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Guillaume Vignat, Preethi Rajendram Soundararajan, Antoine Renaud, Daniel Durox, Sébastien Candel. The flame describing function and flame dynamics under self-sustained oscillations. 5e colloque de l'initiative en combustion avancée (INCA), Safran, Apr 2021, Online, France. hal-03832347

**HAL Id: hal-03832347**

**<https://hal.science/hal-03832347v1>**

Submitted on 27 Oct 2022

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# The flame describing function and flame dynamics under self-sustained oscillations

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Received: date / Accepted: date

**Abstract** Transfer function concepts that appear in many areas and most notably in control systems have been extensively used to represent the flame response in low order models of combustion instability. Much of the theoretical work is based on flame transfer functions (FTF). In recent years the nonlinear extension of the flame transfer function, namely the flame describing function (FDF), was used to get a more accurate representation of the flame response when the level of oscillation becomes large and the system reaches a limit cycle. Despite their wide use, the validity of using FTF/FDF to represent flame response still remains to be experimentally substantiated. This article is aimed at providing a direct assessment of the capacity of the FDF to suitably describe the flame behavior under self-sustained oscillations (SSO). This is accomplished by making use of an experimental combustion configuration which exhibits unstable oscillations but which can also be used to modulate the flame using a set of driver units. The flame dynamics and response are determined under well established oscillations. The chamber length is then modified to obtain a stable regime and the flame is modulated externally at the frequency of the self-sustained oscillation observed in the first stage. The amplitude of incident velocity modulations is then progressively varied until it coincides with that found under self-sustained oscillations. This allows a direct comparison of the flame dynamics in these two situations. Gain and phase of the describing function are measured for the various input levels and found to approximately match those measured under SSO. It is shown that the best match is obtained when the amplitude of external modulation induces a level of velocity oscillations that is closest to that prevailing under SSO demonstrating that the FDF suitably captures the nonlinearity of the flame response, at least in the configuration investigated in this research.

**Keywords** Combustion dynamics · flame describing function · transfer function · limit cycle oscillations

## 1 Introduction

Among the many questions raised by combustion instability that of modeling occupies a central position. Much effort has been devoted to derive models that could guide the analysis of combustion dynamics phenomena and could then be used as a predictive tool. This effort was initiated during the early days of rocket engine development where instability problems were encountered, inducing some spectacular failures. More recently attention has been focused on dynamical phenomena in gas turbine combustors operating in the premixed mode. This has led to the derivation of reduced (or low) order models relying on the transfer function concept borrowed from linear control theory. The aim of the present article is to see if this concept can reflect the complexity of the combustion response. Is it possible to capture the three dimensional flame dynamics into a transfer function or a

set of transfer functions? Is it possible to obtain a direct experimental proof that transfer functions and their extensions (i.e., describing functions) are sufficient to represent the flame behavior and that the reduced order model can be considered to be a suitable description of the multi-dimensional reality? It will not be possible to answer such a broad question in a general way but we wish to report experimental evidence that provides some insight on this fundamental question.

At this point it is worth briefly reviewing the state of the art in low order modeling to place the present investigation in perspective. The early analysis of combustion instability relied on the sensitive time lag (STL) theory. The flame response was represented in terms of an interaction index  $n$  and a time delay  $\tau$  that was assumed to be a function of the state variables in the combustion region. In general these two terms were considered as parameters that could be varied to determine regions of instability. This kind of model assumes, in essence, that a transfer function exists between the state variable disturbances and combustion disturbances such as those of the heat release rate. The gain of this transfer function is a constant  $G_F(\omega) = n$  while the phase is a linear function of the angular frequency  $\varphi_F = \omega\tau$ . More recently, considerable effort was expanded to understand mechanisms controlling instabilities and to represent the flame dynamics in terms of transfer functions (see [1–3] for reviews). The transfer function was introduced to link relative fluctuations of heat release rate in the flame treated as an output to the relative fluctuation in volume flow rate. For experimental convenience, velocity fluctuation is considered as the input instead of volume flow fluctuation. The transfer function may have multiple inputs, and in the present case, one other input could be the perturbations in equivalence ratio. For the case considered in this article, the mode of combustion is quasi-premixed and the primary input will be the disturbance in velocity (representing the disturbance in volume flow rate). The transfer function is given by

$$\mathcal{F}_0(\omega) = \frac{\dot{Q}'(\omega)/\bar{Q}}{u'/\bar{u}} = G_F(\omega)e^{i\varphi_F(\omega)} \quad (1)$$

