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# Stochastic Maintenance Optimization of Complex Industrial Systems under Uncertainty of Repair Time and Resources

Selma Khebbache  
IRT SystemX  
Paris Saclay, France  
selma.khebbache@irt-systemx.fr

Makhlouf Hadji  
IRT SystemX  
Paris Saclay, France  
makhlouf.hadji@irt-systemx.fr

Michel Batteux  
IRT SystemX  
Paris Saclay, France  
michel.batteux@irt-systemx.fr

**Abstract**—This paper focuses on maintenance optimization strategies in complex manufacturing systems under uncertainty of available resources and tasks duration constraints. We propose a deterministic formulation of the maintenance optimization problem and provide a chance constraints modeling to formulate uncertainty constraints. Our stochastic mathematical model is reformulated using approximations providing deterministic equivalent formulations that we solve in negligible times to reach near global solutions. Simulation results are provided to illustrate the efficiency of our mathematical models when highlighting the convergence of the chance constraints formulation to the deterministic form for given confidence levels.

**Index Terms**—Maintenance, Stochastic optimization, Complex systems, Chance constraints

## I. INTRODUCTION

During the past two decades, maintenance in complex industrial systems has drawn the interest of academic and industrial communities as its implication covers various domains such as energy systems, medical equipment, etc. These industries are growing rapidly and are changing considerably—thanks to technology transformation—to be able to consider the data lakes generated within their complex systems (industry 4.0).

Moreover, with the advent of industry 4.0, manufacturing systems are growing in scale and complexity, and the demand for high reliability is increasing. This necessitates new efficient planning strategies since various dependencies (economic, structural, stochastic, and/or resources) arise within such complex systems.

To achieve a high production level in the aforementioned manufacturing systems, when guaranteeing a set of constraints such as limited number of resources, we need cost efficient constrained maintenance planifications to optimize the different maintenance tasks. In addition, and to exploit efficiently the collected data from these different manufacturing complex systems, it is important to propose new approaches based on the relevant information that can be retrieved in these data lakes. Hence, we will investigate Data Driven approaches able to derive cost efficient maintenance optimizations according to the harvested data.

However in many complex industrial systems, the number of necessary resources and the maintenance tasks duration are influenced by uncertain environmental factors and may not be known exactly before task execution. Consequently, the available resources and maintenance duration could be stochastic. Therefore, in this paper, we study the stochastic maintenance planification and optimization problem when considering various constraints such as limited available resources, finite horizon time and maintenance duration.

In this paper, we propose to mathematically formulate the problem of maintenance optimization under repair time uncertainty using stochastic programming. Hence, our contributions are summarized as follows:

- We provide a deterministic mathematical formulation of the maintenance optimization problem of complex industrial systems under a set of given constraints (limited resources, finite horizon time, tasks duration, etc.)
- To consider real life scenarios in manufacturing systems, we propose a stochastic optimization based on chance constraints to represent the uncertainty of maintenance repair time and resources availability.
- We provide approximations schemes of the chance constrained formulation to efficiently address the deterministic equivalent formulations to attend near optimal solutions of our constrained maintenance optimization problem.

The remainder of this paper is organized as follows: Section II is dedicated to summarize some recent related works in terms of deterministic and stochastic maintenance optimizations. In Section III, we provide the deterministic form of our mathematical maintenance optimization under a set of detailed constraints. Then, we discuss the stochastic model using chance constraints and provide efficient approximations to attend near optimal solutions. Simulation results are provided in Section IV, and we conclude the paper in Section V.

## II. RELATED WORK

In this section, we review recent works to situate our contributions in this paper. We propose to review deterministic

and non-deterministic maintenance works and we will emphasize maintenance optimization solution methods in terms of deterministic and stochastic circumstances.

#### A. Maintenance Optimization in Complex Manufacturing Systems

1) *Deterministic maintenance problems:* Maintenance optimization in complex manufacturing systems can also be combined with different fields allowing to take a global decision. One mainstream topic is to simultaneously consider maintenance and spare part ordering. Authors in [1] and [5] studied corrective maintenance and spare part inventory strategies using prognostic information to minimize failure risk. In addition, considering maintenance and production jointly is another important research topic as the maintenance impacts the productivity in the considered manufacturing systems. Authors of [6] proposed to investigate the maintenance in a series system to optimize the production schedule through a preventive maintenance plan.

2) *Stochastic maintenance problems :* Stochastic maintenance optimization in complex manufacturing systems are often presented as optimizations taking into account the stochastic quality, duration, and number of repair person. For instance, authors of [8] and [10] considered random variables to model the stochastic quality and duration parameters. Authors of [12] considered a multi-component maintenance and repair person assignment problem with stochastic maintenance duration.

