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## 2D Local identification of the elasticity parameters of heterogeneous materials by the finite element model updating method

E Ayeleh<sup>1,\*</sup>, K Atchonouglo<sup>1</sup>, Y Koumekpo<sup>1</sup>, A Germaneau<sup>2</sup>, L Caillé<sup>2</sup>, J-C Dupré<sup>2</sup>

<sup>1</sup> Université de Lomé, Faculté des Sciences, Département de physique, 01BP 1515, Lomé1, Togo

<sup>2</sup> Institut Pprime, UPR 3346 CNRS, Université de Poitiers, ENSMA SP2MI, BP 30179, 86962 Futuroscope Chasseneuil Cedex, France

eayeleh@yahoo.fr

**Abstract.** This work is subjected to an identification of elasticity properties which are the Young's modulus  $E$  and the Poisson's ratio  $\nu$  of a heterogeneous material. We have created the experimental displacements by solving the direct problem. A re-calibration of the experimental and the simulated displacements is done based on the principle of the finite element model updating (FEMU). The minimization of the cost function obtained by the FEMU method in the inverse problem is done by Newton type algorithms. A complete study has been done by studying the sensitivity of the identified values with respect to the refinement of the mesh and with respect to the level of disturbance.

### 1. Introduction

Classical tests are based on homogeneity assumptions to determine the coefficients of the stiffness matrix of a material. But modern materials are more and more complex and do not respect this assumption anymore. During these classical tests, we see heterogeneities that are in contradiction with the previous assumptions. We then resort to methods of non-contact field measurements such as digital image correlation, Moiré and Speckle interferometry used in an inverse process to identify mechanical properties.

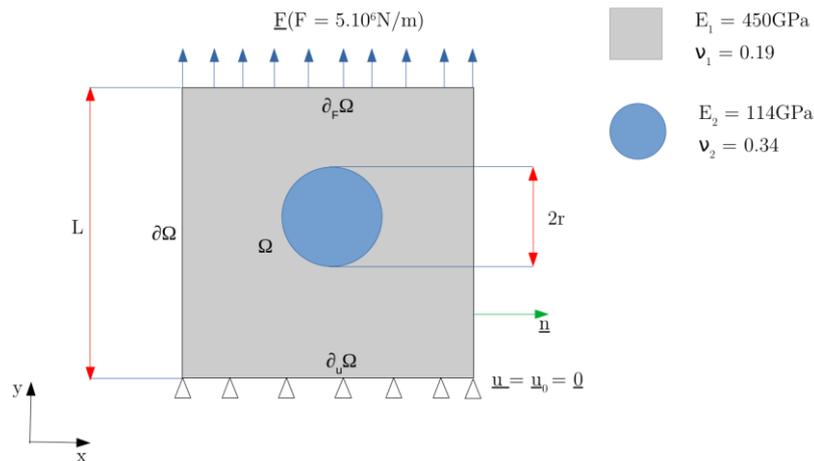
The principal identification methods known today are: the virtual fields method, the equilibrium equation gap method, the constitutive equation gap method and the finite element model updating. An overview of these methods and their principles have been done by Avril et al. [1]. These methods were most often used in cases of global identification. This type of identification considers that the material is homogeneous [2, 3, 4]. But in this last decade, researchers have been interested in a local identification. For this consideration the material is considered as being heterogeneous, which makes even more account composite materials. For the first time in 2019, the finite element model updating (FEMU) was used by Pétureau et al. [5] to identify the local parameters of an isotropic heterogeneous material behavior law.

The objective of this work is to develop a method to identify linear elasticity parameters which are the Young's modulus  $E$  and the Poisson's ratio  $\nu$  of heterogeneous materials.



## 2. Direct problem

The physical problem consists here to solve the linear elastic problem in the case of isotropic heterogeneous material. We consider a square domain with a circular inclusion  $\Omega \in \mathbb{R}^2$  and assume that  $\Omega$  is bounded by a piece-wise smooth boundary  $\partial\Omega$  such that  $\partial\Omega = \partial_F\Omega + \partial_u\Omega$  and  $\partial_T\Omega \cap \partial_u\Omega = \emptyset$ . The size of this square is  $L = 20$  mm and the radius of the circle is  $r = 5$  mm. We impose a load  $\underline{F}$  on the  $\partial_F\Omega$  and a displacement  $\underline{u}_0$  on the  $\partial_u\Omega$  (Figure 1).



