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A Performance Comparison of Classical and Quantum Algorithm for Active User Detection

Muhammad Idham Habibie, Jihad Hamie, Claire Goursaud

Université de Lyon, INSA Lyon, INRIA
CITI EA 3720

F-69621 Villeurbanne, France
muhammad-idham.habibie@insa-lyon.fr

Abstract—The strong growth in the number of connected mobile devices has imposed new challenges in efficiently exploiting the available networks resources. Code Domain *Non-Orthogonal Multiple Access* (NOMA) technique appears as a tremendous efficient solution. Each device uses its assigned code to simultaneously transmit its data along with the user identifier, without any resource reservation exchange, saving precious wireless resources. However, this requires a receiver capable of blindly detecting the active users, which is highly complex. Driven by the promising superposition property of quantum architecture, the goal of this paper is to adapt and apply the quantum Grover algorithm for *Active User Detection* (AUD) purpose in the context of NOMA, to alleviate the search complexity. This adapted Grover’s algorithm is compared with the optimal classical *Maximum Likelihood* (ML) AUD receivers, as well as with the basic classical *Conventional Correlation Receiver* (CCR). A benchmark on the probability of AUD is assessed as a function of the *Signal to Noise Ratio* (SNR) of the received signal. We show that our adapted Grover’s algorithm is very promising in high SNR regime.

Index Terms—NOMA, AUD, Maximum Likelihood, quantum algorithm, Grover’s algorithm

I. INTRODUCTION

Recently, the number of connected mobile devices has been subject to a strong growth, and this will further accelerate within few years to reach billions of devices. Indeed, their use is spread across a variety of new applications in the human daily life such as enhanced multimedia, faster data communication rates and machine type communication [1]. In this context, resource sharing has become a key enabling feature for allowing the simultaneous use of the network resources between several users or devices [2]. Multiple access technologies have been proposed for the existing generations of mobile systems (i.e. GSM, UMTS, LTE, 5G) such as *Time Division Multiple Access* (TDMA), *Frequency DMA* (FDMA), *Code DMA* (CDMA) and *Orthogonal Frequency DMA* (OFDMA). These *Orthogonal Multiple Access* (OMA) schemes permit to assign a specific resource to each of the transmitting node with no (or almost no) inter-user interference. However, the number of active nodes is limited to the number of resources, and a contention-based random access is needed to allocate these resources to the nodes which request to transmit at this time.

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These two constraints lead to resource wasting as each transmitting user may not be able to fully exploit the capacity of its assigned resource, while high signaling overhead (often higher than the data amount to be transmitted) is needed to establish the connection [3]. As a consequence, *Non-Orthogonal Multiple Access* (NOMA) has been proposed and studied by the scientific community to counter these drawbacks [3]. The main idea is to allow the users to share the same resources in a non-orthogonal way so as to relax the constraint due to the limited number of resources, and to overload the network to approach its capacity. The NOMA schemes can mainly be classified into these two categories : power domain NOMA [4] where users are differentiated by *Signal to Noise Ratio* (SNR), and code domain NOMA where each device is identified by its code such as *Sparse Code Multiple Access* (SCMA) [5], *Multi-User Shared Access* (MUSA) [6] and *Pattern Division Multiple Access* (PDMA) [7].

In this paper, we focus on the second category as no prior information is needed before transmission. The devices can transmit their data by using their code assigned once for all at the factory. Besides, code domain NOMA permits to have an all-in-one scheme where the identifier of the transmitting device is provided by the code sequence, while some data mapping can be added. For this setup, the key-enabling feature is the ability to detect, in real time, the active subset among all potential devices, also known as *Active User Detection* (AUD). The device simplicity and the spectrum efficiency is obtained at the cost of pushing the computation burden to the *Base station* (BS) side.