#### B. Optimization Methods for Maintenance Problems

Difference approaches for maintenance problems in complex systems have been proposed, such as analytic, simulation, optimization (see references [13]-[14], for instance). In the following, we succinctly cite references addressing optimization techniques for maintenance problems. We clearly indicate our contributions compared to these approaches.

1) *Deterministic optimization methods.:* Authors of [15] considered a continuous production system with unexpected quality deterioration, with the aim at reducing the occurred cost with a quality constraint. A non-linear programming model was formulated and solved. Authors of [16] proposed an optimal preventive maintenance policy in order to get operational parameters for a production line. A non-linear model and a heuristic are provided to minimize the total cost. Authors of [2] formulated a MILP model for the considered predictive maintenance optimization problem, with the purpose to provide optimal maintenance decisions. However, in order to cope with stochastic maintenance optimization, we need to investigate chance constraints approaches in this paper.

2) *Stochastic optimization methods.:* Stochastic optimization is widely studied and proposed in relevant references such as (Birge [17], [18] and Prékopa [19]). Many stochastic optimization methods such as chance constraints are also used to efficiently model and optimize stochastic problems. In fact, in [11], authors addressed stochastic maintenance optimization based on chance constraints. Authors of [10] and [9] used the scenarios of stochastic

factors for stochastic model approximation.

In our work, we aim at guaranteeing with a confidence level that although resources availability and tasks duration are stochastic, the complex system reduce total costs enough without violating the whole described constraints. We, therefore, concentrate on providing chance constraints using linear approximations, facilitating the problem's resolution.

### III. SYSTEM MODEL AND PROBLEM DESCRIPTION

In the following, we consider (see Figure 1: an example of a complex system with 10 components) a complex manufacturing system composed by various series and parallel components. Our main purpose consists to optimize the maintenance costs of this system when meeting a certain number of constraints (resource limitation and availability, repair time, etc.).

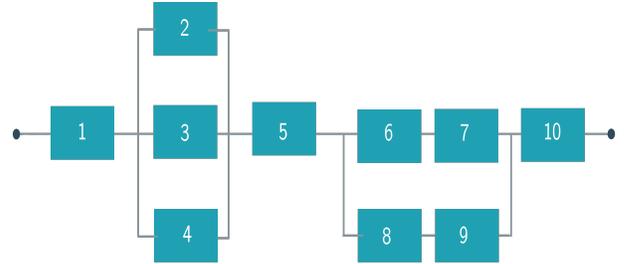


Fig. 1: System model illustrating an example of complex manufacturing system

#### A. Problem's complexity

We address in our work the maintenance optimization problem in complex manufacturing systems. It is already described in [4] that our mathematical formulations are using a non linear objective function that will be detailed in the sequel. Moreover, it is known that addressing non linear optimizations belongs to NP-Complete class. Hence, the two variants (deterministic and stochastic) of the considered problem are also NP-Complete problems.

#### B. Deterministic Formulation

We consider a complex industrial system with  $n$  components to be maintained according to a given strategy. Let  $\mathcal{I} = \{o_1, o_2, \dots, o_n\}$  be the set of these system's components.

To each component  $o_i$ , we associate a number of necessary repairman noted by  $r_i$ , and a necessary repair time  $d_i$ . In this section, we suppose  $d_i \forall i = 1, \dots, n$  deterministic (an input to our mathematical modeling). Next sections are dedicated to uncertain repair time and total available resources through stochastic programming.

Let  $R$  be the total number (we consider  $R$  deterministic first, and stochastic in next sections) of available repairmen in a period  $T$  of maintenance horizon. Let  $\mathcal{T}$  be the set of discrete indices of  $T$ :  $\mathcal{T} = \{1, 2, \dots, T\}$ . Furthermore, for each  $j \in \mathcal{T}$  let  $R_j$  the available resources allocated to

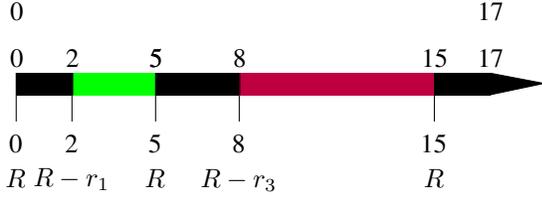


Fig. 2: Maintenance horizon time and resources releasing

the system at time instant  $j$ . We consider a cost function  $F(x)$  representing the total cost of maintenance of a complex industrial system with  $n$  components.  $x^t = (x_1, x_2, \dots, x_n)$  is a vector indicating the maintenance time of each component or object  $i$  of the considered system.

