Modeling the impact of rainfall and temperature on sterile insect control strategies in a Tropical environment

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Abstract

The sterile insect technique (SIT) is a biological control technique that can be used either to eliminate or decay a wild mosquito population under a given threshold to reduce the nuisance or the epidemiological risk. In this work, we propose a model using a differential system that takes into account the variations of rainfall and temperature over time and study their impacts on sterile males releases strategies. Our model is as simple as possible to avoid complexity while being able to capture the temporal variations of an *Aedes albopictus* mosquito population in a domain treated by SIT, located in Réunion island. The main objective is to determine what period of the year is the most suitable to start a SIT control to minimize the duration of massive releases and the amount of sterile males to release, either to reduce the mosquito nuisance, or to reduce the epidemiological risk. Since sterilization is not 100% efficient, we also study the impact of different levels of residual fertility within the released sterile male population. Our study shows that rainfall plays a major role in the dynamics of the mosquito and the SIT control, that the best period to start a massive SIT treatment lasts from July to December, that residual fertility has to be as small as possible, and that increasing the size of the releases is not always necessarily interesting. We also highlight the importance of combining SIT with mechanical control, i.e. the removal of breeding sites, in particular when the initial mosquito population is large.

Keywords: Vector control; *Aedes spp*; Sterile Insect Technique; Temperature; Rainfall; Residual fertility; Nuisance reduction; Epidemiological risk; Mathematical modeling; Numerical simulation.

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1 Introduction

Being vectors of many diseases, like Malaria, Dengue, Lymphatic Filariasis, Zika, Chikungunya, Yellow fever, and Japanese encephalitis, mosquitoes are one of the biggest killers in the world, and particularly in tropical and subtropical areas. Indeed, Female mosquitoes need blood meals to provide the nutriments for egg development. That is why they bite either during the night, like *anopheles spp*, at sunset or sunrise, like *aedes albopictus*, or along the day, like *aedes aegypti*, inside or outside houses. Since they can survive several weeks, they will bite several times depending on their gonotrophic cycle [10]. While opportunistic feeders, Anopheles and Aedes mosquitoes prefer biting mammals and, preferably, humans [11], with sometimes multiple blood feeding [6].

To eradicate mosquito-borne diseases, the initial option was the massive use of chemicals in the 1950's and 1960's, mainly against anopheles. Despite some successes, like in Réunion island where Malaria has been eradicated since the fifties, thanks to the use of DDT [9], we know now that this was a huge mistake: even if the use of adulticides was successful against mosquitoes, the damages on the biodiversity were important. In addition, resistance to some adulticides raised, such that, in several places around the World, there is no option left to fight the mosquitoes, like in the French West Indies [12]. Fortunately, other eco-friendly control methods, more respectful of the biodiversity, have been developed. Among them, the Sterile Insect Technique (SIT) is the most promising one. This is an old control technique, proposed in the 30s and 40s by three key researchers in the USSR, Tanzania and the USA and, first, applied in the field in the 50's [13]. SIT has been used more or less successfully on the field against various kinds of Pests or Vectors (see [4, 13] for various examples). The classical SIT consists of mass releases of males sterilized by ionizing radiation. The released sterile males transfer their sterile sperms to wild females, which will have no viable offspring, resulting in a progressive decay of the targeted population. For mosquitoes, other sterilization techniques have been developed using either genetics (release of insects carrying a dominant lethal technique, in short, RIDL technique [14]) or bacteria (SIT-IIT : cytoplasmic incompatibility thanks to *Wolbachia* bacteria) [15]. Many SIT projects against mosquitoes are now ongoing around the World.

Several mosquito models have been developed, from very simple models to more sophisticated ones, according to the number of biological states that are taken into account. These models consider the mosquito population either at the individual level (see for instance [1]) or at the population level (see for instance [16]), taking eventually into account the spatial component (see for instance [2] for a brief review). Each of these approaches has advantages and disadvantages. However, it is always important to keep in mind that biological parameters are difficult to obtain in the field, such that for too complex models, it might be impossible to set the (right) values of the parameters. In addition, most of the biological parameters are linked to environmental parameters, like rainfall, humidity and temperature. The main goal of the present article will be to take into account these parameters in the mathematical model to find the best control.

