



State observer for water-based hybrid PV/T system with unknown input

Zain Ul Abdin, Ahmed Rachid

► To cite this version:

Zain Ul Abdin, Ahmed Rachid. State observer for water-based hybrid PV/T system with unknown input. 48th Annual Conference of the IEEE Industrial Electronics Society (IECON) 2022, Oct 2022, Brussels, Belgium. ⟨hal-03820435⟩

HAL Id: hal-03820435

<https://hal.science/hal-03820435v1>

Submitted on 19 Oct 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

State observer for water-based hybrid PV/T system with unknown input

Zain Ul Abdin

*Laboratoire des Technologies Innovantes
Université de Picardie Jules Verne
Amiens, France
zain.ul@u-picardie.fr*

Ahmed Rachid

*Laboratoire des Technologies Innovantes
Université de Picardie Jules Verne
Amiens, France
rachid@u-picardie.fr*

Abstract—This paper deals with the observer design problem for bilinear photovoltaic-thermal (PV/T) thermal model to estimate its states. The objective is to construct an observer subjected to an unknown input. This study proposes an improvement in the design compared to the work in the literature on the application of a state observer with unknown input for photovoltaic-thermal systems where \hat{x} is estimated by knowing \dot{T}_{wi} . A Lyapunov approach is used to ensure stability and a Linear Matrix Inequality (LMI) is solved simultaneously in order to find a positive definite symmetric matrix subject to linear equality constraint. Necessary and sufficient conditions for the existence of such approach are given. A design procedure is summarized and the proposed technique is finally illustrated through simulations. Results indicate that the state estimation error converges to zero for all the states, thus displaying effectiveness of the presented scheme.

Index Terms—photovoltaic-thermal systems, unknown input observer, state observation, LMI, Lyapunov stability.

I. INTRODUCTION

A photovoltaic-thermal (PV/T) system combines photovoltaic and solar thermal modules into one integrated unit. These systems provide heat and electricity simultaneously. The measurements of all system states may not be possible in certain systems because of incomplete information on the internal state of the system and for this reason, observers are designed. Observers are the dynamic systems that provide an estimate of the internal state of a plant using its input-output measurements [1]. A real-time Luenberger type state observer is designed by Kicsiny and Varga [2] for solar thermal heating system and the algorithm for the practical use is presented. Wang et al. [3] designed a non-linear observer for solar thermal water heater system considering only four components to estimate the missing state variables. Ouhsaine et al. [4] constructed heat transfer model for PV/T systems in the context of a new consideration of heat conduction with a time fractional-order derivative. The temperature of the nodes are estimated using the fractional-order observer where the observer gain is computed by solving the linear matrix inequality (LMI).

A simple method to design a full-order observer for linear systems with unknown inputs was presented by Darouach et al. [5] along with two examples and necessary conditions for the existence of the observer were provided. A full-order

state observer for bounded inputs bilinear systems subjected to unknown disturbances was designed and investigated by Rafaralahy et al. [6] based on gain optimization of both the linear and the bilinear parts of the observation error dynamics. It was concluded that the proposed observers are applicable to a large class of bilinear systems subjected to unknown disturbances. An unknown input fuzzy bilinear fault diagnosis observer was designed by Saoudi et al. [7] for dynamic systems that can be described by Takagi-Sugeno fuzzy bilinear models subject to unknown disturbance. The effectiveness of the proposed observer has been accompanied by a dynamic system example. A design procedure for full-order and reduced-order observers with unknown inputs was presented using two different approaches by Hui and Žak [8]. The state of the observed system was decomposed into known and unknown components in the first approach whereas the second approach combines sliding modes and the second method of Lyapunov resulting in a non-linear observer. Gao et al. [9] presented an unknown input observer for the augmented system to decouple partial process disturbances and linear matrix inequality optimization technique is implemented to make sure that the estimation error dynamics to be stable. The existence conditions of the proposed observer are addressed.