For instance, Figure 2 provides an example on the usage and release of resources according to different maintenance tasks: in black and red colors, maintenance occurs (resources occupied), in yellow color, resources are released.

We consider binary variables  $m_{ij}$  defined by (for all component  $i$  and time step  $j$ ):  $m_{ij} = 1$  if the maintenance of object  $i$  is planned at time  $j$ , and  $m_{ij} = 0$  otherwise.

We use a matrix  $\mathcal{M}$  to represent the indicated binary variables. The rows of  $\mathcal{M}$  represent the different objects or components of our industrial system, and the columns represent the different maintenance time slots in a given horizon time. Thus, if there exists a solution to  $j = \arg \min(F(x))$  then  $m_{ij} = 1$ .

$$\mathcal{M} = \begin{matrix} & \begin{matrix} 1 & 2 & \dots & T \end{matrix} \\ \begin{matrix} o_1 \\ o_2 \\ \dots \\ o_n \end{matrix} & \begin{pmatrix} m_{11} & m_{12} & \dots & m_{1T} \\ m_{21} & m_{22} & \dots & m_{2T} \\ \dots & \dots & \dots & \dots \\ m_{n1} & m_{n2} & \dots & m_{nT} \end{pmatrix} \end{matrix}$$

Our optimization is under a set of linear constraints that will be described below:

- **Limited horizon time constraints:** Maintenance activities have predefined periods and must fit in a given planning horizon. All maintenance actions must not, therefore, exceed the fixed time window and should be performed accordingly. These constraints are given by inequalities (1) which ensure that each maintenance action with duration  $d_i$  is carried out within the rolling horizon  $[1, T]$ .

$$jm_{ij} + (d_i - 1) \leq T \quad \forall i \in \mathcal{I} \quad \forall j \in \mathcal{T} \quad (1)$$

- **Constraints on the minimum number of maintenance actions per component:** In our system, all components must be replaced when maintenance actions are engaged. Constraints (2) and (3) guarantee that each component  $i$

is maintained at least one time in the allocated horizon time  $[1, T]$ :

$$\sum_{j=1}^T m_{ij} \geq 1 \quad \forall i \in \mathcal{I} \quad (2)$$

$$m_{ij'} \leq 1 - m_{ij} \quad \forall j' > j \in \mathcal{T} \quad \forall i \in \mathcal{I} \quad (3)$$

- **Limited number of resources:** A maintenance action requires a set of resources. To successfully schedule a maintenance activity, a certain number of technicians is expected. As such, Constraints (4) warrant that the number of allocated repairmen for all maintenance actions, initiated at a given period  $j$ , does not exceed the  $R_j$  available resources allocated to our system.

$$\sum_{i=1}^n m_{ij} r_i \leq R_j \quad \forall j \in \mathcal{T} \quad (4)$$

- **Resources release constraints:** (see Figure 2) When technicians accomplish one or several maintenance tasks, they are released and made available. Constraints (5) are formulated to express that used resources are restored at the end of each maintenance action. Hence, several PM (Preventive Maintenance) actions can be executed simultaneously provided that enough repairmen are available in the system. At the end of each maintenance cycle, the system is at its full capacity and all resources are available.

$$m_{ij} r_i \leq R_j - \sum_{\substack{k=1 \\ k \neq i}}^n \sum_{\substack{j' < j \\ j' + d_k > j}}^T m_{kj'} r_k \quad \forall i \in \mathcal{I} \quad \forall j \in \mathcal{T} \quad (5)$$

- **Upper bounds constraints:** To clarify the relationships between the decision variables  $x$  and  $m$ , we provide Constraints (6) illustrating the upper bound of  $x$  using  $m$  ( $\epsilon$  is a negligible positive value):

$$x_i \leq m_{ij} T + \epsilon \quad \forall i \in \mathcal{I} \quad \forall j \in \mathcal{T} \quad \epsilon > 0 \quad (6)$$

Furthermore, the decision variables of the model must be accurately defined and abide by the various considerations of the considered problem. Therefore, Constraints (7) and (8) describe the nature and the ranges of these decision variables.

$$x_i > 0 \quad \forall i \in \mathcal{I} \quad (7)$$

$$m_{ij} \in \{0, 1\} \quad \forall i \in \mathcal{I} \quad \forall j \in \mathcal{T} \quad (8)$$

