

NEW REDUCED ORDER MODELS FOR REAL-TIME DATA ASSIMILATION USING FEW SENSORS

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A. M. Picard, V. Resseguier,
D. Heitz and G. Stabile

CONTENT

1. Context : observer for unsteady aerodynamism
2. Physics, data & reduced order model (ROM)
3. Simulation, measurements & data assimilation
4. Reduced order model under location uncertainty
5. Results



PART I

CONTEXT :
OBSERVER FOR UNSTEADY
AERODYNAMISM

CONTEXT

Observer for unsteady aerodynamism

Application: Real-time estimation and prediction of 3D fluid flow
using strongly-limited computational resources & few sensors

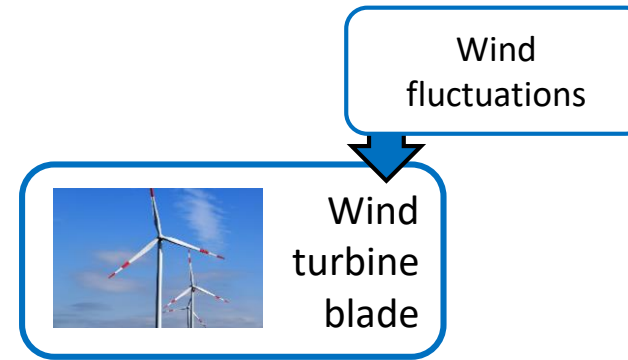


Wind
turbine
blade

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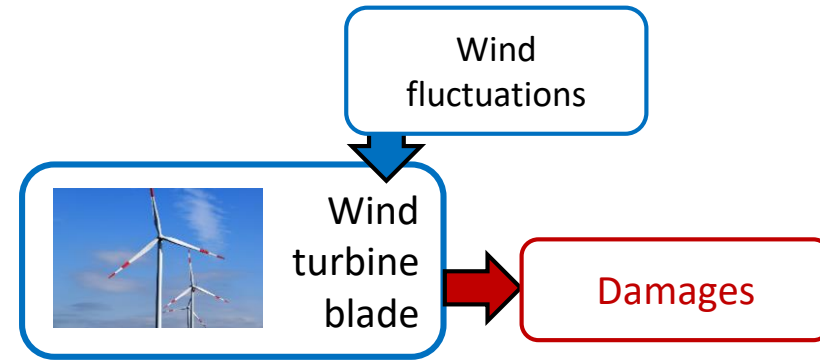
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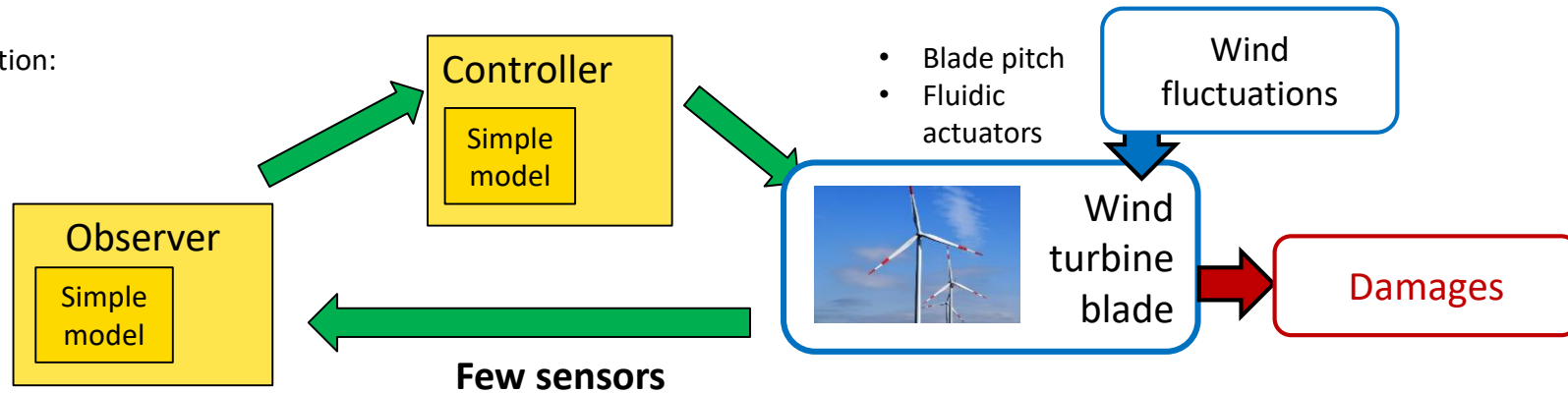
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Estimation and prediction:

- Air flow
- Lift, drag, inflow
- ...



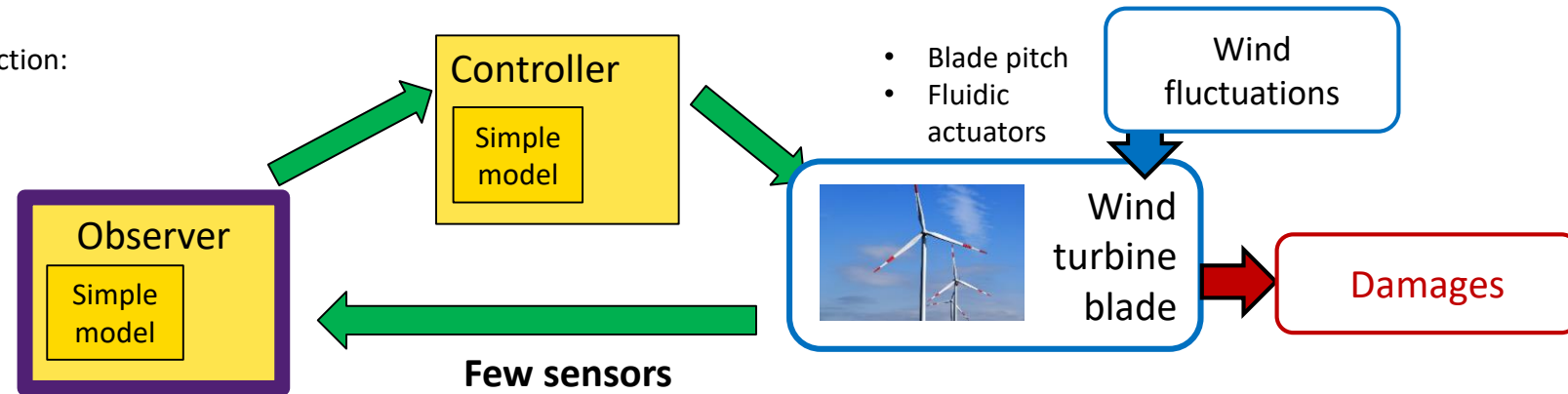
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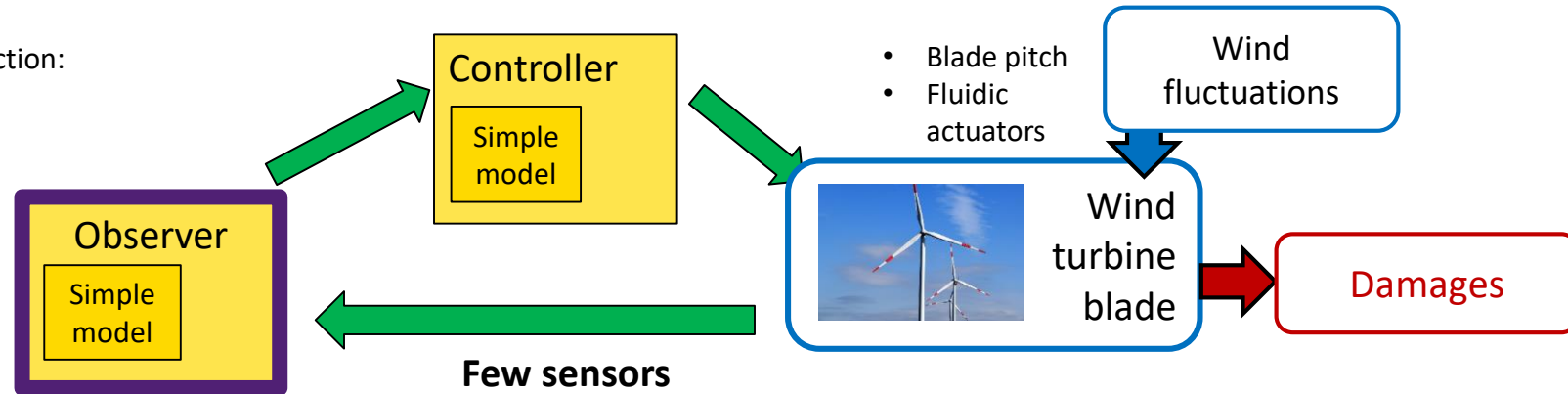
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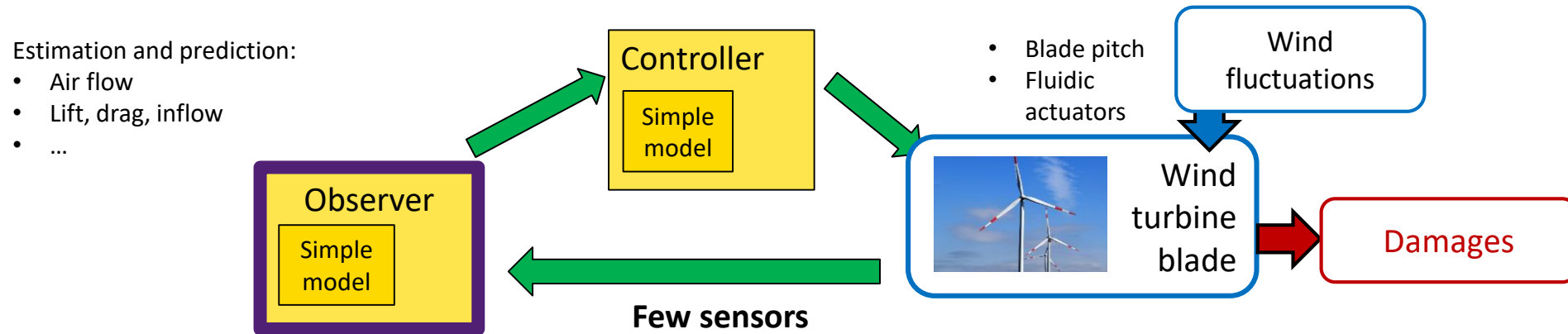


Which simple model? How to combine model & measurements?

