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# An original short proof of the Fermat last conjecture

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**Abstract.** The Fermat last conjecture (see [1] or [2]) states that when  $n$  is an integer  $\geq 3$ , the equation  $x^n + y^n = z^n$  has no non-zero integer solutions for  $x$ ,  $y$  and  $z$  (in other words, no three integers of the form  $x \geq 1$ ,  $y \geq 1$  and  $z \geq 1$  can satisfy the equation  $x^n + y^n = z^n$ , where  $n$  is an integer  $\geq 3$ ). The first proof of the Fermat last conjecture was given by Wiles (see [1]) in an article of at least 105 pages long. Now, in this paper, we give the direct short proof of the Fermat last conjecture by using the immediate reformulation of this conjecture coupled with simple calculus on complex numbers (see [2] for simple calculus on complex numbers), trivial computation, and by reducing the Fermat last conjecture into an equation of three unknowns via the reasoning by reduction to absurd. This new proof is direct, short, very simple, and is different from the first one given in [1]. In addition, this proof clearly shows that the Fermat last conjecture seemed extremely difficult because the parameters used to attack this conjecture were not appropriate for such a discrete conjecture.

**Keywords.** fermat, fermatian, complex numbers, tackle.

**AMS Classification 2000:** 05xx and 11xx.

**Prologue.** In Section.1, we introduce definitions that are not standard and using these definitions, we give the immediate reformulation of the Fermat last conjecture. Curiously, this immediate reformulation is crucial for the direct proof of the Fermat last conjecture. In Section.2, we prove properties linked to simple calculus on complex numbers and computation; and we reduce the Fermat last conjecture into an equation of three unknowns [in Section.2, we let one Proposition unproved and we will prove this Proposition in Section.2' (*Epilogue*)]. In Section.3, using the immediate reformulation of the Fermat last conjecture given in Section.1 and some properties of Section.2, we give the direct proof of the Fermat last conjecture. In Section.2' (*Epilogue*), we end this paper by proving the only Proposition we let unproved in Section.2.

## 1 Non-standard definitions, simple properties, and the immediate reformulation of the Fermat last conjecture

**Definition 1.0.** We say that  $e$  is *fermat*, if  $e$  is an integer  $\geq 3$  and if the equation  $x^e + y^e = z^e$  has no solution in integers  $\geq 1$  [notice (by using the previous Definition) that the *Fermat last conjecture* asserts that every integer  $e \geq 3$  is *fermat*].

**Example 1.0.0.** It is known many years ago that if  $e = 3$  or if  $e = 4$  or if  $e = 5$  or if  $e = 6$  or if  $e = 7$ , then  $e$  is *fermat*; moreover, it is also known many years ago that if  $n = 3k$  [where  $k$  is an integer  $\geq 1$ ], then  $n$  is *fermat*.

That being so, let us define:

**Definition 1.1.** We say that  $e$  is *fermatian*, if  $e$  is an integer  $\geq 3$  and if every integer  $v$  with  $3 \leq v \leq e$  is *fermat* [use Definition 1.0 for the meaning of *fermat*]; in other words,  $e$  is *fermatian* if  $e$  is an integer  $\geq 3$  and  $v$  is an integer of the form  $3 \leq v \leq e$  implies that  $v$  is *fermat* [use Definition 1.0 for the meaning of *fermat*]; in other terms again,  $e$  is *fermatian*, if  $e$  is an integer  $\geq 3$  and if for every integer  $v$  with  $3 \leq v \leq e$ , the equation  $x^v + y^v = z^v$  has no solution in integers  $\geq 1$ .

Using the definition of *fermatian* [use Definition 1.1], then the following remarks are trivial.

**Remark 1.1.0.** 7 is *fermatian*. **Proof.** Indeed observe that every  $v \in \{3, 4, 5, 6, 7\}$  is *fermat* [use Example 1.0.0 of Definition 1.0]; so 7 is an integer  $\geq 3$  and every integer  $v$  with  $3 \leq v \leq 7$  is *fermat*; therefore 7 is *fermatian*.  $\square$

**Remark 1.1.1.** If  $n + 1$  is *fermatian* and if  $x'$  is an integer of the form  $3 \leq x' \leq n + 1$ , then  $x'$  is also *fermatian*. **Proof.** Immediate and is a trivial consequence of the definition of *fermatian* [use Definition 1.1 for the meaning of *fermatian*].  $\square$

**Remark 1.1.2.** If  $2 \leq n \leq 6$ , then  $n + 1$  is *fermatian*. **Proof.** Immediate and is a trivial consequence of Remark 1.1.0 and Remark 1.1.1.  $\square$

**Remark 1.1.3.** If  $n$  is *fermatian* and  $n + 1 \equiv 0 \pmod{3}$ , then  $n + 1$  is *fermatian*. **Proof.** Clearly

$$n + 1 \text{ is } \textit{fermat}$$

[observe that  $n + 1 \equiv 0 \pmod{3}$  and use Example 1.0.0] and clearly

$$n + 1 \text{ is } \textit{fermatian}$$

[observe that  $n + 1$  is fermat (use the previous) and on notice (via the hypotheses) that  $n$  is fermatian; so  $n + 1$  isfermatian (use the preceding and the meaning of fermatian introduced in Definition 1.1)].  $\square$

Note that *fermatian* implies *fermat*; so there is no confusion between *fermatian* and *fermat*. Having defined *fermat* and *fermatian*, then it comes:

**Definitions 1.2.** For every integer  $n \geq 3$ , we define  $\mathcal{W}(n)$ ,  $w_n$ ,  $\mathcal{W}(n + 1)$  and  $w_{n+1}$  as follows:

$\mathcal{W}(n) = \{x; 2 < x \leq n, \text{ and } x \text{ is fermatian}\}$ , and  $w_n = 2 \max_{x \in \mathcal{W}(n)} x$ . Using the definitions of  $\mathcal{W}(n)$  and

$w_n$ , then it becomes trivial to deduce that for every integer  $n \geq 2$ , we clearly have

$\mathcal{W}(n + 1) = \{x; 2 < x \leq n + 1, \text{ and } x \text{ is fermatian}\}$ , and  $w_{n+1} = 2 \max_{x \in \mathcal{W}(n+1)} x$  [It is immediate that

$\mathcal{W}(n) \subseteq \mathcal{W}(n + 1)$  for every integer  $n \geq 3$ , and therefore  $w_n \leq w_{n+1}$  for every integer  $n \geq 3$ ].

We will use  $w_{n+1}$  to give the immediate reformulation of the Fermat last conjecture, from which this conjecture will implicitly follow. Now using the definition of  $(\mathcal{W}(n + 1), w_{n+1})$  [see Definitions 1.2], then we have the following simple Proposition.

**Proposition 1.3.** *Let  $n$  be an integer  $\geq 2$ . We have the following seven properties.*

(1.3.0.)  $w_{n+1}$  is even and  $w_{n+1} \leq 2n + 2$ .

(1.3.1.)  $w_{n+1} = 2n + 2$ , if and only if,  $n + 1$  is fermatian ( in other words,  $w_{n+1} \neq 2n + 2$ , if and only if,  $n + 1$  is not fermatian ).

(1.3.2.) If  $n \geq 3$ , then  $w_n \leq w_{n+1}$ .

(1.3.3.) If  $w_{n+1} < 2n + 2$ , then  $n + 1$  is not fermatian.

(1.3.4.) If  $3 \leq e \leq n + 1$  and if  $n + 1$  is fermatian, then  $e$  is fermatian.

(1.3.5) (An implicit using of the Fermat last conjecture). If  $w_{n+1} < 2n + 2$ , then  $n + 1$  is not fermatian; therefore, there exists an integer  $e$  such that  $3 \leq e \leq n + 1$  and the equation  $x^e + y^e = z^e$  has non-zero integer solutions for  $x$  and  $y$  and  $z$ .

(1.3.6) (An explicit using of the Fermat last conjecture). If  $w_{n+1} = 2n + 2$ , then  $n + 1$  is fermatian; therefore, for every integer  $e$  such that  $3 \leq e \leq n + 1$ , the equation  $x^e + y^e = z^e$  has no non-zero integer solutions for  $x$  and  $y$  and  $z$ . **Proof.** Properties (1.3.0) and (1.3.1) are immediate [it suffices to use the definition of  $w_{n+1}$  and the definition of fermatian]. Property (1.3.2) is trivial [it suffices to use the definition of  $w_{n+1}$  via the definition of  $w_n$ ]; and property (1.3.3) is immediate [indeed observe that  $w_{n+1} \neq 2n + 2$  and use property (1.3.1)]. Property (1.3.4) is a trivial consequence of Remark 1.1.1 of Definition 1.1. Now using the definition of fermatian [see Definition 1.1] and the definition of  $w_{n+1}$  [see Definitions 1.2], then it becomes immediate to see that property (1.3.5) is only the trivial reformulation of property (1.3.3). Using again the definition of fermatian [use Definition 1.1] and the definition of  $w_{n+1}$  [use Definitions 1.2], then it becomes

immediate to see that property (1.3.6) is clearly the contrary of property (1.3.5).  $\square$

Now the following Theorem is the immediate reformulation of the Fermat last conjecture.

**Theorem 1.4.** (The immediate reformulation of the Fermat last conjecture). *The following are equivalent.*

(1) The Fermat last conjecture is true [i.e. no three integers of the form  $x \geq 1$ ,  $y \geq 1$  and  $z \geq 1$  can satisfy the equation  $x^n + y^n = z^n$ , where  $n$  is an integer  $\geq 3$ ].

(2) Every integer  $n \geq 3$  is fermat.

(3) For every integer  $n'' \geq 3$  and for every integer  $n'$  such that  $3 \leq n' \leq n''$ , the equation  $x^{n'} + y^{n'} = z^{n'}$  has no non-zero integer solutions for  $x$ ,  $y$  and  $z$ .

(4) For every integer  $n \geq 2$ ,  $n + 1$  is fermatian.

(5) For every integer  $n \geq 2$ , we have  $w_{n+1} = 2n + 2$ .

**Proof.** (1)  $\Rightarrow$  (2). Immediate [it suffices to use the definition of the Fermat last conjecture and the definition of fermat (use Definition 1.0 for the meaning of fermat)]; (2)  $\Rightarrow$  (3). Immediate [it suffices to use the definition of fermat (see Definition 1.0)]; (3)  $\Rightarrow$  (4). Immediate [indeed observe that property (3) is an obvious reformulation of property (4) via the definition of fermatian (see Definition 1.1 for the meaning of fermatian)]; (4)  $\Rightarrow$  (5). Immediate [indeed, it suffices to use the definition of fermatian (use Definition 1.1) and the definition of  $w_{n+1}$  (use Definitions 1.2)]; (5)  $\Rightarrow$  (1). Immediate [indeed

$$\text{if for every integer } n \geq 2, \text{ we have } w_{n+1} = 2n + 2 \tag{1.0},$$

then clearly

$$\text{for every integer } n \geq 2, n + 1 \text{ is fermatian} \tag{1.1},$$

( use (1.0) and the definition of  $w_{n+1}$  ). So

$$\text{every integer } n \geq 3 \text{ is fermatian} \tag{1.2}$$

( use (1.1) ). It is trivial [by using the definition of fermatian] that (1.2) clearly implies that

$$\text{every integer } n \geq 3 \text{ is fermat} \tag{1.3}.$$

Now using the definition of fermat [see Definition 1.0], then it becomes trivial to deduce that (1.3) clearly says that the Fermat last conjecture is true].  $\square$

Theorem 1.4 is the immediate reformulation of the Fermat last conjecture; curiously, this trivial Theorem is not related to all the investigations that have been done on the Fermat last conjecture in the past (see [1]). Indeed, Theorem 1.4 clearly says that: **if for every integer  $n \geq 2$ , we have  $w_{n+1} = 2n + 2$ , then the Fermat last conjecture immediately follows.** This is what we will do, by using only elementary combinatorics, simple calculus on complex numbers, trivial computation and the reasoning by reduction to absurd via the reduction of the Fermat last conjecture into an equation of three unknowns. Before, we need the following remark.

