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Aircraft Multidisciplinary Design Optimization Under Both Model and Design Variables Uncertainty

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Low-fidelity analytical models are often used at the conceptual aircraft design stage. Because of uncertainties on these models and their corresponding input variables, deterministic optimization may achieve under-design or over-design. Therefore it is important to already consider these uncertainties at the conceptual design stage in order to avoid inefficient design and then costly time over runs due to re-design. This paper presents a procedure for reliable and robust optimization of an aircraft at the conceptual design phase. Uncertainties on model and design variables are taken into account in a probabilistic setting. More precisely, at each point of the optimization process uncertainties are modeled by an adaptive normal law strategy in order to fit the historical aircraft database. The statistical parameters are adjusted depending on the available information at the current point of the optimization process. To improve computational cost, response surface approximations are constructed to represent reliability constraints. The developed methodology is applied to the conceptual design of a short range aircraft. Compared to standard deterministic optimization without design margins, the result shows a modest increase on weight, which allows however to ensure a desired reliability and robustness of the design compared to the unreliable and sensitive deterministic optimum.

Nomenclature

A_{pref}	=	target cruise altitude
A_r	=	wing aspect ratio
C_{buf}	=	minimum altitude for buffeting ceiling
C_{clb}	=	minimum altitude for climb ceiling
C_{crz}	=	minimum altitude for cruise ceiling
C_{oei}	=	minimum altitude for one engine inoperative ceiling
C_{tk}	=	central tank fuel
F_{margin}	=	difference between the amount of fuel required for a nominal mission and the fuel capacity
h_{fus}	=	fuselage height
L_{Tofl}	=	maximum take off field length
l_{fus}	=	fuselage length
N_e	=	number of engines
N_{pax}	=	total number of passengers
R	=	nominal range
R_{BPR}	=	engine by pass ratio
S_{wing}	=	wing area
V_{app}	=	maximum approach speed
V_{Cruise}	=	Cruise number
wA_{fus}	=	fuselage gross wetted area
w_{fus}	=	fuselage width

I. Introduction

THE conceptual aircraft design is a multidisciplinary optimization problem which consists of finding a set of parameters defining the performances subjected to various constraints [1–3]. At this stage of the design, knowledge is limited (for example on material properties, weights data, propulsion data, etc.), the analysis is thus based on low-fidelity models from empirical equations or simplified physical models [4]. In this context, modeling the uncertainties is important because at this stage deterministic optimization can lead to over-design (if arbitrary design margins are taken into account) or under-design (without design margins) during detailed phases of the design [5,6]. Efficient uncertainty modeling allows us to construct a robust and reliable design, thus reducing the time and cost of engineering development.

Conceptual aircraft design involves multiple disciplines (propulsion, aerodynamics, structure, etc.) that can be closely linked: the inputs of a discipline may be the output of another one and vice versa, thus creating feedback loops [7,8]. Multidisciplinary optimization (MDO) has been the subject of various studies [9–11]. It has been shown that these methods can significantly enhance conceptual aircraft optimizations, notably by taking into account the link between the different disciplines. These MDO methods are more efficient than successive monodisciplinary design optimizations, avoiding multiple loops between the different disciplines (decrease computation time), by simultaneously manipulating variables in every disciplines of the problem and determining rapidly the best design [12]. However, conventional methods of aircraft multidisciplinary design optimization are deterministic and any uncertainties introduced by manipulating simplified analytical models are neglected or taken into account by arbitrary safety factors (which consist of replacing the random data by more reliable values than the means). Thus, it is necessary to develop specific methods to propagate uncertainty in a conceptual aircraft sizing problem to ensure both reliability and robustness. Early robust approaches aimed at minimizing a dual-objective optimization problem. Generally, the objective is to minimize the mean value of a performance while reducing or controlling the variance. Green et al. proposed a single objective problem, introducing weights to obtain a compromise between variance and expectation [13]. Delaurentis and Mavris proposes a probabilistic method [8]. This technique uses a measure of the probability of success which is based on the cumulative distribution function (CDF) of the objective function. This method also uses metamodels to minimize the number of functions evaluations (objective or constraints) usually required in a robust optimization problem.

Recent works have been conducted to develop techniques to take into account the uncertainties in conceptual aircraft design. In this context Daskilewicz et al. [14] highlight the impact of disciplinary

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uncertainties. They show that the impact of uncertainties on the optimal aircraft design depends on the amplitude of the uncertainties and on the location of the design parameters in the search space. These conclusions are based on the study of Pareto fronts, visualized for different scenarios of uncertainty (variation of the mean and of the standard deviation of the modeling uncertainties). The authors do not define an optimal design but provide elements for the designer to find regions of the search space where a compromise is established between robustness and performance.

In their work, Neufeld et al. [15] have developed reliability-based design optimization methods to design an aircraft that is both robust and reliable. Their techniques allow taking into account the modeling uncertainties and quantifying the reliability of the conceptual design. Modeling uncertainties are determined from a historical database of the different parameters for commercial aircraft. According to the available information, the modeling uncertainties are modeled by a uniform probability law or by a normal probability law. The mean and standard deviation characterizing this model uncertainty is constant over the entire range of the design variables.

To continue improving the approaches of conceptual aircraft design under uncertainty, further developments appear worthwhile to adjust the magnitude of the uncertainty to the design region where it is used. Indeed, the setup of normal uncertainties with constant standard deviation may in some regions not reflect the real accuracy of analytical models and can cause a loss of performance (the uncertainties may sometimes be lower in some regions and higher in others). Thus, we propose in the present article a novel approach to considering model uncertainties in conceptual aircraft design. The approach is based on the construction of the uncertainty distributions whose standard deviation is variable and adapted to the current point of the optimization.

This article is organized as follows. In Sec. II, we will describe the conceptual aircraft sizing problem. First, we will present the deterministic formulation and then the formulation of the problem under uncertainties. In Sec. III, we will explain the method of sensitivity analysis to determine the relevant uncertain variables causing the greatest sensitivity of the objective function and/or constraints. Once these relevant uncertain variables determined, we will detail in Sec. IV the probabilistic method that has been developed. This novel method takes into account the modeling uncertainties and the uncertain design variables. It is based on the development of adaptive modeling of the uncertainty: the value of the standard deviation of the uncertainty distribution is adjusted to the available information. It reflects the fact that in different parts of the model the uncertainty can be quite different. Because the conceptual aircraft design is a complex, computationally expensive problem, response surfaces approximations are constructed based on Monte Carlo simulations to represent the reliability constraints at low computational cost. The construction of these surrogate models will be presented in Sec. V. We will then examine the different results by comparing the deterministic optimization and the optimization under uncertainty. Moreover, in Sec. VI we take into account the variation between the conceptual and final design parameters and show their effect on optimization. Finally, in Sec. VI, conclusions and future work are presented.

II. Conceptual Aircraft Design

The role of the conceptual aircraft design is to propose aircraft configurations that can best meet a set of needs, then to identify several design alternatives [1]. The objective of this section is to present the deterministic aircraft multidisciplinary design optimization that is used to date. The conceptual design of a short-range aircraft (e.g., Airbus A320) will be considered, which allows us to define the main features of the aircraft. Conceptual sizing is the first step in aircraft design. It is a multidisciplinary optimization problem that consists of finding a set of parameters defining the aircraft. Performances that are subjected to constraints are also determined [7]. In this problem, the maximum takeoff weight of the aircraft must be minimized while satisfying various operating performance constraints g_i: the maximum approach speed, the maximum takeoff field length, flight ceilings (minimum altitude for climb ceiling, minimum altitude for cruise ceiling, minimum altitude for one engine inoperative ceiling, and the minimum altitude for buffeting ceiling) and the fuel capacity. This optimization problem is composed of several disciplines: geometry, propulsion, weights, operating performances, aerodynamics, etc. There are dozens of main parameters for the conceptual design. Among these parameters, the most influential ones are the wing area S_{wing} and the maximum engine thrust sea level static thrust (*SLST*), which are thus defined as the optimization variables. The conceptual sizing is a step in the overall aircraft design, which turns out to be crucial but complex for several reasons: decisions are taken in a context where few features of the plane are defined and the problem data are not yet very well known, the search space solutions must remain general enough not to exclude potential solutions, and finally the models used are often relatively rough. At this stage of the design, the uncertainties in the problem may be significant and are mainly modeling uncertainties due to the simplicity of the used models. Risk control becomes an important element of the project. Identifying risks and uncertainties from the beginning of the design process (i.e., conceptual aircraft design) will allow us to control the reliability and robustness of the design early on, thus preventing cost overruns due to iterations late in the design cycle and redesign. In this section, we present the deterministic and probabilistic formulation.