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Which simple model? How to combine model & measurements?

Scientific problem :

Simulation & data assimilation under severe dimensional reduction

typically, $10^7 \rightarrow O(10)$ degrees of freedom



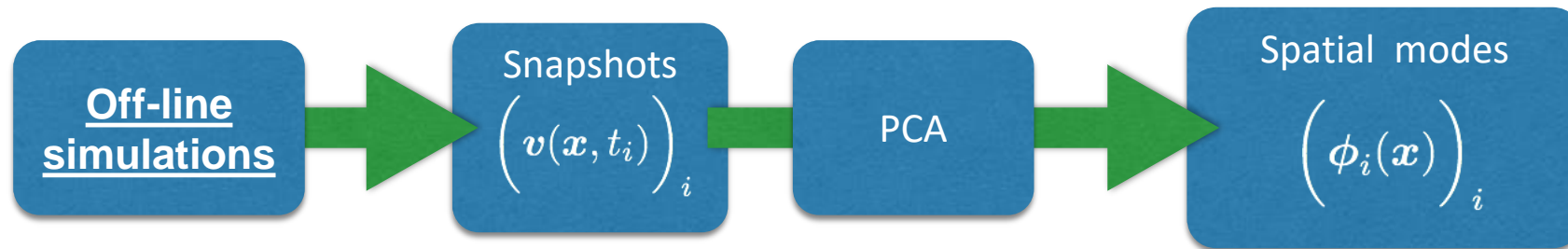
PART II

PHYSICS, DATA & REDUCED ORDER MODEL

INTRUSIVE REDUCED ORDER MODEL

Combine physical models and learning approaches

- Principal Component Analysis (PCA) on a *dataset* to reduce the dimensionality:



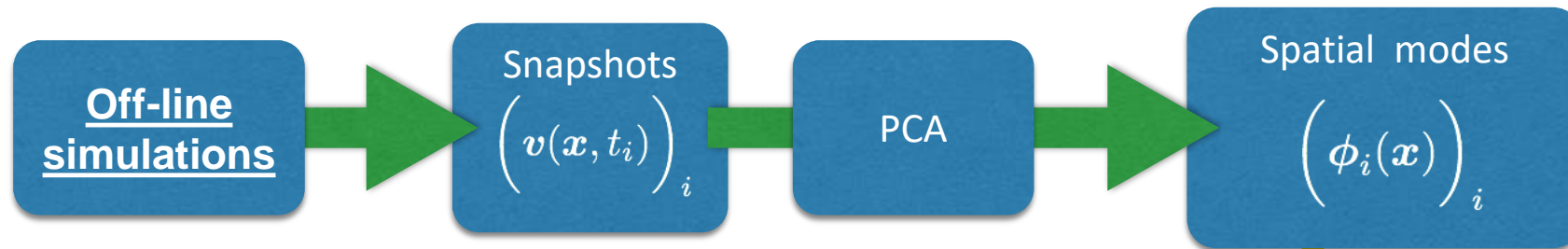
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$$v(x, t) \approx \sum_{i=0}^n b_i(t) \phi_i(x)$$

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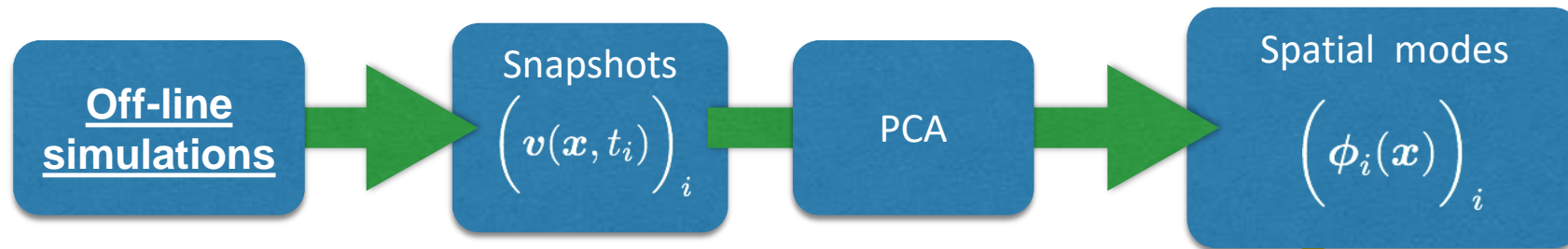
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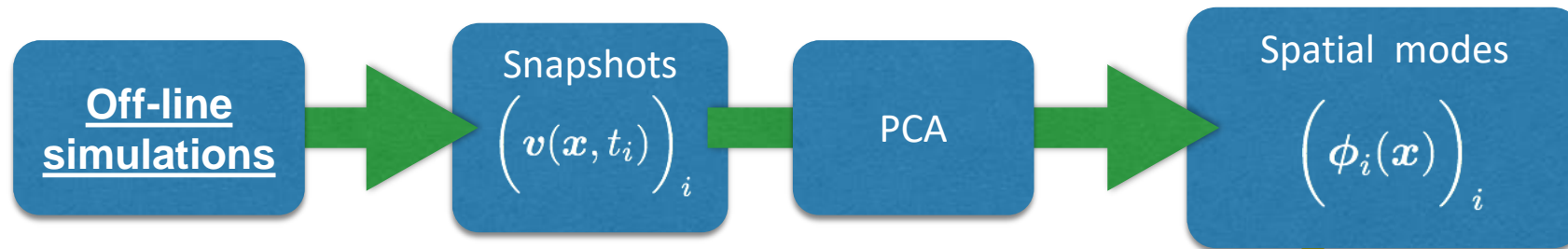
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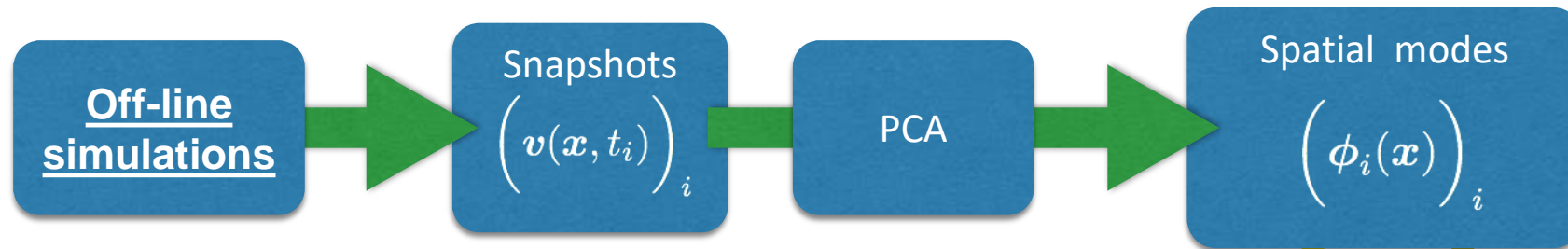
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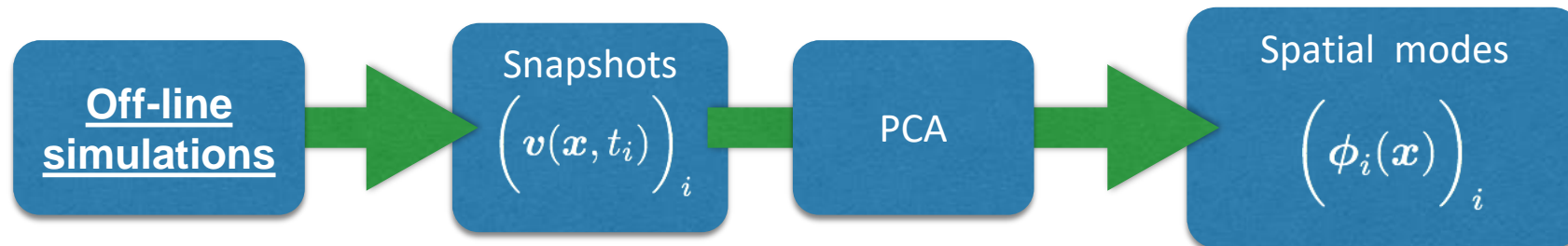
- Projection of the “*physics*”
onto the spatial modes
(POD-Galerkin)