**Remark 1.5.** Let  $n$  be an integer  $\geq 3$ ; consider  $\mathcal{W}(n + 1)$  and  $w_{n+1}$  (use Definitions 1.2). We have the following four properties.

(1.5.0.)  $8 \leq w_{n+1} \leq 2n + 2$ .

(1.5.1.) **If  $3 \leq n \leq 6$  then  $w_{n+1} = 2n + 2$ .**

(1.5.2.) **If  $w_{n+1} \neq 2n + 2$ , then:  $n > 6$  and  $w_{n+1} = w_n$ .**

(1.5.3) (An implicit using of the Fermat last conjecture). **If  $w_{n+1} \neq 2n + 2$ , then:  $n > 6$  and  $w_{n+1} = w_n$ ; therefore there exists an integer  $e$  such that  $3 \leq e \leq n + 1$  and the equation  $x^e + y^e = z^e$  has non-zero integer solutions for  $x$  and  $y$  and  $z$ .** **Proof.** (1.5.0). 4 is fermatian [ use Remark 1.1.2 of Definition

1.1] and  $4 = (3 + 1)$ , so  $4 \in \mathcal{W}(n + 1)$  and therefore,  $w_{n+1} \geq 8$ . It is trivial that  $w_{n+1} \leq 2n + 2$  [ use the definition of  $w_{n+1}$ ]. Now using the previous two inequalities, then it becomes immediate to deduce that  $8 \leq w_{n+1} \leq 2n + 2$ . Property (1.5.0) follows. (1.5.1). **If  $3 \leq n \leq 6$ , then**

$$n + 1 \text{ is fermatian} \tag{1.4}$$

[use Remark 1.1.2 of Definition 1.1] and therefore  $w_{n+1} = 2n + 2$  [use (1.4) use the meaning of  $w_{n+1}$  given in Definitions 1.2].

(1.5.2). Indeed, **if  $w_{n+1} \neq 2n + 2$ , clearly  $n > 6$  [ use property (1.5.1)], and clearly**

$$n + 1 \notin \mathcal{W}(n + 1) \tag{1.5}$$

[ use the meaning of  $\mathcal{W}(n + 1)$  given in Definitions 1.2 and the fact that  $w_{n+1} \neq 2n + 2$ ]; consequently

$$\mathcal{W}(n + 1) = \mathcal{W}(n) \tag{1.6}$$

[ use (1.5) ]. Now using (1.6), then it becomes trivial to deduce that  $w_{n+1} = w_n$ ; property (1.5.2) follows.

(1.5.3). Indeed using the meaning of  $w_{n+1}$  and  $w_n$  [ use Definitions 1.2], then it becomes immediate to see that property (1.5.3) is only the trivial reformulation of property (1.5.2). Remark 1.5 immediately follows.  $\square$

Remark 1.5 will help us in Section.2 and in Section.3 and in Section.2' [Epilogue].

## 2 Properties linked to simple calculus on complex numbers and trivial computation; the reduction of the Fermat last conjecture into an equation of three unknowns

In this section, we prove properties linked to simple calculus on complex numbers and trivial computation; and we reduce the Fermat last conjecture into an equation of three unknowns [in this Section, we let one Proposition unproved and we will prove this Proposition in Section.2' (Epilogue)]. In this section, the definition of *fermat* [ use Definition 1.0], the definition of *fermatian* [ use Definition 1.1], the definitions of  $\mathcal{W}(n + 1)$  and  $w_{n+1}$  [ use Definitions 1.2] are crucial.

**Recalls 2.1** ( *Real numbers,  $\mathcal{R}$ , complex numbers and  $\mathcal{C}$*  ). Recall that  $\mathcal{R}$  is the set all *real numbers* and  $\theta$  is a *complex number* if  $\theta = x + iy$ , where  $x \in \mathcal{R}$ ,  $y \in \mathcal{R}$  and  $i^2 = -1$ ;  $\mathcal{C}$  is the set of all *complex numbers*. That being said, let us define.

**Definition 2.2 ( Fundamental).** ( *tackle* ). Recall ( use Recalls 2.1 ) that  $\mathcal{R}$  is the set all *real numbers* and  $\mathcal{C}$  is the set of all *complex numbers*. Clearly

$$\mathcal{C}^2 = \{(x, y); x \in \mathcal{C} \text{ and } y \in \mathcal{C}\} \text{ and } \mathcal{C}^2 \times \mathcal{R} = \{(x, y, k); (x, y) \in \mathcal{C}^2 \text{ and } k \in \mathcal{R}\}$$

and

$$\mathcal{C}^3 = \{(x, y, z); (x, y) \in \mathcal{C}^2 \text{ and } z \in \mathcal{C}\}.$$

Now let  $n$  be an integer  $\geq 2$  and look at  $w_{n+1}$  [ use Definitions 1.2]; we say that  $(\phi(n), \nu(n), \epsilon(n)) \in \mathcal{C}^3$  tackles  $(1, 3iw_{n+1}^{-1})$  around  $6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i$ , if there exists  $(x, y, k) \in \mathcal{C}^2 \times \mathcal{R}$  such that

$$x + 3iyw_{n+1}^{-1} + k(6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i) + \phi(n) = 0 \text{ where } x + y - \nu(n) = 0 \text{ and } x - y - \epsilon(n) = 0.$$

We will see that the definition of tackle introduced above helps to reduce the Fermat last conjecture into an equation of three unknowns via the reasoning by reduction to absurd. Before, let us define:

**Definitions 2.3 (Fundamental).** Let  $n$  be an integer  $\geq 2$  and let  $w_{n+1}$  [ use Definitions 1.2]; now consider  $(\phi_n, \nu_n, \epsilon_n)$ , where

$$\phi_n = \sum_{j=1}^4 \phi_{n,j} \quad (2.0)$$

and where

$$\phi_{n.1} = \frac{126301iw_{n+1}^{-3} + 2273418iw_{n+1}^{-4} - 23837w_{n+1} - 357566w_{n+1}^{-1} + 2010800w_{n+1}^{-2} + 2399719w_{n+1}^{-4} - 757806w_{n+1}^{-3}}{1331} \quad (2.1),$$

$$\phi_{n.2} = ((2n+1)^2 - 1 - w_{n+1}^2 - 2w_{n+1})(16w_{n+1}^{-3} + 50w_{n+1}^{-2} + 11iw_{n+1}^{-3} - 13i + 5iw_{n+1}) + \frac{(w_{n+1} - 2n)}{2}(8w_{n+1} - 8i - 16) \quad (2.2),$$

$$\phi_{n.3} = ((2n+1)^2 - 1 - w_{n+1}^2 + 2w_{n+1})(34iw_{n+1}^{-2} - 70iw_{n+1}^{-3} - 11i + 5 + 6iw_{n+1}) + \frac{289080}{1331} \quad (2.3),$$

and

$$\phi_{n.4} = ((2n+1)^2 - 1 - w_{n+1}^2 + 7w_{n+1})(11iw_{n+1} + 7 - 50w_{n+1}^{-1} + 23iw_{n+1}^{-3} - 54w_{n+1}^{-3}) - \frac{1003112i}{1331} \quad (2.4);$$

$$\nu_n = (iw_{n+1} - 4in - 4i + 1)^2 - 1 + w_{n+1}^2 \quad (2.5),$$

and

$$\epsilon_n = 4((2n+1)^2 - 1 - w_{n+1}^2 + w_{n+1}) + (iw_{n+1} - 2in + 1)(4iw_{n+1} + 4 - 8i) \quad (2.6).$$

It is immediate that for every integer  $n \geq 2$ ,  $(\phi_n, \nu_n, \epsilon_n)$  is well defined and gets sense. Now using the notion of *tackle* [ use Definition 2.2 ], then the following Theorem immediately implies the Fermat last conjecture.

**Theorem.F.** Let  $n$  be an integer  $\geq 2$  and let  $w_{n+1}$  [use Definitions 1.2]; look at  $(\phi_n, \nu_n, \epsilon_n)$  introduced in Definitions 2.3. Then at least one of the following three properties is satisfied by  $n$ .

(A<sub>1</sub>).  $n + 1$  is fermatian and  $n + 2 \equiv 0 \pmod{3}$ .

(A<sub>2</sub>). For every integer  $e$  such that  $3 \leq e \leq n + 3$ , the equation  $x^e + y^e = z^e$  has no non-zero integer solutions for  $x, y$  and  $z$ .

(A<sub>3</sub>).  $(\phi_n, \nu_n, \epsilon_n)$  tackles  $(1, 3iw_{n+1}^{-1})$  around  $6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i$  [ use Definition 2.2 for the meaning of *tackle* ].

We will simply prove Theorem.F in Section.3 by reasoning by reduction to absurd using equation  $x^n + y^n = z^n$  (where  $n$  is an integer  $\geq 3$ ). But before, let us remark.

**Remark 2.4 (fundamental).** Let  $n$  be an integer  $\geq 2$  and let  $w_{n+1}$  [ use Definitions 1.2]; now look at  $(\phi_n, \nu_n, \epsilon_n)$  introduced in Definitions 2.3. If  $w_{n+1} = 2n + 2$ , then  $(\phi_n, \nu_n, \epsilon_n)$  tackles  $(1, 3iw_{n+1}^{-1})$  around  $6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i$  [ use Definition 2.2 for the meaning of *tackle* ].

