. Around 15 surgeons share these rooms according to a planning indicating the days when they operate, and the list of rooms that each surgeon can use each day.
Each year, approximately 2500 elective surgeries of 211 different types are performed in this block. Some of these surgeries can be scheduled anytime during the opening hours of the rooms, whereas others (ambulatory surgeries) have to be completed before 15:00 because the patient must stay at least 2 hours in the recovery room (which closes at 17:00) before being discharged. The other surgeries can end up to the closing time of the ORs. However, due to the large number of surgeries to perform, each OR can exceptionally stay open up to 3 hours after its official closing time. Each of these surgeries is characterized by an estimated duration time and requires a list of surgical instruments which are organized in small boxes called kits. These kits are available in limited quantities.

### 2.1 Work process in the OSB

Currently, at the OSB, the schedule of the surgeries is constructed as follows. A work shift schedule generated 6 months in advance indicates the full days (shifts) assigned to each surgeon and the ORs in which they can be affected for each of these days. Some shifts, affected to OR 3, are reserved exclusively for ambulatory surgeries. Any surgeon can operate in this OR during these shifts. Usually, there are around 14 OR-day shifts each week, with 2 reserved for ambulatory surgeries.
Then, during his/her consultations, the surgeon decides if his/her patient needs a surgery, and, where relevant, fixes the date of the surgery among his/her shifts (generally between 3 weeks to 3 months later). This date, the type of the surgery (ambulatory or not), the estimated duration of the surgery and the required kits are then recorded in the OSB information system by a secretary.

On a more short term, each Monday, a specialized nurse sequences the surgeries for each OR-day shift of the next week, adding 20 minutes between two consecutive surgeries to clean the room. This nurse also checks if there are
enough kits of each type to perform all the surgeries planned each day. She may cancel a surgery if she discovers that the number of kits is not sufficient.

After each surgery, the used kits are kept in water during 30 minutes at the OSB. After that, they can be collected and sent to the SU. Several shuttles are planned every day at fixed hours to collect and deliver the kits. The schedule is given Table 1. In the SU , the kits are processed in a First In First Out (FIFO) order. Once sterilized, they are brought back to the OSB by a shuttle, and can be used again.

Table 1. SU's pickups and deliveries to the OSB.

| Pick-up | $07: 00$ | $11: 30$ | $13: 00$ | $14: 30$ | $16: 00$ | $17: 30$ | $18: 30$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Delivery | $07: 00$ | - | - | $14: 30$ | - | $17: 30$ | - |

At the SU , the sterilization process is being performed in several steps : the instruments are first cleaned by automatic washers, then reassigned in their corresponding kit before being processed through sterilization machines. Finally, the kits are kept at the SU to cool off before being returned to the block. On average, when a kit arrives at the SU , the whole sterilization process takes around 4 h 30 .

### 2.2 Dysfunctions detected

Currently, the CHU is faced with three major dysfunctions. First, the OSB staff is often forced to work overtime. On average, the monthly number of overtime hours is 28. The problem can be attributed to the lack of global planning: each surgeon decides the date of the surgeries of his/her patients without considering the planning of the other surgeons, which can be a problem when two surgeons share the same OR. Another explanation is the uncertainties concerning the duration of the surgeries. Indeed, for more than $50 \%$ of the surgeries, the real duration differs from the estimated one by more than 20 minutes. And this difference can reach 3 hours.