**Figure 1.** Studied specimen

Since the volume forces are assumed to be zero, the equilibrium equations are given by

$$\begin{cases} \operatorname{div}(\sigma(\underline{M})) = \underline{0} & \text{in } \Omega \\ \sigma(\underline{M}) \cdot \underline{n}(\underline{M}) = \underline{F} & \text{in } \partial_F\Omega \end{cases} \quad (1)$$

where  $\sigma(\underline{M})$  denotes the Cauchy stress tensor and  $\underline{n}(\underline{M})$  the outward unit normal vector at a point  $\underline{M} \in \partial_F\Omega$ .

The strain tensor  $\varepsilon(\underline{M})$  is related to the displacement gradient tensor  $\nabla \underline{u}$  by the kinematic relations:

$$\begin{cases} \varepsilon(\underline{M}) = \frac{1}{2} (\nabla \underline{u}(\underline{M}) + \nabla^t \underline{u}(\underline{M})) & \text{in } \Omega \\ \underline{u}(\underline{M}) = \underline{u}_0 = \underline{0} & \text{in } \partial_u\Omega \end{cases} \quad (2)$$

The Cauchy stress tensor  $\sigma(\underline{M})$  is related to the strain tensor  $\varepsilon(\underline{M})$  through the constitutive law, namely:

$$\sigma(\underline{M}) = \underline{C} : \varepsilon(\underline{M}) \quad \text{in } \Omega \quad (3)$$

The components of  $\underline{C}$ , can, either be constant (homogeneous material), or depend on the points of the space (heterogeneous material).

To solve equations (1), (2), (3), we used finite element method [6] with three-node triangular elements (T3) and implemented it with FEniCs software [7]. This was made because of the powerful libraries it has.

## 3. Inverse problem

After the simulation of the direct problem, we obtain the calculated displacements  $\underline{u}^{EF}$ . With the help of these calculated displacements we artificially create experimental displacements fields  $\underline{u}^{exp}$ . The

finite element model updating method consists in comparing the simulated displacements with experimental ones. To do this, we formulated a cost function  $f(\underline{x})$  as shown in equation (4).

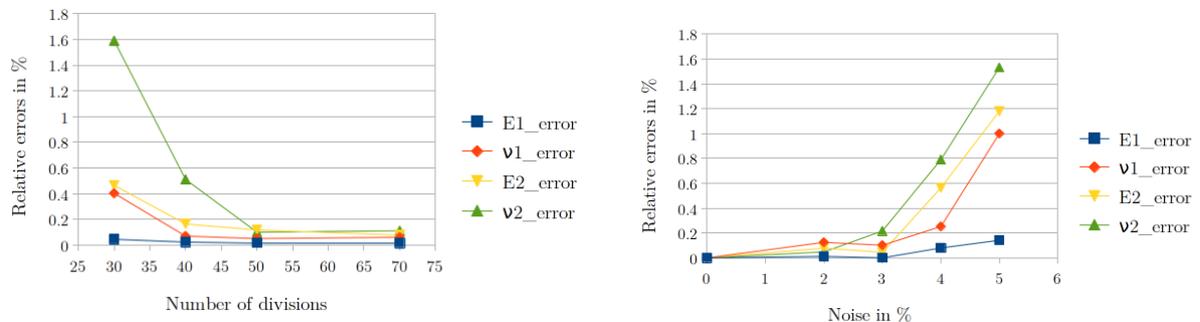
$$f(\underline{x}) = \frac{1}{2} \sum_{i=1}^m \left\| \frac{u_i^{EF}(\underline{x}) - u_i^{exp}}{\max_i(u_i^{exp})} \right\|^2 \quad (4)$$

where  $m$  is the total number of measurement points,  $u_i^{EF}$  are the simulated displacements obtained by the finite element method and  $u_i^{exp}$  are the experimental displacements.  $\underline{x}(E_1, \nu_1, E_2, \nu_2)$  represents the vector of parameters or properties of the specimen considered with  $E_i$  and  $\nu_i$  respectively the Young's modulus and the Poisson's ratio of the material  $i$ . The minimisation problem will be solved by a newton type algorithm [8] implemented with python language.

## 4. Identification results

### 4.1 Sensitivity to the mesh refinement and to the noise levels

In this section we are studying the sensitivity of the parameters identified to the refinement of the mesh (Figure 2 (a)) and their sensitivity to the noise levels (Figure 2 (b)).  $E_i$ \_error and  $\nu_i$ \_error are respectively the relative errors du to Young's modulus and Poisson's ratio of an  $i$  material. White Gaussian noise has been used to perturb the displacements data. Therefore 1% noise level was added to the data before the sensitivity to the refinement of the mesh. We remark that as the number of divisions increases, the relative errors decrease. Also when the noise level increases, the relative errors increase. The maximum error, both in the two cases is around 1.6%, so the parameters are well identified.