**Proof.** Indeed, observing (via the hypotheses) that  $w_{n+1} = 2n + 2$  and using the preceding equality, we easily deduce that

$$w_{n+1} - 2n = 2; iw_{n+1} - 4in - 4i + 1 = 1 - iw_{n+1}; 2n + 1 = w_{n+1} - 1; \text{ and } iw_{n+1} - 2in + 1 = 2i + 1 \quad (2.7).$$

Now look at  $(\phi_n, \nu_n, \epsilon_n)$  introduced in Definitions 2.3 and let  $(\phi_{n.2}, \phi_{n.3}, \phi_{n.4})$  explicited in Definitions 2.3. Clearly

$$\phi_{n.2} = -4w_{n+1}(16w_{n+1}^{-3} + 50w_{n+1}^{-2} + 11iw_{n+1}^{-3} - 13i + 5iw_{n+1}) + 8w_{n+1} - 8i - 16 \quad (2.8)$$

[ use (2.2) of Definitions 2.3 and the third equality of (2.7) (observe [via the third equality of (2.7)] that  $(w_{n+1} - 1)^2 = w_{n+1}^2 - 2w_{n+1} + 1$ ); and the first equality of (2.7) (observe [via the first equality of (2.7)] that  $\frac{(w_{n+1}-2n)}{2} = 1$ ) ], and

$$\phi_{n,3} = \frac{289080}{1331} \quad (2.9)$$

[ use (2.3) of Definitions 2.3 and the third equality of (2.7) (observe [via the third equality of (2.7)] that  $(w_{n+1} - 1)^2 = w_{n+1}^2 - 2w_{n+1} + 1$ ) ], and

$$\phi_{n,4} = 5w_{n+1}(11iw_{n+1} + 7 - 50w_{n+1}^{-1} + 23iw_{n+1}^{-3} - 54w_{n+1}^{-3}) - \frac{1003112i}{1331} \quad (2.10)$$

[ use (2.4) of Definitions 2.3 and the third equality of (2.7) (observe [via the third equality of (2.7)] that  $(w_{n+1} - 1)^2 = w_{n+1}^2 - 2w_{n+1} + 1$ ) ]. That being said let  $(\nu_n, \epsilon_n)$  explicited in Definitions 2.3; clearly

$$\nu_n = (1 - iw_{n+1})^2 - 1 + w_{n+1}^2 \quad (2.11)$$

[ use (2.5) of Definitions 2.3 and the second equality of (2.7) ] and clearly

$$\epsilon_n = 4((w_{n+1} - 1)^2 - 1 - w_{n+1}^2 + w_{n+1}) + (2i + 1)(4iw_{n+1} + 4 - 8i) \quad (2.12)$$

[ use (2.6) of Definitions 2.3 and the last two equalities of (2.7) ]. That being so, let  $(x, y, k)$  such that

$$x = 10 - 6w_{n+1} + iw_{n+1}; y = -10 + 6w_{n+1} - 3iw_{n+1}; \text{ and } k = -\frac{35}{11} - \frac{618w_{n+1}^{-1}}{121} + \frac{7494w_{n+1}^{-2}}{121} + \frac{126301w_{n+1}^{-4}}{1331} \quad (2.13),$$

and let  $(\phi_n, \nu_n, \epsilon_n, x, y, k)$  where  $(x, y, k)$  is explicited in (2.13) and where  $(\phi_n, \nu_n, \epsilon_n)$  is introduced in Definitions 2.3. Now consider  $(\nu_n, \epsilon_n, x, y)$ ; using (2.11) and (2.12) and the first two equalities of (2.13), we easily check [by elementary computation and the fact that  $i^2 = -1$ ] that

$$x + y - \nu_n = 0 \text{ and } x - y - \epsilon_n = 0 \quad (2.14).$$

That being so, look again at  $(\phi_n, \nu_n, \epsilon_n, x, y, k)$  and let  $(\phi_n, x, y, k)$ ; using the three equalities of (2.13), it becomes very easy to check (by elementary computation and the fact that  $i^2 = -1$ ) that

$$x + 3iyw_{n+1}^{-1} + k(6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i) + \phi_n = 0; k \in \mathcal{R} \text{ and } \phi_n = \sum_{j=1}^4 \phi_{n,j} \quad (2.15)$$

[ use the three equalities of (2.13) (for  $x$  and  $y$  and  $k$ ); and (2.1) for  $\phi_{n,1}$ ; and (2.j+6) for  $\phi_{n,j}$  (where  $2 \leq j \leq 4$ ); and (2.0) for  $\phi_n$  ]. Clearly  $(\phi_n, \nu_n, \epsilon_n)$  tackles  $(1, 3iw_{n+1}^{-1})$  around  $6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i$  ( use (2.14) and (2.15) and the notion of tackle introduced in Definition 2.2 [observe that  $(x, y) \in \mathcal{C}^2$  and  $k \in \mathcal{R}$ ; so  $(x, y, k) \in \mathcal{C}^2 \times \mathcal{R}$ ] ). Remark 2.4 follows.

Remark 2.4 clearly says that, if  $w_{n+1} = 2n + 2$  [i.e. **If** for every integer  $e$  such that  $3 \leq e \leq n + 1$  the equation  $x^e + y^e = z^e$  has no non-zero integer solutions for  $x$  and  $y$  and  $z$ ], we will have a simple equation of three unknowns which implies that  $(\phi_n, \nu_n, \epsilon_n)$  tackles  $(1, 3iw_{n+1}^{-1})$  around  $6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i$ . We will use Remark 2.4 in the Theorem which implies the Fermat last conjecture.

**Remark 2.5 (fundamental): reduction of the Fermat last conjecture into an equation of three unknowns via the reasoning by reduction to absurd.** *Let  $n$  be an integer  $\geq 3$  and let  $w_{n+1}$  [ use Definitions 1.2]; now look at  $(\phi_n, \nu_n, \epsilon_n)$  introduced in Definitions 2.3. **If**  $w_{n+1} = 2n$ , then  $(\phi_n, \nu_n, \epsilon_n)$  does not tackle  $(1, 3iw_{n+1}^{-1})$  around  $6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i$  ( use Definition 2.2 for the meaning of tackle ).*

**Proof.** Indeed, observing (via the hypotheses) that  $w_{n+1} = 2n$  and using the preceding equality, we easily deduce that

$$w_{n+1} - 2n = 0; iw_{n+1} - 4in - 4i + 1 = 1 - 4i - iw_{n+1}; 2n + 1 = w_{n+1} + 1; \text{ and } iw_{n+1} - 2in + 1 = 1 \quad (2.16).$$

Now look at  $(\phi_n, \nu_n, \epsilon_n)$  introduced in Definitions 2.3 and let  $(\phi_{n,2}, \phi_{n,3}, \phi_{n,4})$  explicited in Definitions 2.3. Clearly

$$\phi_{n,2} = 0 \quad (2.17)$$

[ use (2.2) of Definitions 2.3 and the third equality of (2.16) (observe [via the third equality of (2.16)] that  $(w_{n+1} + 1)^2 = w_{n+1}^2 + 2w_{n+1} + 1$ ); and the first equality of (2.16) (observe [via the first equality of (2.16)] that  $\frac{(w_{n+1}-2n)}{2} = 0$ ) ], and

$$\phi_{n.3} = 4w_{n+1}(34iw_{n+1}^{-2} - 70iw_{n+1}^{-3} - 11i + 5 + 6iw_{n+1}) + \frac{289080}{1331} \quad (2.18)$$

[ use (2.3) of Definitions 2.3 and the third equality of (2.16) (observe [via the third equality of (2.16)] that  $(w_{n+1} + 1)^2 = w_{n+1}^2 + 2w_{n+1} + 1$ ) ], and

$$\phi_{n.4} = 9w_{n+1}(11iw_{n+1} + 7 - 50w_{n+1}^{-1} + 23iw_{n+1}^{-3} - 54w_{n+1}^{-3}) - \frac{1003112i}{1331} \quad (2.19)$$

[ use (2.4) of Definitions 2.3 and the third equality of (2.16) (observe [via the third equality of (2.16)] that  $(w_{n+1} + 1)^2 = w_{n+1}^2 + 2w_{n+1} + 1$ ) ]. That being said, let  $(\nu_n, \epsilon_n)$  explicited in Definitions 2.3; clearly

$$\nu_n = (1 - 4i - iw_{n+1})^2 - 1 + w_{n+1}^2 \quad (2.20)$$

[ use (2.5) of Definitions 2.3 and the second equality of (2.16) ] and clearly

$$\epsilon_n = 4((w_{n+1} + 1)^2 - 1 - w_{n+1}^2 + w_{n+1}) + 4iw_{n+1} + 4 - 8i \quad (2.21)$$

[ use (2.6) of Definitions 2.3 and the last two equalities of (2.16) ]. That being so, let  $(x, y)$  such that

$$x = 2w_{n+1} + iw_{n+1} - 6 - 8i \text{ and } y = -10 - 10w_{n+1} - 3iw_{n+1} \quad (2.22),$$

and let  $(\phi_n, \nu_n, \epsilon_n, x, y)$  where  $(x, y)$  is explicited in (2.22) and where  $(\phi_n, \nu_n, \epsilon_n)$  is introduced in Definitions 2.3 [ use (2.0) for  $\phi_n$ ; and (2.5) for  $\nu_n$ ; and (2.6) for  $\epsilon_n$ ; and (2.22) for  $(x, y)$  ]. Using (2.20) and (2.21) and the two equalities of (2.22), then we easily check [by elementary computation and the fact that  $i^2 = -1$ ] that

$$x + y - \nu_n = 0 \text{ and } x - y - \epsilon_n = 0 \quad (2.23).$$

That being said, we have this **Fact**.

**Fact.0.**

there exists not  $k \in \mathcal{R}$  such that  $x + 3iyw_{n+1}^{-1} + k(6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i) + \phi_n = 0$ .

**Otherwise** [ we reason by reduction to absurd ]

$$\text{Let } k \in \mathcal{R} \text{ such that } x + 3iyw_{n+1}^{-1} + k(6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i) + \phi_n = 0 \quad (2.24).$$

It is immediate to see that (2.24) says that

$$x + 3iyw_{n+1}^{-1} + \phi_n = -k(6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i); k \in \mathcal{R} \text{ and } \phi_n = \sum_{j=1}^4 \phi_{n.j} \quad (2.25)$$

[ use (2.0) for  $\phi_n$  ]. Now let  $(\phi_n, \nu_n, \epsilon_n, x, y)$  explicited above; consider  $(\phi_n, x, y)$  and look at (2.25), using elementary computation and elementary divisibility coupled with the fact that  $i^2 = -1$  and  $k \in \mathcal{R}$ , it becomes very easy to check that (2.25) immediately implies that

$$k = k_{n.1} = k_{n.2} \quad (2.26),$$

where

$$k_{n.1} = \frac{-23837w_{n+1} - 357566w_{n+1}^{-1} + 2010800w_{n+1}^{-2} + 2399719w_{n+1}^{-4} - 757806w_{n+1}^{-3} + 289080}{1331(19 - 6w_{n+1})} + \frac{85w_{n+1} - 447 - 486w_{n+1}^{-3}}{19 - 6w_{n+1}} \quad (2.27),$$

and

$$k_{n.2} = \frac{2273418w_{n+1}^{-4} + 126301w_{n+1}^{-3} - 1003112}{1331(w_{n+1} - 11w_{n+1}^2 + 18)} + \frac{123w_{n+1}^2 + 106w_{n+1}^{-1} - 38 - 43w_{n+1} - 73w_{n+1}^{-2}}{w_{n+1} - 11w_{n+1}^2 + 18} \quad (2.28)$$

[ via (2.25), use the two equalities of (2.22) (for  $x$  and  $y$ ); and (2.1) for  $\phi_{n.1}$ ; and (2.j+15) for  $\phi_{n.j}$  (where  $2 \leq j \leq 4$ ); and (2.0) for  $\phi_n$  ]. That being so, using (2.26) and (2.27) and (2.28), then we immediately deduce [via elementary computation and the fact that  $k_{n.1} = k_{n.2}$ ] that

$$10996722w_{n+1}^{-2} - 11643588w_{n+1}^{-3} - 7115526 + 7762392w_{n+1}^{-1} = 0 \quad (2.29).$$