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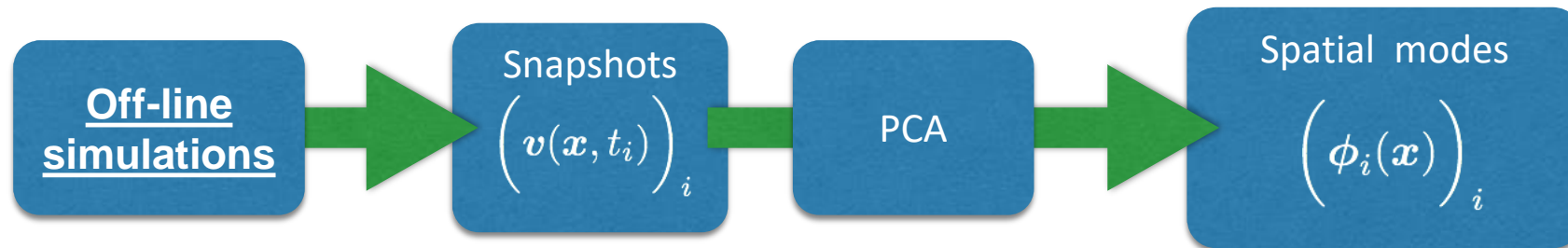
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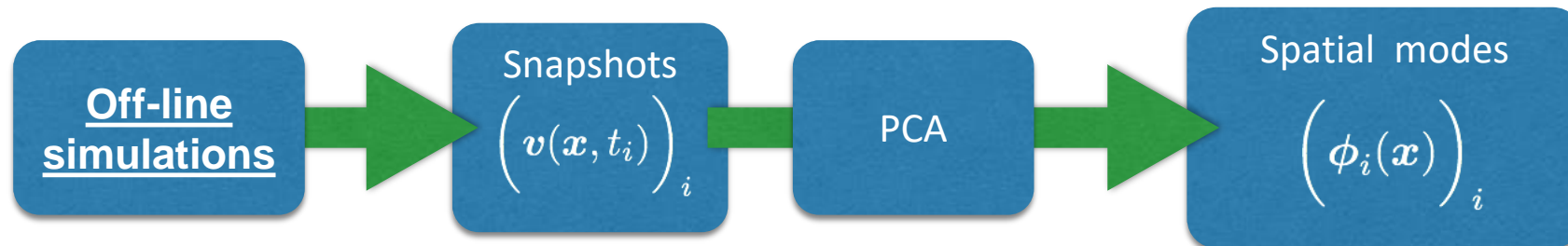
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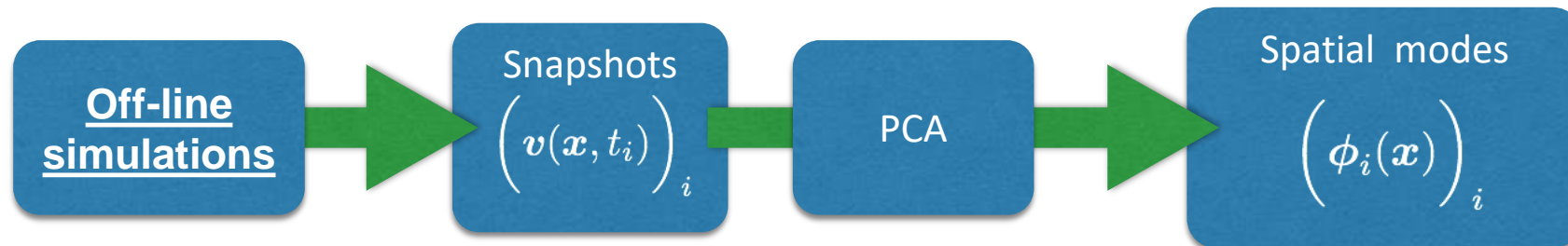
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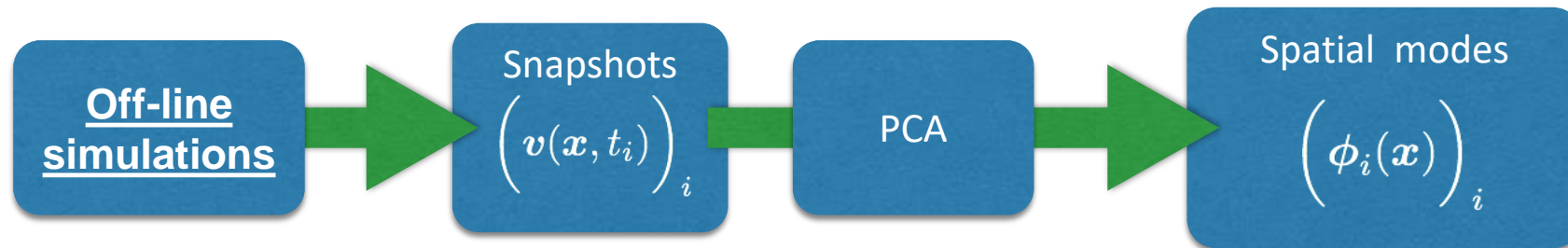
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SPOILER

- Projection of the "physics" onto the spatial modes (POD-Galerkin)

$$\int_{\Omega} dx \phi_i(x) \cdot \text{(Randomized Navier-Stokes)}$$

→ ROM for very fast simulation of temporal modes



PART III

SIMULATION, MEASUREMENTS & DATA ASSIMILATION

DATA ASSIMILATION

= Coupling simulations and measurements y

**Numerical
Simulation
(ROM)**

→ erroneous

**On-line
measurements**

→ incomplete
→ possibly noisy

Velocity

DATA ASSIMILATION

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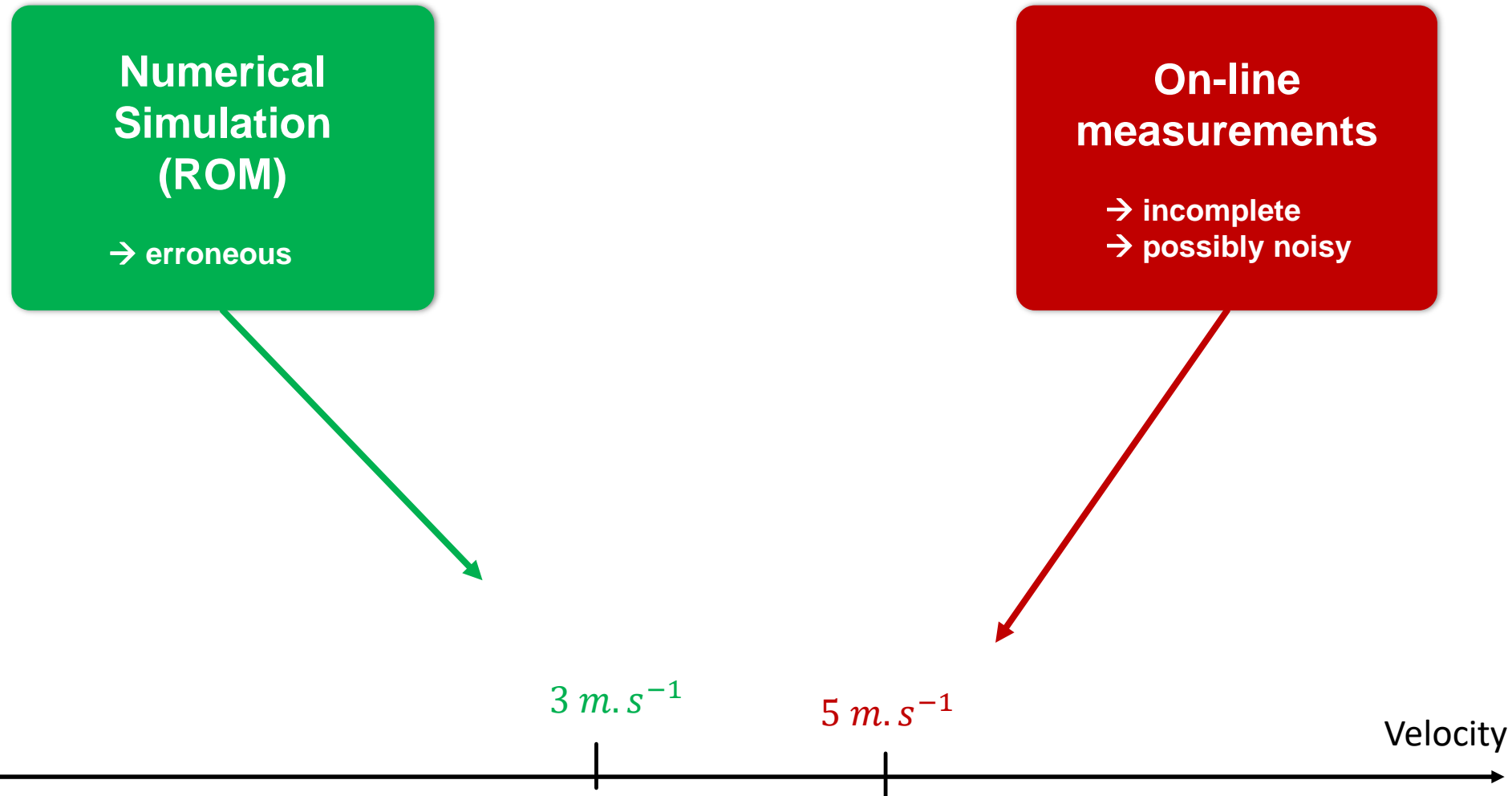
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$3 \text{ m} \cdot \text{s}^{-1}$