As it is already described in our previous work (see reference [4] for more details), we assume for our complex system, the degradation of each component is following a *2-parameter Weibull distribution*. For each component  $i$ , the shape and scale parameters are  $\alpha_i, \beta_i \in \mathbb{R}_+$  where  $\alpha_i > 0$ ,  $\beta_i \geq 1$  for all  $i \in \mathcal{I}$ . We can thus analytically determine the expression of the expected cost over a given time period. This expression is derived from the failure rate  $\lambda_i(t)$  of component  $i \in \mathcal{I}$  that is given by:

$$\lambda_i(t) = \frac{\beta_i}{\alpha_i} \left( \frac{t}{\alpha_i} \right)^{\beta_i - 1} \quad (9)$$

$$\text{cost}(x_i) = \frac{C_i^{\text{prev}} + C_i^{\text{correc}} \left(\frac{x_i}{\alpha_i}\right)^{\beta_i}}{x_i}, \quad x_i > 0 \quad (10)$$

where  $C_i^{\text{prev}}$  (respect.  $C_i^{\text{correc}}$ ) is the preventive maintenance (respect. corrective maintenance) cost of component  $i$ .

Using the aforementioned non-linear objective function (10) under the provided valid inequalities, our optimization is considered as a Mixed Non Linear Programming (MNLPP) approach. The former is given by the following formulation:

$$(DET) : \begin{cases} \min_x C(x) = \sum_{i=1}^n \text{cost}(x_i) \\ S.T. : \\ jm_{ij} + (d_i - 1) \leq T, \forall i \in \mathcal{I}, \forall j \in \mathcal{T} \\ \sum_{j=1}^T m_{ij} \geq 1, \forall i \in \mathcal{I} \\ \sum_{i=1}^n m_{ij} r_i \leq R_j, \forall j \in \mathcal{T} \\ m_{ij} r_i \leq R_j - \sum_{k=1; k \neq i}^n \sum_{j' < j, j' + d_k > j}^T m_{kj'} r_k; \\ \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{T} \\ m_{ij'} \leq 1 - m_{ij}, \forall i \in \mathcal{I}; \forall j' > j \in \mathcal{T} \\ x_i \leq m_{ij} T + \epsilon, \forall i \in \mathcal{I} \quad \forall j \in \mathcal{T}, \epsilon > 0 \\ x_i > 0, \forall i \in \mathcal{I} \\ m_{ij} \in \{0, 1\}, \forall i \in \mathcal{I}, \forall j \in \mathcal{T} \end{cases}$$

Our model (DET) is a deterministic optimization with a non-linear objective function under linear constraints. We propose to use GEKKO solver [7] to obtain a solution  $x$  satisfying the whole described constraints.

When considering uncertainty of total available resources (noted by  $R$ ) and maintenance tasks duration (noted by  $d$ ) then we propose stochastic optimization approaches to cope with this problem.

### C. Stochastic Optimization

As indicated in the previous section, the repair time for a maintenance operation and the necessary available resources can be unknown in advance due to different factors. Hence, we propose to consider these parameters as random variables with a Gaussian distribution that will be detailed in the sequel. Then, our mathematical formulation which consists to optimize (10) under the constraints (1) to (8) is a non-linear stochastic optimization problem to be solved.

Before investigating stochastic modeling and addressing different approaches, and for sake of clarity, we propose to define chance constraints programming and how to use them efficiently to cope with stochastic optimization in a non-linear constrained program.

**Definition III.1.** We assume the underlying problem (P1) with an objective function that does not contain a random variable:

$$\min h(x)$$

subject to:

$$(P1) : \begin{cases} \mathbb{P}(g_1(x, \zeta) \geq 0, g_2(x, \zeta) \geq 0, \dots, g_r(x, \zeta) \geq 0) \geq p \\ x \in D \end{cases}$$

where  $p$  is a prescribed probability, and  $\zeta$  a random variable.

Problem (P1) is called chance constraints stochastic programming problem. The probability  $p$  may reflect the **reliability** of the system and ensures that the state of this system remains within a subset of all possible states, where its functioning is undisturbed by major failures.

In our stochastic approach, we suppose the repair time of each component of our complex system, is unknown and can be represented by a random variable. Moreover, and W.L.O.G. we suppose that:

- Each random variable  $d_i$  has a Gaussian probabilistic distribution with two parameters  $\mu_i$  (the mean value) and  $\sigma_i^2$  (the variance value):  $\forall o_i \in O, d_i \sim \mathcal{N}(\mu_i(d_i), \sigma_i^2(d_i))$
- At each time step  $j$ , we assume  $R_j$  as a random variable following a Gaussian distribution with two parameters:  $\forall j \in \mathcal{T}, R_j \sim \mathcal{N}(\mu_j(R_j), \sigma_j^2(R_j))$

We now introduce the chance constraints in our mathematical formulation to cope with the uncertainty of the repair time and resources of each considered component or object of the complex system. We propose the following mathematical stochastic formulation for the chance constraints that depend on the random variable  $d_i$  and  $R_j$ .

