



**HAL**  
open science

## Automated detection of contextuality proofs with intermediate numbers of observables

Henri de Boutray, Frédéric Holweck, Alain Giorgetti, Pierre-Alain Masson

### ► To cite this version:

Henri de Boutray, Frédéric Holweck, Alain Giorgetti, Pierre-Alain Masson. Automated detection of contextuality proofs with intermediate numbers of observables. 18th International Conference on Quantum Physics and Logic (QPL 2021), Jun 2021, Gdańsk, Poland. . hal-03814543

**HAL Id: hal-03814543**

**<https://hal.science/hal-03814543>**

Submitted on 19 Oct 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Henri de Boutray, Frédéric Holweck, Alain Giorgetti and Pierre-Alain Masson

## Contextual geometries

$$\begin{array}{c} X \otimes I - I \otimes X - X \otimes X \\ | \qquad | \qquad || \\ I \otimes Y - Y \otimes I - Y \otimes Y \\ | \qquad | \qquad || \\ X \otimes Y - Y \otimes X - Z \otimes Z \end{array}$$

Finite geometries  $G = (\mathcal{O}, L)$  parametrized by observables  $O \in \mathcal{O}$  with  $\forall l \in L, l \neq \emptyset, l \subset \mathcal{O}$  such that  $\forall l \in L, (\forall \{O_1, O_2\} \subset l, O_1 O_2 = O_2 O_1) \wedge (\prod_{O \in l} O = \pm I)$ .

They are contextual if  $\nexists f : \mathcal{O} \rightarrow \{-1, 1\} / \forall l \in L, \prod_{O \in l} f(O)I = \prod_{O \in l} O$ . (1)

## A linear problem

- $(\{-1, 1\}, \times) \rightarrow (\mathbb{Z}_2, +)$
- Let  $A$  be the incidence matrix of  $G$ .
  - Let  $e$  be the evaluation vector with  $|L|$  entries such that, for the entry  $e_l$  corresponding to  $l \in L, e_l I = \prod_{O \in l} O$
- Eq.(1) can be rewritten as  $\nexists x \in (\mathbb{Z}_2)^n / Ax = e$
- Here  $x$  is in relation with  $f$  as follows: the entry corresponding to  $l \in L$  would be  $x_l = \prod_{O \in l} f(O)$ .

## Symplectic space

A space encoding the Pauli group

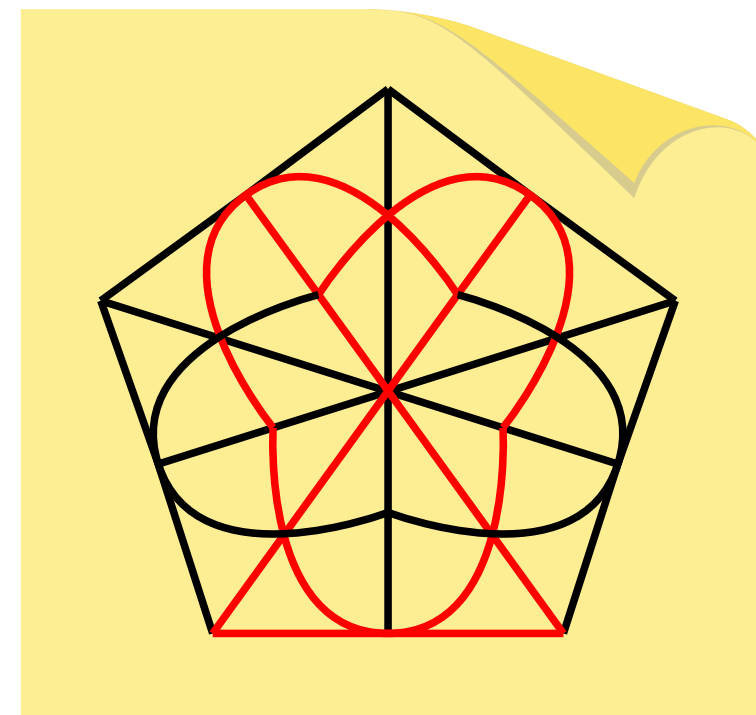
$$(0, 1, 1, 1) \leftrightarrow X \otimes Y$$

$$((0, 0) \leftrightarrow I, (0, 1) \leftrightarrow X, (1, 0) \leftrightarrow Z, (1, 1) \leftrightarrow Y)$$

With an inner product encoding commutativity

$$\left. \begin{array}{l} p_1 \leftrightarrow O_1 \\ p_2 \leftrightarrow O_2 \end{array} \right\} \Rightarrow (\langle p_1 | p_2 \rangle = 0 \Leftrightarrow O_1 O_2 = O_2 O_1)$$

We chose the lines of the geometry  $G$  to be subspaces of the symplectic space.



## Results

A Magma program [**dB21**] generates all elements of five families of geometries and automatically establishes their contextuality:

Geometries	$n = 2$	$n = 3$	$n = 4$	$n = 5$
Lines	C(1)	C(1)	C(1)	C(1)
Generators	C(1)	<b>N(1)</b>	<b>N(1)</b>	<b>N(1)</b>
Hyperbolics	C(10)	<b>C(36)</b>	<b>C(136)</b>	<b>C(528)</b>
Elliptics	N/A (6)	<b>C(28)</b>	<b>C(120)</b>	<b>C(496)</b>
Perpsets	N(15)	<b>N(63)</b>	<b>N(255)</b>	<b>N(1023)</b>

$n$ : Number of qubits of the system  
N: Non-contextual  
C: Contextual  
( $k$ ): There are  $k$  instances in this family  
N/A: Not applicable  
**bold**: New results

## References

- [**dB21**] H. de Boutray Magma-contextuality: Automated detection of contextuality proofs with intermediate numbers of observables. *QuantCert*, <https://quantcert.github.io/Magma-contextuality/>.
- [**dHGM21**] H. de Boutray, F. Holweck, A. Giorgetti, and P.-A. Masson, Automated synthesis of contextuality proofs from subspaces of symplectic polar spaces. *arXiv:2105.13798 [math-ph, physics:quant-ph]*, May 2021.
- [Hol21] F. Holweck. Testing Quantum Contextuality of Binary Symplectic Polar Spaces on a Noisy Intermediate Scale Quantum Computer. *arXiv:2101.03812 [math-ph, physics:quant-ph]*, January 2021.

Contact: [henri.de\\_boutray@univ-fcomte.fr](mailto:henri.de_boutray@univ-fcomte.fr)