Second, despite frequent overtime, some OR-day shifts are not fully utilized: indeed, the minimal occupancy rate of OR-day shifts is only $9.9 \%$, while the average rate is $78.8 \%$. This is due to the myopic process followed to fix the date of the surgeries at each consultation, without knowing the coming ones nor the planning of the other surgeons.
The third dysfunction concerns the availability of the kits. As said before, in the current work process, each Monday, a nurse checks if the number of kits of each type required each day of the next week does not exceed the number of kits available, but without considering the surgeries of the past days. This situation leads the SU to regularly have to treat kits urgently. One can identify two situations of emergency: priority kits, and urgent kits as follows:
(1) A kit collected at 11:30, 13:00 or 14:30 on day $(t)$ can be used in the morning of day $(t+1)$ if it is treated as a priority at the SU (priority kit, case 1 ). If it is not treated as a priority, it is considered that it cannot be used before 14:30 on day $(t+1)$.
(2) A kit collected at 16:00 on day $(t)$ can be used in the morning of day $(t+1)$ if it is treated urgently at the SU (urgent kit). If it is not treated urgently, it can be used from 14:30 on day $(t+1)$.
(3) A kit collected at 17:30, 18:30 on day $(t)$ or $7: 00$ on day $(t+1)$ can be used on day $(t+1)$ from 14:30 if it is treated as a priority at the SU (priority kit, case 2). If it is not treated as a priority, it will be available on day $(t+1)$ from 17:30.

Other emergencies may arise when the delay between the 2 surgeries that use the same kit is large enough for the sterilizing process ( $\geq 4 \mathrm{~h} 30$ ), but the fixed pickup and delivery hours of the shuttle does not allow to deliver the kit at the SU for sterilization and then back in time at the OSB before its next planned usage. This situation can happen for example if a surgery ends at 10:30 and one (or more) of its kits must be reused at 14:30 for another surgery although it cannot be collected before the shuttle of 11:30. Even if these cases must be strictly avoided, they can occur in practice as the SU can send a special shuttle outside the collecting hours just to collect these kits so that there will be enough time to sterilize them before sending them back, possibly by another special shuttle outside the delivery hours. We will refer to these kits by Not allowed kits.

On average, more than 60 problems of kits are listed every month, which implies a lot of stress at the SU and has a negative impact on the work flow.
The CHU would like to improve these dysfunctions while keeping the same level of service in terms of number of surgeries scheduled each month. The objectives are then, in the order of priority given by the CHU, to schedule as many surgeries as possible, to minimize the overtime and to minimize the number of problems of kits (priority and urgent). In order to handle these problems, we propose a rolling horizon approach based on a robust MILP formulation.

## 3. THE ROLLING HORIZON APPROACH

The Rolling Horizon approach proposed is iterative, and consists, at each iteration, in fixing a one week schedule obtained by solving a robust MILP. We first present the rolling horizon framework. Then a formulation of the deterministic problem solved at each iteration is described. We then propose an adaptation of this formulation to obtain a robust MILP building solutions more resistant to the fluctuations of the durations of the surgeries.

### 3.1 Rolling horizon framework

We propose to use a rolling horizon approach in which the date of the surgeries are not fixed during the consultation, but instead, the surgeons define a due date for the surgeries, and the surgeries are added to a waiting list. Then, at the end of each week $t$, the surgeries in the waiting list are scheduled starting from week $t+3$. This 3 weeks gap assures that patients have enough time to prepare and organize their hospitalization. In the schedule obtained, only week $t+3$ is kept, and the surgeries scheduled after are put back in the waiting list (see Figure 1).
Thus, at the end of each week $t$, the process is as follows:
(1) Add the surgeries consulted at the current week $t$ to the waiting list.
(2) Calculate the horizon length $H_{s}$ for each surgeon based on his/her surgeries load and shifts.
(3) For each surgeon $s$, schedule the surgeries in the list over the horizon starting from week $t+3$ and ending at $t+H_{s}$ using a MILP.
(4) Fix the surgeries that were scheduled at week $t+3$ and return the surgeries that were scheduled after $t+3$ back to the list.


Fig. 1. Rolling horizon scheme
This procedure requires to estimate an horizon length $H_{s}$ for each surgeon $s$ sufficient to schedule all his/her surgeries of the waiting list. The value of $H_{s}$ is calculated as the minimal number of shifts needed to cover the load of surgeon $s$ (sum of the planned durations of his/her surgeries), plus 4 additional shifts. These 4 supplementary shifts are a safety margin that takes into account the fact that we can't know for sure the exact amount of shifts needed for each surgeon as surgeons share some ORs occasionally and because of limited quantities of kits.