A. Deterministic Aircraft Multidisciplinary Design Optimization

Conceptual design, based on a comprehensive description of the plane and a set of performance requirements, is discussed when the overall configuration of the aircraft (location of engines, aspect ratio of the wings, etc.) is fixed (see Table 1, design assumption and performances). The problem is to define the right aircraft characteristics to meet the performances and thus to minimize our objective function, which is the mass of the airplane at takeoff (*MTOW*). A conceptual design framework called simple models for conceptual aircraft design (SIMCAD) is used for this purpose [16].

Table 1 Data for the conceptual aircraft design problem

Performances imposed		Design assumption		Constraints	
N_{pax} Total number of passengers	150	N_e Number of engines	2	F_{margin} difference between the amount of fuel required for a nominal mission and the fuel capacity (l); it is the fuel reserve for a nominal mission	≥ -2000
R Nominal range, km	5556	R_{BPR} BPR Engine by-pass ratio	6	C_{clb} Minimum altitude for climb ceiling, ft	$\geq 10,668$
V_{Cruise} Cruise Mach number	0.78	A_r Wing aspect ratio	9.5	C_{crz} Minimum altitude for cruise ceiling, ft	$\geq 10,668$
A_{pref} Target cruise altitude, m	10,668	C_{tk} Central tank fuel ($C_{tk} = 1$ to include a central tank, 0 otherwise)	1	C_{buf} Minimum altitude for buffeting ceiling, ft	$\geq 11,277.6$
		T_{tk} Tail tank fuel ($T_{tk} = 1$ to include a central tank, 0 otherwise)	1	C_{oei} Minimum altitude for one engine inoperative ceiling, ft	≥ 5791.2
				L_{Tofl} Maximum takeoff field length, m	≤ 2100
				V_{app} Maximum approach speed, kt	≤ 41.4528

This approach allows us to build the objective function *MTOW* from the assembly of disciplinary models. The SIMCAD algorithm is in line with a multidisciplinary context in which the different disciplines are provided in the following list:

1) Environment: determination of the characteristics of the environment in which the aircraft is evolving (air density, sound velocity, Mach number, air pressure, etc.).

2) Fuselage: determination of the fuselage geometry (the width, the length, and the fuselage gross wetted area), the mass, the fuselage shape, and the drag coefficient. The general size and shape of the fuselage are determined according to the payload.

3) Wing: determination of the wing geometry (area, wing gross wetted area, wing fuel tank capacity, and wing sweep angle), the mass and aerodynamics of the wing (length, drag coefficient, wing-induced drag, compressibility drag, buffeting lift, divergence Mach number, etc.).

4) Propulsion: determination of the engine nacelle geometry (the nacelle design must satisfy both the geometric constraints and the engine requirements), aerodynamics (friction coefficient), the pylon mass, and the engine thermodynamics (fuel consumption, engine thrust, etc.).

5) Tail: determination of the tail geometry (fuel capacity and area are determined for both the horizontal and vertical parts of the tail, volume, etc.), the masses (mass of the horizontal and vertical part of the tail), and aerodynamic coefficients.

6) Landing gear: calculation of the number of wheels and the landing gear mass.

7) Cabin and cargo: determination of the passenger number in the different classes and the total number of passengers. Mass calculation (payload, furnishing mass, operator item mass, and container and pallet masses).

8) Plane: determination of the design parameters (wing surface and engine size), the objective function (maximum takeoff weight), mass (weight of the aircraft without fuel, mission payload, etc.), aerodynamics (pressure drag, induced drag, drag coefficient, maximum lift at landing, etc.) and the cost of the mission.

9) Performance: determination of the different constraints and the performances, which are listed in Table 1.

The environment block allows us to simulate the influence of external factors: gravity, atmosphere, pressure, etc. The blocks fuselage, wing, propulsion, tail, landing gear, and cargo are used to calculate certain parameters such as the weight, the width, etc. of the corresponding component (fuselage, etc.). The aircraft discipline and the performances discipline are used to obtain general information about the plane: its operational weight when it is empty and its aerodynamic performance. These models are used to calculate a typical configuration of an aircraft from any inputs in terms of required performances and only by running a loop of calculations called "mass-mission." This loop is nested in the constrained optimization algorithm, and so it allows us to model the constraints imposed by the process weight/performance and to provide the set of parameters satisfying all the disciplines.

This design process is highly interdependent: to achieve a mission, a certain amount of fuel is needed, but this quantity affects the maximum takeoff weight, the structure of the aircraft, and therefore the mission. These constraints require a model that allows us to evaluate the mass and know the range of the aircraft, and vice versa, i.e., a model that allows us to evaluate the mission (and thus the range) to know the mass of the aircraft. Thus, one gets a feedback loop. The nature of the mission (reflected by the range) is both a calculation hypothesis to provide as an input to the function and an evaluation result. Therefore, the system is consistent if the value of the range provided in input is equal to the output value obtained by the algorithm. This constraint is ensured by the "mass-mission" loop (see Fig. 1).

During the complete design of the aircraft, the number of parameters involved in the various disciplinary models is very high, and it is thus necessary to start the optimization with a simplified aircraft model. This is what is done during conceptual design, in which all models are analytical models derived from either simplified physical models or simple regression models based on aircraft databases.

In the SIMCAD program, some models that allow us to determine the system's parameters are derived from simplified physical models. For example, the sound velocity is calculated assuming that air is a perfect gas. The specific parameters of the environment block, such as the standard temperature and pressure, are also determined thanks to simplified physical models. All the other models are derived from simple regression models based on aircraft databases. Measurements were made on 47 commercial aircrafts (aircrafts built in recent decades) that lead to a historical database. An example of such a regression model is given after, with the fuselage gross wetted area, wA_{fus}. The regression tries to make a simple model (linear or quadratic) fitting an aircraft parameter, a performance or an operational characteristic, y, based on a number of other parameters, x. These parameters, x, are either the aircraft parameters directly or a combination of parameters resulting from physical considerations. The models that describe the physical aircraft behavior are represented by a linear regression function:

$$Y = \beta_0 + \beta_1 X(1) + \beta_2 X(2) + \ldots + \beta_p X(p) \tag{1}$$

where Y is the output and $X(1), X(2), \ldots, X(p)$ are the input parameters. Note, however, that each input parameter can be a nonlinear combination of others variables characterizing the aircraft. For example variable $X(1)$ can be modelled as $X(1) = X_1 \times X_2$ where X_1 and X_2 are some other variables characterizing the system. To illustrate our explanations, we take the example of the wA_{fus} parameter (fuselage gross wetted area).

A linear model that allows us to evaluate wA_{fus} is deduced from the linear regression:

$$wA_{fus} = \beta_1 \sqrt{w_{fus} \times h_{fus}} \times l_{fus} + \beta_0 \tag{2}$$

This model depends on the variable $x = l_{fus}\sqrt{w_{fus} \times h_{fus}}$ where l_{fus} is the fuselage length, w_{fus} is the fuselage width and h_{fus} is the fuselage height (x is in m^2): $wA_{fus} = 2.3x + 66.6$ (see Fig. 2). The coefficients β_i are the coefficients determined by the linear regression.

Among the small group of high-level parameters that are handled during the conceptual aircraft design, three have a significant impact on the design of the aircraft: the wing area (S_{wing}), the engine size (guided by the variable *SLSThrust*: engine thrust at sea level), and the maximum takeoff weight (*MTOW*). This study will focus on determining these three parameters for an aircraft configuration. The deterministic process is based on a classical constrained optimization (where the constraints are a set of high-level requirements on aircraft). There are only two design variables: the wing surface and the engine size, which are the main variables of our optimization problem. These design variables associated with the fixed parameters (design assumption and performances) allow us to determine the overall parameters describing the aircraft (160 parameters). The formulation of the deterministic optimization problem based solely on these two design parameters is the formulation advocated by Airbus as part of this research project [16]. However, the methodology can be straightforwardly extended to more optimization design variables. The objective function f and the constraints g_i are constructed based on analytical models as described above [16,17]. For some given design assumptions and functional characteristics of the product (Top level aircraft requirements) the optimization formulation can be written as:

$$\begin{aligned} &\mathrm{Find}(S^*_{\mathrm{wing}}, \mathrm{SLST}^*) \text{ solution of} \\ &\min_{S_{\mathrm{wing}},\mathrm{Slst}} \mathrm{MTOW} = f(S_{\mathrm{wing}}, \mathrm{SLST}) \\ &\text{where } S_{\mathrm{wing}}, \mathrm{SLST} \in S,\ S \text{ being the search space} \\ &\text{subjected to: } g_i(S_{\mathrm{wing}}, \mathrm{SLST}) \le c_i,\ i = 1, \ldots, 7 \end{aligned} \tag{3}$$

where the operational performance constraints, g_i, are the maximum approach speed, the maximum takeoff field length, the flight ceilings (minimum altitude for climb ceiling, minimum altitude for cruise

