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# Secondary students' experience using 3D pen in spatial geometry: affective states while problem solving

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*One's affective state can change our actions and thought process while learning. The interplay of student's emotion and problem solving in spatial geometry has not been thoroughly studied. We present qualitative analysis of individual and collaborative problem solving of spatial geometry tasks by middle school students (8 students in a lab setting, and 21 students in group interviews in a classroom setting). We use the concept of embodied learning design, with the 3D printing pen as a medium, to make the process of converting 2D sketches to 3D models more explicit. Findings revealed that the students' affective state significantly influenced the way they solve the problems in spatial geometry. 3D sketching environment allows students to build a bond (intimacy) with the material and use their emotions as signals for heuristic changes (integrity). The discrepancy between 2D and 3D visualization in spatial geometry tasks may lead to students' emotional tension.*

*Keywords: Spatial geometry, secondary school mathematics, embodied learning, 3D sketching affective states, problem solving*

## Introduction and Theoretical Background

### Spatial geometry

Grasping spatial geometry calls for command of five different types of mathematical thinking: spatial structuring, measurement, representing 3D objects, spatial ability, and conceptualizing mathematical properties (Pittalis & Christou, 2010). Students and teachers have historically relied on traditional 2D media and materials and their resulting “flat” perspectives (Alsina, 2010). Students struggle with spatial geometry tasks, especially those requiring several reasoning steps (Fujita et al., 2020). Applying a dynamic geometry environment (DGE) to spatial geometry fosters innovation in didactic and research methods and may stimulate geometrical reasoning (Mithalal & Balacheff, 2019). However, even 3D DGE are limited to the two-dimensional medium (screen) and thus may create perceptual problems (Dimmel & Bock, 2017). Therefore, we are seeking a teaching method that can provide students with a three dimensional and multimodal experience and facilitate the development of the five types of thinking mentioned above.

### 3D pen—a novel technology for freehand drawing of 3D models

The 3D pen is a relatively novel technology which application to spatial geometry instruction has not been significantly studied. The pen excretes hot plastic which allows one to draw in three dimensions free hand. The use of the 3D pen in mathematics education was presented by Ng, et al., (2020) in which elementary students were studied using the pen in comparison to dynamic geometry programs. It was found that long term retention of the group using the 3D pen was significantly better.

Our study focuses on cognitive and affective states of middle school students to find the potential of this novel medium in learning environment that promotes interaction between the object and the person (via gestures, movements and embodiment of the physical world) which is not possible when using a 2D medium (paper or screen). Assuming that using a 3D pen will make the 2D to 3D conversion visible and therefore easier and more natural for learners, we aim to characterize learners' emotions when solving non-standard spatial geometry problems.

### **Embodied learning of geometry**

Embodied learning, an educational approach based on the role of the body — through movement, action, and gesture as a powerful tool for understanding and learning school subjects — is gaining ground in educational research (Abrahamson et al., 2020; Kim et al., 2011). Embodied learning is based on embodied cognition which suggests there is some level of interaction between the body's physical actions and movements in its environment and our cognition (Wilson, 2002).

The activities in which students solve spatial geometry questions by constructing tangible models may potentially bridge the gap between intuitive and disciplinary ways of mathematical reasoning (Palatnik & Abrahamson, 2021). Tasks in which students sketch with the 3D pen to visualize a problem can be considered as a sub-genre of embodied design activity—compelling students to use movements and gestures (c.f. Abrahamson & Lindgren, 2014). Research that combines the theory of embodied design with the effect on students' affective state is relatively rare (Sinclair & Heyd-Metzuyanim, 2014). This study focuses on students' emotions and meta-affective abilities while solving nonroutine for them problems (spatial geometry) using non-routine medium (3D sketching pen).

### **Theoretical framework for influence of affect on problem solving**

While solving non-routine problems, many students are overcome by their emotions and unable to continue to solve the problem, however other students can maneuver their emotions, think of an alternative solution, and therefore solve the problem (McLeod, 1988). When students are aware of the feeling of frustration, for example, they can use it as a sign to give up and move on to the next problem, or as a sign to reevaluate their solution and find an alternative solution (Debellis & Goldin, 2006; McLeod 1988; Hannula, 2015).

Debellis and Goldin (2006) define mathematical intimacy as the students' emotional engagement with mathematics and mathematic integrity as their sense of “fundamental commitment to mathematical truth” and pursuit of understanding (Debellis & Goldin, 2006, p.132). The individual's ability to solve problems is influenced by their control of their emotions, their awareness, context and situation, mathematical education, attitudes, values, beliefs held by themselves and the environment and normative emotional expectation (Debellis & Goldin, 2006). Educators, curriculum developers, instructional coordinators, and textbook writers who are aware of students' emotions and the effects on their learning patterns, can use this to their advantage when searching for methods to create optimal learning environments.