In this study, an observer is designed for the PV/T system subjected to an unknown input, an improvement in the design compared to our previous studies. The idea is to estimate the states of the system even in the presence of unknown inputs and they can represent faults, disturbances or neglected dynamics in the system. To our knowledge there is no work in the literature on the application of a state observer with unknown input for photovoltaic-thermal systems. An important aspect of this study is that it is possible to construct an observer for systems affected by unmeasurable disturbances. The designed observer is efficient in estimating the states of the system and the error converges to zero for all the states.

The paper is organized as follows: section II provides system description and a multiple model of a PV/T system with an unknown input. Next, an observer is designed in section III for the system subjected to an unknown input. Sufficient conditions for the existence of this observer are given. Section IV is devoted to simulation results and the paper ends with conclusions and are found in section V.

II. THE MODEL WITH UNKNOWN INPUT

The main aim of the system is to generate electricity and provide warm water by harnessing the solar radiation collected from the sun. The cross-sectional view of a water-based PV/T collector is presented in Figure 1 and consists of; a glass that is open to the surrounding environment, a PV cell layer which is followed by a tedlar film, an absorber layer, a metallic tube pressed between multiple layers through which fluid (water) flows, and an insulated frame to avoid thermal losses. Few assumptions have been considered for the thermal model such as: pressure losses, dust and partial shading are neglected. Temperature is assumed to be homogeneous for each layer. $T_g, T_c, T_t, T_r, T_m, T_w$ and T_i are the temperatures of various components of the collector.

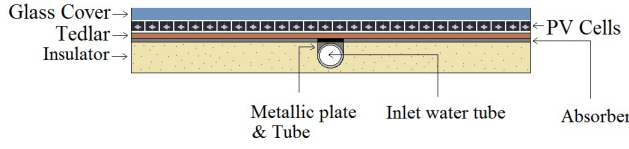


Fig. 1: Sectional view of water-based photovoltaic-thermal (PV/T) collector.

A multiple thermal model is constructed by Rachid and Ul Abdin [10] using the boundedness of nonlinear terms, based on a convex polytopic transformation of non-linear scalar functions. The thermal model can be expressed in state space form as:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^2 \mu_i A_i x(t) + Bx(t)u(t) + Gv(t) \\ y(t) = Cx(t) - T_{wi} \end{cases} \quad (1)$$

where x is defined as;

$$x = [T_g \ T_c \ T_t \ T_r \ T_m \ T_w \ T_i]^T$$

The output y is the outlet water temperature T_{wo} , T_{wi} is water inlet temperature, mass-flow rate \dot{m}_w is defined as controlled input u , v contains non-controlled inputs and can be written as:

$$v = [T_{am} \ I_{sun} \ T_{wi}]^T$$

The matrices B and G are provided in previous studies, which are omitted here for brevity. Temperature of the fluid T_w is respectively given by:

$$T_w = \frac{T_{wi} + T_{wo}}{2}$$

A new sixth state θ_w is introduced, and is given by:

$$\theta_w = T_w - T_{wi}, \quad (2)$$

As a result, an equivalent system with change of variable can be constructed as follows:

$$\begin{cases} \dot{\bar{x}}(t) = \sum_{i=1}^2 \mu_i A_i \bar{x}(t) + \bar{B}\bar{x}(t)u(t) + \bar{G}v(t) - P\dot{T}_{wi} \\ y(t) = C\bar{x}(t) + T_{wi} \end{cases} \quad (3)$$

where \dot{T}_{wi} is unknown input, \bar{x} contains state variables and is defined as;

$$\bar{x} = [T_g \ T_c \ T_t \ T_r \ T_m \ \theta_w \ T_i]^T$$

The variation of T_{wi} is unknown and the noise is highly amplified when make a derivative. High frequency gives high values and taking a derivative of it will give much higher values and considering it as unknown input could avoid the trouble. It should be noted that the only dissimilar term in x and \bar{x} is the sixth state and the actual state x_6 can be calculated using the relation:

$$x_6 = \bar{x}_6 + T_{wi}$$

The new measured output $Y(t)$ can be written as:

$$Y(t) = y(t) - T_{wi} \quad (4)$$

For the corresponding model, $Y(t)$ is given by:

$$Y(t) = C\bar{x}(t) \quad (5)$$

For the A_i matrix, the only varying term is a_{22} :

$$a_{22} = h(I_{sun}) = b_2 + b_3 I_{sun}, \quad (6)$$

where $h(I_{sun})$ is bounded: $b_2 + \underline{I_{sun}}b_3 \leq h(I_{sun}) \leq b_2 + \overline{I_{sun}}b_3$ where $\underline{I_{sun}} = \min\{I_{sun}\}$, $\overline{I_{sun}} = \max\{I_{sun}\}$.

The 2 possible local models available with A_1 containing a_{22} as $b_2 + \underline{I_{sun}}b_3$ whereas for A_2 , it is $b_2 + \overline{I_{sun}}b_3$ and A_i is described as follows

$$A_i = \begin{bmatrix} a_{11} & a_3 & 0 & 0 & 0 & 0 & 0 \\ b_1 & a_{22} & b_4 & 0 & 0 & 0 & 0 \\ 0 & c_1 & a_{33} & c_2 & c_3 & 0 & 0 \\ 0 & 0 & d_1 & a_{44} & d_2 & 0 & d_3 \\ 0 & 0 & e_1 & e_2 & a_{55} & e_3 & e_4 \\ 0 & 0 & 0 & 0 & f_1 & -f_1 & 0 \\ 0 & 0 & 0 & g_1 & g_2 & 0 & a_{77} \end{bmatrix}$$

$$C = [0 \ 0 \ 0 \ 0 \ 0 \ 2 \ 0]$$

\bar{B} , \bar{G} , and P are constant matrices and are given as:

$$\bar{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -f_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{G} = \begin{bmatrix} k_{11} & 0 & 0 & 0 & 0 & 0 & g_3 \\ a_5 & b_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e_3 & -f_1 & 0 \end{bmatrix}^T$$

$$P = [0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0]^T$$

The parameters a, b, c, d, e, f, g , and k are defined in the appendix section.

III. DESIGN OF THE OBSERVER

In this section, an observer is designed for estimating the states of the multiple model. Designing observers is a challenging problem due to its importance in automatic control design. Necessary and sufficient conditions for the existence of such approach are given by [5], [6], [11].

Theorem 1:

If there exist D_i , E , L_i , J , and H matrices with appropriate dimensions satisfying the constraints, the observer error is exponentially stable such that D_i is Hurwitz. If one can make the following constraints hold:

$$E^T X + X E = 0 \quad (7)$$

$$D_i^T X + X D_i < 0 \text{ for } i = 1, 2 \quad (8)$$

$$D_i S - S A_i + L_i C = 0 \quad (9)$$

$$J - S G = 0 \quad (10)$$

$$S P = 0 \quad (11)$$

$$E S - S B + H C = 0 \quad (12)$$

$$S = I_7 - M C \quad (13)$$

then \hat{x} converges to \bar{x} .

The existing conditions for the observer were given by [5], [6], [11]. A constant gain unknown input observer for the system (3) is described with the following structure

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^2 \mu_i D_i z(t) + E z(t) u(t) + \sum_{i=1}^2 \mu_i L_i Y(t) \\ \quad + J v(t) + H u(t) Y(t) \\ \hat{x}(t) = z(t) + M Y(t) \end{cases}, \quad (14)$$

where z is the state vector of dynamic system (14) and \hat{x} represents the estimation of states. The system is unknown input state observer if and only if

$$\lim_{t \rightarrow \infty} \|\hat{x}(t) - x(t)\| = 0$$

holds irrespective of the initialisations, the measured input and the unknown input.