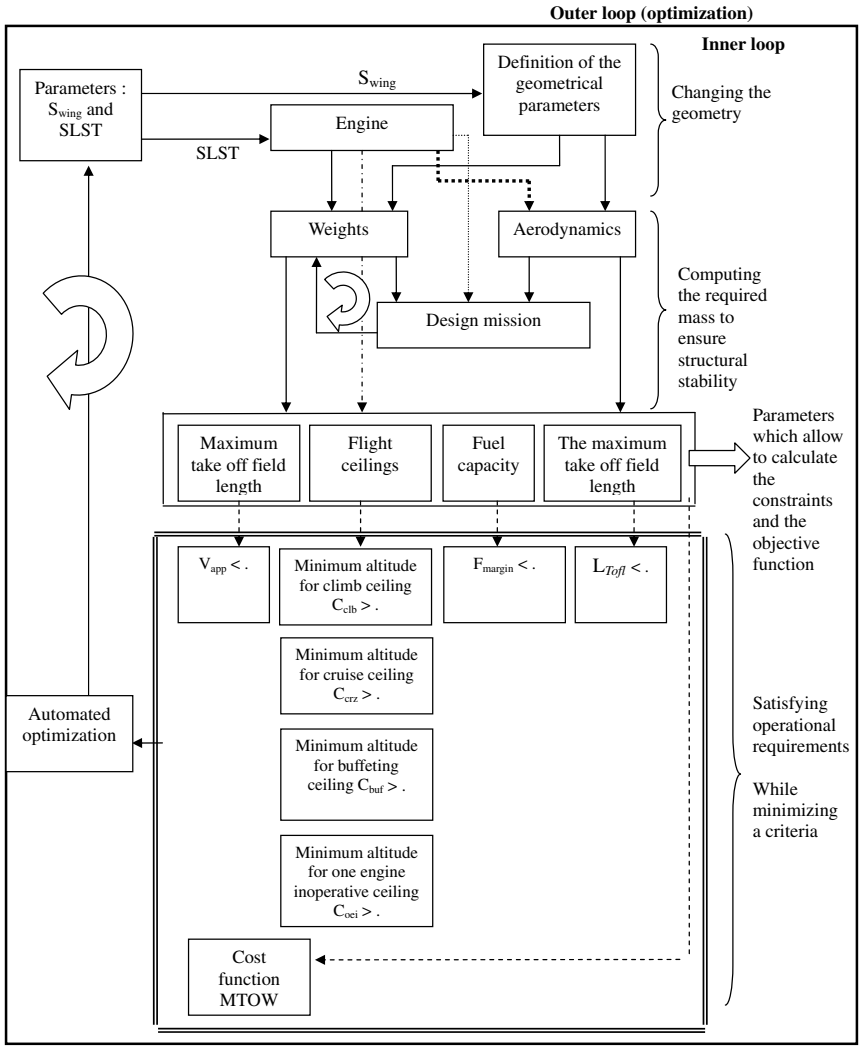

Fig. 1 Description of the conceptual aircraft design test case problem SIMCAD.

ceiling, minimum altitude for one engine inoperative ceiling, and the minimum altitude for buffeting ceiling), and the fuel capacity (see Table 1).

The various steps in the calculation used to define a typical configuration of an aircraft from any arbitrary input data desired are listed below (see Fig. 1, inner loop). Note that in Fig. 1, the arrows represent the interdependencies between the different disciplines, the exchange of variables (a model output parameter is the input parameter of another model). The small loop represents the mass-mission loop and the largest one represents the optimization process:

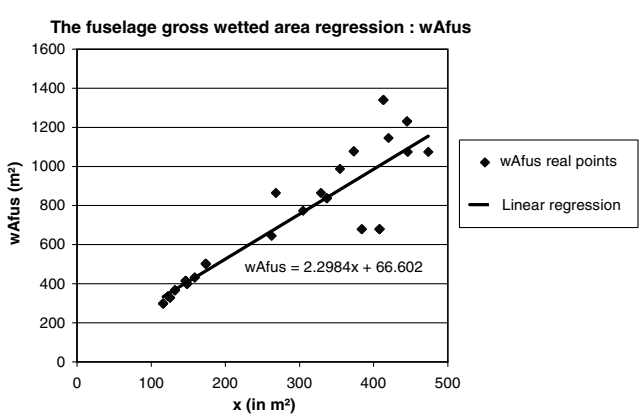

Fig. 2 Example $wA_{fus} = f(x)$.

1) Design assumption and performances are decided (the performances become the operational constraints that one wishes to meet during the optimization calculation, see Fig. 1). The design assumptions and the performances (top level aircraft requirements or functional characteristics of the designed aircraft) are fixed input parameters, which are not modified during the optimization process. These data provide the context of the study, and they allow us to characterize the aircraft and so to define the fuselage, the propulsion, the geometry, and the flight conditions.

2) Calculations are made of the different parameters and the aircraft performances for each discipline by using simple analytical models (these disciplines are represented by a box inside the inner loop of Fig. 1). The set of parameters that define the conceptual aircraft design (160 parameters) are computed from the input data set of the optimization problem: the design assumptions, the performance, and also the design parameters. Therefore, there are no presumed design assumptions about other parameters.

3) Once the parameters are determined, the "mass-mission" loop is executed for obtaining the parameters satisfying all disciplines (this is the loop between the "Weights" box and the "Design mission" box of the inner loop of Fig. 1).

4) Following the "mass-mission" loop, the disciplines that are impacted by this coupling are recalculated.

5) The aircraft takeoff weight *MTOW* is determined.

6) This calculation is then embedded in an optimization loop to solve the problem of Eq. (1) and determine the configuration of the aircraft with the minimum mass that also meets the constraints that have been set.

The optimization algorithm is of pattern search type [18], which is an exploration method.

In the following paragraph, an application of the described approach is presented for the deterministic design optimization of a short-range type aircraft (e.g., A320). The design assumptions and the operational performances of the considered problem are given in Table 1.

Firstly, one calculates the optimum of the problem without uncertainties. Thus, Eq. (1) is rewritten as

$$\begin{aligned} &\min_{S_{\text{wing}},\text{Slst}} \text{MTOW}(S_{\text{wing}}, \text{SLST}) \\ &\text{Where } S_{\text{wing}}, \text{SLST} \in S, S \text{ the search space} \\ &\text{subjected to: } g_i(S_{\text{wing}}, \text{SLST}) \leq 0, i = 1, \ldots, 7 \end{aligned} \tag{4}$$

The deterministic optimum found for this problem has the following parameters: $\text{MTOW} = 75,415$ kg for a wing area $S_{\text{wing}} = 129.35\ \text{m}^2$ and an engine thrust $\text{SLST} = 89,474$ N (see Fig. 3). In Fig. 3, the takeoff weight of the aircraft, *MTOW*, is represented in the background by the color map: the darker the background, the higher the mass. The optimum is indicated by a black dot in Fig. 3. The active constraints at the optimum are the minimum altitude for climb and the minimum altitude for buffeting (these constraints operate as equalities at the optimum point). Note that only the active constraints are represented in Fig. 3.

The problem of conceptual aircraft sizing is characterized by limited knowledge and important uncertainty. To develop effective conceptual design that is both robust and reliable and that fares well in later design phases, there is a need to develop a probabilistic optimization approach taking into account various uncertainties present in the problem.

Uncertainty can manifest itself under multiple forms. A distinction is usually made between aleatory uncertainty and epistemic uncertainty [19]. Aleatory uncertainty is also known as stochastic uncertainty, irreducible uncertainty, inherent uncertainty, variability, or type I uncertainty. It can stem from environmental stochasticity, inhomogeneity of materials, fluctuations in time, variation in space, heterogeneity, or other intrinsic differences in the features of a system. Epistemic uncertainty on the other hand, sometimes called reducible uncertainty or type II uncertainty, stems from a lack of knowledge. This kind of uncertainty is usually related to scientific ignorance, measurement uncertainty (e.g., sensor uncertainty), insufficient experimental data, phenomena inobservability, or censoring, thus in general terms, lack of knowledge. Note that after reducing the epistemic uncertainty, aleatory uncertainty may still remain and become the preponderant, irreducible uncertainty.

