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# BEM-FEM formulation based on magnetic vector and scalar potentials for eddy current problems

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**Abstract**—The hybrid boundary element - finite element method (BEM-FEM) has been developed for several decades and has shown many advantages since air media has not to be meshed. In this paper, a new BEM-FEM formulation is presented. It is based on the modified magnetic vector potential  $\mathbf{A}^*$  as unknown in active regions and on the reduced magnetic scalar potential  $\varphi$  as unknown in the air media. The formulation and its numerical validation is proposed in this paper.

**Index Terms**—eddy currents, quasi-static, finite element method, boundary element method

## I. INTRODUCTION

We address the problem of eddy-currents computation under the quasi-static assumption. In the BEM-FEM approach, FEM method is applied to the material domains and the BEM method is applied to the free domain. In the literature, FEM equations are based on vector quantities such as the magnetic field  $\mathbf{H}$ , the electric field  $\mathbf{E}$  or the magnetic potential  $\mathbf{A}$ , while BEM equations are mainly based on magnetic [1], [2], [3] or electric [5] scalar quantities. BEM equations for the free domains can also be based on the quantities  $\mathbf{E}$  or  $\mathbf{H}$  like proposed in [4] and [6]. These two last formulations are suitable for solving multiply connected regions problems. In the next section, we present a new BEM-FEM formulation based on  $\mathbf{A}^*$  and  $\varphi$ . A key point of the formulation is that it does not need a costly matrix inversion like it is often encountered in the literacy.

## II. $\mathbf{A}^* - \varphi$ BEM-FEM HYBRID FORMULATION

Let us consider a simply connected region  $\Omega$  bounded by the  $\Gamma$ , characterised by a permeability  $\mu$  and a conductivity  $\sigma$ . The excitation magnetic field is created by a coil located outside  $\Omega$  and denoted  $\mathbf{H}_{ex}$ . From Maxwell's equations under eddy current assumption and the relation  $\mathbf{E} = -j\omega\mathbf{A}^*$ , we have equation for  $\mathbf{A}^*$  in the  $\Omega$  region:

$$\nabla \times \nabla \times \mathbf{A}^* + j\omega\mu\sigma\mathbf{A}^* = 0 \quad (1)$$

Applying the Galerkin's method to equation above with the projection function being the edge shape functions  $\mathbf{w}$ . After some vector algebras, we get:

$$\int_{\Omega} (\nabla \times \mathbf{w}) \cdot (\nabla \times \mathbf{A}^*) d\Omega + \int_{\Gamma} \mathbf{w} \cdot (\mathbf{n} \times (\nabla \times \mathbf{A}^*)) d\Gamma + j\omega\mu\sigma \int_{\Omega} \mathbf{w} \cdot \mathbf{A}^* d\Omega = 0 \quad (2)$$

The magnetic field in the air can be expressed as:

$$\mathbf{H}_{air} = \mathbf{H}_{ex} - \nabla\varphi \quad (3)$$

$\mathbf{H}_{air}$  is a solenoidal field, so the magnetic scalar potential is governed by the Laplace equation  $\Delta\varphi = 0$ . Applying Green's identity, we have the following integral equation:

$$h\varphi = \int_{\Gamma} (\mathbf{n} \cdot \nabla\varphi) G d\Gamma - \int_{\Gamma} \varphi (\mathbf{n} \cdot \nabla G) d\Gamma \quad (4)$$

where  $h = 0.5$  if the target point is a regular point of the border  $\Gamma$  and  $G = \frac{1}{r}$  is the Green's function with  $r$  is the distance between integration and observation point. The boundary conditions of magnetic field allow us to link the surface quantities  $\mathbf{A}^*$  and  $\varphi$  as follows:

$$\mathbf{n} \times (\nabla \times \mathbf{A}^*) = \mu(\mathbf{n} \times \mathbf{H}_{ex} - \mathbf{n} \times \nabla\varphi) \quad (5)$$

$$\mathbf{n} \cdot \nabla\varphi = \mathbf{n} \cdot \mathbf{H}_{ex} - \frac{1}{\mu_0} \mathbf{n} \cdot (\nabla \times \mathbf{A}^*) \quad (6)$$

Let us replace (5), (6) in (2), (4) respectively and then perform one more algebra to separate  $\varphi$  from the gradient operator:

$$\int_{\Gamma} \mathbf{w} \cdot (\mathbf{n} \times \nabla\varphi) d\Gamma = \int_{\Gamma} \mathbf{n} \cdot (\nabla \times \mathbf{w}) \varphi d\Gamma \quad (7)$$

Finally, a set of coupled integral equations is obtained:

$$\begin{aligned} - \int_{\Gamma} \mathbf{w} \cdot (\mathbf{n} \times \mathbf{H}_{ex}) d\Gamma &= \frac{1}{\mu} \int_{\Omega} (\nabla \times \mathbf{w}) \cdot (\nabla \times \mathbf{A}^*) d\Omega \\ &+ j\omega\sigma \int_{\Omega} \mathbf{w} \cdot \mathbf{A}^* d\Omega - \int_{\Gamma} \mathbf{n} \cdot (\nabla \times \mathbf{w}) \varphi d\Gamma \end{aligned} \quad (8)$$

$$\begin{aligned} \int_{\Gamma} \mathbf{n} \cdot \mathbf{H}_{ex} G d\Gamma &= h\varphi + \frac{1}{\mu_0} \int_{\Gamma} \mathbf{n} \cdot (\nabla \times \mathbf{A}^*) G d\Gamma \\ &+ \int_{\Gamma} \varphi (\mathbf{n} \cdot \nabla G) d\Gamma \end{aligned} \quad (9)$$