Proof:

Let estimation error e be the difference between estimated and the actual states, and using the output equation in (14), one has

$$e(t) = \hat{x}(t) - \bar{x}(t) = z(t) + M Y(t) - \bar{x}(t) \quad (15)$$

the dynamics of the state estimation error is as follows:

$$\begin{aligned} \dot{e} = & \sum_{i=1}^2 \mu_i D_i e + \sum_{i=1}^2 \mu_i (D_i S - S A_i + L_i C) \bar{x} + (J - S G) v \\ & - S P \dot{w}_i + E u e + u (E S - S B + H C) \end{aligned} \quad (16)$$

If constraints (9)-(12) hold, state estimation error dynamics (16) reduces to

$$\dot{e}(t) = \sum_{i=1}^2 \mu_i D_i e(t) + E u(t) e(t) \quad (17)$$

To prove the stability, let a function $V = e^T X e$ be a Lyapunov candidate for error dynamic system (17) and X be a positive definite symmetric matrix. The time derivative of the lyapunov function, respectively gives

$$\dot{V} = e^T \left(\sum_{i=1}^2 \mu_i (D_i^T X + X D_i) + (E^T X + X E) u(t) \right) e(t), \quad (18)$$

or equivalently

$$\dot{V} = e^T T(u) e(t), \quad (19)$$

where

$$T(u) = \sum_{i=1}^2 \mu_i X_i + X_o u(t), \quad (20)$$

with $X_i = D_i^T X + X D_i$ and $X_o = E^T X + X E$.

The asymptotic stability is ensured with constraints (7) and (8) if the derivative of the Lyapunov function is negative if and only if $T(u)$ is negative definite for all admissible $u(t)$. The constraint (8) is satisfied if D_i is made Hurwitz, that is, $\Re(\lambda(D_i)) < 0$ then there exists X which is a symmetric positive definite matrix. The proof is completed.

Now, let us find the unknown matrices of (14) using the constraints provided in Theorem 1. Taken into account the constraint (13), (11) yields:

$$P - M C P = 0 \quad (21)$$

the solution of this equation depends on the rank of matrix CP , M exists if $\text{rank}(CP) = \text{rank}(P)$. The general solution of (21) exists and can be written as:

$$M = P(CP)^+ + \xi(I - (CP)(CP)^+),$$

where ξ is an arbitrary matrix with appropriate dimension and $(CP)^+$ is the generalized inverse of (CP) . [5] shown that ξ must be choosen such that M is of maximal rank and the obvious choice of matrix is $\xi = 0$ and this yields

$$M = P(CP)^+ \quad (22)$$

Theorem 2:

The necessary conditions for the existence of structure (14) are

i) $\text{rank}(CP) = \text{rank}(P)$

if it holds then, (21) is solvable otherwise observer cannot be constructed.

ii) The pair $(C, S A_i)$ is detectable (observable) if and only if the rank condition holds

$$\text{rank} \begin{bmatrix} s I_7 - A & P \\ C & 0 \end{bmatrix} = 7 + \text{rank}(P)$$

for all s with $\Re(s) \geq 0$, but $s \neq 0$.

Suppose if these conditions are satisfied, then matrix D_i can be made stable and observer (14) exists.

Using (13) in (9), the equation reduces to

$$D_i = SA_i - K_{oi}C, \quad (23)$$

where

$$K_{oi} = L_i - D_iM \text{ for } i = 1, 2 \quad (24)$$

and D_i should be Hurwitz, for that, (SA_i, C) should be detectable. K_{oi} can be computed by pole placement for the pair (SA_i, C) and (24) can be written as:

$$L_i = K_{oi} + D_iM \quad (25)$$

Substituting (23) in (8), LMI is solved for X and has a positive definite solution X if D_i is Hurwitz.