The theory of probabilities has historically provided the first framework for modeling and quantifying uncertainty [20]. Currently, it is generally agreed that aleatory uncertainty can be appropriately handled by the theory of probabilities. The probability method is the most mature due to the large number of developments it has benefited from. It is also the best known in industry. While the probabilistic approach can also be used to model epistemic uncertainty, other alternative representations for this type of uncertainty are also available. With interval analysis [21], the uncertain variable is described as an interval and the width represents the extent of the uncertainty. This is especially useful in cases where we lack information for determining an exact probability distribution that accurately represents the reality and when only the worst case needs to be considered. While epistemic uncertainty was quantified by bounds in interval analysis, there are often cases where more information is available than just bounds. The information provided by the expert can be modeled by fuzzy set theory [22,23]. For example, when asking an expert what he can tell about a quantity x, he may be able to provide approximate bounds on a quantity. In [24], the authors worked on a method coupling both aleatory and epistemic uncertainties to develop fuzzy sets for improved pattern recognition and decision making. Specifically, they address the challenge of using both dense and sparse data. Finally, possibility theory [25] and evidence theory [26] provide bounds on the actual unknown probability. This can be very useful in situations where, due to lack of knowledge, we cannot derive accurate probability density functions. Note that both evidence and possibility theories provide a bounding of the actual probability by belief and plausibility (evidence theory) and necessity and possibility (possibility theory). However, because belief and plausibility can be simultaneously different from 0 and 1, the bounding in evidence theory can be much tighter, and thus less conservative, than in possibility theory.

In this paper we chose to model the uncertainties entirely by probabilistic approaches for two main reasons: one, probability theory is the approach that has benefited from the largest number of theoretical developments, and two, compared to alternative approaches it is also the best known in industry. Next, we will present the probabilistic optimization approach developed.

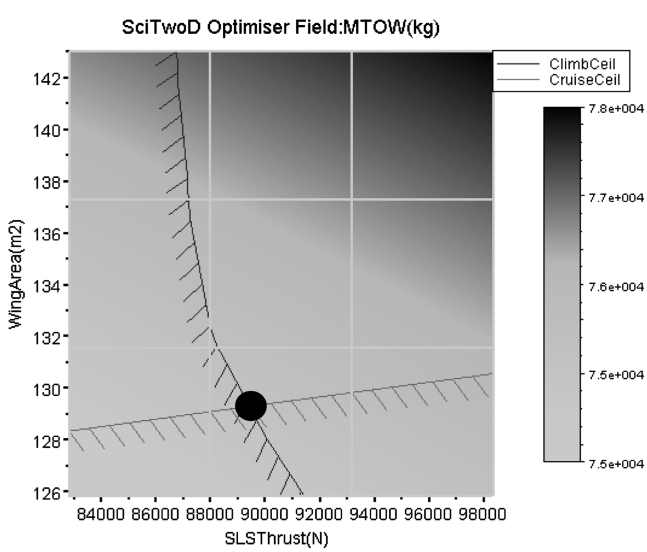

Fig. 3 Deterministic optimization.

B. Probabilistic Aircraft Multidisciplinary Design Optimization

The optimization under uncertainty aims at designing a system insensitive to variations, having statistically the best performances and satisfying a given reliability level. To do this, the deterministic problem is modified, to include constraints of robustness and reliability as follows [17,27]:

$$\begin{aligned} &\min_{S_{\text{wing}},\text{Slst}} E(\text{MTOW}(S_{\text{wing}}, \text{SLST}, \delta)) \text{with } S_{\text{wing}}, \text{SLST} \in S \\ &\text{such as:} \\ &\text{COV}(\text{MTOW}(S_{\text{wing}}, \text{SLST}, \delta)) \leq \alpha \text{ and} \\ &\text{Prob}(g_i(S_{\text{wing}}, \text{SLST}) \leq 0) \geq \alpha_i^g \text{ with } i = 1, \ldots, 7 \end{aligned} \tag{5}$$

where δ represents the uncertainties involved in the problem and α is the variation tolerated on the coefficient of variation (COV), representing the robustness constraint. In practice, an acceptable α is taken as 2% here, and $(1 - \alpha_i^g)$ is the risk of violating the constraints provided in Table 2. The imposed reliability thresholds are based on the risk that the manufacturer is willing to take at the conceptual aircraft design. In this article, these are the ones desired by Airbus at the end of the conceptual aircraft design stage. Taking into account uncertainties allows us to introduce margins to ensure robustness and reliability.

Note that because the COV is the ratio of the standard deviation to the mean, the higher the value of the COV, the greater the dispersion around the mean. Being dimensionless, it allows for the comparison of different distributions when the scales of the uncertain parameters are not comparable. We deal with physical parameters of very different scales and the COV is a relative value, so it is found to be more suitable than the standard deviation. Because for this probabilistic optimization problem a pattern search algorithm was not efficient enough, we used a search algorithm based on the simplex method (Nelder–Mead).

Table 2 Ceiling values not to exceed

Constraint	Minimum reliability of constraint satisfaction (%)
Maximum approach speed	98
Maximum takeoff field length	90
Minimum altitude for climb	95
Minimum altitude for cruise	90
Minimum altitude for buffeting	90
Minimum altitude for one engine inoperative	95
Fuel capacity	90

Note that it would be possible to make a robust optimization based on a weighting of mean and variance, but the aim here is to minimize the performance of the system while controlling the risk [28,29].

In our problem, we found the objective function was close to linearity with respect to the uncertain parameters and because the uncertainties are centered, the equality $E(\mathrm{MTOW}(S_{\mathrm{wing}}, \mathrm{SLST}, \delta)) = \mathrm{MTOW}(S_{\mathrm{wing}}, \mathrm{SLST})$ is approximately verified, which allows us to simplify the calculation of the objective function. Moreover, this approximation was verified by experiments: for each step of the optimization, the calculations $E(\mathrm{MTOW}(x))$ and $\mathrm{MTOW}(E(x))$ were carried out and the mean relative error between the two terms was found to be less than 0.0006%.

Carrying out the optimization with all the uncertain parameters involved in the problem is possible but computationally very expensive, whereas some parameters would have only minor effect. Thus, in the next section we will implement a sensitivity analysis that aims to describe how much system output values (objective function and constraints) are affected by changes in model input values. This study allows us to determine the model inputs with the highest influence in the modeling process, on which we will implement uncertainties.

III. Selection of Relevant Modeling Uncertainties

The conceptual aircraft design problem involves more than 160 parameters that are the outputs of simple analytical models (note that a model output parameter is often the input parameter of another model). Therefore, all of the parameters can potentially have uncertainty. Assuming that uncertainties would be introduced on each of these parameters, the evaluation of the constraints would become very costly in computing time. In addition, all the uncertainties are not significant and some of them can be eliminated. Prevalent uncertain parameters have to be chosen to reduce the number of uncertain parameters and also the number of simulations in the optimization process.

To do this, selection of a local sensitivity analysis around the deterministic optimum point is performed. The goal is to seek information about the impact of the change of a certain factor X_i at a local level, i.e., in the vicinity of the deterministic optimum value X_0. In a local sensitivity analysis, we make the assumption that the local behavior is enough to characterize the effect of the variable over the entire search space. Because the conceptual design models are almost linear, local sensitivity gives significant quantitative information of the influence of each parameter.

The aim of sensitivity analysis is to estimate the rate of change in the output of a model with respect to changes in model inputs. To do this, the impact of each uncertain parameter on the objective function and on the constraints is measured by the formula [30]

$$S_i = \frac{x_i^{\mathrm{opt}}}{y^{\mathrm{opt}}} \frac{\partial f}{\partial X_i}\bigg|_{X_i = x_i^{\mathrm{opt}}} \tag{6}$$

where S_i is the sensitivity of the variable x_i on the function $f(x) = y$, where f may be the objective function (aircraft mass) or any of the constraints. The partial derivative is calculated by finite difference $\Delta X = 1/100 x_i^{\mathrm{opt}}$. Note that the calculation of the sensitivity is dimensionless and that it is normalized to obtain an objective value (one estimates what is the value of the percentage variation of the objective function and constraints when one considers an uncertain parameter X varying by a certain percentage). The model parameters are thus classified according to their degree of influence on the variability of the objective function and on the constraints. Sensitivity analysis on a short-range aircraft leads to 15 significant parameters (less than 10% of the initial parameters) that are the parameters that cause the greatest sensitivities (in %) of the objective function and/or constraints. These parameters are the uncertain parameters of our optimization under uncertainty. They are: the fuselage section width, the fuselage section height, the total fuselage length, the wing mean aerodynamic chord, the fuel volume in the wing, the fuselage gross wetted area, the wing gross wetted area, the maximum lift factor in takeoff configuration, the maximum lift factor in landing configuration, the maximum equipment weight, the operational empty weight, the payload of nominal mission, the maximum zero fuel weight, the approach speed for landing case 1, and the range of nominal mission.