The *Maximum Likelihood* (ML) is the optimal AUD. Its principle is to evaluate the likelihood of any possible active subset and identify the most likely one [8]. This AUD suffers from high complexity which is not compatible with real time implementation. Indeed, for a set of N devices, 2^N combinations have to be tested. Thus, iterative versions of the optimal detector have been proposed and studied [8], [9] However, quantum computation offers a promising perspective to overcome the classical algorithm complexity and to implement the ML in a more time-efficient way. Indeed, quantum algorithm benefits from the superposition property which permits to evaluate all cases simultaneously. Quantum algorithms have already been proposed for wireless communications [10] to

jointly decode the transmitted symbols of all transmitting users. But in [10], Multi-User Detection (MUD) was done to recover data, based on the knowledge of the active nodes set, while in this paper we consider the activity detection.

The contribution of this paper is the adaptation of the quantum Grover algorithm [11] for detecting the set of active users in a non-orthogonal CD NOMA communication scheme, where the unipolar or the bipolar families of codes are addressed. Besides, our proposed quantum algorithm will be compared with the optimal classical ML as well as with the *Conventional Correlation Receiver (CCR)*. We further evaluate the probability of success in detecting the active users as a function of the SNR of the received signal.

Section II presents the basic principles of quantum and the Grover's algorithm. Section III talks about the adaptation of Grover's algorithm for AUD purposes and some classical AUD receivers such as ML and CCR. Section IV presents the simulation setup and the obtained results. Finally, section V concludes the paper.

II. OVERVIEW ON QUANTUM COMPUTING

A. Quantum Principles

Thanks to their superposition property allowing to reduce the complexity, quantum algorithms have gained wide interest in the recent years. In particular, quantum Grover's algorithm needs only $\mathcal{O}(\sqrt{N})$ [12] [13] for searching a value in a data base of size N , while classical approach needs $\mathcal{O}(N)$.

Indeed, a superposition of states in quantum allows to have two different bits values simultaneously in a single state $|\psi\rangle$. Thus, a new type of bits, so-called *qubit* is introduced, whose notation $|\psi\rangle$ is written as follows:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad (1)$$

where α and β are normalized complex numbers which verify $\alpha^2 + \beta^2 = 1$. α^2 and β^2 are the probability of being in state $|0\rangle$ and $|1\rangle$ respectively. For example, in order to have 100% probability of $|0\rangle$, we should have $\alpha = 1$ and $\beta = 0$.

Even if α and β can both be complex values, the usual convention considers the normalized case where α is a real, with the equation as follows :

$$|\psi\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\varphi} \sin \frac{\theta}{2} |1\rangle \quad (2)$$

where $0 \leq \theta \leq \pi$ and $0 \leq \varphi \leq 2\pi$ [12]. This quantum state can be represented with the *Bloch Sphere* [12].

B. Grover's Algorithm

Grover Algorithm is the key quantum engine for searching a value in an unsorted database. It is based on two main parts; 1) *Oracle* and 2) *Diffuser* as shown on Fig. 1. The *Oracle* aims to mark the states which verify a given constraint (for example address that contains the desired value (δ) for a database, or the solution of a function), whereas the diffuser aims at amplifying the marked states by the use of inverse mean [11].

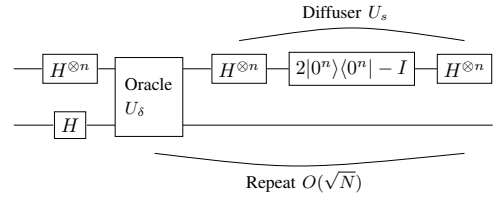


Fig. 1. Grover Scheme

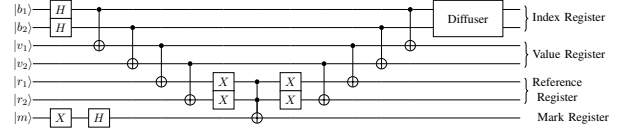


Fig. 2. General Grover Circuit with of 2 qubits

The function U_δ denotes the oracle action and can be modeled as :

$$U_\delta|x\rangle = \begin{cases} -|x\rangle & \text{if } f(x) = \delta \\ |x\rangle & \text{if } f(x) \neq \delta \end{cases} \quad (3)$$