Substituting (13) in (7), the equation reduces to

$$E = SB - KC, \quad (26)$$

where

$$K = H - EM \quad (27)$$

Using (26) in (7), yields

$$(SB - KC)^T X + X(SB - KC) \quad (28)$$

or equivalently

$$(SB)^T X + X(SB) = (KC)^T X + X(KC) \quad (29)$$

if a new variable ζ is introduced that is equal to (29) and particular solution of left hand side of (29) is $XKC = (XKC)^T$ then ζ can be written as:

$$\zeta = 2XKC, \quad (30)$$

where the solution of (30) exists only if $\text{rank}(C^T) = \text{rank}[C^T \ (X^{-1}\zeta)^T]$ and K is then given by:

$$K = \frac{1}{2}X^{-1}\zeta C^+, \quad (31)$$

where (C^+) is the generalized inverse of C .

If the above assumption does not holds, the solution can be obtained by minimizing $\| KC - \frac{1}{2}X^{-1}\zeta \|_2$, given by (31) with $(C^+) = C^T(CC^T)^{-1}$.

Remark 1:

The solar radiation I_{sun} and ambient temperature T_{amb} are not chosen as unknown inputs because assumption (i) in theorem 2 is not satisfied. It is necessary condition to ensure the existence of such observer. Infact, the product of matrices C and P becomes zero.

Remark 2:

Fluid inlet temperature T_{wi} cannot be taken as unknown input because measured output $Y(t)$ contains it and will make impossible for us to design such scheme for it.

Remark 3:

In order to simplify B , we made a change of variable θ_w as given in (2) and this change results in an introduction of time derivative of inlet water temperature. Since, \dot{T}_{wi} in (3) cannot be found directly so $\hat{\hat{x}}$ is estimated without knowing it.

The unknown input observer of the form (14) can be constructed by following the design procedure provided in section below.

Algorithm:

The design procedure can be summarised as follows:

- 1) Verify if assumption (i) is satisfied
- 2) Compute M using (22)
- 3) Compute S using (13)
- 4) Compute D_i in (23) by pole placement
- 5) Compute L using (25)
- 6) Using (8) and (23), compute X using LMI toolbox
- 7) Compute K using (31)
- 8) Compute E using (26)
- 9) Compute H using (27)

IV. SIMULATION RESULTS

The multiple thermal model of a photovoltaic-thermal collector (1) is simulated using MATLAB/Simulink by randomly choosing systems initial condition $x_0 = [23 \ 25 \ 23 \ 23 \ 23 \ 3 \ 20]^T$ and observers initial condition $z_0 = [23 \ 27 \ 23 \ 23 \ 24 \ 5 \ 20]^T$. Figure 2 show respectively the evolution of the estimation error between the actual system variables and their corresponding observer ones. It can be seen that the state estimation error converges asymptotically tends to zero for all of the states even though, the unknown input is present.

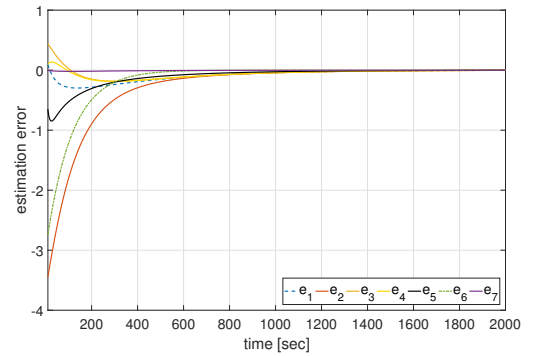


Fig. 2: State estimation errors: e_1 through e_7 .

To estimate the states of the system, the system was initially simulated without any unknown input. In Figure 3, it can be observed that there is a reasonable agreement between the true and estimated states of the system. The system real states (x) are tracked by the observer states (\hat{x}). It is obvious that the PV component has a higher temperature than other components. We consider now the same previous system subject to a

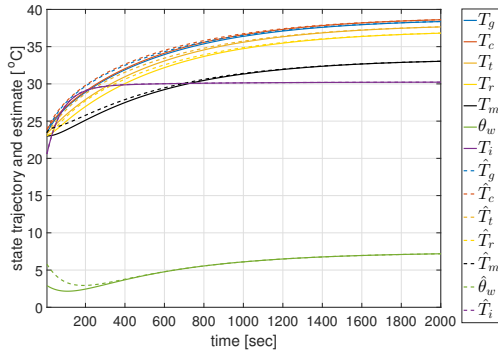


Fig. 3: True states and their estimations without unknown input.