Once the significant uncertain parameters are defined, uncertainty quantification is an important challenge in multidisciplinary optimization. This problem will be addressed with both model and design variables in the next section.

IV. New Approach for Uncertainty Quantification

To consider the effect of uncertainties on the physical phenomenon, we opted for a probabilistic modeling. Each model output variable of the conceptual aircraft design problem is supposed to be expressed as follows, to include the modeling uncertainty:

$$Y = \beta_0 + \beta_1 X(1) + \beta_2 X(2) + \ldots + \beta_p X(p) + \varepsilon \tag{7}$$

where Y is the model output related to the explanatory variables $X(i)$ through the coefficients β. Y is also considered to be one of the uncertain variables of the optimization problem and the uncertainty, ε, represents modeling uncertainty due to the relative simplicity of the model form. This uncertainty is modeled by the random variable, ε, following a normal distribution with mean 0, so that $E(Y) = \beta_0 + \beta_1 X(1) + \beta_2 X(2) + \ldots + \beta_p X(p)$. In general terms, this is a linear regression problem for which the parameters β are obtained by minimizing the sum of the residual squared:

$$L = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} (y_i - \beta_0 - \beta_j X_{ij})^2 \tag{8}$$

To minimize the L function, the partial derivatives of this function with respect to the parameters β are set equal to 0:

$$\frac{\partial L}{\partial \beta_j}\bigg|_{b_0, b_1, \ldots, b_p} = -2 \sum_{i=1}^{n} (y_i - \beta_0 - \beta_j X_{ij}) X_{ij} = 0 \tag{9}$$

In matrix form this can be written as

$$\beta = (X'X)^{-1} X'y \tag{10}$$

with X the X_{ij} matrix and y the response vector at the point of the design of experiments.

The modeling uncertainties that are considered are due to the fact that there is a difference between the real points stemming from the historical database and the model predictions stemming from the linear regression model. The corresponding errors between actual and predicted values are highlighted on an example of a model (see Fig. 4), wA_{fus} which is the fuselage gross wetted area.

One way to take into account these modeling uncertainties and to characterize ε would be to define them from the residuals of the linear regression. This method results in a normal uncertainty, ε, with a 0 mean and a standard deviation, σ, calculated from the residuals: it is the least squares error between the real points and the model points of the simple model (linear or quadratic). The standard deviation thus calculated represents the residuals on the whole variation range of the function. This can be problematic, because in relative terms to the function's value, the standard deviation can be very important and

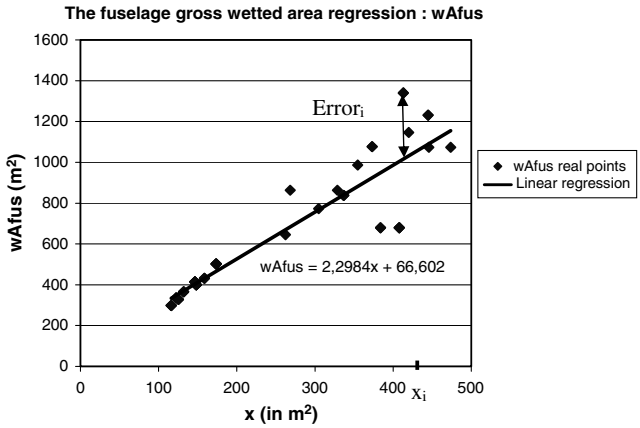

Fig. 4 Comparison between the real points and the model points.

the confidence interval very large. This behavior is illustrated by the following example: the empirical standard deviation calculated for the F_{wing} function (fuel volume in the wing) in relative terms is very large for low values of F_{wing} (cf. Fig. 5). So there is a strong probability that for low values of F_{wing}, random draws can give negative points and therefore nonphysical points. This is illustrated in Fig. 5, in which it can be seen that for values of the abscissa below 100, the function value less one standard deviation becomes negative. Such a high confidence interval may also result in designs that are unnecessarily conservative, thus detrimental to the performance of the airplane (the optimum airplane will be robust to uncertainties but the weight will be too high). Hence, we propose in the next paragraphs an improved representation of the modeling uncertainty.

As a first approach for avoiding the use of too large uncertainties, which can lead to nonphysical results for some variables, the standard deviations were truncated to ± 2 standard deviations. The implementation of this first approach still led to an unacceptable configuration (excessive weight) due to a large confidence interval on the constraints. Next, we provide some theoretical background on the method we eventually implemented for modeling variable uncertainty depending on the abscissa of the regression.

Consider the regression model $Y_i = \beta_i X_i + \varepsilon_i$, $1 \le i \le n$, with nonrandom design variables (X_i) and measurements (Y_i) for the unknown regression function $g(X_i) = \beta_i X_i$. The values of Y_i are contaminated with errors (ε_i), which are assumed to have an expectation value of zero and are independent but can be not identically distributed. The data, ε_i, are called heteroscedastic [31,32] if $\text{Var}(\varepsilon_i) = \sigma_i^2$ is nonconstant (see for example Fig. 4). The function σ_i such as $\sigma_i^2(X_i) = \sigma^2$ can be determined by kernel smoothers [33–35]. The principle of the kernel smoother is a generalization of the histogram estimation. It is a nonparametric method of estimating the probability density of a random variable. It is based on a statistical

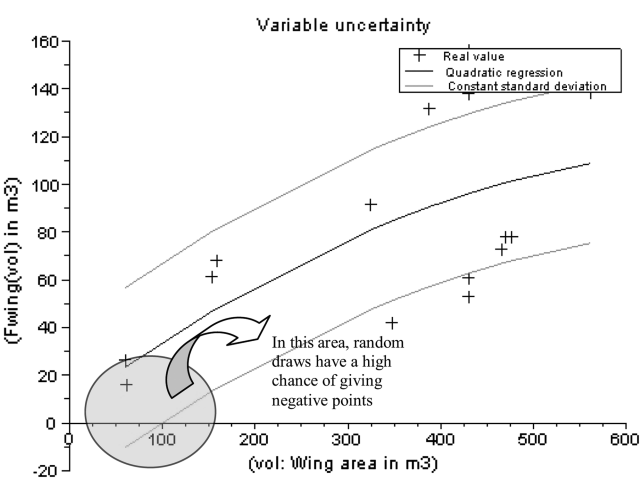

Fig. 5 Constant standard deviation for the F_{wing} parameter.

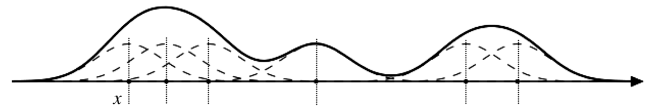

Fig. 6 Density function built from a kernel smoother.

population sample and allows us to estimate the density at any point of the support. The density at a point x is estimated by the proportion of data $x_1, x_2, \ldots x_N$, that are close to x. For this, a box is drawn around x, whose width is governed by a smoothing parameter h. Then the number of observations into this box is counted. This estimate, which depends on the smoothing parameter h, has good statistical properties but is noncontinuous by construction. The method of Parzen-Rozenblatt [36] (kernel method) allows continuity. For this, the box centered at x is replaced by a curve centered at x, which is a Gaussian function (reduced Gaussian distribution); this is shown by the dashed curve in Fig. 6. The more a data point is close to the support point x, the greater the value of the weight is. On the other hand, observations too far from x are assigned with a negligible weight. The estimator of the density function is formed by the mean of the curves [37], which is the curve represented by a solid line in Fig. 6. The resulting final estimator of the local error variance is uniformly consistent [32]. These methods are popular in many different areas, such as econometrics [33,35,38]. In [38], a relationship between electricity sales and temperature is estimated using a regression and the distribution of model uncertainty is based on kernel smoothing. An equivalent method is used in [35] to determine financial data. For our conceptual aircraft design problem we propose a method that is based on an adaptation of the kernel smoothers method. It allows us to both account for the fact that some areas have a higher density of points and also that some points are not well predicted by the analytical model.

To accurately model the uncertainties, the standard deviation needed to vary according to the current point on the model is evaluated. The variable uncertainty model will take into account two phenomena:

1) Phenomenon 1 is where the points of the database are sometimes concentrated in certain regions.

2) Phenomenon 2 is that in which some regions of the model can be much better than in others.