Equality (2.29) is clearly impossible [ since it is trivial to see that  $w_{n+1} > 4$  (use property (1.5.0) of Remark 1.5) and therefore

$$10996722w_{n+1}^{-2} - 11643588w_{n+1}^{-3} - 7115526 + 7762392w_{n+1}^{-1} < 0$$

]. So assuming that

$$\text{there exists } k \in \mathcal{R} \text{ such that } x + 3iyw_{n+1}^{-1} + k(6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i) + \phi_n = 0$$

gives rise to a serious contradiction; therefore

$$\text{there exists not } k \in \mathcal{R} \text{ such that } x + 3iyw_{n+1}^{-1} + k(6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i) + \phi_n = 0.$$

**Fact.0.** follows.

Clearly  $(\phi_n, \nu_n, \epsilon_n)$  does not tackle  $(1, 3iw_{n+1}^{-1})$  around  $6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i$  [ use (2.23) and **Fact.0** and the notion of tackle introduced in Definition 2.2]. Remark 2.5 follows.  $\square$

Remark 2.5 reduces the Fermat last conjecture into a simple equation of three unknowns (via the reasoning by reduction to absurd) and is the heart of the Fermat last conjecture. Indeed Remark 2.5 clearly says that, if  $w_{n+1} = 2n$  [i.e. **If** the equation  $x^{n+1} + y^{n+1} = z^{n+1}$  has non-zero integer solutions for  $x$  and  $y$  and  $z$ , and if for every integer  $e$  such that  $3 \leq e \leq n$  the equation  $x^e + y^e = z^e$  has no non-zero integer solutions for  $x$  and  $y$  and  $z$ ], we will have a simple equation of three unknowns which implies that  $(\phi_n, \nu_n, \epsilon_n)$  does not tackle  $(1, 3iw_{n+1}^{-1})$  around  $6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i$ . *It is also clear to see that Remark 2.5 says that: " for the first  $n$  such that the equation  $x^{n+1} + y^{n+1} = z^{n+1}$  has non-zero integer solutions for  $x$  and  $y$  and  $z$ , then we will have a simple equation of three unknowns which implies that  $(\phi_n, \nu_n, \epsilon_n)$  does not tackle  $(1, 3iw_{n+1}^{-1})$  around  $6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i$ ".* We will use Remark 2.5 in Section.3 to immediately deduce the Fermat last conjecture. That being said, the following remark is only an immediate reformulation of Remark 2.5.

**Remark 2.5'** (fundamental). *Let  $n$  be an integer  $\geq 3$  and let  $w_{n+1}$  [ use Definitions 1.2]; now look at  $(\phi_n, \nu_n, \epsilon_n)$  introduced in Definitions 2.3. **If** the equation  $x^{n+1} + y^{n+1} = z^{n+1}$  has non-zero integer solutions for  $x$  and  $y$  and  $z$ , and if for every integer  $e$  such that  $3 \leq e \leq n$  the equation  $x^e + y^e = z^e$  has no non-zero integer solutions for  $x$  and  $y$  and  $z$ , then  $(\phi_n, \nu_n, \epsilon_n)$  does not tackle  $(1, 3iw_{n+1}^{-1})$  around  $6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i$  ( use Definition 2.2 for the meaning of tackle ).*

**Proof.** Indeed observing (via the hypotheses) that the equation  $x^{n+1} + y^{n+1} = z^{n+1}$  has non-zero integer solutions for  $x$  and  $y$  and  $z$ , and for every integer  $e$  such that  $3 \leq e \leq n$  the equation  $x^e + y^e = z^e$  has no non-zero integer solutions for  $x$  and  $y$  and  $z$ ; then using the previous and the definition of  $w_{n+1}$ , we immediately deduce that  $w_{n+1} = 2n$ . So  $(\phi_n, \nu_n, \epsilon_n)$  does not tackle  $(1, 3iw_{n+1}^{-1})$  around  $6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i$  (use the last equality and Remark 2.5). Remark 2.5' follows.  $\square$

Now using Definitions 2.3, then we have the following Proposition.

**Proposition 2.6.** *Let  $n$  be an integer  $\geq 3$  and let  $w_{n+1}$  ( use Definitions 1.2); now look at  $(\phi_n, \nu_n, \epsilon_n)$  introduced in Definitions 2.3, and via  $(\phi_n, \nu_n, \epsilon_n)$ , consider  $(\phi_{n-1}, \nu_{n-1}, \epsilon_{n-1})$  ( this consideration gets sense, since  $n \geq 3$ , and therefore  $n-1 \geq 2$  ). **If**  $w_{n+1} \leq 2n$ , then we have the following two properties.*

(2.6.0).  $w_{n+1} = w_n$ ; therefore there exists an integer  $e$  such that  $3 \leq e \leq n+1$  and the equation  $x^e + y^e = z^e$  has non-zero integer solutions for  $x$  and  $y$  and  $z$ .

(2.6.1).  $(\phi_{n-1} - \phi_n, \nu_{n-1} - \nu_n, \epsilon_{n-1} - \epsilon_n)$  tackles  $(1, 3iw_{n+1}^{-1})$  around  $6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i$ .

We will use Proposition 2.6 in the Theorem which implies the Fermat last conjecture. Proposition 2.6 is not very difficult to prove and we will prove Proposition 2.6 in Section.2' (Epilogue). Now we are ready to give the short complete proof the Fermat last conjecture.

### 3 The short proof of the Fermat last conjecture

In this Section, using the elementary reformulation of the Fermat last conjecture proved in Section.1 and simple properties of Section.2, we give the direct simple proof of the Fermat last conjecture. In this section, the definition of fermatian [use Definition 1.1], the definitions of  $\mathcal{W}(n+1)$  and  $w_{n+1}$  [use Definitions 1.2], the definition of *tackle* [use Definition 2.2] and the definition of  $(\phi_n, \nu_n, \epsilon_n)$  [ use Definitions 2.3], are *fundamental and crucial*. Now the following Theorem immediately implies the Fermat last conjecture.

**Theorem 3.1.** *Let  $n$  be an integer  $\geq 2$  and let  $w_{n+1}$  [use Definitions 1.2]; look at  $(\phi_n, \nu_n, \epsilon_n)$  introduced in Definitions 2.3. Then at least one of the following three properties is satisfied by  $n$ .*  
*(A<sub>1</sub>).  $n + 1$  is fermatian and  $n + 2 \equiv 0 \pmod{3}$ .*  
*(A<sub>2</sub>). For every integer  $e$  such that  $3 \leq e \leq n + 3$ , the equation  $x^e + y^e = z^e$  has no non-zero integer solutions for  $x, y$  and  $z$ .*  
*(A<sub>3</sub>).  $(\phi_n, \nu_n, \epsilon_n)$  tackles  $(1, 3iw_{n+1}^{-1})$  around  $6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i$  [ use Definition 2.2 for the meaning of tackle ].*

It is immediate that Theorem 3.1 is exactly Theorem.F stated after Definitions 2.3. We are going to prove simply Theorem 3.1. But before doing so, let us remark.

**Remark 3.2.** *Let  $n$  be an integer  $\geq 2$  and look at  $w_{n+1}$  [use Definitions 1.2]. We have the following three properties.*

(3.2.1.) **If  $w_{n+1} = 2n + 2$ , then Theorem 3.1 is satisfied.**

(3.2.2.) **If  $2 \leq n \leq 6$ , then Theorem 3.1 is satisfied.**

(3.2.3) [ *The explicit using of the Fermat last conjecture*]. **If for every integer  $e$  with  $3 \leq e \leq n + 1$ , the equation  $x^e + y^e = z^e$  has no non-zero integer solutions for  $x$  and  $y$  and  $z$ , then Theorem 3.1 is satisfied.**

**Proof.** Property (3.2.1) is immediate [indeed, let  $n$  be an integer  $\geq 2$ ; observing (by the hypotheses) that  $w_{n+1} = 2n + 2$ , then

$$(\phi_n, \nu_n, \epsilon_n) \text{ tackles } (1, 3iw_{n+1}^{-1}) \text{ around } 6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i \quad (3.1)$$

( use Remark 2.4). (3.1) clearly says that property (A<sub>3</sub>) of Theorem 3.1 is satisfied by  $n$ ; therefore Theorem 3.1 is satisfied. Property (3.2.1) follows].

Property (3.2.2.) is immediate [ indeed, observing (by the hypotheses) that  $2 \leq n \leq 6$ , clearly

$$n + 1 \text{ is fermatian} \quad (3.2)$$

(use Remark 1.1.2 of Definition 1.1); consequently

$$w_{n+1} = 2n + 2 \quad (3.2')$$

(use (3.2) and the meaning of  $w_{n+1}$  introduced in Definitions 1.2). Clearly Theorem 3.1 is satisfied (use equality (3.2') and property (3.2.1).]

Property (3.2.3) is trivial [ indeed using the definition of  $w_{n+1}$  and property (1.3.6) of Proposition 1.3, then it becomes trivial to see that Property (3.2.3) is the immediate reformulation of Property (3.2.1); so Theorem 3.1 is satisfied (use the previous and Property (3.2.1))].  $\square$

**Remark 3.3.** *Suppose that Theorem 3.1 is false. Then there exists an integer  $n$  which is a **minimum counter-example** to Theorem 3.1.*

**Proof.** Immediate.  $\square$

The previous simple remarks made, we now prove simply Theorem 3.1.

**Proof of Theorem 3.1.** Otherwise [ we reason by reduction to absurd], let  $n$  be a minimum counter-example to Theorem 3.1 [ such a  $n$  exists, by using Remark 3.3]. We observe the following.

*Observation.3.1.i.* *Look at  $n$  [recall  $n$  is a minimum counter-example to Theorem 3.1]. Then*

$$(\phi_n, \nu_n, \epsilon_n) \text{ does not tackle } (1, 3iw_{n+1}^{-1}) \text{ around } 6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i.$$

Immediate, since in particular,  $n$  is a counter-example to Theorem 3.1.

*Observation.3.1.ii.* *Look at  $n$  [recall  $n$  is a minimum counter-example to Theorem 3.1]. Then for every integer  $j \in \{1, 2\}$ , property (A<sub>j</sub>) of Theorem 3.1 is not satisfied by  $n$ .*

Immediate, since in particular,  $n$  is a counter-example to Theorem 3.1.

*Observation.3.1.iii.* *Look at  $n$  [recall  $n$  is a minimum counter-example to Theorem 3.1]. Then  $n > 6$ .*

Otherwise  $n \leq 6$  and clearly Theorem 3.1 is satisfied [ use previous inequality and property (3.2.2) of Remark 3.2]; a contradiction, since in particular  $n$  is a counter-example to Theorem 3.1.