Transfer functions were introduced, in particular, to derive active control methods and help interpret their experimental demonstrations. Transfer function expressions were obtained for many simple flames like premixed conical and “V” flames and for swirling premixed flames and were compared with experimental data. It was then recognized that the flame response depended not only on frequency but also on the amplitude of oscillation. This led to replace the FTF by a describing function, i.e., a family of transfer functions with each of these functions depending on the amplitude of the input.

$$\mathcal{F}(\omega, u') = \frac{\dot{Q}'(\omega, u')/\bar{Q}}{u'/\bar{u}} = G_F(\omega, u')e^{i\varphi_F(\omega, u')} \quad (2)$$

This was initially employed in a theoretical analysis of the dynamics of a ducted flame by [4], which indicated only a gain saturation with the amplitude of velocity fluctuation. The concept of flame describing function (FDF) was generalized by [5] to also consider phase dependence on the amplitude of perturbations and was shown to provide an understanding of many nonlinear features observed experimentally like frequency shifting during oscillation growth, mode switching (frequency jumping during oscillation), instability triggering and hysteresis. With respect to the transfer function, the FDF allowed to render some of the complex behavior of practical systems and more specifically those linked to finite amplitude oscillations. This has been a considerable advance because the transfer function could only reflect the linear behavior of small perturbations. Being a linear concept, the FTF does not provide information on finite amplitude effects. On the other hand, models using the FDF yield amplitude dependent results that allow direct comparisons with experimental data since most instability experiments are carried out when the oscillations are established and have a finite value. It was however found that the FDF has some shortcomings and could not easily handle complex limit cycles sustained by multiple modes. Despite these difficulties, the FDF accounts for some essential nonlinear features. It is then interesting to use this concept and see if it is representative of the combustion dynamics under self-sustained oscillations (SSO). This question is represented schematically in Fig. 1(a) and (b).

This diagram shows on the left, a representation of the system when it is executing SSO and features a limit cycle at an angular frequency  $\omega_0$ . The flame dynamical response is  $\mathcal{H}(\omega_0, u')$ . The inclusion of  $u'$  in this expression is to indicate that the flame behavior is also controlled by the

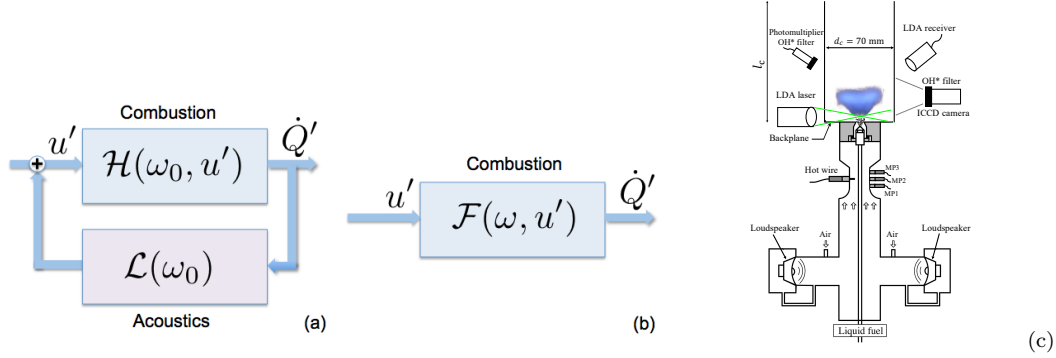


Fig. 1: (a) Closed loop representation of a self-sustained instability at an angular frequency  $\omega_0$  and an amplitude of oscillation  $u'$ . The function  $\mathcal{H}$  represents the flame response in the self-sustained oscillation. (b) Open loop determination in terms of a flame describing function  $\mathcal{F}(\omega, u')$ . (c) Schematic of the experimental setup SICCA-spray.