At each iteration, the problem of scheduling all the surgeries in the waiting list from week $t+3$ is solved using a MILP. In the next section, we present a formulation for the deterministic case of the problem, assuming that the duration of the surgeries is known with certainty. A robust version for the non-deterministic case will be then proposed.

### 3.2 MILP formulation for the deterministic case

In order to model this problem, we propose a mathematical formulation similar to the one presented in Al Hasan et al. (2019) based on the decomposition of the day in four periods (period 1 from 8:15 to 14:00, period 2 from 14:00 to 14:30, period 3 from 14:30 to $15: 30$ and period 4 from $15: 30$ to 17:00). This division is obtained from several key hours in the schedule of the shuttle, and the opening and closing hours of the ORs.

In line with the requirements of the CHU, the objective of this model is to schedule as many surgeries as possible while minimizing the total overtime and the total penalties of emergencies and priorities in the SU . In order to take into account the new process in which the surgeons provide
a due date for each surgery during the consultation, we also add as objective the minimization of the total tardiness of the surgeries.
The reader is referred to Al Hasan et al. (2019) for a full view of the deterministic MILP model. We present in this section only the parameters, variables and constraints that we modified in order to obtain a robust formulation.

The concerned parameters are:

| $H$ | total number of days in the horizon |
| :--- | :--- |
| $J$ | number of periods in day $(=4)$ |
| $K$ | total number of kit types |
| $O$ | total number of surgeries |
| $R$ | total number of operating rooms |
| $S$ | total number of surgeons |
| $c u$ | urgent kit penalty |
| $c p$ | priority kit penalty |
| $p_{i}$ | duration of surgery $i$ |
| $\Delta_{r t}^{\beta \gamma}$ | the total duration from period $\beta$ to period $\gamma$ in room $r$ <br> on day $t$ |
| $\Delta_{r t}^{j}$ | the total duration of period $j$ in room $r$ on day $t$ <br> $a_{i}$ |
| binary parameter equal to 1 if surgery $i$ is ambulatory,  <br> $A_{m a x}$ 0 otherwise <br> $u_{\gamma}$ latest time for ambulatory surgeries to be performed at |  |
|  |  |

And the concerned decisions variables are:

$$
\begin{array}{cl}
w_{i t r} & \begin{array}{l}
\text { binary variable equal to } 1 \text { if surgery } i \text { is scheduled on day } \\
t \text { in OR } r
\end{array} \\
x_{i t r}^{b f} & \begin{array}{l}
\text { binary variable equal to } 1 \text { if surgery } i \text { begins at period } b \\
\text { and finishes at } f, \text { on day } t, \text { in OR } r
\end{array} \\
E_{t k} & \begin{array}{l}
\text { integer variable representing the total urgent kits of type } \\
k \text { on day } t
\end{array} \\
Y_{t k}^{1} & \begin{array}{l}
\text { integer variable representing the total priority kits (case } \\
1) \text { of type } k \text { used on day } t
\end{array} \\
Y_{t k}^{2} & \begin{array}{l}
\text { integer variable representing the total priority kits (case } \\
2) \text { of type } k \text { used on day } t
\end{array} \\
\varepsilon_{t r} & \begin{array}{l}
\text { real variable representing the total overtime in OR } r \text { on } \\
\text { day } t
\end{array} \\
T_{i} & \begin{array}{l}
\text { real variable representing the tardiness of surgery } i
\end{array}
\end{array}
$$

The constraints that need to be modified to obtain a robust formulation are the two capacity constraints below:

- the workload of surgeries for each period of the day and each room must not exceed the duration of the period (an overtime of $\varepsilon_{t r}$ is allowed in the last period in each room $r$ and day t) (1):

$$
\begin{gather*}
\sum_{i \in O} \sum_{b=\beta}^{\gamma} \sum_{f=b}^{\gamma} p_{i} x_{i t r}^{b f}+\sum_{i \in O} \sum_{b=1}^{\beta-1} \sum_{f=\gamma+1}^{J} \Delta_{r t}^{\beta \gamma} x_{i t r}^{b f} \leq \Delta_{r t}^{\beta \gamma}+u_{\gamma} \varepsilon_{t r} \\
\forall \beta \in\{1, \ldots, J\}, \gamma \in\{\beta, \ldots, J\}, t \in\{1, \ldots, H\}, r \in R \tag{1}
\end{gather*}
$$