*Observation.3.1.iv.* *Look at  $n$  [recall  $n$  is a minimum counter-example to Theorem 3.1]; then  $w_{n+1} \leq 2n$ .*

Otherwise,

$$w_{n+1} > 2n \tag{3.3};$$

remarking that  $w_{n+1}$  and  $2n$  are even [  $w_{n+1}$  is even (use the definition of  $w_{n+1}$ ) and  $2n$  is trivially even ], then inequality (3.3) immediately implies that  $w_{n+1} \geq 2n + 2$ . It is trivial [by the definition of  $w_{n+1}$ ] that  $w_{n+1} \leq 2n + 2$ . Now using the previous two inequalities, then it becomes trivial to deduce that  $w_{n+1} = 2n + 2$ ; so Theorem 3.1 is satisfied [use the preceding equality and property (3.2.1) of Remark 3.2]; we have a contradiction, since in particular  $n$  is a counter-example to Theorem 3.1. So  $w_{n+1} \leq 2n$  and Observation.3.1.iv follows.

*Observation.3.1.v ( The implicit using of the Fermat last conjecture).* Look at  $n$  [recall  $n$  is a minimum counter-example to Theorem 3.1]; then  $w_{n+1} = w_n$  and therefore, there exists an integer  $e$  such that  $3 \leq e \leq n + 1$  and the equation  $x^e + y^e = z^e$  has non-zero integer solutions for  $x$  and  $y$  and  $z$ .

Indeed, observing [by Observation.3.1.iv] that  $w_{n+1} \leq 2n$  and noticing [by Observation.3.1.iii] that  $n > 6$ , then using the previous two inequalities, it becomes trivial to deduce that all the hypotheses of Proposition 2.6 are satisfied, therefore, all the conclusions of Proposition 2.6 are satisfied; in particular property (2.6.0) of Proposition 2.6 is satisfied, and consequently  $w_{n+1} = w_n$  and there exists an integer  $e$  such that  $3 \leq e \leq n + 1$  and the equation  $x^e + y^e = z^e$  has non-zero integer solutions for  $x$  and  $y$  and  $z$ . Observation.3.1.v follows.

*Observation.3.1.vi.* Let  $(\phi_n, \nu_n, \epsilon_n)$  [use Definitions 2.3], and via  $(\phi_n, \nu_n, \epsilon_n)$ , consider  $(\phi_{n-1}, \nu_{n-1}, \epsilon_{n-1})$  [ this consideration gets sense, since  $n > 6$  (by Observation.3.1.iii), and so  $n - 1 > 5 > 2$  ]. Then

$$(\phi_{n-1} - \phi_n, \nu_{n-1} - \nu_n, \epsilon_{n-1} - \epsilon_n) \text{ tackles } (1, 3iw_{n+1}^{-1}) \text{ around } 6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i.$$

Indeed, observing [by Observation.3.1.iv] that  $w_{n+1} \leq 2n$  and noticing [by Observation.3.1.iii] that  $n > 6$ , then using the previous two inequalities, it becomes trivial to deduce that all the hypotheses of Proposition 2.6 are satisfied, therefore, all the conclusions of Proposition 2.6 are satisfied; in particular property (2.6.1) of Proposition 2.6 is satisfied; consequently  $(\phi_{n-1} - \phi_n, \nu_{n-1} - \nu_n, \epsilon_{n-1} - \epsilon_n)$  tackles  $(1, 3iw_{n+1}^{-1})$  around  $6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i$ . Observation.3.1.vi follows.

*Observation.3.1.vii.* Look at  $n$  [recall  $n$  is a minimum counter-example to Theorem 3.1] and consider  $n - 1$  [ this consideration gets sense, since  $n > 6$  (use Observation.3.1.iii), and so  $n - 1 > 5 > 2$  ].

*Then property  $(A_1)$  of Theorem 3.1 is not satisfied by  $n - 1$ .*

Otherwise [ we reason by reduction to absurd ]

$$(n - 1) + 1 \text{ is fermatian and } (n - 1) + 2 \equiv 0 \text{ mod } [3] \tag{3.4}$$

[ since property  $(A_1)$  of Theorem 3.1 is satisfied by  $n - 1$  ]. It trivial to see that (3.4) says that

$$n \text{ is fermatian and } n + 1 \equiv 0 \text{ mod } [3] \tag{3.4'}$$

Clearly

$$n + 1 \text{ is fermatian} \tag{3.5}$$

[ observe that  $n > 6$  (use Observation.3.1.iii) and (3.4') and Remark 1.1.3 of Definition 1.1 ). Clearly  $w_{n+1} = 2n + 2$  [ use (3.5) and the meaning of  $w_{n+1}$  introduced in Definitions 1.2 ]; so Theorem 3.1 is satisfied by  $n$  [ use the previous equality and property (3.2.1) of Remark 3.2 ], and we have a contradiction, since in particular  $n$  is a counter-example to Theorem 3.1. Observation.3.1.vii follows.

*Observation.3.1.viii.* Look at  $n$  [recall  $n$  is a minimum counter-example to Theorem 3.1] and consider  $n - 1$  [ this consideration gets sense, since  $n > 6$  (use Observation.3.1.iii), and so  $n - 1 > 5 > 2$  ].

*Then property  $(A_2)$  of Theorem 3.1 is not satisfied by  $n - 1$ .*

Otherwise [ (we reason by reduction to absurd).

For every integer  $e$  such that  $3 \leq e \leq (n-1) + 3$ , the equation  $x^e + y^e = z^e$  has no solution in integers  $\geq 1$  (3.6)

[ since property  $(A_2)$  of Theorem 3.1 is satisfied by  $n-1$  ]. It is immediate that (3.6) clearly says that

For every integer  $e$  such that  $3 \leq e \leq n+2$ , the equation  $x^e + y^e = z^e$  has no solution in integers  $\geq 1$  (3.6').

It trivial to deduce that (3.6') implies that

For every integer  $e$  such that  $3 \leq e \leq n+1$ , the equation  $x^e + y^e = z^e$  has no solution in integers  $\geq 1$  (3.7).

Clearly

$n+1$  is fermatian (3.7').

[ use (3.7) and the meaning of fermatian introduced in Definition 1.1 ]. So  $w_{n+1} = 2n+2$  [ use (3.7') and the meaning of  $w_{n+1}$  introduced in Definitions 1.2 ]. Clearly Theorem 3.1 is satisfied [ use the previous equality and property (3.2.1) of Remark 3.2 ], and we have a contradiction, since in particular  $n$  is a counter-example to Theorem 3.1. Observation.3.1.viii follows.

Observation.3.1.ix. Look at  $(\phi_n, \nu_n, \epsilon_n)$  ( use Definitions 2.3 and recall  $n$  is a minimum counter-example to Theorem 3.1 ), and via  $(\phi_n, \nu_n, \epsilon_n)$ , consider  $(\phi_{n-1}, \nu_{n-1}, \epsilon_{n-1})$  [ this consideration gets sense, since  $n > 6$  (use Observation.3.1.iii), and so  $n-1 > 5 > 2$  ]. Then

$(\phi_{n-1}, \nu_{n-1}, \epsilon_{n-1})$  tackles  $(1, 3iw_{n+1}^{-1})$  around  $6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i$ .

Indeed look at  $n$  ( recall  $n$  is a minimum counter-example to Theorem 3.1 ), and via  $n$ , consider  $n-1$  (this consideration gets sense, since  $n > 6$  (use Observation.3.1.iii), and so  $n-1 > 5 > 2$ ); then, by the minimality of  $n$ , we immediately deduce that  $n-1$  is not a counter-example to Theorem 3.1; so

at least one of properties  $(A_j)$  of Theorem 3.1 is satisfied by  $n-1$ ;  $j \in \{1, 2, 3\}$  (3.8).

Clearly

property  $(A_3)$  of Theorem 3.1 is satisfied by  $n-1$  (3.9)

[ use (3.8) and Observation.3.1.viii and Observation.3.1.vii ]. So

$(\phi_{n-1}, \nu_{n-1}, \epsilon_{n-1})$  tackles  $(1, 3iw_n^{-1})$  around  $6w_n - 19 - iw_n + 11iw_n^2 - 18i$  (3.10)

( use (3.9) and property  $(A_3)$  of Theorem 3.1 for  $n-1$  ). Now observing [ by Observation.3.1.v ] that  $w_{n+1} = w_n$  and using the preceding equality, then we easily deduce that (3.10) clearly says that

$(\phi_{n-1}, \nu_{n-1}, \epsilon_{n-1})$  tackles  $(1, 3iw_{n+1}^{-1})$  around  $6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i$ .

Observation.3.1.ix follows.

Observation.3.1.x. Look at  $n$  and let  $(\phi_n, \nu_n, \epsilon_n)$  [ use Definitions 2.3 ]. Then

$(\phi_n, \nu_n, \epsilon_n)$  tackles  $(1, 3iw_{n+1}^{-1})$  around  $6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i$ .

Indeed using Observation.3.1.vi and the definition of tackle [ use Definition 2.2 ], then we immediately deduce that

there exists  $(x, y, k) \in \mathcal{C}^2 \times \mathcal{R}$  such that  $x + 3iyw_{n+1}^{-1} + k(6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i) + (\phi_{n-1} - \phi_n) = 0$  (3.11)

where

$$x + y - (\nu_{n-1} - \nu_n) = 0 \text{ and } x - y - (\epsilon_{n-1} - \epsilon_n) = 0 \quad (3.12).$$

That being so, using Observation.3.1.ix and the definition of *tackle* [ use Definition 2.2 ], then we immediately deduce that

$$\text{there exists } (x', y', k') \in \mathcal{C}^2 \times \mathcal{R} \text{ such that } x' + 3iy'w_{n+1}^{-1} + k'(6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i) + \phi_{n-1} = 0 \quad (3.13)$$

where

$$x' + y' - \nu_{n-1} = 0 \text{ and } x' - y' - \epsilon_{n-1} = 0 \quad (3.14).$$

Now using equality of (3.13), then we immediately deduce that equality of (3.11) clearly says that

$$x - x' + 3i(y - y')w_{n+1}^{-1} + (k - k')(6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i) - \phi_n = 0 \quad (3.15).$$

It is trivial to see that equality (3.15) clearly says that

$$x' - x + 3i(y' - y)w_{n+1}^{-1} + (k' - k)(6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i) + \phi_n = 0 \quad (3.16).$$

Using the two equalities of (3.14), then we immediately deduce that the two equalities of (3.12) clearly say that

$$(x - x') + (y - y') + \nu_n = 0 \text{ and } (x - x') - (y - y') + \epsilon_n = 0 \quad (3.17).$$

It is trivial to see that (3.17) clearly says that

$$(x' - x) + (y' - y) - \nu_n = 0 \text{ and } (x' - x) - (y' - y) - \epsilon_n = 0 \quad (3.18).$$

Now observing that  $(x, y, k) \in \mathcal{C}^2 \times \mathcal{R}$  ( use (3.11) ) and since  $(x', y', k') \in \mathcal{C}^2 \times \mathcal{R}$  ( use (3.13) ), then it becomes trivial to deduce that (3.16) and (3.18) clearly say that

$$\text{there exists } (x'', y'', k'') \in \mathcal{C}^2 \times \mathcal{R} \text{ such that } x'' = x' - x \text{ and } y'' = y' - y \text{ and } k'' = k' - k \quad (3.19)$$

where

$$x'' + y'' - \nu_n = 0, x'' - y'' - \epsilon_n = 0 \text{ and } x'' + 3iy''w_{n+1}^{-1} + k''(6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i) + \phi_n = 0 \quad (3.19').$$

Clearly

$$(\phi_n, \nu_n, \epsilon_n) \text{ tackles } (1, 3iw_{n+1}^{-1}) \text{ around } 6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i$$

( use (3.19) and (3.19') and the definition of *tackle* introduced in Definition 2.2 ). Observation.3.1.x follows.