Velocity

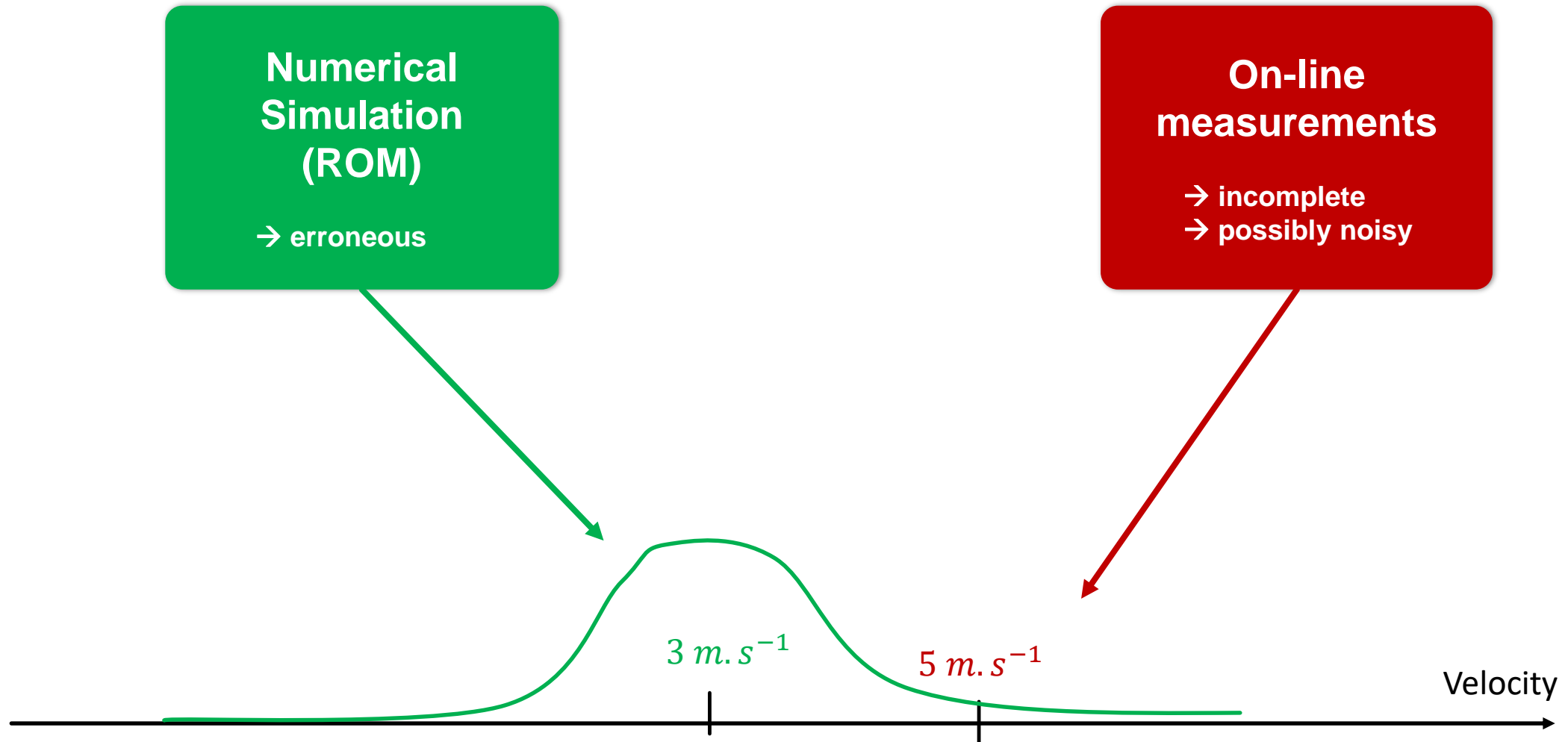
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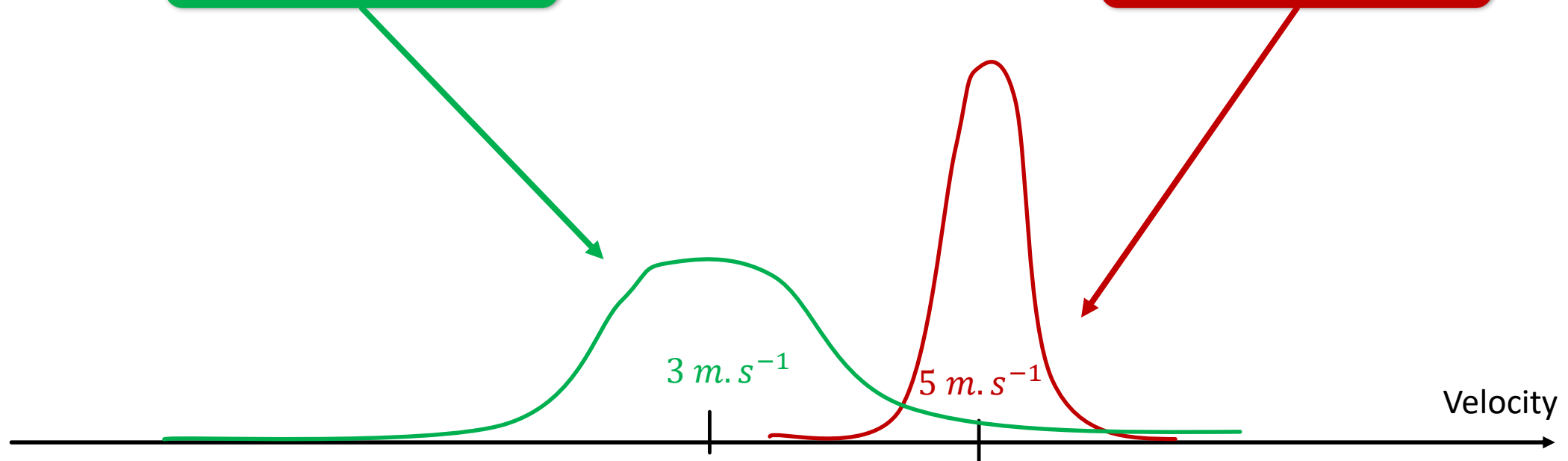
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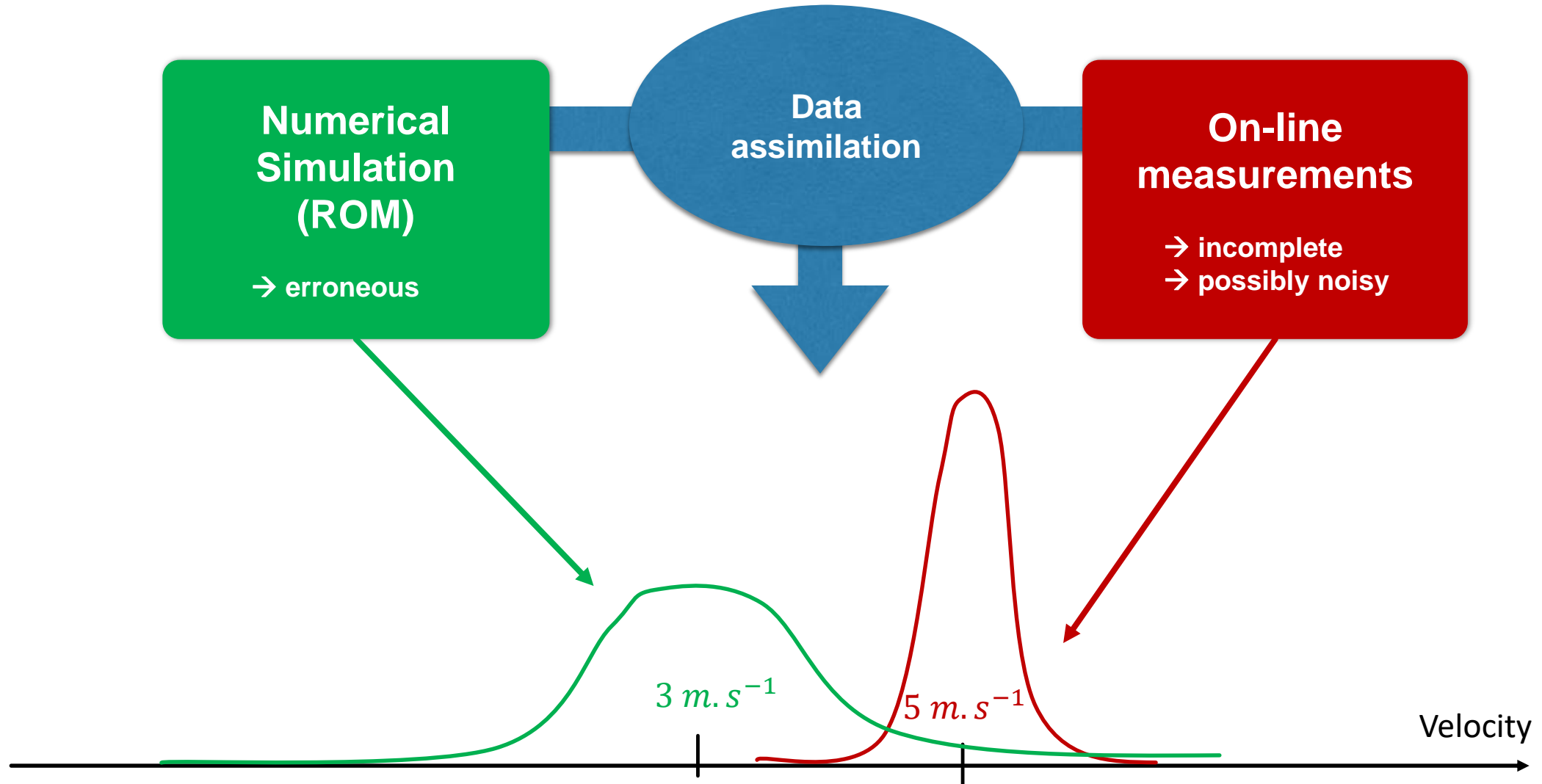
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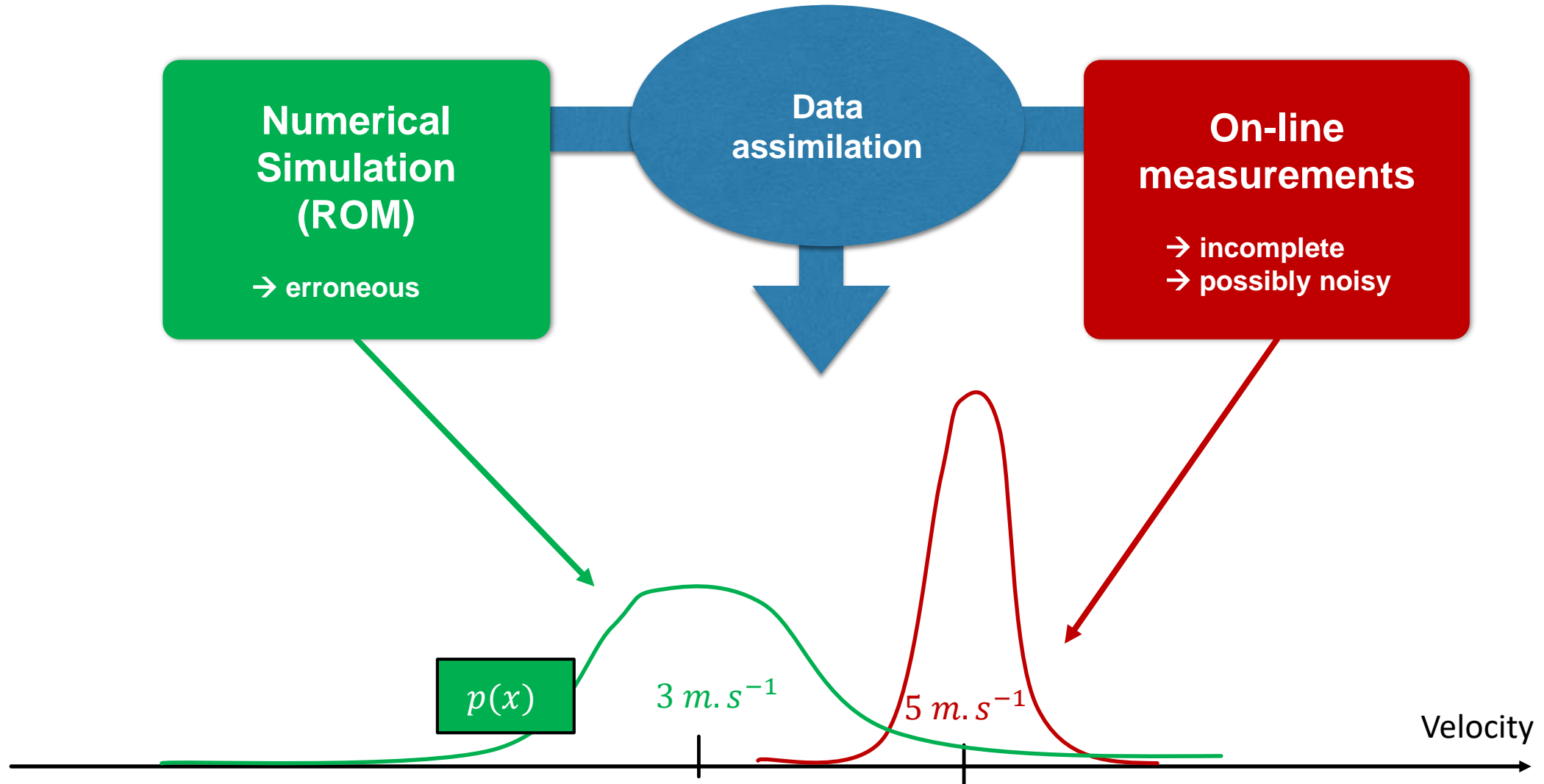