level of incident disturbances. In the center, the diagram shows the combustor being modulated externally to measure the FDF designated as  $\mathcal{F}(\omega, u')$ . For this measurement one has to reduce the acoustical coupling so that the system operates in a steady regime. In practice, this may be accomplished by changing the combustion chamber size to move the resonant frequencies out of the range of interest manifested under SSO. One may then see if

$$\mathcal{F}(\omega_0, u') \simeq \mathcal{H}(\omega_0, u') \quad (3)$$

A good match between these two functions will indicate that a low order model using a measured FDF may suitably represent the real system and will provide reasonable predictions of self-sustained oscillations. However, one cannot be certain that the flame behavior has not been modified when the loop is closed and when a strong acoustic coupling takes place. A modification of this type is not considered in control systems where the transfer function or describing function of the “plant” does not depend on the feedback path. Here the situation is different because the flame is a result of a complex flow where exothermic reactions take place. The dependence on amplitude of the FDF captures some of the modifications in flame dynamics but there are perhaps other changes that arise under SSO but are not accounted for by the FDF.

This article begins with a presentation of the experimental set-up (section 2). Flame dynamics are then examined using  $\text{OH}^*$  chemiluminescence images in section 3 under SSO and compared to those corresponding to external modulation (referred to as open loop modulation or OLM). A comparison between the flame response  $\mathcal{H}$  and the FDF  $\mathcal{F}$  is then carried out in section 4.

## 2 Experimental set-up

Experiments are carried out in a generic single injector set-up (SICCA-spray). This configuration, shown schematically in Fig. 1(c), comprises a plenum, a spray-swirl injector, and a cylindrical chamber. Liquid heptane fuel is delivered as a hollow cone spray by a simplex atomizer producing a dispersion of fine fuel droplets. The mass flow rate of fuel is set by a Bronkhorst CORI-flow controller with a relative accuracy of 0.2%. The air flow rate measured by a Bronkhorst EL-FLOW mass flow controller with a relative accuracy of 0.6% is injected at the bottom of the plenum. The stream of air enters the chamber through an injection unit described in detail in a recent publication [6]. This unit comprises an air distributor leading to a tangential swirler with six channels. This element induces a clockwise rotation of the flow. The air and the spray of fuel are delivered to the combustor through a conical section having an 8 mm diameter outlet. A swirler, namely 716, is used in the present investigation. The swirl number determined experimentally, by integrating the velocity profiles at the outlet of the injector at a height of 2.5 mm above the backplane is  $S=0.70$  (refer to [6] for swirler characteristics). The burner is operated at a global equivalence ratio of  $\phi = 0.95$  which corresponds to an airflow rate of  $2.3 \text{ g s}^{-1}$  and a fuel flow rate of  $520 \text{ g h}^{-1}$ . The combustion chamber is formed by a fully transparent cylindrical quartz tube providing complete

optical access to the combustion zone. For examining SSO of the system, the chamber length  $l_c$  is varied to achieve different frequencies and amplitudes of longitudinal instabilities. During the measurement of FDFs, a chamber length of  $l_c = 150$  mm is chosen to operate the system under stable conditions. For these measurements, two driver units located at the bottom of SICCA-spray are excited to achieve different levels of fluctuations. At first, a frequency ramp at the rate of  $4.5 \text{ Hz s}^{-1}$  and at different amplifier voltages (0.5 to 3.5 V peak to peak) is performed to obtain the FDF as a function of amplitude and frequency of incident perturbations. Next, the driver units are modulated at the same frequencies as those of SSO and at an amplifier voltage that almost matches the level of relative fluctuations observed under limit cycle oscillations. When the system is operating under SSO, the driver units are left inactive. An estimate of heat release rate integrated over the flame volume is obtained by measuring the  $\text{OH}^*$  chemiluminescence (with a filter centered at 308 nm) from the flame using a photomultiplier tube (PMT). The mode of combustion in the system is quasi-premixed, a result demonstrated numerically in [7] and confirmed through experiments under steady conditions by [8], validating the use of  $\text{OH}^*$  chemiluminescence as a heat release rate indicator. The measurement of axial velocity fluctuations is achieved by laser Doppler anemometry (LDA) performed on the spray of heptane droplets. The system used for the LDA measurement is a Dantec FlowExplorer from Dantec Dynamics. The measurement of velocity at the injector outlet is carried out at a height of 2.5 mm from the backplane and at a distance of 4 mm from the injector axis.