If $f(x)$ corresponds to the desired value (δ), it marks $|x\rangle$ with (-1) . Otherwise, i.e. if $f(x) \neq \delta$, the state amplitude remains constant through the calculation. The oracle output feeds the diffuser, which amplifies the marked states. Eq.(4) models the diffuser functionality where $|s\rangle$ is the equally superposition of states that consists of a phase shifter ($2|0^n\rangle\langle 0^n|$) and the identity matrix I .

$$U_s = H^{\otimes n}(2|0^n\rangle\langle 0^n| - I)H^{\otimes n} = 2|s\rangle\langle s| - I \quad (4)$$

To do so, as illustrated in Fig. 2 for the two qubits case, Grover's Oracle [14] relies on four different registers. *Index register* contains the argument of the function f , and will store the solution at the end of the algorithm. The *value register* contains the results of the function applied to the *Index register* states. The *reference register* corresponds to the targeted value δ . Finally, the *mark register* provides the negative sign into the computation, to mark the valid states.

These operations amplify the searched states, but the expected solutions are only partially highlighted. Thus several iterations of the *Oracle* and *Diffuser* are needed. Authors in [15] have expressed the optimum number of iterations (L_{opt}) to find a specific solution in a given database. It depends on the number of valid solutions (S) and on the database size N , which is given by :

$$L_{opt} = \lceil \pi/4(\sqrt{N/S}) \rceil \quad (5)$$

III. GROVER'S CIRCUIT DESIGN FOR MULTI-USER DETECTION SCHEME

A. System Model

The main goal of this paper is to adapt the quantum Grover's algorithm for AUD purposes in a non-orthogonal multiple access (NOMA) wireless communication systems. In Code Domain NOMA systems, the transmitted data is spread with a

code containing SF chips, where SF is the spreading factor. Herein, we consider a network with N users communicating with the Base Station (BS). These users are mainly on sleep mode, and initiate the transmission only when needed. We focus on the initialization of the transmission, where the users send their code for identification, and nothing otherwise. We consider that each user i is assigned a unique code c_i . We model that a user is active by setting b_i to 1 and to 0 when it is idle. Since all the users are sharing the same medium, the resulting received signal is the summation of all the transmitted code from active users. We point out here that the transmission scheme is still classical and the only quantum part lies in the algorithm performed at the receiver side. For simplicity purposes, we consider that the channel is modeled as a perfect one with a gain $h = 1$, along with a normalized *Additive White Gaussian Noise* (AWGN) denoted by n . Hence, the received signal \tilde{y} can be given by the following equation:

$$\tilde{y} = \sum_{i=0}^N b_i c_i + n \quad (6)$$

where N is the number of users, i is the user's index, $b_i \in \{0, 1\}$ is the activity status of user i , c_i is the corresponding codeword and n is the Gaussian noise following $\mathcal{N}(0, \sigma^2)$. For comparison purposes, two families of codes will be retained. The first one is formed by the bipolar codes, where the component of each code c_i belongs to the set $\{-1, 1\}$. The second family is formed by unipolar codes, where the components of each user's code belong to the set $\{0, 1\}$.

Given the received signal and the set of users codes, the problem that the receiver wants to solve is to detect the set of active users, i.e. the set users verifying $b_i = 1$.

B. Conventional Correlation Receiver

The *Conventional Correlation Receiver* (CCR) is the simplest receiver and is widely used for AUD in wireless networks [16]. This receiver correlates the received signal with the targeted signature code's sequence. The CCR detector can be shortly described as follows:

- The sampled received $\tilde{y} = [y_1, y_2, \dots, y_{SF}]$ is multiplied with code $c_i = [c_{i1}, c_{i2}, \dots, c_{iSF}]$ of the desired user i .
- The result of the product is integrated over all slots (SF). The output value corresponds to the decision variable.
- At the end, the decision variable is compared with a predefined threshold T . Hence, a user is considered active if the decision variable value is greater than T .