These simple observations made, then it becomes trivial to see that Observation.3.1.x clearly contradicts Observation.3.1.i. Theorem 3.1 follows.  $\square$ .

Theorem 3.1 and Remark 2.5 of Section.2 immediately imply the Fermat last conjecture.

**Theorem 3.4** (The using of Remark 2.5 of Section.2). *For every integer  $n \geq 2$ , we have  $w_{n+1} = 2n + 2$  [i.e. For every integer  $n \geq 2$ ,  $n + 1$  is fermatian].*

**Proof.** Otherwise, let  $n$  be a minimum counter-example; we have

$$w_{n+1} \neq 2n + 2 \quad (3.20).$$

Then we observe the following.

*Observation.3.4.1.*  $w_{n+1} \leq 2n$ .

Otherwise, clearly  $w_{n+1} > 2n$ ; now observing that  $w_{n+1}$  and  $2n$  are even [  $w_{n+1}$  is even (by the definition of  $w_{n+1}$ ) and  $2n$  is trivially even ], then the previous inequality immediately implies that

$$w_{n+1} \geq 2n + 2 \quad (3.21).$$

Remarking that

$$w_{n+1} \leq 2n + 2 \quad (3.22)$$

[use the definition of  $w_{n+1}$  explicited in Definitions 1.2], clearly

$$w_{n+1} = 2n + 2 \quad (3.23)$$

[use (3.21) and (3.22)] and equality (3.23) contradicts (3.20). Observation.3.4.1 follows.

*Observation.3.4.2.*  $n > 6$ .

Otherwise, clearly

$$2 \leq n \leq 6 \quad (3.24).$$

So

$$n + 1 \text{ is fermatian} \quad (3.25)$$

[use (3.24) and Remark 1.1.2 of Definition 1.1], clearly

$$w_{n+1} = 2n + 2 \quad (3.26)$$

[use (3.25) and the defintion of  $w_{n+1}$  explicited in Definitions 1.2]. Equality (3.26) contradicts (3.20). Observation.3.4.2 follows.

*Observation.3.4.3.* [an implicit using of the Fermat last conjecture].  $w_{n+1} = w_n$ ; therefore there exists an integer  $e$  such that  $3 \leq e \leq n + 1$  and the equation  $x^e + y^e = z^e$  has non-zero integer solutions for  $x$  and  $y$  and  $z$ .

Immediate [use (3.20) and property (1.5.3) of Remark 1.5]. Observation.3.4.3 follows.

*Observation.3.4.4.* Look at  $n$ ; then  $w_{n+1} = 2n$  and  $n + 1 \not\equiv 0 [3]$ .

Indeed, look at  $n$  [recall  $n$  is a minimum counter-example], and via  $n$ , consider  $n - 1$  [ this consideration gets sense, since  $n > 6$  (by Observation.3.4.2), and therefore  $n - 1 > 5 > 2$  ]. Then, by the minimality of  $n$ ,  $n - 1$  is not a counter-example; consequently  $w_{(n-1)+1} = 2(n-1)+2$  and the previous equality clearly says that

$$w_n = 2n \quad (3.27).$$

Clearly

$$n \text{ is fermatian} \quad (3.28)$$

[ use (3.27) and the meaning of  $w_n$  explicited in Definitions 1.2 ]. Now notice that

$$w_{n+1} = w_n \quad (3.29)$$

[ use Observation.3.4.3] and clearly

$$w_{n+1} = 2n \quad (3.30)$$

[use equalities (3.27) and (3.29)]. That being said, observe that

$$n + 1 \not\equiv 0 [3] \quad (3.31)$$

[ otherwise

$$n + 1 \equiv 0 [3] \quad (3.31').$$

Clearly

$$n + 1 \text{ is fermatian} \quad (3.32)$$

(use congruence (3.31') and (3.28) and Remark 1.1.3 of Definition 1.1). So  $w_{n+1} = 2n + 2$  ( use (3.32) and the definition of  $w_{n+1}$ ) and the preceding equality contradicts (3.20). So  $n + 1 \not\equiv 0 [3]$  ]. Observation.3.4.4 follows [use (3.30) and (3.31)].

*Observation.3.4.5* [the using of Remark 2.5 of Section.2].

$$(\phi_n, \nu_n, \epsilon_n) \text{ does not tackle } (1, 3ig_{n+1}^{-1}) \text{ around } 6g_{n+1} - 19 - ig_{n+1} + 11ig_{n+1}^2 - 18i.$$

Indeed, observing [by Observation.3.4.4] that  $w_{n+1} = 2n$  and using Remark 2.5 of Section.2 (notice that  $n > 6$ , by Observation.3.4.2), then it becomes trivial to deduce that

$$(\phi_n, \nu_n, \epsilon_n) \text{ does not tackle } (1, 3ig_{n+1}^{-1}) \text{ around } 6g_{n+1} - 19 - ig_{n+1} + 11ig_{n+1}^2 - 18i.$$

Observation.3.4.5 follows.

These simple observations made, let  $n$  and look at  $(\phi_n, \nu_n, \epsilon_n)$ ; remarking that  $n > 6$  [use Observation.3.4.2], then using the previous, it becomes immediate to deduce that all the hypotheses of Theorem 3.1 are satisfied, therefore, the conclusion of Theorem 3.1 is satisfied; consequently

$$\text{At least one of properties } (A_j) \text{ of Theorem 3.1 is satisfied by } n; j \in \{1, 2, 3\} \quad (3.33).$$

That being said, we have the following four **Claims**.

**Claim.1. Property  $(A_1)$  of Theorem 3.1 is not satisfied by  $n$ .**

Otherwise [ we reason by reduction to absurd ]

$$n + 1 \text{ is fermatian and } n + 2 \equiv 0 \text{ mod } [3] \quad (3.33')$$

[ since property  $(A_1)$  of Theorem 3.1 is satisfied by  $n$  ]. It is immediate that  $(3.33')$  implies that

$$n + 1 \text{ is fermatian} \quad (3.34).$$

Clearly  $w_{n+1} = 2n + 2$  [ use  $(3.34)$  and the meaning of  $w_{n+1}$  introduced in Definitions 1.2 ] and the preceding equality contradicts  $(3.20)$ . **Claim.1** follows.

**Claim.2. Property  $(A_2)$  of Theorem 3.1 is not satisfied by  $n$ .**

Otherwise [ (we reason by reduction to absurd).

$$\text{For every integer } e \text{ such that } 3 \leq e \leq n + 3, \text{ the equation } x^e + y^e = z^e \text{ has no solution in integers } \geq 1 \quad (3.35)$$

[ since property  $(A_2)$  of Theorem 3.1 is satisfied by  $n$  ]. It is immediate that  $(3.35)$  implies that

$$\text{For every integer } e \text{ such that } 3 \leq e \leq n + 1, \text{ the equation } x^e + y^e = z^e \text{ has no solution in integers } \geq 1 \quad (3.36).$$

Now using the meaning of fermatian introduced in Definition 1.1, then it becomes trivial to deduce that  $(3.36)$  clearly says that

$$n + 1 \text{ is fermatian} \quad (3.37).$$

So  $w_{n+1} = 2n + 2$  ( use  $(3.37)$  and the meaning of  $w_{n+1}$  introduced in Definitions 1.2 ) and the preceding equality contradicts  $(3.20)$ . **Claim.2** follows.

**Claim.3. Property  $(A_3)$  of Theorem 3.1 is satisfied by  $n$ .**

Immediate [use  $(3.33)$  and **Claim.1** and **Claim.2** ]. **Claim.3** follows.

**Claim.4.  $(\phi_n, \nu_n, \epsilon_n)$  tackles  $(1, 3ig_{n+1}^{-1})$  around  $6g_{n+1} - 19 - ig_{n+1} + 11ig_{n+1}^2 - 18i$ .**

Immediate [use **Claim.3** and the meaning of property  $(A_3)$  of Theorem 3.1 ]. **Claim.4** follows.

The previous four **Claims** made, then it becomes trivial to deduce that **Claim.4** contradicts Observation.3.4.5. Theorem 3.4 follows.  $\square$

**Theorem 3.5** (*The Proof of the Fermat last conjecture*). *The Fermat last conjecture is true.*

**Proof.** Observe [by using Theorem 3.4] that

$$\text{For every integer } n \geq 2 \text{ we have } w_{n+1} = 2n + 2 \quad (3.38);$$

consequently, the Fermat last conjecture is true ( use  $(3.38)$  and Theorem 1.4).  $\square$

## 2' Epilogue

Our article clearly shows that simple calculus on complex numbers coupled with trivial computation via the reduction of the Fermat last conjecture into an equation of three unknowns and the trivial reformulation of this conjecture, help to give the short complete simple proof of the Fermat last conjecture. Now we end this article by proving the only Proposition that we let unproved in Section.2.

**Proposition 2.6.** Let  $n$  be an integer  $\geq 3$  and let  $w_{n+1}$  ( use Definitions 1.2); now look at  $(\phi_n, \nu_n, \epsilon_n)$  introduced in Definitions 2.3, and via  $(\phi_n, \nu_n, \epsilon_n)$ , consider  $(\phi_{n-1}, \nu_{n-1}, \epsilon_{n-1})$  ( this consideration gets sense, since  $n \geq 3$ , and therefore  $n-1 \geq 2$  ). **If**  $w_{n+1} \leq 2n$ , then we have the following two properties.

(2.6.0).  $w_{n+1} = w_n$ ; therefore there exists an integer  $e$  such that  $3 \leq e \leq n+1$  and the equation  $x^e + y^e = z^e$  has non-zero integer solutions for  $x$  and  $y$  and  $z$ .

(2.6.1).  $(\phi_{n-1} - \phi_n, \nu_{n-1} - \nu_n, \epsilon_{n-1} - \epsilon_n)$  tackles  $(1, 3iw_{n+1}^{-1})$  around  $6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i$ .

To prove Proposition 2.6, we need the following elementary remarks.

**Remark 2.7.** Let  $n$  be an integer  $\geq 2$  and let  $w_{n+1}$  [ use Definitions 1.2]. **If**  $w_{n+1} \leq 2n$ , then  $w_{n+1} = w_n$ . **Proof.** Indeed observing [via the hypotheses] that

$$w_{n+1} \leq 2n \quad (2'.0)$$

clearly

$$w_{n+1} \neq 2n+2 \quad (2'.1)$$

[ use (2'.0) ]. So  $w_{n+1} = w_n$  [ use (2'.1) and property (1.5.2) of Remark 1.5 ]. Remark 2.7 follows.  $\square$

**Remark 2.8.** Let  $n$  be an integer  $\geq 3$  and let  $w_{n+1}$  [ use Definitions 1.2 ]. Look at  $(\phi_{n.1}, \phi_{n.2}, \phi_{n.3}, \phi_{n.4})$  introduced in Definitions 2.3, and via  $(\phi_{n.1}, \phi_{n.2}, \phi_{n.3}, \phi_{n.4})$ , consider  $(\phi_{n-1.1}, \phi_{n-1.2}, \phi_{n-1.3}, \phi_{n-1.4})$  [ this consideration gets sense, since  $n \geq 3$ , and therefore  $n-1 \geq 2$  ]. **If**  $w_{n+1} \leq 2n$ , then we have the following four properties.