Mathematically, it can make sense to consider constant or periodic parameters to derive the main dynamics of the mosquito population. However, for practical reasons, i.e. anticipating when the mosquito population is growing or decaying, it is better to have a mosquito model that takes into account the most important parameters, like rainfall, humidity and temperature, to adapt the control, i.e. for instance the size of the releases or/and the period of the releases.

Since our work takes place within an ongoing SIT feasibility program (TIS 2B) against *Aedes albopictus* in La Réunion, a tropical French island located in the Indian Ocean, we will focus on this species in the rest of the paper. In La Réunion, *Aedes albopictus* has become the main vector of Dengue and Chikungunya [17]. However, we believe that our approach is sufficiently generic to be applied to other mosquito species.

While links between *Aedes albopictus* parameters and temperature have been studied in laboratory [7], for a fixed (laboratory) humidity, it is more challenging to take into account the effect of rainfall (and humidity).

Some weather-dependent mosquito models have been developed, mainly with Temperature-dependent parameters (see for instance [8, 18] and references therein) and very few with temperature and rainfall-dependent parameters (see [19] and references therein). However, in general, these last models are quite complex: they relied on statistical approaches and on the user's subjective choices, such that the calibration (of many parameters), with respect to the environmental parameters, is not generic and might not be able to provide a unique set of valuable values. We firmly believe that simple (but not too simple) models can rapidly provide useful and reliable information to help field experts to manage vector control campaigns.

That is why we consider a minimalistic model (minimal in terms of stages and thus parameters), based on [3], to build an approximation of the mosquito population over many years. Then, we show that the density variation (in time) of the mosquito population can help to improve vector-control strategies combining SIT with mechanical control, i.e. the removal of breeding sites to reduce the larvae carrying capacity.

It is well known that sterilization does not necessarily induce that the sperm of the sterilized males is 100% sterile (see for instance [20, 27]): it depends on the radiation source, the dose-rate, and the container. That is why quality control after sterilization is set up to control that the sterile males remain competitive and efficient. Efficiency is thus related to the effective sterility or the residual fertility, i.e. the percentage of sperms that is still fertile despite the irradiation, that is equivalent to say, at the population level, that a small proportion, ε,

of the sterile males is fertile. In our model, we will take into account the fact that residual fertility can occur within the sterile male population to study its impact on SIT control.

In general, in most of the SIT models (except in [4] and the present paper), residual fertility is never taken into account, while it can have a strong impact on SIT efficiency. Indeed, in [4], using a 2-dimensional mosquito model with constant parameters, the authors showed that the residual fertility, ε, is strongly linked to the basic offspring number, $\mathcal{N}$, of the wild population: when

$$\varepsilon < \frac{1}{\mathcal{N}}, \tag{1}$$

then SIT is efficient and the mosquito population can be lowered under any given threshold, provided that enough sterile males are released. However, when ε is below but close to $\frac{1}{\mathcal{N}}$, the amount of sterile males increases almost exponentially [4]. On the contrary, when $\varepsilon > \frac{1}{\mathcal{N}}$, then, whatever the size of the sterile males releases, the wild population cannot be lowered under a certain threshold value, that can be roughly estimated [4]. In fact, we suspect that a too-large residual fertility (RF, shortly) could partly explain failures in some SIT programs (see for instance [27]).

The paper is organized as follows: In section 2, we build a temperature and rainfall-dependent entomological model and an impulsive periodic SIT model. Then, in section 3, we provide several numerical simulations to discuss the impact of the temperature, the rainfall, the residual fertility and the mechanical control on SIT starting period and duration to reduce either the nuisance or the epidemiological. Finally, in section 4, we end the paper with some conclusions and perspectives.

2 Rainfall, humidity, and temperature dependent SIT model

The first aim of the present work is to develop a temperature-rainfall-dependent entomological model, to take into account real field data, including mean daily rainfall and mean daily temperature.