sinusoidal input T_{wi} and is injected with similar conditions. Figure 4 illustrates that it affects the state θ_w whereas there is a small effect on the state x_5 . However, the estimation of the state remained, which shows the robustness of the observer. The estimation error converges to zero even if the

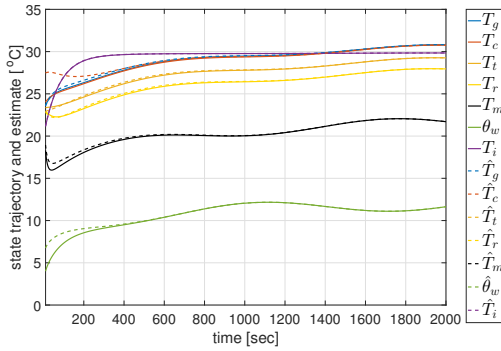


Fig. 4: True states and their estimations with sinusoidal input T_{wi} .

initial conditions for the observer are changed.

V. CONCLUSIONS

Water-based photovoltaic-thermal system continue to gain popularity as the reliance on fossil fuels decreases. In this study, an observer is designed subjected to unknown input for PV/T system thermal multiple model. The existence conditions are given using Lyapunov stability approach. The estimation error converges to zero for all of the states, thus displaying the effectiveness of the proposed approach. The observer showed high robustness against unknown input and estimation of the states remained very accurate. Experimental investigations are in progress to validate the concrete applicability of the proposed design.

NOMENCLATURE

Symbols

A	area
A_{mod}	module area of a PV panel
C	specific heat capacity
d_i	inner diameter of the tube
d_o	outer diameter of the tube
I_{sun}	solar radiation
M	mass
T	temperature
h_c	conduction heat transfer coefficient between two components
h_r	radiation heat transfer coefficient between two components
h_v	convective heat transfer coefficient between two components

Greek letters

α	absorptivity
β	packing factor
β_p	temperature coefficient
η_{ref}	reference efficiency
τ	transmissivity

Subscripts

am	ambient
c	PV cell
g	glass cover
i	thermal insulator
m	tube
r	absorber
s	sky
t	tedlar
w	fluid (water)

APPENDIX

$$\begin{aligned}
 a_1 &= \frac{A_{mod}}{M_g C_g}, a_2 = \frac{A_{mod} h_{v,am}}{M_g C_g}, a_3 = \frac{A_{mod} h_{c,gc}}{M_g C_g}, \\
 a_4 &= \frac{0.0552 A_{mod}}{M_g C_g}, a_5 = \frac{A_{mod} \alpha_g}{M_g C_g} \\
 b_1 &= \frac{A_{mod} h_{c,gc}}{M_c C_c}, b_2 = \frac{A_{mod} (-h_{c,gc} - h_{c,ct})}{M_c C_c}, \\
 b_3 &= \frac{A_{mod} \eta_{ref} \beta_p}{M_c C_c}, b_4 = \frac{A_{mod} h_{c,ct}}{M_c C_c}, \\
 b_5 &= \frac{A_{mod} \tau_g \alpha_c \beta - \eta_{ref} - \eta_{ref} \beta_p T_{c,ref}}{M_c C_c}, \\
 c_1 &= \frac{A_{mod} h_{c,ct}}{M_t C_t}, c_2 = \frac{A_{tr} h_{c,tr}}{M_t C_t}, c_3 = \frac{A_{tm} h_{c,tm}}{M_t C_t}, \\
 d_1 &= \frac{A_{tr} h_{c,tr}}{M_r C_r}, d_2 = \frac{A_{rm} h_{c,rm}}{M_r C_r}, d_3 = \frac{A_{ri} h_{c,ri}}{M_r C_r}, \\
 e_1 &= \frac{A_{tm} h_{c,tm}}{M_m C_m}, e_2 = \frac{A_{rm} h_{c,rm}}{M_m C_m}, e_3 = \frac{A_{mw} h_{v,mw}}{M_m C_m}, \\
 e_4 &= \frac{A_{mi} h_{c,mi}}{M_m C_m}, f_1 = \frac{A_{mw} h_{v,mw}}{M_w C_w}, f_2 = \frac{2C_w}{M_w C_w},
 \end{aligned}$$