(2.8.1)  $\phi_{n-1.1} - \phi_{n.1} = 0$ .

(2.8.2)  $\phi_{n-1.2} - \phi_{n.2} = -8n(16w_{n+1}^{-3} + 50w_{n+1}^{-2} + 11iw_{n+1}^{-3} - 13i + 5iw_{n+1}) + 8w_{n+1} - 8i - 16$ .

(2.8.3)  $\phi_{n-1.3} - \phi_{n.3} = -8n(34iw_{n+1}^{-2} - 70iw_{n+1}^{-3} - 11i + 5 + 6iw_{n+1})$ .

(2.8.4)  $\phi_{n-1.4} - \phi_{n.4} = -8n(11iw_{n+1} + 7 - 50w_{n+1}^{-1} + 23iw_{n+1}^{-3} - 54w_{n+1}^{-3})$ .

**Proof.** (2.8.1) Indeed let  $\phi_{n-1.1}$ ; clearly

$$\phi_{n-1.1} = \frac{126301iw_n^{-3} + 2273418iw_n^{-4} - 23837w_n - 357566w_n^{-1}}{1331} + \phi'_{n-1.1} \quad (2'.2)$$

where

$$\phi'_{n-1.1} = \frac{2010800w_n^{-2} + 2399719w_n^{-4} - 757806w_n^{-3}}{1331} \quad (2'.3)$$

[ use (2.1) of Definitions 2.3 for  $n-1$  ]. So

$$\phi_{n-1.1} = \frac{126301iw_{n+1}^{-3} + 2273418iw_{n+1}^{-4} - 23837w_{n+1} - 357566w_{n+1}^{-1} + 2010800w_{n+1}^{-2} + 2399719w_{n+1}^{-4} - 757806w_{n+1}^{-3}}{1331} \quad (2'.4)$$

( use (2'.2) and (2'.3) and notice [ by observing that  $w_{n+1} \leq 2n$  and by using Remark 2.7 ] that  $w_{n+1} = w_n$  ). Clearly

$$\phi_{n-1.1} = \phi_{n.1} \quad (2'.5)$$

[ use (2.1) of Definitions 2.3 and (2'.4) ] and so  $\phi_{n-1.1} - \phi_{n.1} = 0$  [ use (2'.5) ]. Property (2.8.1) immediately follows.

(2.8.2) Indeed let  $\phi_{n-1.2}$ ; clearly

$$\phi_{n-1.2} = ((2(n-1)+1)^2 - 1 - w_n^2 - 2w_n)(16w_n^{-3} + 50w_n^{-2} + 11iw_n^{-3} - 13i + 5iw_n) + \gamma_n \quad (2'.6)$$

where

$$\gamma_n = \frac{(w_n - 2(n-1))}{2}(8w_n - 8i - 16) \quad (2'.7)$$

[ use (2.2) of Definitions 2.3 for  $n-1$  ]. So

$$\phi_{n-1.2} = ((2(n-1)+1)^2 - 1 - w_{n+1}^2 - 2w_{n+1})(16w_{n+1}^{-3} + 50w_{n+1}^{-2} + 11iw_{n+1}^{-3} - 13i + 5iw_{n+1}) + \gamma'_n \quad (2'.8)$$

where

$$\gamma'_n = \frac{(w_{n+1} - 2(n-1))}{2}(8w_{n+1} - 8i - 16) \quad (2'.8')$$

( use (2'.6) and (2'.7) and notice [ by observing that  $w_{n+1} \leq 2n$  and by using Remark 2.7 ] that  $w_{n+1} = w_n$  ). Clearly

$$\phi_{n-1.2} = ((2n+1)^2 - 1 - w_{n+1}^2 - 2w_{n+1})(16w_{n+1}^{-3} + 50w_{n+1}^{-2} + 11iw_{n+1}^{-3} - 13i + 5iw_{n+1}) + \rho_n \quad (2'.9)$$

where

$$\rho_n = \frac{(w_{n+1} - 2n)}{2}(8w_{n+1} - 8i - 16) + \phi'_{n-1.2} \quad (2'.9')$$

and where

$$\phi'_{n-1.2} = -8n(16w_{n+1}^{-3} + 50w_{n+1}^{-2} + 11iw_{n+1}^{-3} - 13i + 5iw_{n+1}) + 8w_{n+1} - 8i - 16 \quad (2'.10)$$

( use the first member of (2'.8) and observe [by elementary computation and by using the first member of (2'.8)] that

$$((2(n-1)+1)^2 - 1 - w_{n+1}^2 - 2w_{n+1}) = ((2n+1)^2 - 1 - w_{n+1}^2 - 2w_{n+1}) - 8n;$$

and remark [by elementary computation and by using (2'.8')] that

$$\frac{(w_{n+1} - 2(n-1))}{2} = \frac{(w_{n+1} - 2n)}{2} + 1$$

). So

$$\phi_{n-1.2} = \phi_{n.2} - 8n(16w_{n+1}^{-3} + 50w_{n+1}^{-2} + 11iw_{n+1}^{-3} - 13i + 5iw_{n+1}) + 8w_{n+1} - 8i - 16 \quad (2'.11)$$

[ use (2.2) of Definitions 2.3 and (2'.9) and (2'.9') and (2'.10) ]. Clearly

$$\phi_{n-1.2} - \phi_{n.2} = -8n(16w_{n+1}^{-3} + 50w_{n+1}^{-2} + 11iw_{n+1}^{-3} - 13i + 5iw_{n+1}) + 8w_{n+1} - 8i - 16$$

[ use (2'.11) ]. Property (2.8.2) immediately follows.

(2.8.3) Indeed let  $\phi_{n-1.3}$ ; clearly

$$\phi_{n-1.3} = ((2(n-1)+1)^2 - 1 - w_n^2 + 2w_n)(34iw_n^{-2} - 70iw_n^{-3} - 11i + 5 + 6iw_n) + \frac{289080}{1331} \quad (2'.12)$$

[ use (2.3) of Definitions 2.3 for  $n-1$  ]. So

$$\phi_{n-1.3} = ((2(n-1)+1)^2 - 1 - w_{n+1}^2 + 2w_{n+1})(34iw_{n+1}^{-2} - 70iw_{n+1}^{-3} - 11i + 5 + 6iw_{n+1}) + \frac{289080}{1331} \quad (2'.13)$$

( use (2'.12) and notice [ by observing that  $w_{n+1} \leq 2n$  and by using Remark 2.7 ] that  $w_{n+1} = w_n$  ). Clearly

$$\phi_{n-1.3} = ((2n+1)^2 - 1 - w_{n+1}^2 + 2w_{n+1})(34iw_{n+1}^{-2} - 70iw_{n+1}^{-3} - 11i + 5 + 6iw_{n+1}) + \frac{289080}{1331} + \phi'_{n-1.3} \quad (2'.14)$$

where

$$\phi'_{n-1.3} = -8n(34iw_{n+1}^{-2} - 70iw_{n+1}^{-3} - 11i + 5 + 6iw_{n+1}) \quad (2'.15)$$

( use (2'.13) and observe [by elementary computation] that  $((2(n-1)+1)^2 - 1 - w_{n+1}^2 + 2w_{n+1}) = ((2n+1)^2 - 1 - w_{n+1}^2 + 2w_{n+1}) - 8n$  ). So

$$\phi_{n-1.3} = \phi_{n.3} - 8n(34iw_{n+1}^{-2} - 70iw_{n+1}^{-3} - 11i + 5 + 6iw_{n+1}) \quad (2'.16)$$

[ use (2'.14) and (2'.15) and (2.3) of Definitions 2.3 ] and clearly

$$\phi_{n-1.3} - \phi_{n.3} = -8n(34iw_{n+1}^{-2} - 70iw_{n+1}^{-3} - 11i + 5 + 6iw_{n+1})$$

[ use (2'.16) ]. Property (2.8.3) immediately follows.

(2.8.4) Indeed let  $\phi_{n-1.4}$ ; clearly

$$\phi_{n-1.4} = ((2(n-1)+1)^2 - 1 - w_n^2 + 7w_n)(11iw_n + 7 - 50w_n^{-1} + 23iw_n^{-3} - 54w_n^{-3}) - \frac{1003112i}{1331} \quad (2'.17)$$

[ use (2.4) of Definitions 2.3 for  $n - 1$  ]. So

$$\phi_{n-1.4} = ((2(n-1)+1)^2 - 1 - w_{n+1}^2 + 7w_{n+1})(11iw_{n+1} + 7 - 50w_{n+1}^{-1} + 23iw_{n+1}^{-3} - 54w_{n+1}^{-3}) - \frac{1003112i}{1331} \quad (2'.18)$$

( use (2'.17) and notice [ by observing that  $w_{n+1} \leq 2n$  and by using Remark 2.7 ] that  $w_{n+1} = w_n$  ). Clearly

$$\phi_{n-1.4} = ((2n+1)^2 - 1 - w_{n+1}^2 + 7w_{n+1})(11iw_{n+1} + 7 - 50w_{n+1}^{-1} + 23iw_{n+1}^{-3} - 54w_{n+1}^{-3}) - \frac{1003112i}{1331} + \phi'_{n-1.4} \quad (2'.19)$$

where

$$\phi'_{n-1.4} = -8n(11iw_{n+1} + 7 - 50w_{n+1}^{-1} + 23iw_{n+1}^{-3} - 54w_{n+1}^{-3}) \quad (2'.20)$$

( use (2'.18) and observe [by elementary computation and by using (2'.18)] that

$$((2(n-1)+1)^2 - 1 - w_{n+1}^2 + 7w_{n+1}) = ((2n+1)^2 - 1 - w_{n+1}^2 + 7w_{n+1}) - 8n$$

). So

$$\phi_{n-1.4} = \phi_{n.4} - 8n(11iw_{n+1} + 7 - 50w_{n+1}^{-1} + 23iw_{n+1}^{-3} - 54w_{n+1}^{-3}) \quad (2'.21)$$

[ use (2'.19) and (2'.20) and (2.4) of Definitions 2.3 ] and clearly

$$\phi_{n-1.4} - \phi_{n.4} = -8n(11iw_{n+1} + 7 - 50w_{n+1}^{-1} + 23iw_{n+1}^{-3} - 54w_{n+1}^{-3})$$

[ use (2'.21) ]. Property (2.8.4) follows and Remark 2.8 immediately follows.  $\square$

**Remark 2.9.** Let  $n$  be an integer  $\geq 3$  and let  $w_{n+1}$  [ use Definitions 1.2 ]. Look at  $(\nu_n, \epsilon_n)$  introduced in Definitions 2.3, and via  $(\nu_n, \epsilon_n)$ , consider  $(\nu_{n-1}, \epsilon_{n-1})$  [ this consideration gets sense, since  $n \geq 3$ , and therefore  $n - 1 \geq 2$  ]. If  $w_{n+1} \leq 2n$ , then we have the following two properties.