The operational correlation receiver is written as follows:

$$b_i = \begin{cases} 1 & \text{if } \sum_{j=1}^{SF} \tilde{y}_j \cdot c_{ij} \geq T \\ 0 & \text{if } \sum_{j=1}^{SF} \tilde{y}_j \cdot c_{ij} < T \end{cases} \quad (7)$$

where b_i reflects the activity status of user i . The CCR has to be done N times to retrieve the complete active user set.

C. Maximum Likelihood (ML)

The ML receiver is the optimal solution for AUD [17]. This detector identifies the most likely active users set b ,

given the the received sequence. For an AWGN channel, the maximum likelihood is obtained by searching the active user set that minimizes the distance between its contribution and the received signal. With \tilde{y} and c_i the received sequence and the set of user's signatures respectively, the ML receiver can be given as follows, in an AWGN channel, and equiprobable activity :

$$\operatorname{argmin}_b \left\| \tilde{y} - \sum_{i=1}^N b_i c_i \right\|^2 \quad (8)$$

where i is the index of a specific user among the N possible ones. The ML solution suffers from a high computation complexity $\mathcal{O}(2^N)$, as it is based on an exhaustive search over all the existing possibilities. Thus, the high complexity of the ML detector makes it intractable with classical processors when the number of users increases.

D. Proposed Quantum Algorithm

Fortunately, quantum computing is quickly evolving and is a promising solution to mitigate the complexity limitations of the ML receivers. Our proposed algorithm is based on Grover's one. Each qubit of the index register corresponds to the activity status of one user. They are all initialized with the superposition state $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$. We then apply quantum gates customized to reflect the system behavior, so that to compute all the possible signal signatures (one for each user activity set). The value register then contains these signatures in a superposed way. Finally, the received signal is fed into the reference register, and compared to the value register's states. The identical state is marked with a negative sign, before amplitude amplification in the diffuser part. This is performed several times (L_{opt}), to reach the best accuracy.

However, before being fed into the reference register, the received signal has to be pre-processed. Indeed, Grover's circuit can only be fed with binary data while our received signal contains real components. To overcome this problem, one simple way is to transform \tilde{y} to the closest integer values and then to convert it to binary numbers [18]. In compliance with the involved families of codes (i.e. unipolar and bipolar), the retained integer part of \tilde{y} is denoted by \tilde{y}_p and illustrated as follows:

$$\tilde{y}_p = \begin{cases} \min(\max(0, \operatorname{round}(\tilde{y})), 2^m - 1) & c \in \{0, 1\} \\ \min(\max(\operatorname{round}(\tilde{y}), -2^m - 1), 2^m - 1) & c \in \{-1, 1\} \end{cases} \quad (9)$$

where m is the number of used bits to represent each \tilde{y}_p 's component, and defines the scale of the processed values.

Note that all these previous operations are still performed with classical devices and the quantum computation starts with the Grover's algorithm. Then, the Grover's algorithm is performed for the corresponding L_{opt} iterations in order to detect the set of active users.

IV. SIMULATION SETUP AND RESULTS

A. System Example 1 : Unipolar Code

In order to obtain the performances of our proposed algorithm, we have run simulations with the dedicated Python

library *Qiskit*. As the quantum behavior is evaluated with a classical circuit, the performance evaluation remains resource consuming. Thus, without loss of generality, we are using reduced networks configurations to validate our approach. We have selected a non-orthogonal code family with $SF = 4$, and $N = 5$, whose codewords are: $c_1 = [0, 1, 0, 1]$, $c_2 = [1, 0, 0, 1]$, $c_3 = [0, 1, 1, 0]$, $c_4 = [1, 1, 1, 0]$ and $c_5 = [0, 0, 1, 1]$. We point out here that we selected a codes family which handle the maximum number of users, while keeping a unique signature for each active user set. With this constraint, we ensure that, in a noiseless channel, the error probability of AUD is null.