### 3 Flame dynamics

Before examining the FDF and the flame response in terms of gain and phase, it is natural to compare the flame dynamics under SSO and under OLM. The  $\text{OH}^*$  chemiluminescence images are presented in Fig. 2. The quartz tube for the unstable case is 265 mm long and the limit cycle features a frequency of 533 Hz. The amplitude of velocity oscillation at  $(r, z) = (4.0, 2.5)$  mm is  $u'/\bar{u} = 9\%$  (RMS), that of the chemiluminescence signal is  $I'_{\text{OH}^*}/\overline{I_{\text{OH}^*}} = 28.9\%$  (RMS). The flame dynamics under open loop forcing is examined in Fig. 2(b-d). In Fig. 2(b), directly underneath of the SSO images, the forcing level is set to match the conditions encountered under SSO. In Fig. 2(c), the forcing level is significantly lower, whilst in Fig. 2(d) it is significantly higher.

The images are captured by a PiMax intensified camera equipped with a Nikon 105 mm UV lens and an Asahi optical bandpass filter (310 nm, emission of  $\text{OH}^*$  radicals in the flame). The camera is triggered with respect to the instability using the PM signal which is low pass filtered with a cut-off frequency of 800 Hz using an analog filter. This improves triggering by reducing the jitter present in the triggering signal. A Tektronix TBS 2000 oscilloscope provides a trigger signal when the filtered PM signal reaches its mean value and is rising. This setup is used to obtain the phase averaged flame images shown in Fig. 2. The exposure is 40  $\mu\text{s}$  long. The images of Fig. 2 are averaged over 1000 individual samples and processed with an Abel inversion algorithm.

The data show a close match between the SSO and OLM flame shapes and intensity when the forcing level of OLM matches the oscillation level of SSO. Only minor differences are observed in the first three phases of the cycle. Although the forcing amplitude has a large impact on the integrated intensity fluctuation level, the changes in flame shape and flame motion are modest: it does not appear that strong acoustic forcing significantly affects flame dynamics at this frequency. The motion of this complex “M”-shaped turbulent spray swirled flame shares some features with simpler laminar “M”-flames, which were studied under intense OLM by Durox et al. [9]. A periodic elongation and widening is also visible, in the present case between phase instants  $\Phi = 3\pi/2$  and  $\pi/2$ , the latter corresponding to the broadest flame during the cycle.

### 4 Comparison of FDF and flame response under limit cycle oscillations

One may now compare the FDF  $\mathcal{F}$  and the flame response  $\mathcal{H}$  as shown in Fig. 3. The measurement under OLM is first performed with a frequency ramp at different levels of amplifier voltages fed to the two driver units. For the measurements under SSO, the chamber length  $l_c$  is varied to achieve the oscillations at different frequencies. Chamber lengths of 215, 250, and 300 mm are used to attain the oscillations at frequencies 606, 558, and 474 Hz respectively. Figure 3(a) & (b) shows the