(2.9.1.)  $\nu_{n-1} - \nu_n = 32n - 8w_{n+1} + 8i + 16$ .

(2.9.2.)  $\epsilon_{n-1} - \epsilon_n = -32n - 8w_{n+1} + 8i + 16$ .

**Proof.** Indeed let  $(\nu_{n-1}, \epsilon_{n-1})$ ; clearly

$$\nu_{n-1} = (iw_n - 4i(n-1) - 4i + 1)^2 - 1 + w_n^2 \quad (2'.22)$$

[ use (2.5) of Definitions 2.3 for  $n - 1$  ] and

$$\epsilon_{n-1} = 4((2(n-1)+1)^2 - 1 - w_n^2 + w_n) + (iw_n - 2i(n-1) + 1)(4iw_n + 4 - 8i) \quad (2'.23)$$

[ use (2.6) of Definitions 2.3 for  $n - 1$  ]. So

$$\nu_{n-1} = (iw_{n+1} - 4i(n-1) - 4i + 1)^2 - 1 + w_{n+1}^2 \quad (2'.24)$$

( use (2'.22) and notice [ by observing that  $w_{n+1} \leq 2n$  and by using Remark 2.7 ] that  $w_{n+1} = w_n$  ) and

$$\epsilon_{n-1} = 4((2(n-1)+1)^2 - 1 - w_{n+1}^2 + w_{n+1}) + (iw_{n+1} - 2i(n-1) + 1)(4iw_{n+1} + 4 - 8i) \quad (2'.25)$$

( use (2'.23) and notice [ by observing that  $w_{n+1} \leq 2n$  and by using Remark 2.7 ] that  $w_{n+1} = w_n$  ). That being said, we now prove easily property (2.9.1) and property (2.9.2).

(2.9.1.) Indeed observing [ by elementary computation and the fact that  $i^2 = -1$  ] that

$$(iw_{n+1} - 4i(n-1) - 4i + 1)^2 - 1 + w_{n+1}^2 = (iw_{n+1} - 4in - 4i + 1)^2 - 1 + w_{n+1}^2 + 32n - 8w_{n+1} + 8i + 16 \quad (2'.26)$$

then clearly

$$\nu_{n-1} = (iw_{n+1} - 4in - 4i + 1)^2 - 1 + w_{n+1}^2 + 32n - 8w_{n+1} + 8i + 16 \quad (2'.27)$$

[ use (2'.24) and (2'.26) ]. So

$$\nu_{n-1} = \nu_n + 32n - 8w_{n+1} + 8i + 16 \quad (2'.28)$$

[ use (2'.27) and (2.5) of Definitions 2.3 ] and clearly

$$\nu_{n-1} - \nu_n = 32n - 8w_{n+1} + 8i + 16$$

[ use (2'.28) ]. Property (2.9.1.) follows.

(2.9.2.) Indeed observing [ by elementary computation and the fact that  $i^2 = -1$  ] that

$$4( (2(n-1)+1)^2 - 1 - w_{n+1}^2 + w_{n+1} ) = 4( (2n+1)^2 - 1 - w_{n+1}^2 + w_{n+1} ) - 32n \quad (2'.29)$$

and

$$(iw_{n+1} - 2i(n-1) + 1)(4iw_{n+1} + 4 - 8i) = (iw_{n+1} - 2in + 1)(4iw_{n+1} + 4 - 8i) + 2i(4iw_{n+1} + 4 - 8i) \quad (2'.30),$$

then clearly

$$\epsilon_{n-1} = 4( (2n+1)^2 - 1 - w_{n+1}^2 + w_{n+1} ) + (iw_{n+1} - 2in + 1)(4iw_{n+1} + 4 - 8i) - 32n + 2i(4iw_{n+1} + 4 - 8i) \quad (2'.31)$$

[ use (2'.25) and (2'.29) and (2'.30) ]. So

$$\epsilon_{n-1} = \epsilon_n - 32n + 2i(4iw_{n+1} + 4 - 8i) \quad (2'.32)$$

[ use (2'.31) and (2.6) of Definitions 2.3 ]. Clearly

$$\epsilon_{n-1} - \epsilon_n = -32n - 8w_{n+1} + 8i + 16$$

( use (2'.32) and observe [ by elementary computation and the fact that  $i^2 = -1$  ] that

$$-32n + 2i(4iw_{n+1} + 4 - 8i) = -32n - 8w_{n+1} + 8i + 16$$

). Property (2.9.2) follows and Remark 2.9 immediately follows.  $\square$

**Remark 2.10.** Let  $n$  be an integer  $\geq 3$  and let  $w_{n+1}$  [ use Definitions 1.2]. Look at  $\phi_n$  introduced in Definitions 2.3, and via  $\phi_n$ , consider  $\phi_{n-1}$  [ this consideration gets sense, since  $n \geq 3$ , and therefore  $n-1 \geq 2$  ]. Then

$$\phi_{n-1} - \phi_n = \sum_{j=1}^4 (\phi_{n-1,j} - \phi_{n,j}).$$

**Proof.** Indeed let  $\phi_n$ ; clearly

$$\phi_{n-1} = \sum_{j=1}^4 \phi_{n-1,j} \quad (2'.33)$$

[ use (2.0) of Definitions 2.3 for  $n-1$  ]. So

$$\phi_{n-1} - \phi_n = \sum_{j=1}^4 \phi_{n-1,j} - \left( \sum_{j=1}^4 \phi_{n,j} \right) \quad (2'.34)$$

[ use (2'.33) and (2.0) of Definitions 2.3 ] and clearly

$$\phi_{n-1} - \phi_n = \sum_{j=1}^4 (\phi_{n-1,j} - \phi_{n,j})$$

[ use (2'.34) ]. Remark 2.10 follows.  $\square$

Now using the previous four Remarks, then Proposition 2.6 becomes elementary to prove.

**Proof of Proposition 2.6.** Property (2.6.0) is immediate [use Remark 2.7 and property (1.5.3) of Remark 1.5]. Now to prove Proposition 2.6, it suffices to prove property (2.6.1).

(2.6.1). Indeed look at  $(\phi_{n-1} - \phi_n, \nu_{n-1} - \nu_n, \epsilon_{n-1} - \epsilon_n)$ . Clearly

$$\phi_{n-1} - \phi_n = \sum_{j=1}^4 (\phi_{n-1,j} - \phi_{n,j}) \quad (2'.35)$$

[ use Remark 2.10 ] , where

$$\phi_{n-1.1} - \phi_{n.1} = 0 \quad (2'.36)$$

[ use property (2.8.1) of Remark 2.8 ] , and

$$\phi_{n-1.2} - \phi_{n.2} = -8n(16w_{n+1}^{-3} + 50w_{n+1}^{-2} + 11iw_{n+1}^{-3} - 13i + 5iw_{n+1}) + 8w_{n+1} - 8i - 16 \quad (2'.37)$$

[ use property (2.8.2) of Remark 2.8 ] , and

$$\phi_{n-1.3} - \phi_{n.3} = -8n(34iw_{n+1}^{-2} - 70iw_{n+1}^{-3} - 11i + 5 + 6iw_{n+1}) \quad (2'.38)$$

[ use property (2.8.3) of Remark 2.8 ] , and

$$\phi_{n-1.4} - \phi_{n.4} = -8n(11iw_{n+1} + 7 - 50w_{n+1}^{-1} + 23iw_{n+1}^{-3} - 54w_{n+1}^{-3}) \quad (2'.39)$$

[ use property (2.8.4) of Remark 2.8 ] , and

$$\nu_{n-1} - \nu_n = 32n - 8w_{n+1} + 8i + 16 \quad (2'.40)$$

[ use property (2.9.1) of Remark 2.9 ] , and

$$\epsilon_{n-1} - \epsilon_n = -32n - 8w_{n+1} + 8i + 16 \quad (2'.41)$$

[ use property (2.9.2) of Remark 2.9 ] . That being so, let  $(x, y, k)$  such that

$$x = -8w_{n+1} + 8i + 16; y = 32n; \text{ and } k = 16nw_{n+1}^{-3} - 16nw_{n+1}^{-2} + 16nw_{n+1}^{-1} \quad (2'.42).$$

Now let  $(\phi_{n-1} - \phi_n, \nu_{n-1} - \nu_n, \epsilon_{n-1} - \epsilon_n, x, y, k)$  where  $(x, y, k)$  is explicited in (2'.42) and where

$$(\phi_{n-1} - \phi_n, \nu_{n-1} - \nu_n, \epsilon_{n-1} - \epsilon_n)$$

is explicited above [ use (2'.35) for  $\phi_{n-1} - \phi_n$ ; and (2'.40) for  $\nu_{n-1} - \nu_n$ ; and (2'.41) for  $\epsilon_{n-1} - \epsilon_n$  ]. Now consider  $(\nu_{n-1} - \nu_n, \epsilon_{n-1} - \epsilon_n, x, y)$ ; using (2'.40) and (2'.41) and the first two equalities of (2'.42), then we easily check [by elementary computation and the fact that  $i^2 = -1$ ] that

$$x + y - (\nu_{n-1} - \nu_n) = 0 \text{ and } x - y - (\epsilon_{n-1} - \epsilon_n) = 0 \quad (2'.43).$$

That being so, look again at  $(\phi_{n-1} - \phi_n, \nu_{n-1} - \nu_n, \epsilon_{n-1} - \epsilon_n, x, y, k)$  and consider  $(\phi_{n-1} - \phi_n, x, y, k)$ ; using the three equalities of (2'.42), then it becomes very easy to check [by elementary computation and the fact that  $i^2 = -1$ ] that

$$x + 3iyw_{n+1}^{-1} + k(6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i) + \phi_{n-1} - \phi_n = 0; k \in \mathcal{R} \quad (2'.44)$$

[ use the three equalities of (2'.42) for  $(x, y, k)$ ; and (2'.36) for  $\phi_{n-1.1} - \phi_{n.1}$ ; and (2'.37) for  $\phi_{n-1.2} - \phi_{n.2}$ ; and (2'.38) for  $\phi_{n-1.3} - \phi_{n.3}$ ; and (2'.39) for  $\phi_{n-1.4} - \phi_{n.4}$ ; and (2'.35) for  $\phi_{n-1} - \phi_n$  ]. Clearly

$(\phi_{n-1} - \phi_n, \nu_{n-1} - \nu_n, \epsilon_{n-1} - \epsilon_n)$  tackles  $(1, 3iyw_{n+1}^{-1})$  around  $6w_{n+1} - 19 - iw_{n+1} + 11iw_{n+1}^2 - 18i$  ( use (2'.43) and (2'.44) and the notion of tackle introduced in Definition 2.2 [observe that  $(x, y) \in \mathcal{C}^2$  and  $k \in \mathcal{R}$ ; so  $(x, y, k) \in \mathcal{C}^2 \times \mathcal{R}$  ] ). Property (2.6.1) follows and Proposition 2.6 immediately follows.  $\square$

## References

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