Based on these codes, we can compute all the possible sequences \tilde{y} that can be received in a noiseless condition. The value observed in each slot depends on the users activity indicator as follows :

$$\tilde{y} = \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \tilde{y}_3 \\ \tilde{y}_4 \end{bmatrix} = \begin{bmatrix} b_2 + b_4 \\ b_1 + b_3 + b_4 \\ b_3 + b_4 + b_5 \\ b_1 + b_2 + b_5 \end{bmatrix} \quad (10)$$

In order to compute the ideal signatures within Grover algorithm, we have to implement an adder with quantum gates [19]. Fig. 3 shows the corresponding circuit when adding qubit $|b_1\rangle$ and $|b_2\rangle$, using ancilla qubits $|v_0\rangle$ and $|v_1\rangle$, providing the sum $S = |b_1 \oplus b_2\rangle$ and the carry $C = |b_1 \cdot b_2\rangle$.

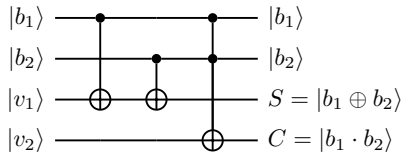


Fig. 3. Quantum Adder $b_1 \oplus b_2$

The user's code is added to the signal when active, and nothing is done otherwise.

B. System Example 2 : Bipolar Codes

The same principle applies for the bipolar codes. The difference is that the codes values belong to the set $\{-1, 1\}$. We consider in this paper a family with $SF = 4$, and $N = 5$, with the codewords $c_1 = [-1, -1, 1, 1]$, $c_2 = [1, 1, 1, -1]$, $c_3 = [-1, -1, -1, -1]$, $c_4 = [1, -1, -1, 1]$ and $c_5 = [1, -1, 1, -1]$.

$$\tilde{y} = \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \tilde{y}_3 \\ \tilde{y}_4 \end{bmatrix} = \begin{bmatrix} -b_1 + b_2 - b_3 + b_4 + b_5 \\ -b_1 + b_2 - b_3 - b_4 - b_5 \\ b_1 + b_2 - b_3 - b_4 + b_5 \\ b_1 - b_2 - b_3 + b_4 - b_5 \end{bmatrix} \quad (11)$$

Contrarily to unipolar codes, bipolar codes lead to negative components. We thus use the binary *two's complement* representation. The *positive components* are directly added with the previous *quantum adder*. Conversely, for the negative contributions, the *two's complement* of the value is computed before the use of the *quantum adder*.

C. Results

We have evaluated and reported on Fig. 4 the network performances for various configurations, as a function of the

SNR of the received signal. In this work, a success is defined as recovering the exact user activity set. Thus, a false alarm, or a misdetection within the set makes the set considered as erroneous.

Fig. 4 illustrates the variation of the average *Probability of Success* (P_s) in detecting the active set of users, as a function of SNR, for the classical receivers as reference (ML, CCR) and our proposed quantum Grover based algorithm, while using unipolar codes (*uni*), as well as bipolar codes (*bip*). We note here that P_s is averaged over 4000 independent realizations of noise.

First of all, we can verify that the performance improves when the SNR increases, whatever the receiver and the used code family. Nevertheless, we can observe that the performance improvement is faster for the quantum algorithm than for the classical algorithms. This is due to the research approach used in our quantum algorithm. Indeed, the oracle searches for states verifying exactly $f(x) = \delta$. However, in case of high noise, the signature is often modified and becomes no longer valid. In this case, the oracle fails to find a solution for $f(x) = \delta$. Grover's algorithm thus leads to the random and equiprobable selection of active users among the set of 2^N possible solutions. This leads to a low probability of success. To solve the problem of search accuracy in quantum in the low SNR regime. Conversely, for low noise, the pre-processing algorithm generally allows to retrieve the emitted sequence, by cancelling the noise contribution. The increase of the SNR allows be in the second situation much more frequently, and thus to improve significantly the performances.