$$\mathbb{P}(jm_{ij} + (d_i - 1) \leq T) \geq \alpha_i, \forall i \in \mathcal{I}, \forall j \in \mathcal{T} \quad (11)$$

$$\mathbb{P}\left(\sum_{i=1}^n m_{ij} r_i \leq R_j\right) \geq \beta_i, \forall j \in \mathcal{T} \quad (12)$$

$$\mathbb{P}\left(m_{ij} r_i \leq R_j - \sum_{k=1, k \neq i}^n \sum_{\substack{j' < j \\ j' + d_k > j}}^T m_{kj'} r_k\right) \geq \gamma_i \quad (13)$$

Using these chance constraints ((11), (12)) and (III-D) in our formulation, we obtain the following non-linear stochastic problem:

$$(PSto) : \begin{cases} \min_x C(x) = \sum_{i=1}^n \text{cost}(x_i) \\ S.T. : \\ \mathbb{P}(jm_{ij} + (d_i - 1) \leq T) \geq \alpha_i, \forall i \in \mathcal{I} \\ \mathbb{P}\left(\sum_{i=1}^n m_{ij} r_i \leq R_j\right) \geq \beta_i, \forall j \in \mathcal{T} \\ \mathbb{P}\left(m_{ij} r_i \leq R_j - \sum_{k=0, k \neq i}^n \sum_{\substack{j' < j \\ j' + d_k > j}}^T m_{kj'} r_k\right) \geq \gamma_i \\ x_i \leq m_{ij} \times T, \forall i \in \mathcal{I} \\ \sum_{j=1}^T m_{ij} \geq 1 \quad \forall i \in \mathcal{I} \\ m_{ij'} \leq 1 - m_{ij}, \forall i \in \mathcal{I} \quad \forall j' > j \in \mathcal{T} \\ m_{ij} = \{0, 1\}^n, x_i > 0, \forall i \in \mathcal{I} \end{cases}$$

Note here  $\alpha_i$ ,  $\beta_i$  and  $\gamma_i$  are fixed probabilities of success (confidence levels) to satisfy chance constraints (given here by (11), (12) and III-D) and one usually chooses  $\alpha_i$ ,  $\beta_i$  and  $\gamma_i$  close to .95 or .99.

#### D. Approximations of Chance Constraints

(PSto) problem is generally non-convex, and a standard approach is to give safe approximations to the chance constraints, that is, to find (convex) functions so that (PSto) guarantees a solution. Hence, we propose the following approximations schemes of the aforementioned chance constraints.

**Proposition III.1.** *Chance constraint (11) has a Deterministic Equivalent Constraint (DEC) given by:*

$$jm_{ij} \leq T + 1 - \mu_i(d) + \phi^{-1}(\alpha_i)\sigma_i(d) \quad (14)$$

Where  $\phi(\cdot)$  is the distribution function of the standard normal distribution, and  $\alpha_i$  is the confidence probability/level.  $\phi(\cdot)$  is tabulated widely as standard normal tables.

*Proof.* W.L.O.G. we assume that,  $\forall i \in \mathcal{I} : \alpha_i = \alpha$ . We have:

$$\mathbb{P}(jm_{ij} + d_i - 1 \leq T) \geq \alpha \iff \mathbb{P}(d_i \leq T - jm_{ij} + 1) \geq \alpha \quad (15)$$

As  $d_i \sim \mathcal{N}(\mu_i, \sigma_i^2)$ , then we use the standardized variable

$$\frac{d_i - \mu_i}{\sqrt{\text{Var}(d_i)}} = \frac{d_i - \mu_i}{\sigma_i}$$

Then, (15) is equivalent to:

$$\mathbb{P}\left(\frac{d_i - \mu_i(d)}{\sigma_i(d)} \leq \frac{T - jm_{ij} + 1 - \mu_i(d)}{\sigma_i(d)}\right) \geq \alpha_i$$

Which is equivalent to:

$$\phi\left(\frac{T - jm_{ij} + 1 - \mu_i(d)}{\sigma_i(d)}\right) \geq \alpha_i$$

where  $\phi$  is the standard distribution of the Gaussian distribution.

Then, we obtain:

$$\frac{T - jm_{ij} + 1 - \mu_i(d)}{\sigma_i(d)} \geq \phi^{-1}(\alpha)$$

which leads to the expected result:

$$jm_{ij} \leq T + 1 - \mu_i(d) + \phi^{-1}(\alpha)\sigma_i(d)$$

■

**Proposition III.2.** *Chance constraint (12) has a Deterministic Equivalent Constraint (DEC) given by:*

$$\sum_{i=1}^n m_{i,j} \times r_i \leq \mu(R_j) + \phi^{-1}(1 - \beta)\sigma(R_j) \quad (16)$$

*Proof.* For sake of simplicity, we assume  $\forall i, \beta_i = \beta$ .