$$g_1 = \frac{A_{ri}h_{c,ri}}{M_iC_i}, g_2 = \frac{A_{mi}h_{c,mi}}{M_iC_i}, g_3 = \frac{A_{mod}h_{v,iam}}{M_iC_i},$$

$$a_{11} = -(a_1h_{r,gs} + a_2 + a_3), a_{22} = b_2 + b_3I_{sun},$$

$$a_{33} = -(c_1 + c_2 + c_3), a_{44} = -(d_1 + d_2 + d_3),$$

$$a_{55} = -(e_1 + e_2 + e_3 + e_4), a_{77} = -(g_1 + g_2 + g_3),$$

ACKNOWLEDGEMENT

The authors gratefully acknowledge the financial support from the European regional development fund under the SOLARISE project of the Interreg 2 Seas with sub-grant contract No. 2S04-004.

REFERENCES

- [1] K. Kalsi, J. Lian, S. Hui, and S. H. Žak, "Sliding-mode observers for systems with unknown inputs: A high-gain approach," *Automatica*, vol. 46, no. 2, pp. 347–353, 2010.
- [2] R. Kicsiny and Z. Varga, "Real-time state observer design for solar thermal heating systems," *Applied mathematics and computation*, vol. 218, no. 23, pp. 11 558–11 569, 2012.
- [3] Z. Wang, R. Rosas, and C. Qian, "Nonlinear observer design for a solar thermal water heater system," in *The 27th Chinese Control and Decision Conference (2015 CCDC)*. IEEE, 2015, pp. 4817–4822.
- [4] L. Ouhsaine, Y. Boukal, M. El Ganaoui, M. Darouach, M. Zasadzinski, A. Mimet, and N. E. Radhy, "A general fractional-order heat transfer model for photovoltaic/thermal hybrid systems and its observer design," *Energy Procedia*, vol. 139, pp. 49–54, 2017.
- [5] M. Darouach, M. Zasadzinski, and S. J. Xu, "Full-order observers for linear systems with unknown inputs," *IEEE transactions on automatic control*, vol. 39, no. 3, pp. 606–609, 1994.
- [6] H. Rafaralahy, M. Zasadzinski, C. Mechmeche, and M. Darouach, "Unknown input observer for bilinear systems with gain optimisation," *IFAC Proceedings Volumes*, vol. 29, no. 1, pp. 4368–4373, 1996.
- [7] D. Saoudi, M. Chadli, and N. B. Braiek, "State estimation of unknown input fuzzy bilinear systems: Application to fault diagnosis," in *2013 European Control Conference (ECC)*. IEEE, 2013, pp. 2465–2470.
- [8] S. Hui and S. Žak, "Observer design for systems with unknown inputs," *International Journal of Applied Mathematics and Computer Science*, vol. 15, pp. 431–446, 2005.
- [9] Z. Gao, X. Liu, and M. Z. Chen, "Unknown input observer-based robust fault estimation for systems corrupted by partially decoupled disturbances," *IEEE Transactions on Industrial Electronics*, vol. 63, no. 4, pp. 2537–2547, 2015.
- [10] A. Rachid and Z. Ul Abdin, "Multiple model approach and observer design for a water-based pv/t collector," in *2022 6th IEEE Conference on Control Technology and Applications*. IEEE, 2022.
- [11] C. Mechmeche, H. Hamdi, and H. Yahoui, "State observer for bilinear systems subjected to unknown inputs," in *2008 5th International Multi-Conference on Systems, Signals and Devices*. IEEE, 2008, pp. 1–6.