The first phenomenon is highlighted by an example. In Fig. 4, representing wA_{fus} (the fuselage gross wetted area), one can see that there is a cloud of points in the low values area and a cloud of points in the high values area. In the middle area, for wA_{fus} values between 500 and 800 m^2 there are relatively few points. One would like the modeling uncertainty to be slightly higher in the areas where there is an absence of points in the database, to reflect the fact that one has little information available to actually assess the modeling uncertainty based on actual database points. One therefore introduces a more complex uncertainty model. To account for phenomenon 1, one introduces the prediction variance at point x_0 [39]:

$$\sigma_1 = \text{Var}[\hat{y}(x_0)] = \sigma_0^2 x_0'(X'X)^{-1} x_0 \tag{11}$$

with σ_0 the standard deviation characterizing the residues. The value of our standard deviation is adjusted to the available information, i.e., the standard deviation will be less important if the region is well defined (many points in the database); however, it will be important when fewer points define the region in the database (the uncertainty in the predicted characteristics can possibly be more important).

The first formulation with a variable standard deviation accounts for phenomenon 1 but does not reflect the fact that certain regions are much better predicted by the model than others (phenomenon 2). This second phenomenon is illustrated again in Fig. 4, where one can see that the low values region is much better predicted by the model than the high values region. This phenomenon will be taken into account by incorporating a mean standard deviation, σ_2, defined only in the vicinity of the studied point (at which one would like to predict the modeling uncertainty). Similarly to kernel smoothing, this deviation is the least squares error calculated in a "Gaussian window" of

variable size. It is the mean errors of the points situated in a window whose dimensions vary so as to maintain a constant number of points within it. A weight is assigned to each point of the database based on a Gaussian weighting function and depending on the region in which our current point is located. The Gaussian function is such that the standard deviation is equal to half the size of the window constructed, and it is normalized so that the weight of the point where it is desired to define the model uncertainty is equal to 1. Thus, the more a point of the database is close to the point where we want to determine the uncertainty, the more its impact on the calculation of the uncertainty is important.

$$\sigma_2 = \sqrt{\frac{\sum_{i=1}^{n} (y_i^{real} - y_i^{model})^2 \times p(y_i^{real})}{\sum_{i=1}^{n} p(y_i^{real})}} \tag{12}$$

where n is the number of points in the database, y^{real} the value of the real point, y^{model} is the value given by the model, and $p(y_i^{\text{real}})$ is the weight that is assigned to each point of the database based on the Gaussian weighting function. The function p is given by Eq. (13):

$$p(y) = \frac{1}{\sigma\sqrt{2\Pi}} e^{-1/2(\frac{y-\mu}{\sigma})^2} \tag{13}$$

The standard deviation of the modeling uncertainty thus becomes

$$\sigma_{\text{tot}} = \sigma_2^2 x_0'(X'X)^{-1} x_0 \tag{14}$$

The resulting uncertainty using the total variance is illustrated in Fig. 7 for the wA_{fus} function. The new formulation of the standard deviation is adjusted to the available information, i.e., it is an adaptive standard deviation. In Fig. 6, we can see that the reduction of the average of the confidence interval is around 50% over the range of variation of wA_{fus} and X (combination of the fuselage dimensions).

The modeling of variable uncertainties from a historical database can give a prediction of the uncertainty value at any point of the search space. This uncertainty represents the variability that can occur in the prediction of the physical phenomena. Note that even if the manufacturer decides to build a plane with the same parameters again, model uncertainty is not zero. This is due to the fact that regression models do not pass exactly through all the points, which translates to the fact that even at a known point, the uncertainty is not equal to zero. Furthermore, when no information is available in a certain area (that is to say, when no plane has yet been designed with such dimensions), it is possible to predict the parameter's value but uncertainty will be greater. Of course, if there is a significant technological breakthrough, the variability in prediction models may be underestimated or overestimated. But it is also possible to introduce subjective information based on expert opinion or calibrate the uncertainty based on previous technological breakthrough.

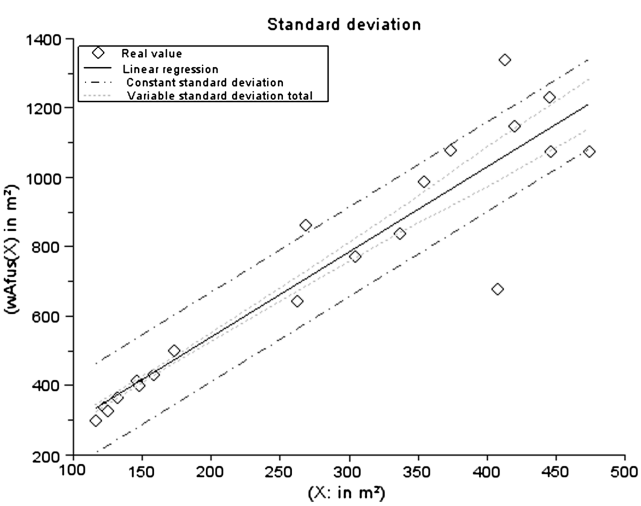

Fig. 7 Comparison of different standard deviations for the wA_{fus} parameter.

V. Calculation of the Probabilistic Constraints

The goal of the probabilistic optimization problem is to design an aircraft robust to the parameters variations and satisfying a certain level of reliability [cf. Eq. (3)]. The probability of failure of the constraints and the robustness of the system can be evaluated by randomly sampling the uncertain parameters and thus determining each times the constraint and the objective function. The probability of failure is estimated by the ratio of failures to the total number of samples. To reduce the number of calls to the computer code and to minimize the computation time, we chose to replace the constraints by surrogate models with a fast response time; these surrogate models provide directly the reliability index, β, which is further detailed below.

The minimum level of reliability is often characterized using the reliability index. The reliability constraints of the Eq. (3) may be expressed as [40]

$$\beta(S_{\text{wing}}, \text{SLST}, \delta) \geq \beta_i^g \tag{15}$$

where the reliability index β is calculated by determining the most probable failure point in random variable space for each limit state equation $g_i(S_{\text{wing}}, \text{SLST}, \delta) = 0$. The symbol β_i^g is the target reliability index corresponding to the allowable violation probability P_i^g. In the case where the uncertain variables are Gaussian variables, we have

$$\beta_i^g = -\Phi^{-1}(P_i^g) \tag{16}$$

where Φ is the cumulative density function of a normal variable.

The probabilistic constraints are estimated in terms of reliability index to build the response surfaces. This is because the reliability index lends itself better to a polynomial fit than the probability of failure itself. The surrogate models are constructed using a quadratic polynomial on a database generated from a design of experiments (two-dimensional grid design). At each point of the design of experiments, Monte Carlo runs are carried out with the proposed modeling uncertainties (3000 runs per point for a satisfactory accuracy given the probabilities thought to be about 90%). Thus, thanks to the construction of response surfaces to model reliability constraints and robustness constraints we are able to efficiently obtain the values of the constraints at any point in the search space (i.e., we now have reasonable computational cost). The computation time of the optimization under uncertainty is thus the same as for the deterministic optimization, whereas the construction time of the response surface approximation reduces the computational cost by a factor of three compared to an approach based on Monte Carlo simulations at each optimization step.

The formulation of the optimization in Eq. (3) is solved using the response surfaces for the constraints and the simplex (Nelder–Mead) algorithm. The graphical illustration of the optimization result and of the active constraints is given in Fig. 8, whereas the numerical results at the optimum are

$$\begin{cases} S_{\text{wing}} = 133.4 \text{ m}^2 \\ \text{SLST} = 92094 \text{ N} \\ \text{MTOW} = 76044 \text{ kg} \end{cases} \tag{17}$$

When comparing the results of the deterministic optimization without design margins with those obtained for the optimizations under uncertainty in Table 3, one notices an increase in the objective function: the takeoff weight of the aircraft is higher for the reliability-based optimum (this is a normal tendency when one sets up a reliability and/or robustness problem: the presence of margins for the reliability-based optimization method has the effect of moving the location of the optimum towards the conservative side). In particular, to take into account the uncertainty and design a system that is reliable and robust, the optimum point has to be moved to the

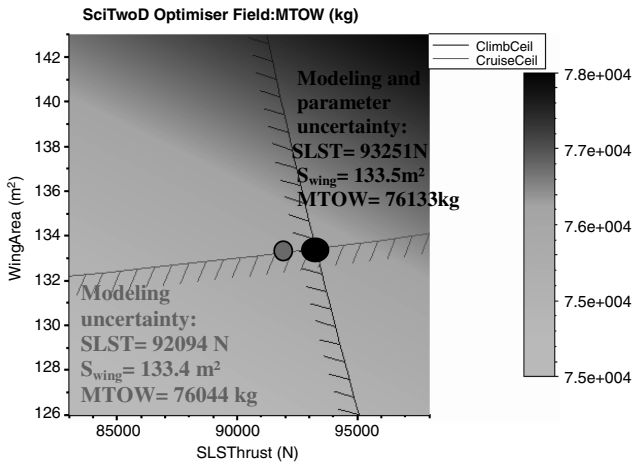

Fig. 8 Comparison of the optimization with and without design variables uncertainty.

conservative domain, generating an increase of about 1.3% of the maximum take of weight compared to the deterministic optimum. However, if we chose to solve the deterministic formulation instead of the probabilistic one, the designer does not control the risk taken; thus, as he goes along in the design process, he may need to undertake multiple redesign iterations if the constraints are not met.