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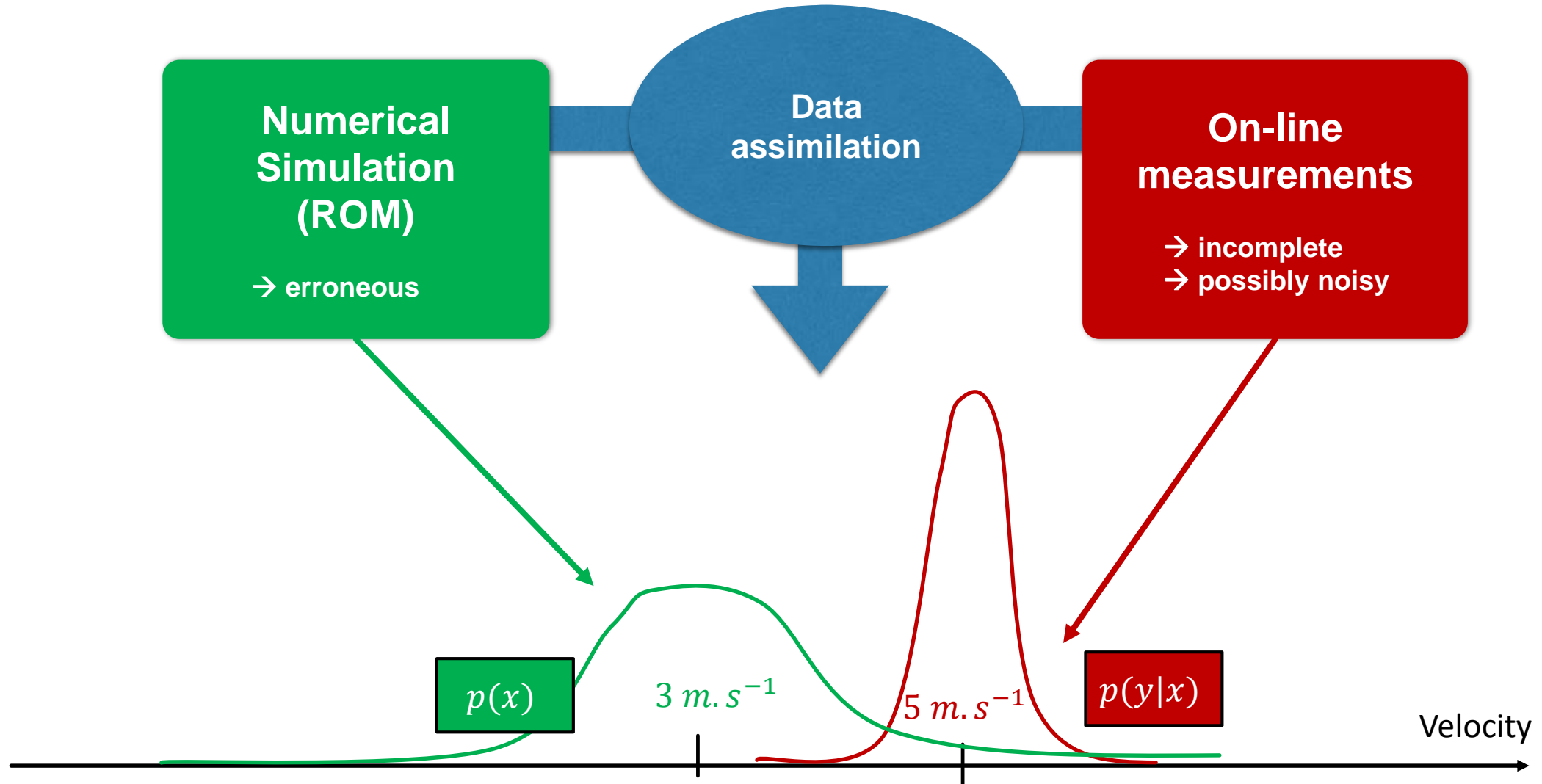
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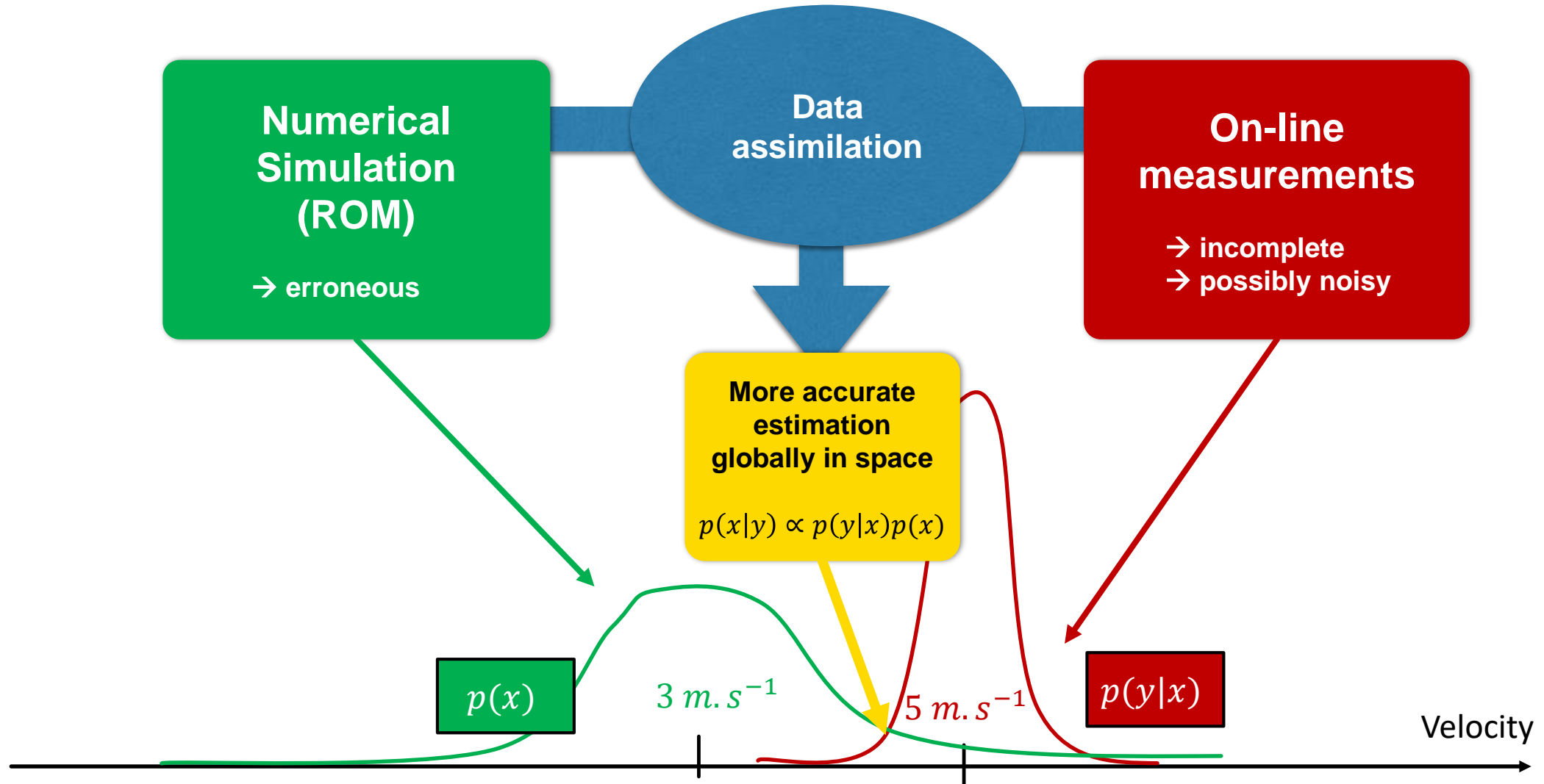
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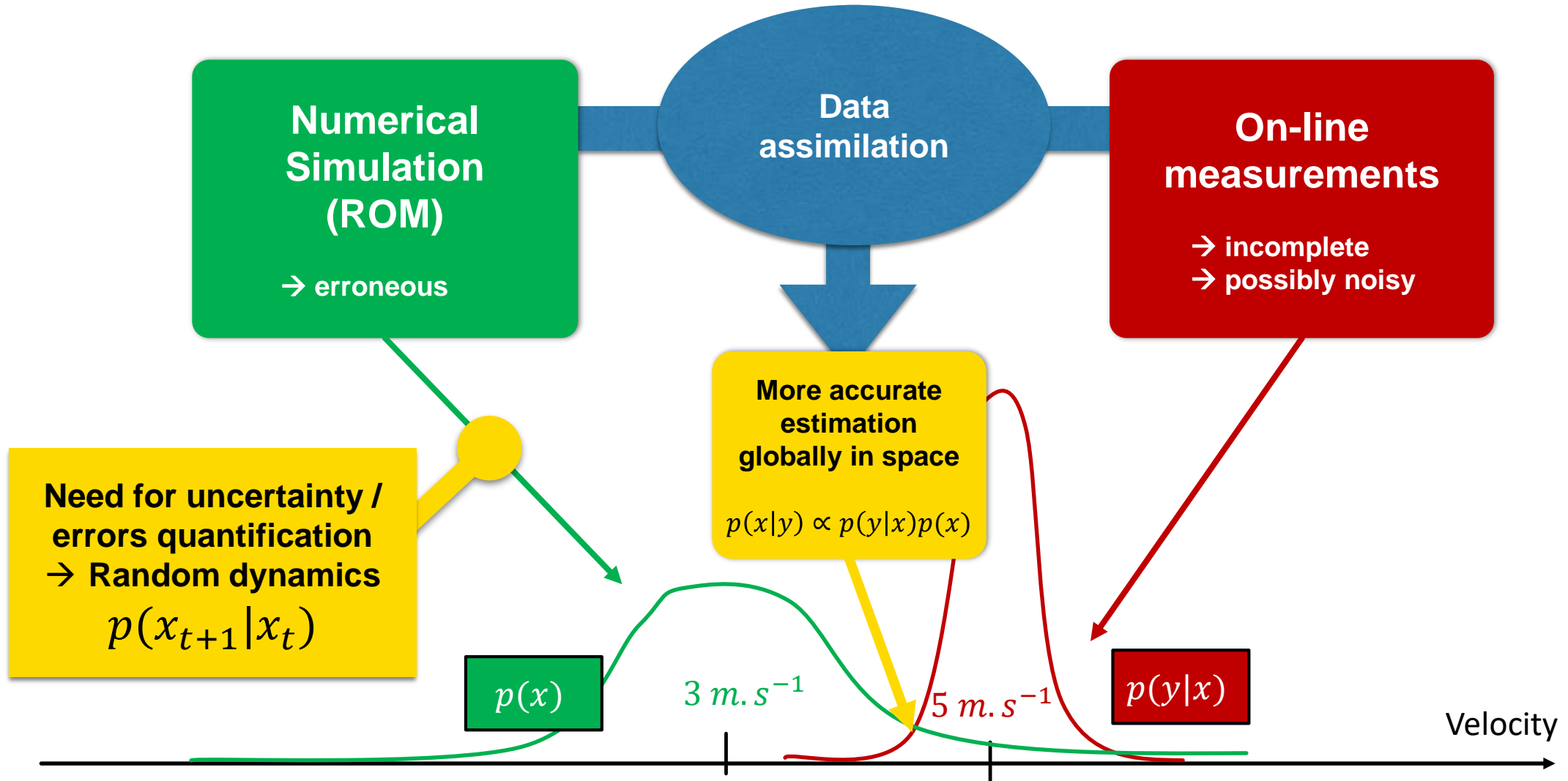
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PART IV