Moreover, by comparing the performance of unipolar and bipolar codes, we can notice that the detection accuracy of the classical algorithms is better when using bipolar codes. This is due to the fact that the Euclidean distance is higher between the components of the bipolar codes, making the signature detection more reliable for the same noise level. However, the performance gap is strongly reduced by using the quantum algorithm. Indeed, as explained above, the detector is perturbed when it receives an invalid sequence. However, the probability of error is the same whatever the difference between the received sequence and the transmitted sequence. The advantage concerning the Euclidean distance for bipolar codes with classical receivers does not apply in this case.

Besides, while comparing the performances of the different receivers, we can observe that our quantum algorithm is less performing than the CCR at low SNR. On the contrary, for high SNR, it largely outperforms it and converges quickly to the optimal ML, for the 2 families of codes. Meanwhile, the CCR reaches its lower ceiling because of the interference effect. Indeed, as the codes are not orthogonal, the CCR makes errors because of the residual interference on the decision variable of each user. On the contrary, the ML, whether it is a classical or a quantum version, manages to get rid of this interference by jointly considering all the users.

Finally, our quantum algorithm is characterized by a lower complexity, since it requires $\mathcal{O}(\sqrt{2^N})$ iterations, where the classical ML requires $\mathcal{O}(2^N)$, (and N iterations for the CRR).

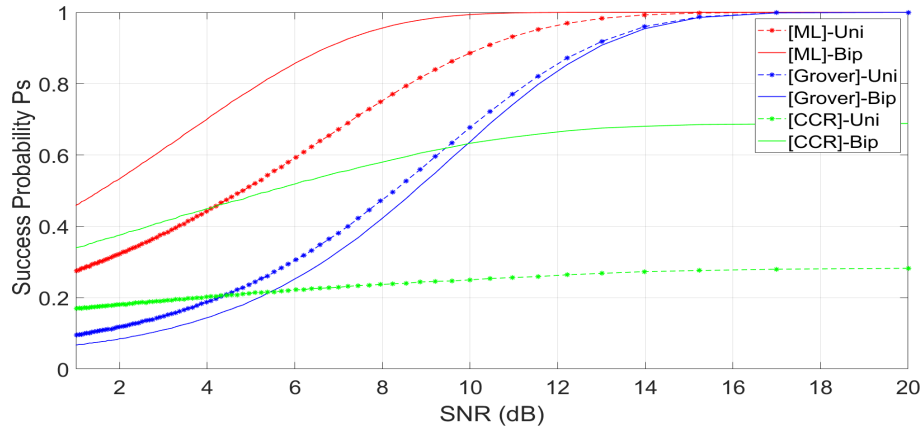


Fig. 4. Probability of Success as a function of the noise variance (σ^2)

This study thus proves that this quantum ML can significantly reduce the complexity compared to classical ML, while keeping the same detection performance if the SNR is sufficient. However, as we have implemented the simplest quantum version of the ML, the performances can be further improved, to comply with the optimal receiver even for lower SNR. This can be done by improving the oracle, and looking for the states that minimize the gap, i.e. verifying $\min ||f(x) - \delta||$ (instead of $f(x) = \delta$ currently), and/or improving the preprocessing. It is also possible to extend this work to any receivers based on ML.

V. CONCLUSION

In this paper, the quantum Grover's algorithm is adapted for AUD purposes for a non-orthogonal CD NOMA communication schemes. This adapted quantum algorithm is fed by a noisy received signal, which has been reshaped in order to be in compliance with the Grover's algorithm constraints. Our adapted Grover's algorithm is compared with the optimal ML and the CCR receivers for both CDMA unipolar and bipolar codes, where the probability of success is evaluated as a function of SNR. The results show that our adapted quantum algorithm is very promising, especially when the SNR is relatively high. Indeed, it reached the same performances than the optimal ML, with a reduced computation complexity. Thus, this works opens the way to quantum processing for AUD in massive communication networks.

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