McLeod (1988) characterizes these affective states based on magnitude (intensity), direction (positive or negative), level of awareness (of their meta-affective processing), duration, and level of control.

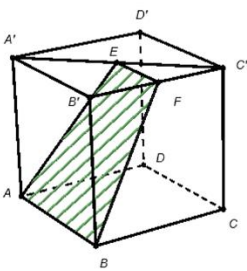
According to Debellis and Goldin (2006) mathematical intimacy refers to a specific, emotional engagement with mathematics. Mathematical integrity relates to one's affective state when determining if a solution is satisfactory or should be praised. This leads to the pursuit of understanding and the students' commitment (or lack thereof) to the mathematical truth. Students with high mathematical integrity recognize their insufficient knowledge or understanding and decide to change their solution or direction when problem solving.

Our research is situated on a nexus of spatial geometry content, embodied pedagogy, and the emotional component of students' learning. This research report focuses on the following RQ:

How does the experience of 3D sketching influence the students' affective state and their awareness of these states while solving spatial geometry problems?

## Methods

The cases reported in this paper are part of a larger research project which explores the effect of the use of embodied learning design on spatial geometry problems (Palatnik & Abrahamson, 2021; Rosenski, 2021). We tested 9 students in individual interviews in a lab setting, and 25 students in group interviews in a classroom setting. Of the 9 students interviewed in the lab setting, 8 were included in the data analysis. The students were chosen based grade level (9<sup>th</sup>-11<sup>th</sup> grade), as they have all studied concepts in planar geometry necessary to solve the problems given. Of the 25 students interviewed in group settings, 21 students were included in the data analysis. The others were omitted for not completing the entire question set.

	Given	Section	Question	True / False	
	<p>ABCD A'B'C'D' represents a cube.</p> <p>E is the middle points of A'C'.</p> <p>F is the middle point of B'C'.</p>	1	B' is a point on AE.	True	False
		2	ABFE is an isosceles trapezoid. The equal edges are _____.	True	False
		3	ABFE is a right-angled trapezoid. The right angles are _____.	True	False
		4	AE is the longest edge of quadrilateral ABFE.	True	False

**Figure 1: Example of question #3**

Before beginning the main activity, the students were asked to answer a computerized test of 30 questions that test spatial ability. The test is based on mental rotation and ability to rotate 3D objects projected on the screen. Subsequently, the students were given a 3D pen and were asked to sketch a pyramid with the 3D pen to practice drawing lines in the air. Most students were able to sketch freely with the pen within 10 minutes. The main activity of the study includes 3 questions in spatial geometry. Figure 1 shows the last question in the series of items. The students answered the questions by sketching a 3D object (shown to them on a 2D sketch) inside a plastic wire model of the cube and

then using mathematical reasoning. Our decision to have the students sketch the drawings, is not only for the use of the 3D pen technology, but also to make the process of converting 2D sketches to 3D models more explicit and potentially overcome the difficulties associated with this process as described by Fujita, et al. (2020). We hypothesize that students struggle to use spatial visualization and property based spatial analytic reasoning, but the physical act of sketching in 3D allows the student to develop new ways to interact with the 3D model using embodied cognition to overcome this issue. After the students have sketched their 3D models and move on to the phase of problem solving. After solving each question, the students were asked to express how confident they were in their answers. This allowed us to see if students who answered incorrectly were sure or unsure of their solutions. The three questions increase in terms of difficulty and familiarize students with the 3D model affordances.

After completing the questions, the students participated in a task-based interview (Goldin, 2000) about the questions themselves along with a semi-structured interview about their overall experience and feelings. Some of the question in the interview were open ended while some questions asked students to 'rate their feelings' on a scale from 1-10. In open ended questions, many students do not elaborate, but in the direct 'rating' questions students felt comfortable answering. The one-on-one interviews allows participants to expand on their experience of answering questions. The entire session was audio and video recorded (the students' face off frame) in order to properly record and analysis the students' actions and arguments and answers.

Our analysis used triangulation of data from the videos, responses, and field notes of the researcher. The data was transferred to detailed transcripts which were coded freely. The student's internal affects are inferred from external factors such as observable behavior in individual children's mathematical problem-solving during task-based interviews (Debellis & Goldin, 2006). The affective states and emotions of the students were characterized as suggested by McLeod (1988) magnitude (intensity), direction (positive or negative), level of awareness (of their meta-affective processing), duration, and level of control. Throughout the coding process, the main themes surfaced, and we were able to categorize the various aspects of the students' experience.