## III. NUMERICAL IMPLEMENTATION

The 0-degree interpolation functions are chosen for scalar potential  $\varphi$  while the vector potential  $\mathbf{A}^*$  is interpolated by using edge shape functions  $\mathbf{w}$ :

$$\mathbf{A}^* = \sum_i \mathbf{w}_i A_i^* \quad (10)$$

where  $A_i^*$  is the circulation of magnetic potential  $\mathbf{A}^*$  along the edge  $e_i$ :

$$A_i^* = \int_{e_i} \mathbf{A}^* \cdot d\mathbf{l} \quad (11)$$

Equation (8) is written in matrix form:

$$[M] \{A^*\} + [B] \{\varphi\} = \{Q\} \quad (12)$$

The term  $\mathbf{n} \cdot (\nabla \times \mathbf{A}^*)$  in (9) is the magnetic flux density, denoted  $\phi$ . Assuming that  $\phi$  is uniform on each facet element of the discretized surface  $\Gamma$ , it can be written as:

$$\phi = \sum_i \Phi_i \frac{1}{s_i} \quad (13)$$

where  $\Phi_i$  is the magnetic flux flowing through the facet element number  $i$  with the area  $s_i$  of border  $\Gamma$ . By applying discretization according to the Galerkin's approach, (9) is written in matrix form as follows:

$$[T] \{\Phi\} + [H] \{\varphi\} = [T] \{S\} \quad (14)$$

The matrices in (12), (14) are not detailed in this abstract. Let us now establish a relationship between  $\Phi$  and  $A^*$ . For the facet number  $i$ , we have:

$$\Phi_i = \int_{\Gamma_i} (\nabla \times \mathbf{A}^*) \cdot \mathbf{n} d\Gamma = \oint_{C_i} \mathbf{A}^* d\mathbf{l} \quad (15)$$

Equations (11) and (15) enable the written of a matrix relation linking  $\Phi$  to  $A^*$ .

$$\{\Phi\} = [C] \{A^*\} \quad (16)$$

where  $[C]$  is an incidence matrix that links the magnetic flux on each mesh element to the edge unknown values  $A^*$ . Equation (14) becomes:

$$[T] [C] \{A^*\} + [H] \{\varphi\} = [T] \{S\} \quad (17)$$

Our final equation system includes (12) and (17).

#### IV. VALIDATION

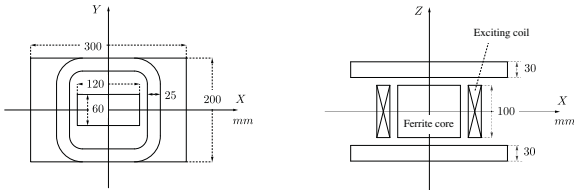


Fig. 1. The model proposed by IEEJ.

In order to validate the proposed formulation, the test case proposed by IEE of Japan [4] is considered (Fig. 1). The conductivity of both aluminum plates is  $3.215E7(S/m)$ . The ferrite magnetic region has a relative permeability of 3000. The excitation coil is fed by an alternating current  $1000A$ ,  $50Hz$ . The mesh is composed of 27860 hexahedron for the material regions. Another simulation was performed by pure FEM method with an adapted mesh. A comparisons of current

density computed on two different lines on the border  $\Gamma$  by our formulation and by FE method is shown in Fig. 2. The Joule losses can be calculated from the current density in  $\Omega$  or from surface quantities  $\mathbf{A}^*$  and  $\varphi$ . The relative error with respect to the result obtained by the FEM method is about 0.36%.

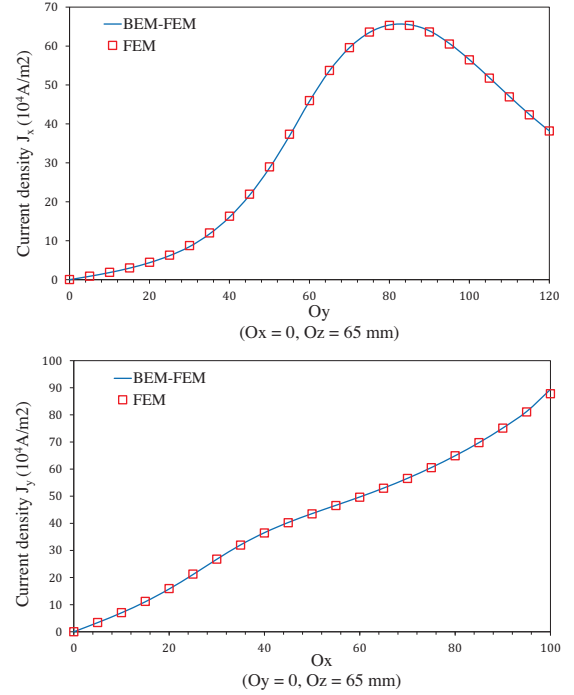


Fig. 2. Comparisons of current density on border.

#### V. CONCLUSIONS

In this paper, we have presented a new hybrid model using the modified vector potential  $\mathbf{A}^*$  and the reduced scalar potential  $\varphi$ . The results calculated by proposed formulation through standard tests shown good agreement with the results obtained by FEM method.

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