REDUCED ORDER MODELS UNDER LOCATION UNCERTAINTY


LOCATION UNCERTAINTY MODELS (LUM)

$$v = \sum_{i=0}^n b_i \phi_i + v'$$

The equation is presented with a blue rounded rectangle around the sum term $\sum_{i=0}^n b_i \phi_i$ labeled "Resolved modes" and a green rounded rectangle around the residual term v' labeled "Residual".

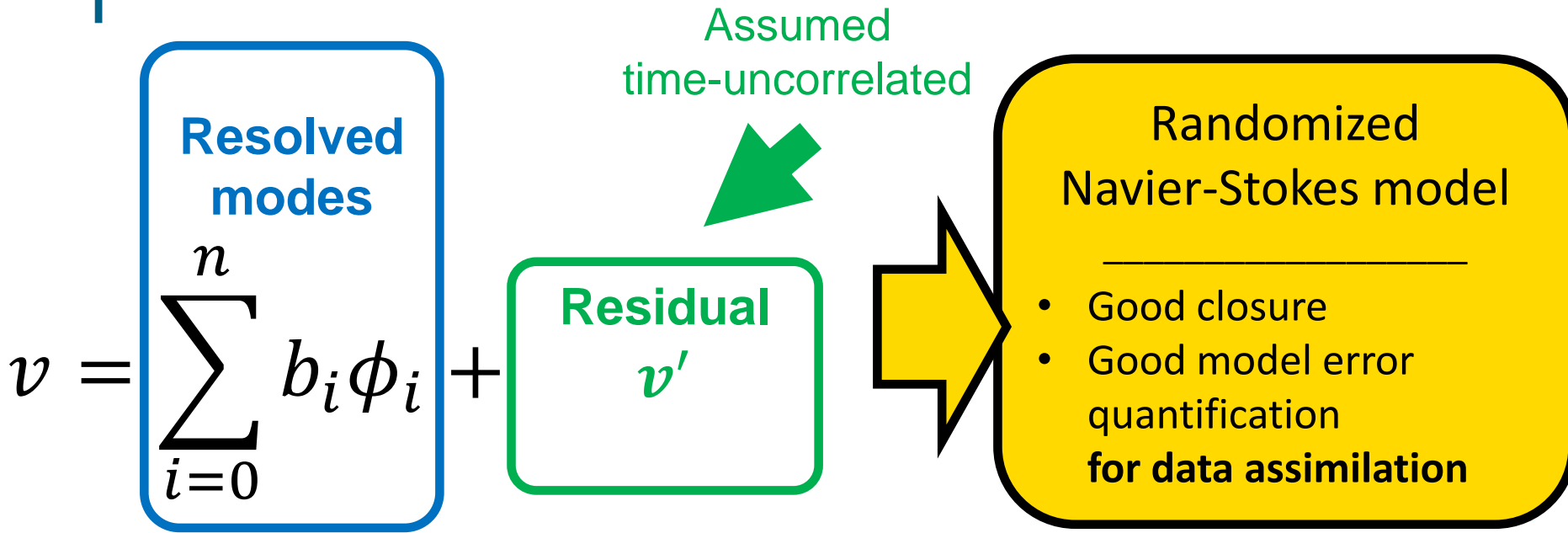
LOCATION UNCERTAINTY MODELS (LUM)

Assumed
time-uncorrelated

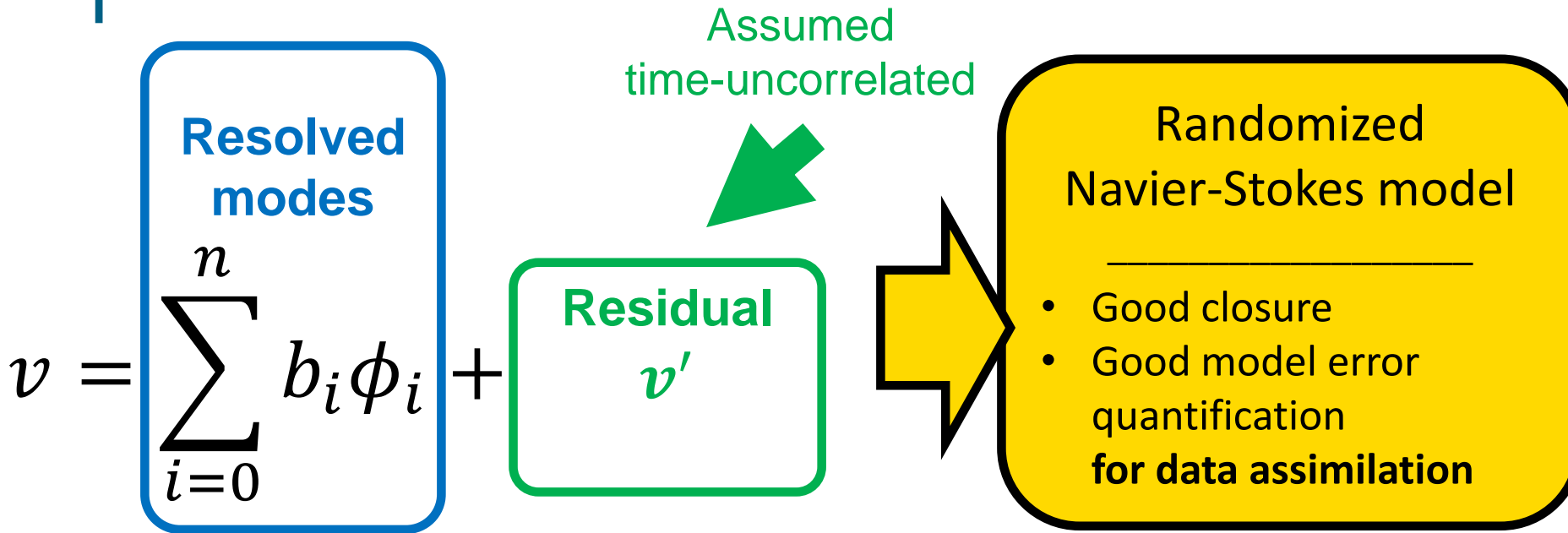

$$v = \sum_{i=0}^n b_i \phi_i + \text{Residual } v'$$

The diagram illustrates the Location Uncertainty Model (LUM) equation. It shows the total displacement v as the sum of resolved modes and a residual. The resolved modes are represented by a summation from $i=0$ to n of $b_i \phi_i$, enclosed in a blue rounded rectangle. The residual v' is enclosed in a green rounded rectangle. A green arrow points from the text 'Assumed time-uncorrelated' to the residual box.

LOCATION UNCERTAINTY MODELS (LUM)