## **Findings**

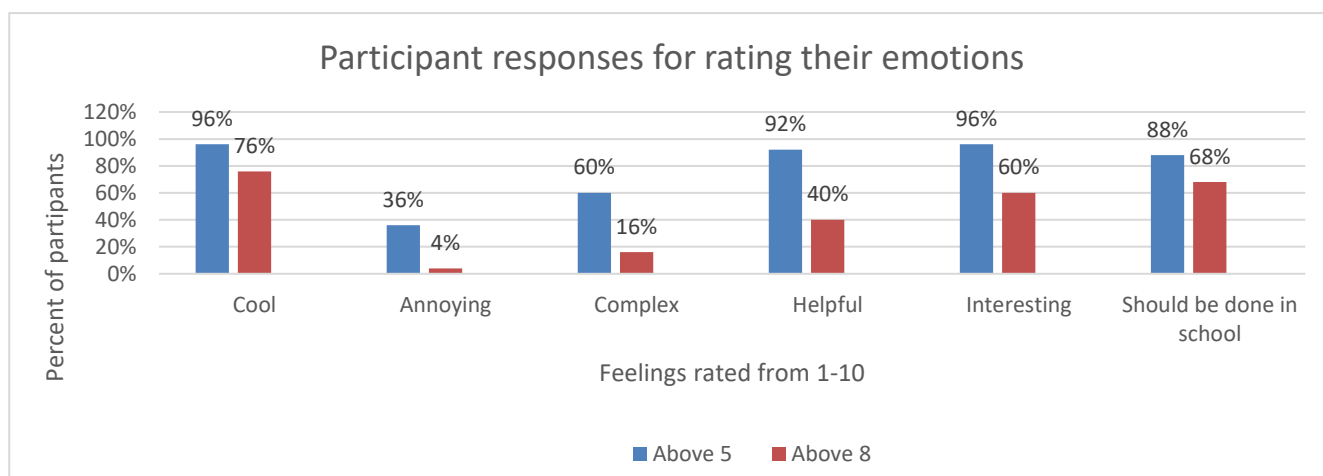
### **Mathematical intimacy**

Mathematical intimacy is the deep engagement of participants while solving the problems (Debellis, & Goldin, 2006). These feelings can be positive or negative and have varying magnitudes. Most of the participants of our study experienced mathematical intimacy while facing the tasks. Namely, the students were emotionally connected to the activity and showed (verbally and via behaviors) their deep care as the activity advanced from question to question. For instance, when answering the questions, a participant C was supposed to have a break for lunch. However, he did not agree to leave the room for the break until he finished the question he was working on. Note that C initially answered most of the questions incorrectly, but then saying aloud, "I'm not sure I sketched it correctly," felt he had made a mistake and corrected all his answers to the correct answer.

Each participant had a different range of emotions at all stages of the study, but there are several "common scenarios." Some participants were excited when they drew a 3D model but sad when they

could not immediately answer the question, were satisfied when answering the question, and described strong positive emotions about the activity during the interview. The other participants were frustrated at their inability to find the correct explanation for their answer, making them give up and move on to another question. These students still got the correct answers and were able to solve the problem. Some participants were restrained when drawing with a 3D pen but expressed extreme excitement when asked about their experience during interviews.

The emotions in the students after finishing the activity were generally positive in fairly high magnitude. Most participants felt the activity was generally good, while others felt deep feelings of excitement and amusement, directly connecting a 3D pen experience with this feeling. For instance, participant O5 claimed it was “a thousand times more fun” than a “traditional math class.” When asked directly to rate their experience of 3D sketching while solving a problem on a scale of 1-10 in one of the emotionally charged characteristics, participants answered in the following way:



**Figure 2: Participant responses for rating their emotions, post activity**

As can be seen in Figure 2, the students felt that the activity and use of the 3D pen was cool, not very annoying, not very complex, quite helpful, interesting, and should be done later in the school year. While asking students to describe the magnitude of their emotions is not necessarily the most accurate measurement, these numbers relate to a general feeling among the students and is corroborated with the findings throughout the interview. When solving the problem, the participant A, after 3 minutes of searching for the solution, suddenly found the answer and screamed “Oh! I know! The diagonal and the side are never equal in a square.” His task-based interview confirmed his deep satisfaction in solving the problem: “It was really fun, there was a lot of hands-on work which is nice. It makes you think in a way you usually don’t”

### **Mathematical integrity.**

Some of the participants of our study demonstrated mathematical integrity related to problem solving in general and spatial reasoning in particular. For instance, M, struggled to understand the characteristics of the 3D figure and when asked if the trapezoid she had drawn was isosceles (shown in Figure 1, question #3, section 2) she said “Based on how it looks I think no [not an isosceles trapezoid] I’m trying to think how I can prove why....um, wait I need to understand this” and later said “wow this is hard.” Her acknowledgement of lack of understanding, eventually led her to the

correct answer. She used this seemingly negative emotion to alter her problem-solving plans and change her thinking. In this case, M's negative feelings were at a low magnitude, and they allowed her to change direction without being overcome by anxiety or despair.