- each ambulatory surgery must end before $A_{\max }(2)$ :

$$
\begin{gather*}
\sum_{i \in O} \sum_{b=\beta}^{2} \sum_{f=b}^{2} p_{i} x_{i t r}^{b f}+\sum_{i \in O} \sum_{b=\beta}^{3} a_{i} p_{i} x_{i t r}^{b 3} \leq A_{\max }-\sum_{j=1}^{\beta-1} \Delta_{r t}^{j} \\
\forall \beta \in\{1, \ldots, J-1\}, t \in\{1, \ldots, H\}, r \in R \tag{2}
\end{gather*}
$$

The multiple objective functions (3) are taken into account by using a lexicographic method. We first maximize $f_{1}$, the
total number of scheduled surgeries. We then minimize $f_{2}$, the total overtime required to schedule at least the number of scheduled surgeries found in $f_{1}$. Next, we minimize $f_{3}$, the total penalty cost of urgent and priority kits, without scheduling less surgeries than what was found for objective $f_{1}$ nor exceeding the total overtime found for objective $f_{2}$. And finally we minimize $f_{4}$, the total tardiness of surgeries, without scheduling less surgeries than what we found in $f_{1}$ nor exceeding the total overtime calculated for $f_{2}$ nor the total penalty cost of urgent and priority kits found for $f_{3}$.
$\operatorname{Lex}\left(\begin{array}{l}f_{1}: \operatorname{Max} \sum_{s \in S} \sum_{i \in O_{s}} \sum_{t=1}^{H_{s}} \sum_{r \in R} w_{i t r} ; \\ f_{2}: \operatorname{Min} \sum_{t=1}^{H} \sum_{r \in R} \varepsilon_{t r} ; \\ f_{3}: \operatorname{Min} \sum_{t=1}^{H} \sum_{k \in K}\left[c u E_{t k}+c p\left(Y_{t k}^{1}+Y_{t k}^{2}\right)\right] ; \\ f_{4}: \operatorname{Min} \sum_{i \in O} T_{i}\end{array}\right)$

### 3.3 Robust formulation

We first proposed a formulation based on the model of Bertsimas and Sim (Bertsimas and Sim (2004)). Although this model provides good results in a static environment (see Al Hasan et al. (2019)), its integration in a rolling horizon approach leads to prohibitive execution times. In our experiments, the model was not able to provide good solutions in reasonable time, due to the higher number of surgeries considered at each iteration (400 on average, against 200 in the static version) and longer horizons (150 days considered at each iteration, instead of 26 days on average for the static version). We thus opted for a robust formulation using redundancy-based technique like the one of Hans et al. (2008). Unlike the problem studied in Hans et al. (2008), our problem consists in both scheduling and sequencing the surgeries in each room at each day. Thus, we insert several slack times in each OR-day shift (one at the end of each scheduled surgery) instead of only one at the end of each OR-day. The slack time added at the end of each surgery corresponds to the standard deviation of its duration. We applied this technique to the MILP modelling the deterministic case of the problem.

Let $z_{\text {trbf }}$ represents the amount of slack added on day $t$ in OR $r$ for surgeries that start in period $b$ and end in period $f$. The value of $z_{t r b f}$ is calculated as follows:

$$
\begin{align*}
& z_{t r b f} \geq B \sum_{i \in O} \hat{p}_{i} x_{i t r}^{b f}, \quad \forall t \in\{1, \ldots, H\}  \tag{4}\\
& \forall r \in R, \forall b \in\{1 \ldots, J\}, \forall f \in\{b, \ldots, J\}
\end{align*}
$$

where:

- $B(\geq 0)$ is a parameter that controls the probability that a surgery will finish on time, i.e, so overtime will not occur.
- $\hat{p}_{i}$ is the standard deviation of the duration of surgery $i$ (based on a statistical analysis).
The robust formulation is then obtained from the deterministic formulation by adding constraint (4), and the following constraint,

$$
\begin{gather*}
z_{\text {trbf }} \geq 0, \quad \forall t \in\{1, \ldots, H\},  \tag{5}\\
\forall r \in R, \forall b \in\{1, \ldots, J\}, \forall f \in\{b, \ldots, J\}
\end{gather*}
$$

and, in order to add the slack times, by replacing Constraints (1) by :

$$
\begin{gather*}
\sum_{i \in O} \sum_{b=\beta}^{\gamma} \sum_{f=b}^{\gamma} p_{i} x_{i t r}^{b f}+\sum_{b=\beta}^{\gamma} \sum_{f=b}^{\gamma} z_{t r b f}+\sum_{i \in O} \sum_{b=1}^{\beta-1} \sum_{f=\gamma+1}^{J} \Delta_{r t}^{\beta \gamma} x_{i t r}^{b f} \\
\quad \leq \Delta_{r t}^{\beta \gamma}+u_{\gamma} \varepsilon_{t r} \\
\forall \beta \in\{1, \ldots, J\}, \gamma \in\{\beta, \ldots, J\}, t \in\{1, \ldots, H\}, r \in R \tag{6}
\end{gather*}
$$

and Constraints (2) by :

$$
\begin{gather*}
\sum_{i \in O} \sum_{b=\beta}^{2} \sum_{f=b}^{2} p_{i} x_{i t r}^{b f}+\sum_{b=\beta}^{2} \sum_{f=b}^{2} z_{t r b f}+\sum_{i \in O} \sum_{b=\beta}^{3} a_{i} p_{i} x_{i t r}^{b 3} \\
\quad+\sum_{b=\beta}^{3} z_{t r b 3} \leq A_{\max }-\sum_{j=1}^{\beta-1} \Delta_{r t}^{j}  \tag{7}\\
\forall \beta \in\{1, \ldots, 3\}, t \in\{1, \ldots, H\}, r \in R
\end{gather*}
$$

## 4. EXPERIMENTAL RESULTS

We tested the method on real data over a horizon of 10 months ( 44 weeks, from September 2014 to June 2015) provided by CHU. During this horizon, 2069 surgeries were performed by 15 different surgeons. 69 of these surgeries were performed outside the block during the 10 concerned months. We however had to consider them due to their impact on the numbers of kits. In order to do so, we created a fourth OR that is exclusive to the surgeries performed outside the block and fixed the dates and times for these surgeries as in the original schedule. Furthermore, 38 surgeries from other blocks were performed at the OSB. Again, we had to consider these surgeries due to their impact on the total operating time at the ORs, and we achieved this by decreasing the total operating hours of the corresponding ORs at the dates of these surgeries by the duration of each of these surgeries. In this data, there is no due date for the surgeries. For the experiments, we considered for each surgery that its due date is equal to its planned date in the schedule constructed by the CHU.

The values of the parameters are as follows:

- Latest time $A_{\max }$ for ambulatory surgeries : 3 p.m.
- Maximum overtime $\varepsilon_{\max }$ allowed each day : 180 minutes (3 hours)
- Penalty cost $c u$ of urgent kits : 5
- Penalty cost $c p$ of priority kits : 1

Note that the urgent and priority penalties have been evaluated by the CHU according to the disturbance and stress perceived at the SU.

In addition, Table 2 shows the duration $\Delta_{r t}^{j}$ of each period $j$ in each OR $r$ for each day $t$ when the OR is open ( 0 if the OR is closed).

We set the execution time limit of the solver to be 4 hours per iteration (1 hour per objective).

Table 2. Periods duration for the ORs.

| $\Delta_{r t}^{j}$ | ORs 1 \& 2 | OR 3 |
| :---: | :---: | :--- |
| $\Delta_{r t}^{1}$ | 5 h 45 | 5 h 45 |
| $\Delta_{r t}^{2}$ | 0 h 30 | 0 h 30 |
| $\Delta_{r t}^{3}$ | 1 h 00 | 0 h 00 |
| $\Delta_{r t}^{4}$ | 2 h 30 | 0 h 00 |

We performed 5 experiments, one with the deterministic MILP in the rolling horizon approach $(D R H)$ and 4 other experiments with the robust MILP in the rolling horizon approach $(R R H)$ using the following values for $B: 0.25$, $0.5,0.75$ and 1 , and compared the results with the schedule that has actually been used at the CHU.