Inequality (12) is equivalent to:

$$\mathbb{P}\left(\frac{R_j - \mu(R_j)}{\sigma(R_j)} \leq \frac{\sum_{i=1}^n m_{i,j}r_i - \mu(R_j)}{\sigma(R_j)}\right) \leq 1 - \beta, \forall j = 1, \dots, T$$

Using the distribution function of a standard Gaussian variable noted by  $\phi$ , we obtain:

$$\phi\left(\frac{\sum_{i=1}^n m_{i,j}r_i - \mu(R_j)}{\sigma(R_j)}\right) \leq 1 - \beta \quad (17)$$

From (17), we deduce:

$$\sum_{i=1}^n m_{i,j} \times r_i \leq \mu(R_j) + \phi^{-1}(1 - \beta)\sigma(R_j)$$

■

**Proposition III.3.** *Chance constraint (III-D) has a Deterministic Equivalent Constraint (DEC) given by:*

$$m_{ij}r_i - \sum_{k=1, k \neq i}^n \sum_{\substack{j' < j \\ j' + d_k > j}}^T m_{kj'}r_k \leq \mu(R_j) + \phi^{-1}(1 - \gamma)\sigma(R_j) \quad (18)$$

*Proof.* For sake of simplicity, we assume  $\forall i, \gamma_i = \gamma$ .

Starting from

$$\mathbb{P}\left(m_{ij}r_i \leq R_j - \sum_{k=1, k \neq i}^n \sum_{\substack{j' < j \\ j' + d_k > j}}^T m_{kj'}r_k\right) \geq \gamma$$

We obtain:

$$\mathbb{P}\left(R_j \geq m_{ij}r_i - \sum_{k=1, k \neq i}^n \sum_{\substack{j' < j \\ j' + d_k > j}}^T m_{kj'}r_k\right) \geq \gamma$$

which is equivalent to:

$$\mathbb{P}\left(R_j \leq m_{ij}r_i - \sum_{k=1, k \neq i}^n \sum_{\substack{j' < j \\ j' + d_k > j}}^T m_{kj'}r_k\right) \leq (1 - \gamma)$$

Using the normal distribution properties, we obtain the following:

$$\phi\left(\frac{m_{ij}r_i - \sum_{k=1, k \neq i}^n \sum_{\substack{j' < j \\ j' + d_k > j}}^T m_{kj'}r_k - \mu(R_j)}{\sigma(R_j)}\right) \leq (1 - \gamma) \quad (19)$$

Then, from 19, we deduce the following DEC:

$$m_{ij}r_i - \sum_{k=1, k \neq i}^n \sum_{\substack{j' < j \\ j' + d_k > j}}^T m_{kj'}r_k \leq \mu(R_j) + \phi^{-1}(1 - \gamma)\sigma(R_j)$$

■

Using the results of the three previous propositions, will lead to construct a Deterministic Equivalent Program (DEP) of the stochastic program (PSto). This is provided by:

$$(PDET) : \begin{cases} \min_x C(x) = \sum_{i=1}^n \text{cost}(x_i) \\ S.T. : \\ jm_{ij} \leq T + 1 - \mu_i(d) + \phi^{-1}(\alpha_i)\sigma_i(d), \forall i \in \mathcal{I} \\ \sum_{i=1}^n m_{i,j} \times r_i \leq \mu(R_j) + \phi^{-1}(1 - \beta)\sigma(R_j); \\ \forall i \in \mathcal{I} \\ m_{ij}r_i - \sum_{k=1, k \neq i}^n \sum_{\substack{j' < j \\ j' + d_k > j}}^T m_{kj'}r_k \leq \mu(R_j) + \\ + \phi^{-1}(1 - \gamma)\sigma(R_j) \\ \sum_{x_i=1}^T m_{ix_i} \leq \theta\sigma_i + \mu_i, \forall i \in \mathcal{I} \\ x_i \leq m_{i,j} \times T, \forall i \in \mathcal{I} \\ \sum_{j=1}^T m_{ij} \geq 1 \quad \forall i \in \mathcal{I} \\ m_{ij'} \leq 1 - m_{ij}, \forall i \in \mathcal{I} \quad \forall j' > j \in \mathcal{T} \\ m_{ij} = \{0, 1\}^n, x_i > 0, \forall i \in \mathcal{I} \end{cases} \quad (20)$$

The obtained model (20) is a deterministic optimization with linearized constraints.