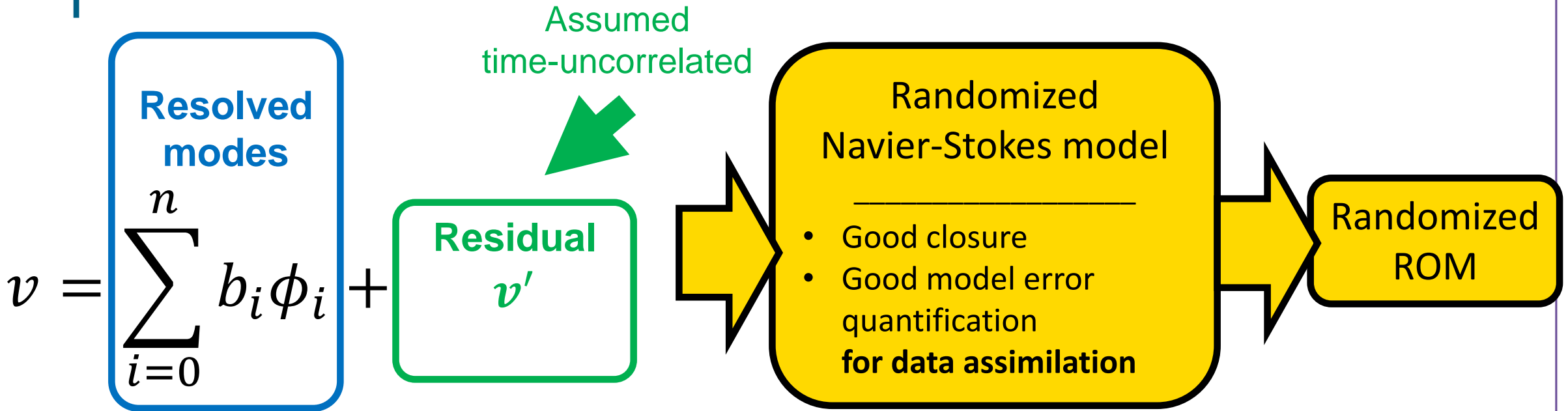


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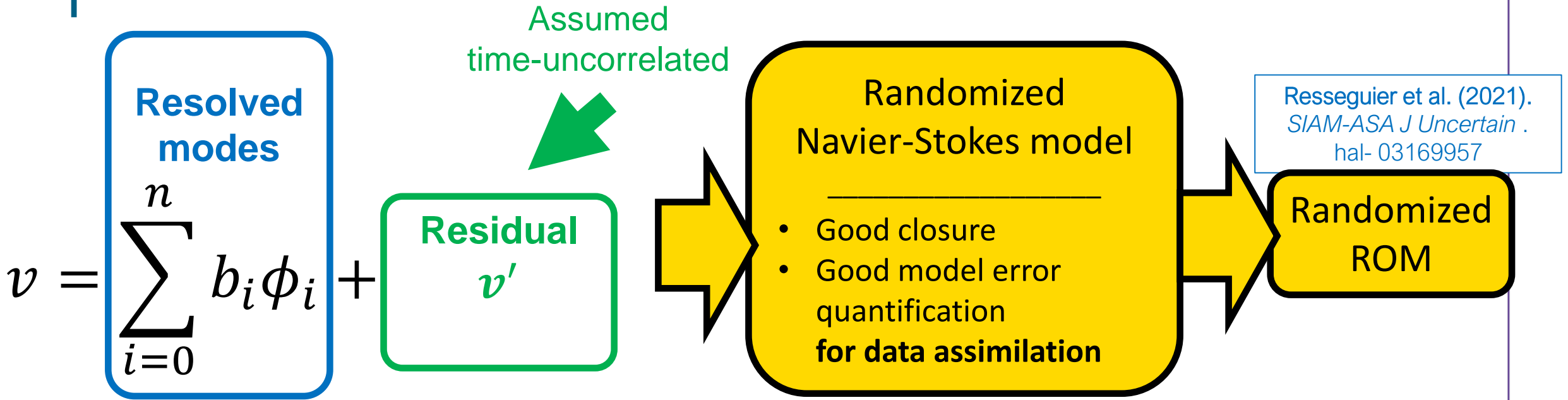
	LUM	SALT
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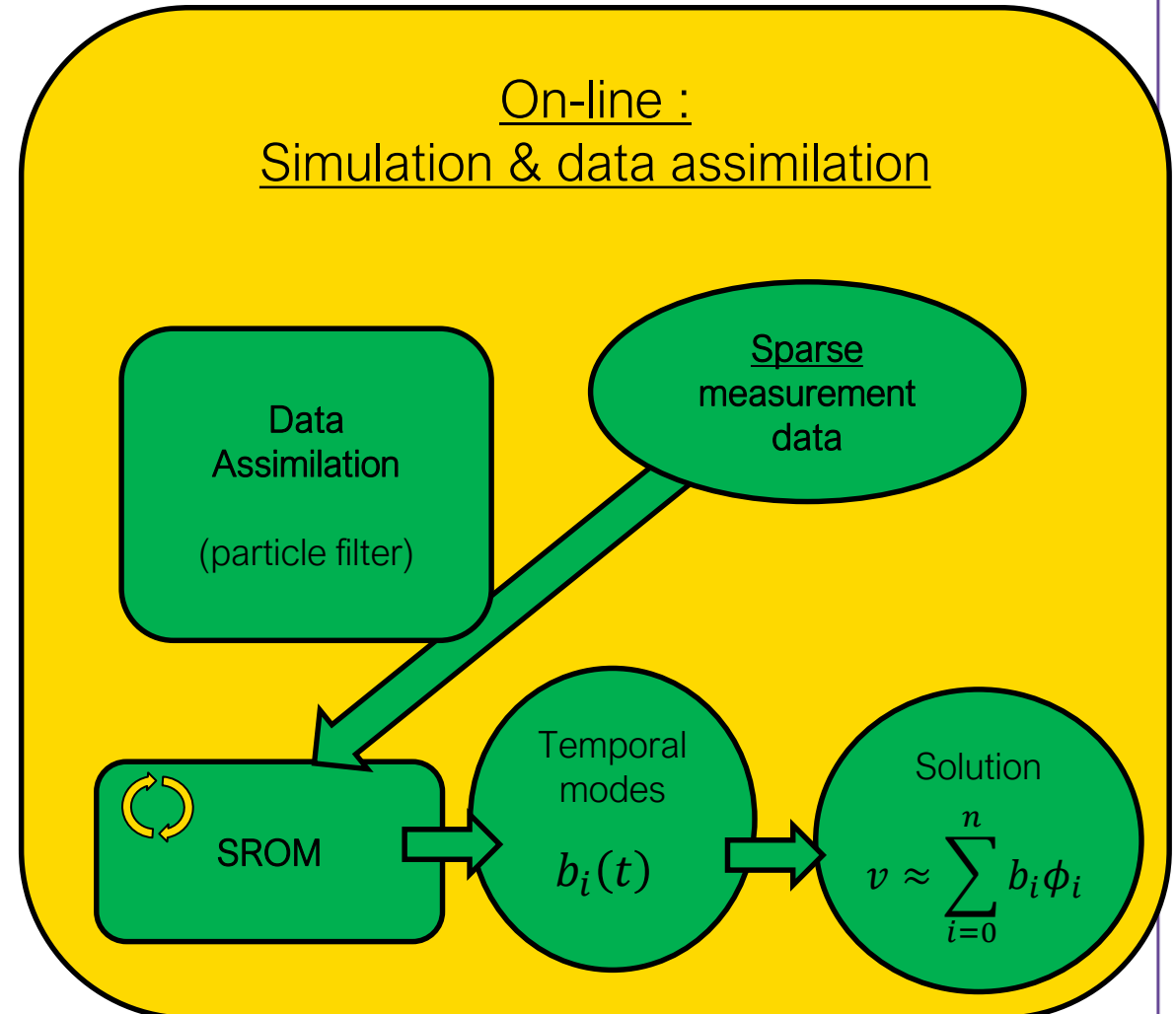
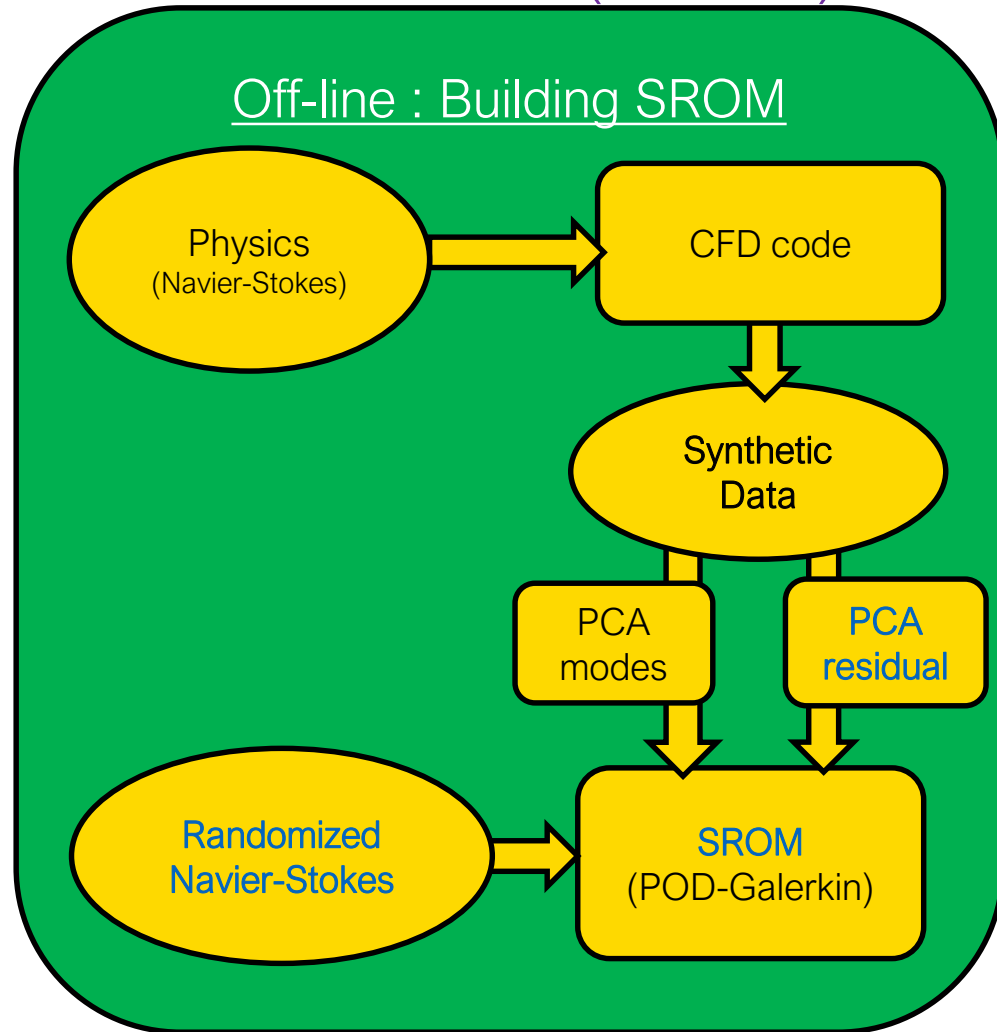
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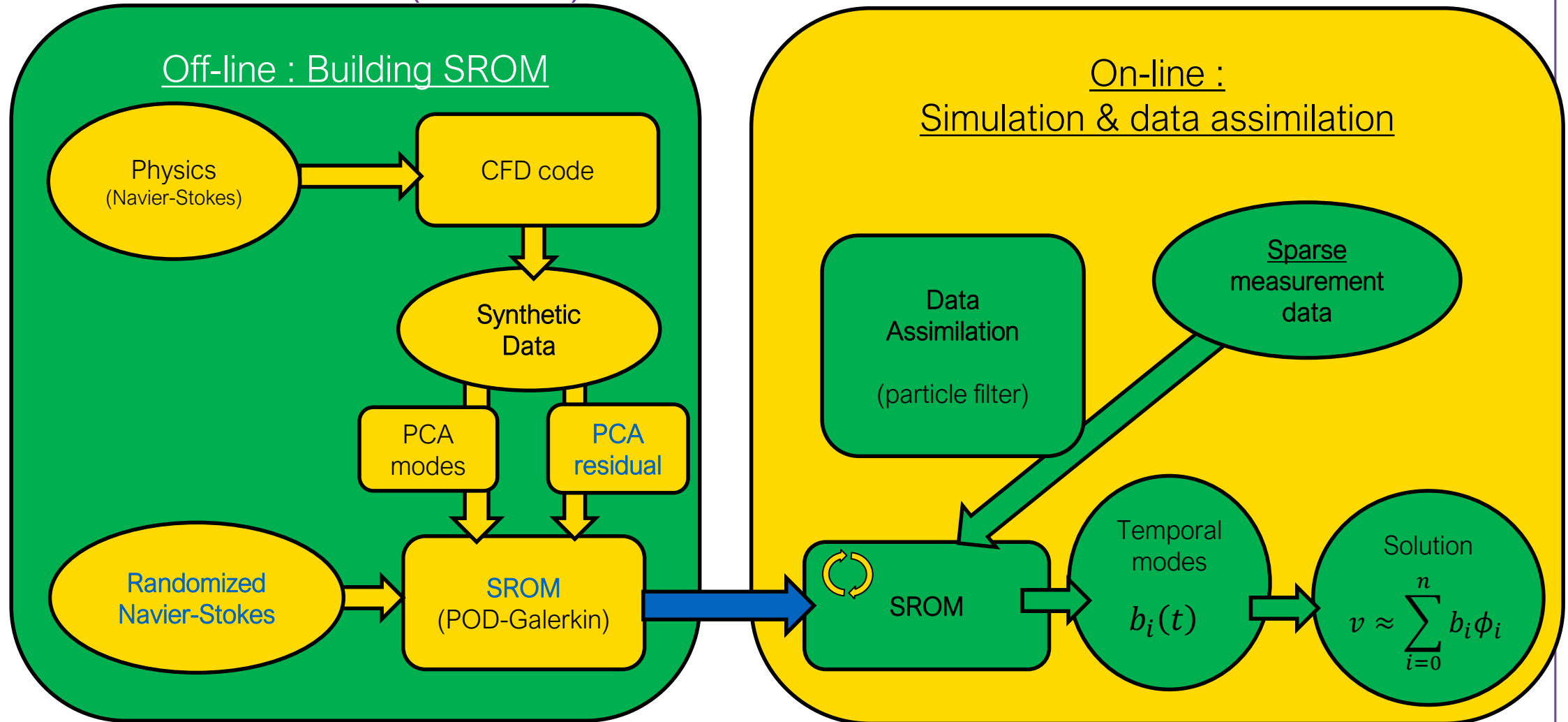
SUMMARY