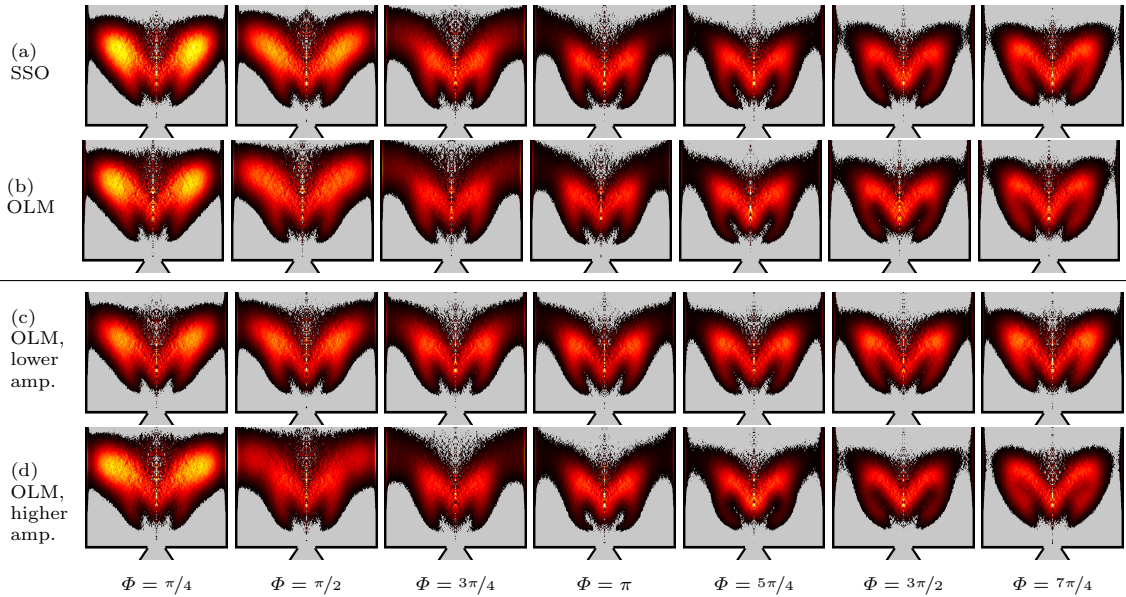


Fig. 2: Phase averaged Abel transformed flame images. Light intensity of  $\text{OH}^*$  chemiluminescence. Seven regularly spaced phase values. (a) Images obtained under self-sustained oscillations (SSO) at a frequency  $f_0 = 533$  Hz.  $I'_{\text{OH}^*}/\overline{I_{\text{OH}^*}} = 28.9\%$  (RMS). (b-d) Images corresponding to open loop modulation at the frequency determined under SSO. (b) The forcing amplitude matches that observed during the self-sustained oscillation ( $I'_{\text{OH}^*}/\overline{I_{\text{OH}^*}} = 30.4\%$  (RMS)). (c) The forcing amplitude is lower than that observed during the self-sustained oscillation ( $I'_{\text{OH}^*}/\overline{I_{\text{OH}^*}} = 14.2\%$  (RMS)). (d) The forcing amplitude exceeds that observed during the self-sustained oscillation ( $I'_{\text{OH}^*}/\overline{I_{\text{OH}^*}} = 37.8\%$  (RMS)).

evolution of FDF with respect to the amplitude and frequency of oscillations. The red diamonds correspond to the gain and phase of  $\mathcal{H}$  (SSO), whereas the different color levels represent the gain and phase of  $\mathcal{F}$  (OLM) at different amplifier voltage levels. The phase does not exhibit any discernible nonlinearity with respect to the level of fluctuation. On comparing the gain, one may notice the presence of nonlinearity with respect to the lowest amplifier voltage at 0.5 V but the velocity fluctuations are quite weak ( $\approx 4\%$ ) and the measured signals are noisy. The gain at higher voltages (1 V and above) remain close together with some difference at higher frequencies. Figure 3(c) shows the comparison of gain and phase only at the frequency of oscillation and at 3.5 V. These measurements, under OLM, are carried out by modulating at the frequency corresponding to SSO at 3.5 V. The data for SSO is the same as in Fig. 3(a) & (b). The corresponding values for heat release rate and velocity fluctuations are shown in Fig. 3(d). It is interesting to observe that the phases of  $\mathcal{H}$  and  $\mathcal{F}$  match quite well. The role of phase on stability analyses (see, for example [3]) is highly critical and the experimentally observed match validates the usage of FTF/FDF in reduced order models. On comparing the gains between  $\mathcal{H}$  and  $\mathcal{F}$ , at 606 Hz and 558 Hz, a reasonable match can be seen. There is a moderate difference ( $\approx 23\%$ ) at 474 Hz even though the level of velocity fluctuation is almost the same between SSO and OLM and the reasoning for this requires further investigation.