#### IV. NUMERICAL RESULTS

We assess and evaluate the performance of (DET) algorithm using GEKKO solver (see [7]) to solve the non linear mathematical model. We summarized in Table I the set of simulation parameters invoking the Weibull distribution's parameters  $s$  and  $t$ , and the preventive and corrective costs. Tasks duration and total number of available resources are also generated randomly in Table I. Note that these two last parameters are considered as random variables in our stochastic optimization.

TABLE I: Simulation parameters

Parameters	$s_i$	$t_i$	$C_i^{Prev}$	$C_i^{Correc}$	$r_i$	$d_i$	$R_j$
Values	[10, 40]	[1.25, 2.75]	[40, 150]	[140, 350]	[1, 10]	[2, 8]	[0, 20]

We assessed the performance of the deterministic algorithm for different horizon time values as illustrated in Table II. For a system with 5 components, one can remark the necessary convergence time of our non linear mathematical model to attend a solution in few seconds (2.89 seconds as the worst case of simulations in Table II).

In Table II), we remark the evolution of the total cost function ( $C(x)$ ) which is higher for small values of  $T$  ( $C(x) = 1126.24$ ) and reaches its minimum value for an average horizon time of  $T = 55$ . This costs is then increasing for higher values of  $T$ .

In the following we provide comparisons of the stochastic and deterministic approaches, for different values of confidence levels ( $\alpha, \beta$ , and  $\gamma$ ). Indeed, we provide three main values of these parameters in the set  $\{0, 0.5, 1\}$  to be used during the following analysis.

Given the following deterministic equivalent constraint of chance constraint (11):

$$jm_{ij} \leq T + 1 - \mu_i(d) + \phi^{-1}(\alpha_i)\sigma_i(d), \forall i \in \mathcal{I} \quad (21)$$

For  $\sigma_i(d) = 1$ , we discuss the following three cases:

- $\alpha = 0$ : Then we have  $\phi^{-1}(\alpha) = -\infty$ . Hence, (21) is indicating that no maintenance is possible with confidence levels very low or close to 0. Theoretically speaking, this is interpreted by the fact that the system will be infeasible for confidence levels close to 0.
- $\alpha = 0.5$ : Then we have  $\phi^{-1}(\alpha) = 0$ . Hence, (21) is converging totally to the original deterministic valid inequality provided in (1) for  $d_i$  values close to the average value  $\mu_i(d)$ .
- $\alpha = 1$ : Then we have  $\phi^{-1}(\alpha) = +\infty$ . Hence, (21) is indicating that maintenance operations are totally relaxed and can be done according to the availability of the repair man. Theoretically, this case indicates that for large values (close to 1) of the considered confidence level, then maintenance operations can occur without limitations in time.

For the next deterministic equivalent constraint of chance constraint (12):

$$\sum_{i=1}^n m_{i,j} \times r_i \leq \mu(R_j) + \phi^{-1}(1 - \beta)\sigma(R_j); \forall j \in \mathcal{T} \quad (22)$$

we propose to assess three cases as follows:

- $\beta = 0$ : Then we have  $\phi^{-1}(1 - \beta) = +\infty$ . Hence, (22) is indicating that for low values of confidence levels (close to zero), the resources available become theoretically large (infinity). This is due to the confidence we put in our theoretical model, as we are considering low confidence levels. Hence, our optimization in providing comfortable conditions in terms of available resources for maintenance operations.
- $\beta = 0.5$ : Then we have  $\phi^{-1}(1 - \beta) = 0$ . Hence, (22) is converging to the original and deterministic valid inequality provided by (4) for  $R_j$  close to the average value  $\mu(R_j)$ .
- $\beta = 1$ : Then we have  $\phi^{-1}(1 - \beta) = -\infty$ . Hence, (22) is indicating that maintenance could not occur and this due to lack of resources (repair man). This result comes from the high confidence level we used in our optimization causing the lack of available resources in our model.

The same analysis and deductions can be derived for different confidence levels values ( $\gamma$ ) concerning the following inequality:

$$m_{ij}r_i - \sum_{k=1, k \neq i}^n \sum_{\substack{j' < j \\ j' + d_k > j}}^T m_{kj'}r_k \leq \mu(R_j) + \phi^{-1}(1 - \gamma)\sigma(R_j) \quad (23)$$

#### V. CONCLUSION AND FUTURE WORK

In this paper, we discussed the constrained maintenance problem of a complex industrial system using a non linear objective function to be optimized. A deterministic mathematical formulation is provided for the scenarios in which resources and maintenance tasks duration are known

TABLE II: Deterministic algorithm performance for complex systems

		Planning Horizon						
		$T = 25$	$T = 35$	$T = 45$	$T = 55$	$T = 65$	$T = 75$	$T = 100$
$C(x)$		<b>1126.24</b>	653.28	626.50	<b>487.51</b>	494.34	814.06	652.95
(DET) Time (s)		0.628	1.178	1.085	1.829	<b>2.289</b>	2.292	3.177

in advance. However in many complex industrial systems, the number of necessary resources and the maintenance tasks duration are influenced by uncertain environmental factors and may not be known exactly before task execution. Consequently, we considered the available resources and maintenance duration as random variables following Gaussian distributions. Therefore, we proposed a stochastic maintenance optimization problem under various constraints. To cope with the stochastic optimization, we provided efficient approximations based on deterministic equivalent formulations leading to obtain cost efficient solutions of the stochastic variant of our maintenance optimization problem.