Stochastic ROM (SROM) + Data assimilation



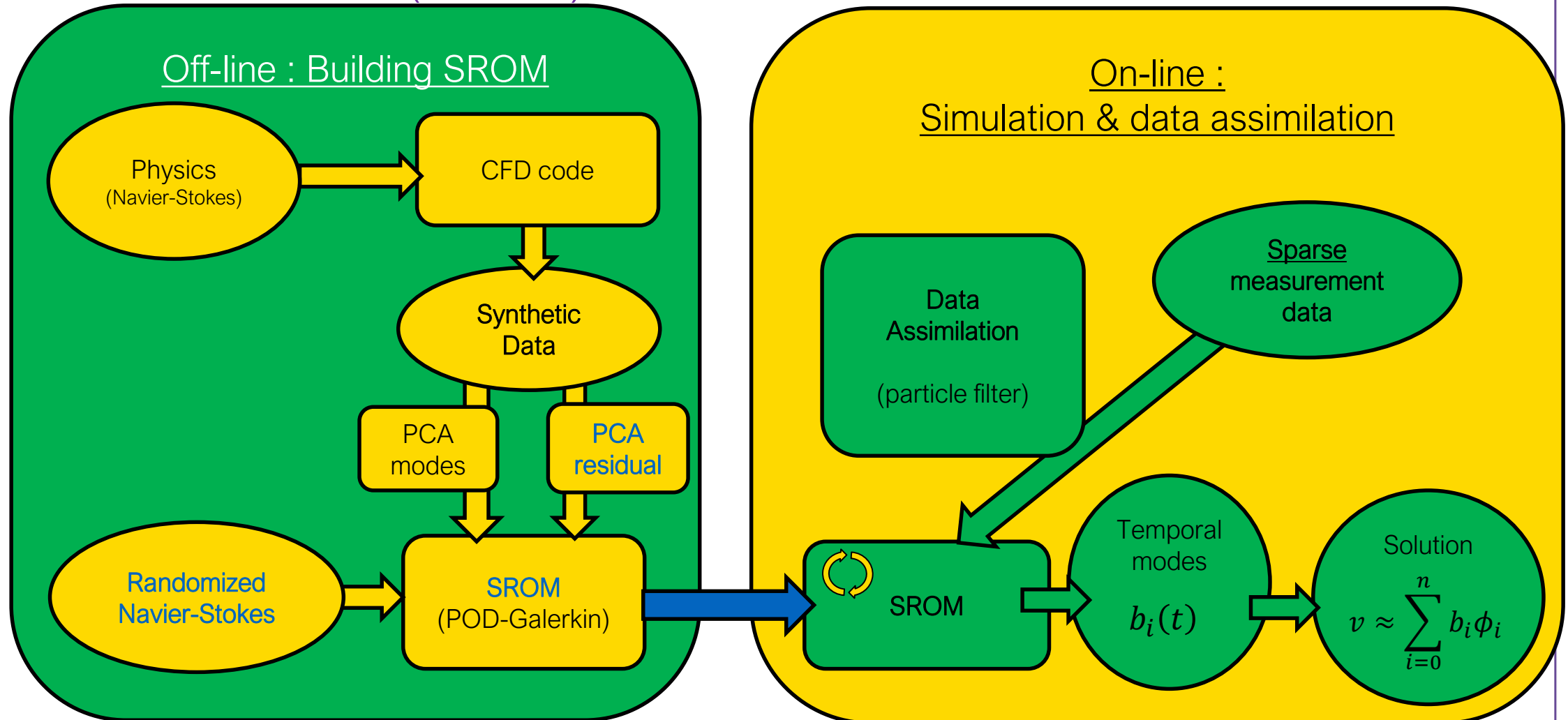
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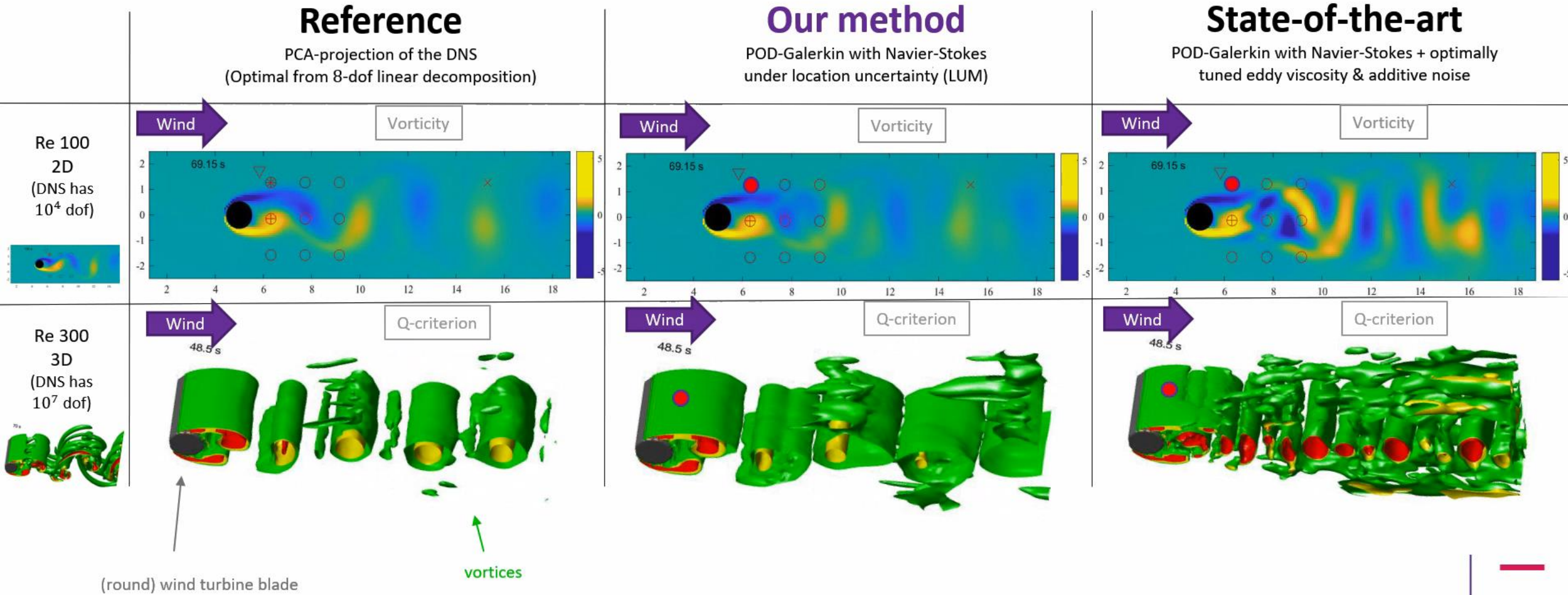
RESULTS :
UQ &
FAST OBSERVER
OF THE FLOW

DATA ASSIMILATION (POSTERIOR)

On-line estimation of the solution

From 10^7 to 8 degrees of freedom

● Single measurement point (blurred & noisy velocity)



CONCLUSION

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- ▶ Reduced order model (ROM) : for very fast and robust CFD ($10^7 \rightarrow 8$ degrees of freedom.)
 - Combine data & physics (built off-line) ▪ Now implemented in OpenFOAM / ITHACA-FV
 - Closure problem handled by LUM
- ▶ Data assimilation : to correct the fast simulation on-line by incomplete/noisy measurements
 - Model error quantification handled by LUM
- ▶ First results
 - Optimal **unsteady 3D flow** estimation/prediction in the whole spatial domain (large-scale structures)
 - Robust far outside the training set (time extrapolation / out of sample)

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WORK IN PROGRESS:

- ▶ Increasing Reynolds (ROM of LES, DDES)
 - Hyperreduction (=interpolation with a POD basis) of turbulence model terms