Our testing environment is:

- Intel Core i3-2120 @ 3.30 GHz
- 8 GB of RAM
- IBM ILOG CPLEX 12.5

The obtained results rely on the estimated durations of the surgeries. In order to obtain the effective schedule based on the real durations, we implemented a procedure that simulates the work process of the OSB to create the corresponding achieved schedule from the planned schedule. This algorithm simulates each day of the horizon by fixing the start of the surgeries one after the other, considering their real durations and adding a break of 20 minutes between two consecutive ones to clean the OR. Note that the procedure allows the possibility to start the next surgery $(i+1)$ in place of $i$ if $i$ is waiting for a kit and $i+1$ is not. But, it is not possible for a further surgery to start in place of $i$ (i.e. $i+3$ cannot start in place of $i$ ). This is consistent with the process of the OSB, as patients are not prepared for their surgery a long time in advance, so later patients in the queue $(>i+1)$ won't be ready to start directly.

The results are presented in Table 3. The first row (Scheduled) contains the number of surgeries scheduled over the horizon of 44 weeks, and the second (Late ambs) contains the number of ambulatory surgeries that are planned to end after the time limit $A_{\max }$. Row Overtime and Max. overtime indicate respectively the total overtime in minutes, and the maximal overtime of the OR-day shifts. The minimal and average occupation rate of OR-day shifts (OR opened) are presented in Rows Min. Occ. rate and Avg. Occ. rate respectively. Finally, the number of urgent, priority and not allowed kits are given in Rows Urgent, Priority and Not allowed.

Among the four versions of $R R H, R R H_{0.25}$ remains preferred as the number of surgeries scheduled is higher than for the other versions, with a competitive overtime. Indeed, increasing the value of $B$ increases the robustness of the method, but it also leads to less scheduled surgeries in total. This can be observed clearly in the last two versions $\left(R R H_{0.75}\right.$ and $\left.R R H_{1}\right)$ as they insert slack times that are finally not used and lead to being too conservative and to a lower average occupation rate of the ORs.
$R R H_{0.25}$ allows a reduction of $54 \%$ of the overtime of the schedule of the CHU (from 17174 to 7893 minutes, that is from 28,6 hours per month on average to 13,15 hours), and of $30.4 \%$ of the $D R H$ schedule, and the maximum overtime (238 minutes) stays close to the time limit of 180 minutes,

Table 3. Achieved schedules comparison

| Objective | OSB schedule | $D R H$ | $R R H_{0.25}$ | $R R H_{0.5}$ | $R R H_{0.75}$ | $R R H_{1}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Scheduled | 2000 | 1981 | 1962 | 1937 | 1864 | 1771 |
| Late ambs | 101 | 12 | 8 | 4 | 4 | 2 |
| Overtime | 17174 | 11346 | 7893 | 6501 | 6217 | 5494 |
| Max overtime | 326 | 408 | 238 | 198 | 195 | 181 |
| Min. Occ. rate | 9.9 | 39.3 | 38.7 | 34.2 | 24.9 | 28.1 |
| Avg. Occ. rate | 78.8 | 88.5 | 86.4 | 83.8 | 82.1 | 80.6 |
| Urgent | 79 | 2 | 4 | 3 | 4 | 10 |
| Priority | 494 | 181 | 79 | 48 | 43 | 39 |
| Not allowed | 92 | 0 | 0 | 0 | 0 | 0 |

which is not the case for the CHU (326 minutes) nor the DRH approach ( 408 minutes). As regards the occupation of the ORs, the average occupation rate obtained with $R R H_{0.25}(86.4 \%)$, similar to the one of DRH approach, is more satisfactory than the one of the CHU (78.8\%), and the minimal occupation rate ( $38.7 \%$ against $9.9 \%$ for the CHU) shows a better load balancing on the different shifts. It should be noted that $R R H_{0.25}$ schedules a little less surgeries than the CHU, although there are still free slots in some rooms as all shifts are not fully occupied. This situation is due to the fact that these slots do not belong to the shifts or ORs assigned to the surgeons who must perform the remaining surgeries. Finally, there are only 83 problems of kits with $R R H_{0.25}$, instead of 635 for the CHU, and 183 for $D R H$.