For future work, we will consider stochastic maintenance optimization for distributed multi-sites of a plenty of complex systems. Routing and logistic constraints will then be jointly optimized.

#### REFERENCES

- [1] De Jonge, B. Scarf, P. A.: A review on maintenance optimization. *European Journal of Operational Research* **285**(3), 805-824 (2020)
- [2] He, J., Anjos, F. M., Hadji, M., Khebbache, S.: Prognostic-based Maintenance Optimization in Complex Systems with Resource Limitation Constraints. In *Proceedings of the 11th International Conference on Operations Research and Enterprise Systems*, Feb, 3th-5th (2022).
- [3] Camci, F., Medjaher, K., Atamuradov, V., Berdinyazov, A.: Integrated maintenance and mission planning using remaining useful life information. *Engineering optimization* **51**(10), 1794-1809 (2019)
- [4] Hanini, M., Khebbache, S., Bouillaut, L., and Hadji, M.: "Dynamic and adaptive grouping maintenance strategies: New scalable optimization algorithms. *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, (2021)
- [5] Camci, F.: System maintenance scheduling with prognostics information using genetic algorithm. *IEEE Transactions on Reliability* **58**(3), 539-552 (2009)
- [6] Xiao, L., Song, S., Chen, X., David W. C.: Joint optimization of production scheduling and machine group preventive maintenance. *Reliability Engineering & System Safety* **146**, 68-78 (2016)
- [7] GEKKO., <https://gekko.readthedocs.io/en/latest/>. (2022)
- [8] Khatab, A., Aghezzaf, E. H.: Selective maintenance optimization when quality of imperfect maintenance actions are stochastic. *Reliability engineering & system safety* **150**, 182-189 (2016)
- [9] Shahraki, A. F., Yadav, O. P., Vogiatzis, C.: Selective maintenance optimization for multi-state systems considering stochastically dependent components and stochastic imperfect maintenance actions. *Reliability Engineering & System Safety* **196**, 106738 (2020)
- [10] Ghorbani, M., Nourelfath, M., Gendreau, M.: A two-stage stochastic programming model for selective maintenance optimization. *Reliability Engineering & System Safety* **223**, 108480 (2022)
- [11] Khatab, A., Aghezzaf, E. H., Djelloul, I., Sari, I.: Selective maintenance optimization for systems operating missions and scheduled breaks with stochastic durations. *Journal of manufacturing systems* **43**, 168-177 (2017)
- [12] Liu, L., Yang, J., Kong, X., Xiao, Y.: Multi-mission selective maintenance and repair persons assignment problem with stochastic durations. *Reliability Engineering & System Safety* **219**, 108209 (2022)
- [13] Xenos, D. P., Kopanos, G. M., Ciccioiti, M., Thornhill, N. F.: Operational optimization of networks of compressors considering condition-based maintenance. *Computers & Chemical Engineering* **84**, 117-131 (2016)
- [14] Ye, Y., Grossmann, I. E., Pinto, J. M., Ramaswamy, S.: Modeling for reliability optimization of system design and maintenance based on Markov chain theory. *Computers & Chemical Engineering* **124**, 381-404 (2019)
- [15] Rivera-Gómez, H., Gharbi, A., Kenné, J. P., Montaño-Arango, O., Corona-Armenta, J. R.: Joint optimization of production and maintenance strategies considering a dynamic sampling strategy for a deteriorating system. *Computers & Industrial Engineering* **140**, 106273 (2020)
- [16] Zhou, H., Wang, S., Qi, F., Gao, S.: Maintenance modeling and operation parameters optimization for complex production line under reliability constraints. *Annals of Operations Research*, 1-17 (2019)
- [17] Birge, J. R., Louveaux, F.: *Introduction to stochastic programming*. Springer Science & Business Media (2011)
- [18] Charnes, A, Cooper, W. W.: Chance-constrained programming. *Management Science* **5**, 73-79 (1959)
- [19] Prékopa A.: Contributions to the theory of stochastic programs. *Mathematical Programming* **4**, 202-221 (1973)