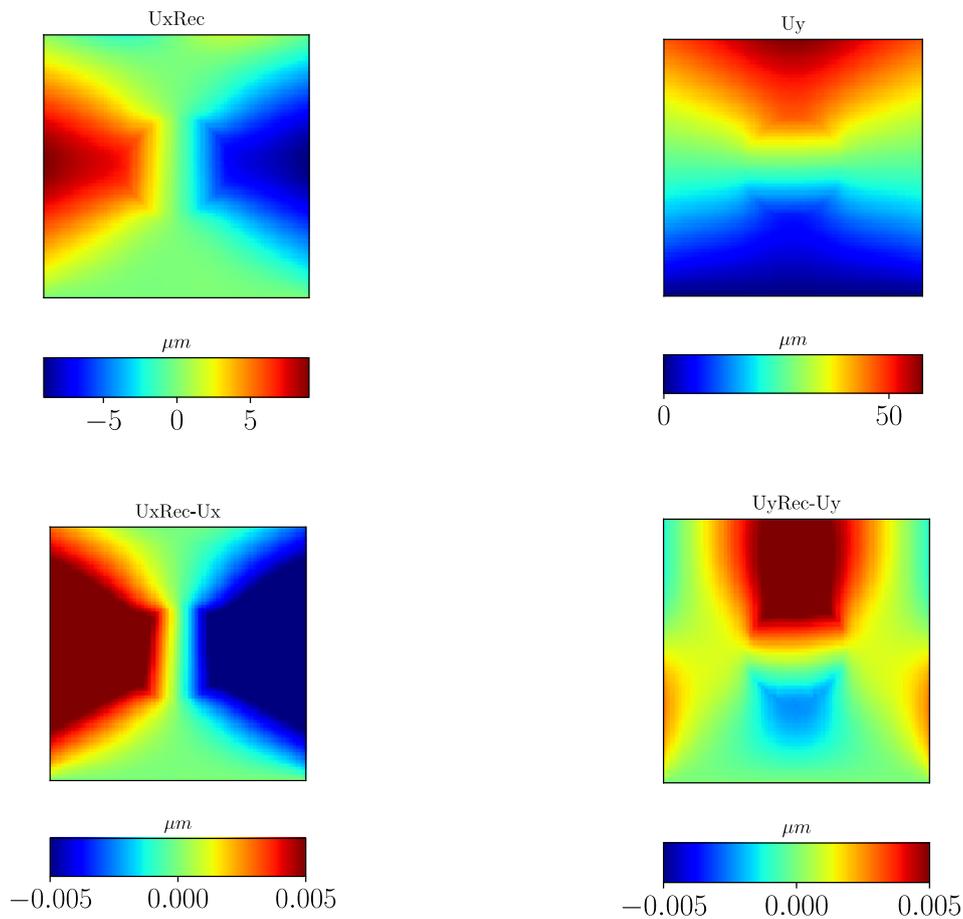
(a) Sensitivity to the mesh refinement (The maximum number of elements here is  $70 \times 70 \times 2 = 9,800$ )

(b) Sensitivity to noise levels for 9,800 triangular elements

**Figure 2.** Sensitivity to the mesh refinement and to the noise levels

### 4.2 Reconstruction of displacements fields

With the values, which have been identified at a noise level of 3%, we reconstructed the displacement fields and calculated the differences between the reconstructed and the reference displacements (Figure 3). Considering the displacements values, we observe that the differences are very small and closed to zero. Thus the parameters are well identified. Our algorithm is then powerful enough to identify heterogeneous material parameters. The non-zero differences are du to the fact that the material is heterogeneous and that the identified parameters are not equal to the reference ones.



**Figure 3.** Reconstructed displacements and differences between reference and reconstructed displacements when the specimen is meshed with 9,800 triangular elements.

## 5. Conclusion

The finite element model updating method is one of the identification methods we used in this paper. This method is based on the comparison of measured and calculated displacement fields. In our case we have created the measured displacements by solving the direct problem. The chosen specimen is a square specimen with a circular inclusion. This choice was made to represent biomechanical and civil engineering materials. We have identified the parameters of each part, studied the sensitivity of the parameter to the mesh refinement and their sensitivity to noise levels. We have remarked that when the number of elements increases in the specimen, the relative errors decrease and when the noise level increases, the relative errors increase. We reconstructed the displacement fields with the identified parameters at 3% noise level and calculated the differences between them and the reference displacements. The differences were very closed to zero, which prove that the parameters were identified with a good accuracy. Future works will focus on the experimental study of this work because here we have made a numerical study.

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