## 5 Conclusion

Although many theoretical models in combustion instability rely on transfer function or describing concepts, it was important to see if these concepts are effectively applicable. This central point is investigated by comparing two situations, the first of which corresponds to a well established limit cycle self-sustained oscillation (SSO) while in the second, the flame is externally modulated (namely, open loop modulation or OLM). It is shown that the flame dynamics observed using  $\text{OH}^*$  light intensity Abel transformed images are best matched when the level of acoustic oscillation in the two situations are equal. It is then found that the gain and phase of the flame describing

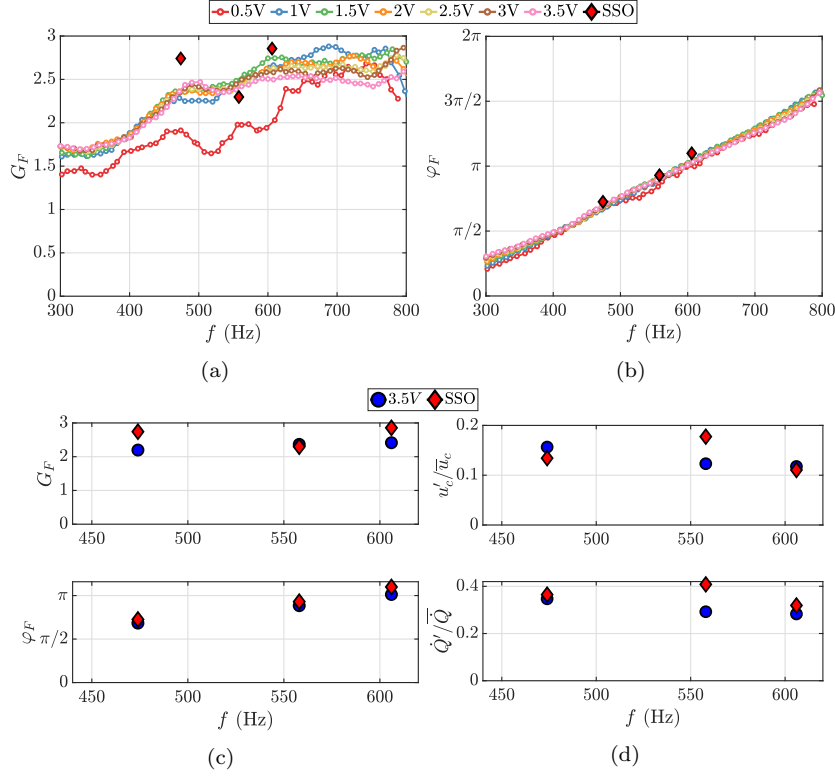


Fig. 3: (a) Gain of  $\mathcal{F}$  and modulus of  $\mathcal{H}$ , (b) Phase of  $\mathcal{F}$  and argument of  $\mathcal{H}$  plotted as a function of the amplitude and frequency of oscillation. The red diamonds on the plot represent  $\mathcal{H}$  during SSO. The different color levels on the plots (a) & (b) represent the FDF  $\mathcal{F}$  during OLM obtained with a frequency ramp at different levels of amplifier voltages. In (c), the results are shown at three frequencies for SSO (red diamond) and the corresponding values during OLM (blue circle) obtained by modulating at the specific frequency of SSO (instead of a frequency ramp) and at an amplifier voltage of 3.5 V. (d) Fluctuation levels of heat release rate (integrated over the flame volume), and axial velocity (measured at  $(r, z) = (4.0, 2.5)$  mm) at the injector outlet.

function are also close to those of the flame response measured under SSO when the level of oscillation in the externally modulated flame equals that found under self-sustained oscillation. These two elements confirm that the FDF framework is applicable and that it is important to take into account the dependence of the flame response on the level of incident perturbations. These experiments provide a direct validation of this concept in the analysis of combustion instabilities leading to limit cycle oscillations.

**Acknowledgements** This work benefited from the support of project FASMIC ANR16-CE22-0013 of the French National Research Agency (ANR) and by the European Union’s Horizon 2020 research and innovation programme, Annulight with grant agreement no. 765998.

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