Table 3 allows us to highlight the fact that the deterministic approach leads to a probability of satisfying the active constraints of the order of 50% when Monte Carlo experiments are done in the vicinity of the deterministic optimum by considering uncertainties on the same variables as in the optimization under uncertainty. Such low values are not satisfactory because they would in many cases imply redesign iterations. On the other hand, the probability of satisfying the active constraints are, as required, 90 or 95% in the case of the optimization under uncertainty.

VI. Design Parameter Uncertainty

Another possible source of uncertainty in model output results from modeling inaccuracies on the optimization design variables themselves. These inaccuracies stem from differences between the idealized model used in the conceptual design and final design of the aircraft after all the design phases. For example, the wing area in conceptual design is derived from simple geometry formulas on a simple polyhedron. On the built airplane, however, the wing area is calculated of course on the much more complicated geometry of the actual wing. So, we will also take into account now the uncertainties on the design variables: the wing area S_{wing} and the engine thrust at sea level *SLST*. The probabilistic problem to solve is the following one:

$$\begin{cases} \text{Find}(S^*_{\text{wing}}, \text{SLST}^*)\text{solution of} \\ \min\limits_{S_{\text{wing}},\text{Slst}} E(\text{MTOW}(S_{\text{wing}}, \text{SLST}, \delta)) \text{ with } S_{\text{wing}}, \text{SLST} \in S \\ \text{such as:} \\ \text{COV}(\text{MTOW}(S_{\text{wing}}, \text{SLST}, \delta)) \leq \alpha \\ \text{Prob}(S_{\text{wing}}, \text{SLST}, \delta) \leq 0) \geq \alpha_i^g \text{ with } i = 1, \ldots, 7 \end{cases} \tag{18}$$

where δ represents the uncertainties on both the design parameters and the disciplinary models. We consider here an additive uncertainty, δ, i.e., $f(x, \delta) = f(x) + \delta$ where δ is a normal probability and variable standard deviation determined as described in Sec. IV. This problem is solved by a double-loop procedure: at each point of the optimization process, a reliability and robustness study is conducted. To do this, we performed simulations of Monte Carlo: draws of 15 uncertain parameters that have been selected according to the sensitivity study (Sec. III) and draws of the design parameters (wing surface and engine thrust at sea level). The set of uncertain design parameters are modeled as follows:

$$x^{\text{incert}} = x^{\text{opt}} + \varepsilon(x^{\text{opt}}) \tag{19}$$

where x^{opt} is the parameter value at the current point of the optimization process. The value $\varepsilon(x^{\text{opt}})$ is the uncertainty determined using the method presented in Sec. IV, and it follows a normal distribution with mean zero and a variable standard deviation (dependent on the location of the current point x^{opt} in the search space) determined from a historical data base (same database for all the uncertain parameters, including uncertainties of design parameters). Note that the uncertainties of the design are determined from the database as well. More precisely, the design parameters can themselves be roughly expressed as regression models of a certain number of design assumptions (see Table 1 and Sec. IV). These coarse models are consistent with the other models used in the conceptual design, in terms of modeling fidelity. In comparison with the real data derived from the historical database, we can thus estimate the variability of these design variables. The estimate of this variability shows that the dispersion is less than 5% for the S_{wing} parameter and 8% for the *SLST* parameter. Once the simulations are done, we can estimate the reliabilities of the constraints and the robustness of the objective function.

New response surfaces are constructed for the probability of failure taking into account the modeling uncertainties and the design variables uncertainties. The result of the probabilistic optimization is provided in Fig. 8.

It is shown by comparing the optimal designs that the design parameter uncertainties have a nonnegligible effect on the solution, i.e., when the design parameter uncertainties are taken into account the location of the optimum point has moved towards the conservative side (weight increased by a further 0.4%). The uncertainties are larger, thus introducing an additional margin to ensure that the aircraft is robust. However, we can see that the impact of the design parameter uncertainties on both the active constraints is different: one of the constraints is much more sensitive to the parameter uncertainty than the other constraint. Indeed, the minimum altitude for climb constraint becomes more critical, i.e., the translation towards the conservative side is important for this constraint (cf. Fig. 8). On the other hand, the minimum altitude for buffeting constraint changes relatively little. This means that to assure the robustness and the reliability of the aircraft, it is important to take into account the design variable uncertainties at the conceptual design level as well.

VII. Conclusion

The present paper addresses the problem of optimization under uncertainty applied to the conceptual aircraft design. Conceptual aircraft design involves low-fidelity models that are characterized by significant modeling uncertainty. In this paper we proposed a novel approach which allows us to update uncertainties from the historical database at each step of the optimization process. This more accurate adaptive uncertainty quantification allowed us to take into account two phenomenon:

Table 3 Comparison between the probabilistic optimization and the deterministic optimization

Variable	Deterministic optimization	Optimization under uncertainties
MTOW	75415.36 kg	76123 kg
S_{wing}	129.35 m^2	132.9 m^2
SLST	89474.97 N	94022.7 N
Success probability (active constraints)	$\text{Prob}(g_{\text{climbceil}}(S_{\text{wing}}, \text{SLST}) \leq 0) = 54.9\%$	$\text{Prob}(g_{\text{climbceil}}(S_{\text{wing}}, \text{SLST}) \leq 0) = 95\%$
	$\text{Prob}(g_{\text{buffetceil}}(S_{\text{wing}}, \text{SLST}) \leq 0) = 51.05\%$	$\text{Prob}(g_{\text{buffetceil}}(S_{\text{wing}}, \text{SLST}) \leq 0) = 90\%$

1) The output models are subjected to varying uncertainty depending on the region in which the model is queried (the amount of data available varies).

2) Some regions are more poorly predicted by the regression than others. This adaptive uncertainty quantification is based on a combination of the regression variance prediction and of least squares errors on Gaussian windows, similar to kernel smoothing.

Through this framework, which takes into account uncertainties on both model and design variables, an optimization problem under constraints of robustness and reliability applied to the conceptual aircraft design was presented. The computational cost was reduced by replacing the probabilistic constraints with approximate models (response surfaces) built using Monte Carlo simulations. The resolution of the problem allowed us to define the additional margins needed to ensure the levels of robustness and reliability required. It was shown that design variable uncertainties have an important impact on the location of the optimum point. Compared to standard deterministic optimization without a design margin, the results show quite different design variables and a light increase on weight. This modest increase in weight allows us to ensure a high reliability and robustness of the design compared to the unreliable and sensitive deterministic optimum. Future work involves integrating the optimization problem under uncertainty within a multi-agent optimization framework [41], which is expected to solve more efficiently highly multidisciplinary optimization problems.

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References

[1] Roskam, J., "Airplane Design: Part I, Preliminary Sizing of Airplanes," *Airplane Design*, 3rd ed., Design, Analysis, and Research Corp. (DARcorp.), Lawrence, Kansas, 2003, p. 202.

[2] Torenbeek, E., *Synthesis of Subsonic Airplane Design*, Delft Univ. Press, Amsterdam, 1976, p. 4.

[3] Howe, D., *Aircraft Design Synthesis*, Wiley, New York, 2000.

[4] Raymer, D., *Aircraft Design: A Conceptual Approach*, 3rd ed., AIAA, Reston, VA, 1999, p. 923.

[5] Wagner, R. L., "Science, Uncertainty and Risk: The Problem of Complex Phenomena," *APS News*, Vol. 12, No. 1, 2003, p. 8.

[6] Oberkampf, W. L., DeLand, S. M., Rutherford, B. M., Diegert, K. V., and Alvin, K. F., "Error and Uncertainty in Modeling and Simulation," *Reliability Engineering and System Safety*, Vol. 75, No. 3, 2002, pp. 333–357. doi:10.1016/S0951-8320(01)00120-X

[7] Perez, R., Liu, H., and Behdinan, K., "Evaluation of Multidisciplinary Optimization Approaches for Aircraft Conceptual Design," *10th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, Albany, New York, NY, 2004; also Paper 2004-4537.