In conclusion, the quality of the obtained results makes $R R H_{0.25}$ very suitable and attractive for the CHU: first, for approximately the same number of surgeries scheduled, it allows a decrease of $54 \%$ of the overtime, a reduction of 1 OR-day shift on average per month, a better use of the ORs (better repartition of the surgeries in the room, and less idle times), and far less stress in the $\mathrm{SU}(86,9 \%$ less problems). It then follows less costs and higher quality of labour for the staff.

## 5. CONCLUSION

This paper addresses the surgical case scheduling of the orthopaedic surgical block (OSB) of a French university hospital (CHU). We have first presented the procedure used to construct the schedule of the surgeries, and then the dysfunctions detected in the OSB and in the Sterilization Unit (SU) which is in charge of the sterilization of all the medical instruments used for the surgeries of the CHU. These dysfunctions are: a lot of overtime for the staff, unbalanced operating rooms (ORs) occupancy rates, and lots of stress in the SU to sterilize all the instruments on time. Then we have proposed a modification in the schedule construction process and a rolling horizon approach based on the resolution of a robust MILP in order to allow a better coordination between surgeons, and to obtain a schedule less sensitive to the uncertain duration of the surgeries. We tested the approach on a real 10 months horizon instance provided by CHU. The experiments show a reduction of more than $50 \%$ of over-
time, higher occupation rates of the ORs, and more than $85 \%$ less stress situations in the SU. Despite these good results, we noted that several ORs are underutilized due to the fact that if an OR is reserved for a surgeon on a given day, it can not be used by another one who would be available. An interesting avenue for future research would be to jointly address the Master Surgical Schedule problem (which assigns surgeons to OR-day shifts) and this surgical case scheduling problem, in order to obtain an assignment of the OR-days shifts to the surgeons as close as possible to the needs.

## ACKNOWLEDGEMENTS

The authors would like to thank L. Hubert, A.V. Lebelle and A. Robelet from the CHU for submitting us this challenging problem and kindly providing us with real instances. This research is partially founded by Angers Loire Metropole (ALM) and IMT Atlantique.

## REFERENCES

Al Hasan, H., Guéret, C., Lemoine, D., and Rivreau, D. (2019). Surgical case scheduling with sterilising activity constraints. International Journal of Production Research, 57(10), 2984-3002.
Beroule, B., Grunder, O., Barakat, O., Aujoulat, O., and Lustig, H. (2016). Operating room scheduling including medical devices sterilization: towards a transverse logistic. IFAC-PapersOnLine, 49(12), 1146-1151.
Bertsimas, D. and Sim, M. (2004). The price of robustness. Operations research, 52(1), 35-53.
Cardoen, B., Demeulemeester, E., and Beliën, J. (2009). Sequencing surgical cases in a day-care environment: an exact branch-and-price approach. Computers $\mathcal{\xi}$ Operations Research, 36(9), 2660-2669.
Hans, E., Wullink, G., Van Houdenhoven, M., and Kazemier, G. (2008). Robust surgery loading. European Journal of Operational Research, 185(3), 1038-1050.
Jackson, R. (2002). The business of surgery. Health Management Technology, 23(7).
Macario, A., Vitez, T., Dunn, B., and McDonald, T. (1995). Where are the costs in perioperative care?: Analysis of hospital costs and charges for inpatient surgical care. The Journal of the American Society of Anesthesiologists, 83(6), 1138-1144.
Zhu, S., Fan, W., Yang, S., Pei, J., and Pardalos, P.M. (2018). Operating room planning and surgical case scheduling: a review of literature. Journal of Combinatorial Optimization, 1-49.