We also observed interesting manifestations of mathematical integrity pertinent to the material and visual aspects of 3D sketching activities. One participant explained that the 3D pen's biggest disadvantage is not being "exact" enough, the lines drawn are "not straight" and it's too bad that they did not use a ruler. This may relate to the participants' values or previous beliefs about mathematics in general, but also shows his search for 'exact' and 'true' mathematical representations, rather than ones that are not exact. Another student, A, consistently looked to the given 2D sketch when solving the problem instead of using the 3D model (c.f. Palatnik & Abrahamson, 2021 for the similar phenomenon). He explained in the interview that "I used the sketch because it's more exact." It seems that A feels that in order to align with his own 'mathematical truth' all the sketches must be exact. Throughout the problem-solving process, he decided to use calculations with Pythagorean theorem, despite not needing them to prove his answer. However, for him, it may have been the best way to solve the problem. When solving the trapezoid question (shown in Figure 1) section 4, he used complex calculations and an auxiliary line drawn with the 3D pen to explain his answer. When trying to thoroughly explain the calculations to the researcher he laughed and seemed to realize the excess calculations. He took this struggle with explanation 'lightly' and was able to solve the problem.

The participants of the study used their mathematical integrity to solve non-routine problems successfully without allowing their affective states of frustration or anxiety overcome their ability to make heuristic decisions and changes. This allowed them to describe their experience of using 3D pen and a tangible model to solve spatial geometry problems as positive. Students who can feel comfortable in a state of 'unknown' while searching for the right solution, will be able to solve non-routine problems (Debellis, & Goldin, 2006).

## **Discussion.**

We conducted a study of middle-school students facing the spatial geometry tasks while sketching with a 3D pen. We sought to find out how does the experience of 3D sketching influence the students' affective state and their awareness of these states while geometry problem solving. The participants demonstrated a high level of mathematic intimacy—connection and strong emotions and mathematic integrity, the ability to use the intimacy to their favor. The students showed strong positive emotions such as excitement, satisfaction, and awe. The nature of the activity in the study gives students the ability to sketch 3D figures by themselves, rotate the figures, and change or add auxiliary lines. The two-dimensional diagrams on which the school's spatial geometry curriculum is based represent three-dimensional bodies in a distorted form (Widder, et al., 2019). This quality can create a sense of alienation in students. However, the affordances given by the 3D sketching develops the students' bond with a problem. This bond gave them general positive feelings about the activity, despite being a nonroutine and challenging problem to solve.

Students in the study showed a high level of mathematic integrity as they were able to recognize their own lack of spatial understanding or geometric knowledge which led them to reassess their problem-solving strategy. Most of the students navigated their emotions of dissatisfaction, lack of confidence,

or frustration and were able to successfully solve the problems. In certain cases, we observed a type of mathematical integrity specific to 3D representation and sketching. Students are accustomed to using 2D sketches of 3D figures, and therefore some of them felt that the 2D sketch was more exact or closer to their own 'mathematic truth.' Educators and educational designers should be aware that the discrepancy between 2D and 3D visualization in spatial geometry tasks can lead to emotional tension for students. Some secondary students may have feelings of antagonism towards the novel medium of 3D sketches and require mediation before beginning the activity.

The participants' level of intimacy and integrity shows a high level of comfort and openness throughout the activity. Our study used a learning environment based on the principles of embodied learning. Abramson and Lindgren (2014) suggest that one may create a deep understanding by physical interaction with the environment. The 3D model of the cube allows students to "off load" at least some of the cognitive processes (mental rotations, imaginary auxiliary elements) needed to complete the task (Wilson, 2012). The use of the 3D pen affords students a physical visualization of the conversion process from a 2D sketch to a 3D model. In its "purely mental" form, this process proves to be very challenging for students of all ages (Fujita, et al., 2020). Throughout the intervention phases, the participants' affective states can be seen as 'pathways' that may guide them to various directions and changes in the heuristic decisions (Debellis, & Goldin, 2006). The students in our study used their recognition of inadequate justifications as a signal to make a heuristic change (McLeod, 1988). In many cases this recognition was directly connected to the manipulation of 3D model and in particular 3D sketching.

In this study we found that the students' affective state significantly influenced the way they solve problems. The awareness of the students' emotions and its effect on how they solve problems can allow both students and teachers to mitigate the negative emotions and reroute them to an alternate solution. This topic demands further examination with research using a variety of geometry tasks and populations. The high school curriculum in Israel is focused on having students successfully pass matriculation exams and spatial geometry tasks are part of the exams on all the levels. Thus, drawing on our findings on secondary students' experience while using 3D pen in spatial geometry, we suggest that teacher professional development programs consider recent advancements of embodied learning, novel medium (as 3D pen) ,and student's affective states as well as the connection between these fields.

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