[8] DeLaurentis, D. A., and Mavris, D. N., "Uncertainty Modeling and Management in Multidisciplinary Analysis and Synthesis," AIAA Paper 2000-0422, Jan. 2000.

[9] Sobieszczanski-Sobieski, J., and Haftka, R. T., "Multidisciplinary Aerospace Design Optimization Survey of Recent Developments," *Structural and Multidisciplinary Optimization*, Vol. 14, No. 1, Aug. 1997, pp. 1–23. doi:10.1007/BF01197554

[10] Sobieszczanski-Sobieski, J., and Kodiyalam, S., "Multidisciplinary Design Optimization—Some Formal Methods, Framework Requirements, and Application to Vehicle Design," *International Journal of Vehicle Design*, Vol. 25, Nos. 1–2/2001, online Feb. 2004, pp. 3–22.

[11] Tores, R., and Bes, C., "Uncertainty Tolerance of Multiobjective Environmentally Friendly Departure Procedures for Civil Aircraft," *Journal of Aircraft*, Vol. 48, No. 6, 2011, pp. 2169–2172. doi:10.2514/1.C031427

[12] Das, I., "Robustness Optimization for Constrained, Nonlinear Programming Problems," Department of Computational & Applied Mathematics, Rice Univ., Technical Rept. 97-01, Houston, TX, 1997.

[13] Green, L., Lin, H. Z., and Khalessi, M., "Probabilistic Methods for Uncertainty Propagation Applied to Aircraft Design," AIAA Paper 2002-3140, 2002.

[14] Daskilewicz, M. J., German, B. J., Takahashi, T. T., Donovan, S., and Shajanian, A., "Effects of Disciplinary Uncertainty on Multi-Objective Optimization in Aircraft Conceptual Design," *Structural and Multidisciplinary Optimization*, Vol. 44, No. 6, July 2011, pp. 831–846. doi:10.1007/s00158-011-0673-4

[15] Neufeld, D., Chung, J., and Behdinan, K., "Aircraft Conceptual Design Optimization Considering Fidelity Uncertainties," *Journal of Aircraft*, Vol. 48, No. 5, 2011, pp. 1602–1612. doi:10.2514/1.C031312

[16] Birman, J., and Druot, T., "Robustness Assessment of a Margin-Setting Process in Preliminary Aircraft Sizing," *12e congrès annuel de la société française de recherche opérationnelle et d'aide à la décision*, Saint Etienne, France, 2011.

[17] Le Riche, R., Roustant, O., Bay, X., and Pujol, G., "Formulation de l'optimisation avec incertitudes dans le projet RNTL/OMD," Ecole de Mines de Saint Etienne, Technical Report, Saint Etienne, France, 2007.

[18] Torczon, V., "On the Convergence of Pattern Search Algorithms," *SIAM Journal on Optimisation*, Vol. 7, No. 1, 1997, pp. 1–25. doi:10.1137/S1052623493250780

[19] Ferson, S., Joslyn, C. A., Helton, J. C., Oberkampf, W. L., and Sentz, K., "Summary from the Epistemic Uncertainty Workshop: Consensus Amid Diversity," *Reliability Engineering and System Safety*, Vol. 85, Nos. 1–3, 2004, pp. 355–369. doi:10.1016/j.ress.2004.03.023

[20] Dekking, M., Kraaikamp, C., and Lopuhaa, H. P., *A Modern Introduction to Probability and Statistics*, Springer, London, 2005.

[21] Daney, D., Andreff, N., Chabert, G., and Papegay, Y., "Interval Method for Calibration of Parallel Robots: Vision-Based Experiments," *Mechanism and Machine Theory*, Vol. 41, No. 8, 2006, pp. 929–944. doi:10.1016/j.mechmachtheory.2006.03.014

[22] Zadeh, L. A., "Information and Control Fuzzy Sets," *Fuzzy Sets*, Vol. 8, 1965, pp. 338–353. doi:10.1016/S0019-9958(65)90241-X

[23] Verma, P., Singh, P., George, K. V., Singh, H. V., Devotta, S., and Singh, R. N., "Uncertainty Analysis of Transport of Water and Pesticide in an Unsaturated Layered Soil Profile Using Fuzzy Set Theory," *Applied Mathematical Modelling*, Vol. 33, No. 2, 2009, pp. 770–782. doi:10.1016/j.apm.2007.12.004

[24] Altunok, E., Reda Taha, M. M., Epp, D. S., Mayes, R. L., and Baca, T. J., "Damage Pattern Recognition for Structural Health Monitoring Using Fuzzy Similarity Prescription," *Computer-Aided Civil and Infrastructure Engineering*, Vol. 21, No. 8, Nov. 2006, pp. 549–560. doi:10.1111/j.1467-8667.2006.00457.x

[25] Nikolaidis, E., Chen, S., Cudney, H., Hatftka, R. T., and Rosca, R., "Comparison of Probability and Possibility for Design Against Catastrophic Failure Under Uncertainty," *Journal of Mechanical Design*, Vol. 126, No. 3, 2004, pp. 386–394. doi:10.1115/1.1701878

[26] Shafer, G., *A Mathematical Theory of Evidence*, Princeton Univ. Press, Princeton, NJ, 1976, p. 302.

[27] Beyer, H. G., and Sendhoff, B., "Robust Optimization–A Comprehensive Survey," *Computer Methods in Applied Mechanics and Engineering*, Vol. 196, Nos. 33–34, 2007, pp. 3190–3218. doi:10.1016/j.cma.2007.03.003

[28] Mulvey, J., Vanderbei, R., and Zenios, S., "Robust Optimization of Large-Scale Systems," *Operations Research*, Vol. 43, No. 2, 1995, pp. 264–281. doi:10.1287/opre.43.2.264

[29] Lee, K. H., and Park, G. J., "Robust Optimization Considering Tolerances of Design Variables," *Computers and Structures*, Vol. 79, 2001, pp. 77–86. doi:10.1016/S0045-7949(00)00117-6

[30] Jacques, J., Lavergne, C., and Devictor, N., "Analyse de sensibilité globale," *36ème journée de Statistique*, Montpellier, France, 2004.

[31] Gasser, T., Sroka, L., and Jennen-Steinmetz, C., "Residual Variance and Residual Pattern in Nonlinear Regression," *Biometrika*, Vol. 73, No. 3, 1986, pp. 625–633. doi:10.1093/biomet/73.3.625

[32] Müller, H. G., and Stadtmüller, U., "Estimation of Heteroscedasticity in Regression Analysis," *Annals of Statistics*, Vol. 15, No. 2, 1987, pp. 610–625. doi:10.1214/aos/1176350364

[33] Linton, O., and Mammen, E., "Estimating Semiparametric Arch (∞) Models by Kernel Smoothing Methods," *Econometrica*, Vol. 73, No. 3, 2005, pp. 771–836. doi:10.1111/ecta.2005.73.issue-3

[34] Herrmann, E., "Local Bandwidth Choice in Kernel Regression Estimation," *Journal of Computational and Graphical Statistics*, Vol. 6,

No. 1, March 1997, pp. 35–54.
doi:10.1080/10618600.1997.10474726

[35] Linton, O., Mammen, E., Perch, N., and Tanggaard, C., "Yield Curve Estimation by Kernel Smoothing Methods," *Journal of Econometrics*, Vol. 105, No. 1, Nov. 2001, pp 185–223.
doi:10.1016/S0304-4076(01)00075-6

[36] Parzen, E., "On Estimation of a Probability Density Function and Mode," *Annals of Mathematical Statistics*, Vol. 3, No. 3, 1962, pp. 1065–1076.

[37] Duda, R., and Hart, P., *Pattern Classification and Scene Analysis*, John Wiley and Sons, Inc., New York, 1973, p. 512.

[38] Engle, R. F., Granger, C. W. J., Rice, J., and Weiss, A., "Semiparametric Estimates of the Relation Between Weather and Electricity Sales," *Journal of the American Statistical Association*, Vol. 81, No. 394, 1986, pp. 310–320.

[39] Myers, R. H., Montgomery, D. C., and Anderson-Cook, C. M., "Process and Product Optimization Using Designed Experiments," *Response Surface Methodology*, 3rd ed., Wiley, Hoboken, NJ, 2009, pp. 13–72.

[40] Lemaire, M., Chateauneuf, A., and Mitteau, J. C., *Fiabilité des Structures: Couplage Mécano-Fiabiliste Statique*, Hermes Science Publication, London, 2005.

[41] Jorquera, T., Georgé, J.-P., and Régis, C., "Self-Organizing Multi-Agent System for MDO," *12e Congrès Annuel de la Société Française de Recherche Opérationnelle et d'Aide à la Décision (ROADEF)*, St-Etienne, France, 2011.