Following [3], we will consider the following model

$$\begin{cases} \dfrac{dA}{dt} = \phi(Temp)F - (\gamma(Temp) + \mu_{1,A}(Temp) + \mu_{A,2}(Temp, Rain)A)\, A, \\ \dfrac{dM}{dt} = (1 - r(Temp))\gamma(Temp)A - \mu_M(Temp)M, \\ \dfrac{dF}{dt} = r(Temp)\gamma(Temp)A - \mu_F(Temp)F, \end{cases} \tag{2}$$

where A, M, and F represent, respectively the aquatic (larvae, pupae) stage, the adult (male and female) stages. This model, for constant parameters, has been studied in [3], where the authors developed a new strategy to maintain the wild population under a certain threshold, using a permanent and sustainable low level of SIT control, thanks to a massive-small releases strategy. We set $\mathcal{N}_{\max} = \max_{t\in[0,+\infty)} \mathcal{N}(t)$, where $\mathcal{N}$ represents the basic offspring number, defined as follows

$$\mathcal{N} = \frac{r\phi\gamma}{(\gamma + \mu_{A,1})\,\mu_F}.$$

In Réunion island, a network of Weather stations (from Météo France but also from CIRAD) allows us to estimate some weather parameters all around the island, and in particular where real SIT field experiments started in July 2021, in the site of Duparc, a 20-hectare urban area located within the commune of Sainte Marie in the northern district of La Réunion [21]. These releases consist of manually weekly releases of 150 000 to 250 000 sterile males (once a week) produced and irradiated in La Réunion. The efficacy of sterile males releases is assessed over time by monitoring the induced sterility in eggs using ovitraps and the subsequent population suppression using adult traps, and, from time to time, Mark-Release-Recapture experiments. This experiment lasted until September 2022.

From the Weather station located in La Mare, close to the site of Duparc (1km), we are able to obtain the following daily data: the rainfall, the average daily temperature and also the humidity. Thus, following [22], we first define the breeding site carrying capacity to define the density death-rate $\mu_{A,2}$. Obviously, the persistence of breeding sites is a key factor for the mosquito population's survival. Indeed, rainfall creates breeding sites, while evapotranspiration tends to shrink them. Following [22], we define the variable $H(t)$ as the amount of water available at day t and defined as follows

$$H(t+1) = \begin{cases} 0 & \text{if } H(t) + \Delta(t) \leq 0, \\ H_{max} & \text{if } H(t) + \Delta(t) \geq H_{max}, \\ H(t) + \Delta(t) & \text{otherwise,} \end{cases}$$

with

$$H_{\max} := \max_{t\in[t_0,t_0+T]}\{Rain(t)\}$$

and

$$\Delta(t) := Rain(t) - Evap(t),$$

where $Rain(t)$ is the daily rainfall and $Evap(t)$ the daily evaporation, t_0 is the starting date of the simulations, and T the total duration. Following [22], the evaporation function is defined as follows

$$Evap(t) = k \times \left(25 + Temp(t)^2\right) \times (100 - Hum(t)),$$

where $Temp(t)$ is the average temperature and $Hum(t)$ the humidity. Finally, the carrying capacity is defined

$$K(t) = K_{max}\frac{H(t)}{H_{max}} + K_0,$$

where $K_0 > 0$ can be seen as the fixed artificial carrying capacity, i.e. rainfall-independent, human-made, by watering, for instance, plants around houses (flower pots, plates, and vases), and $K_{\max}$, the natural maximal carrying capacity.

The initialization in time of the variables (in particular of H) is explained in the simulations section, page 5. We notice that, in general, in the literature, the question of initialization is not always taken into account, while it can have a strong influence over several months on the population dynamic. We illustrate this fact in Figure 2, page 7.

In order to estimate $\mu_{2,A}$, we consider the positive equilibrium related to the carrying capacity, like in [8,23,24]. Thus, for a fixed value of K, the aquatic stage at equilibrium is given by

$$A_K^* = \left(1 - \frac{1}{\mathcal{N}}\right)K. \tag{3}$$

In our model, the aquatic stage at equilibrium is defined as follows

$$A^* = \frac{\gamma + \mu_{A,1}}{\mu_{A,2}}(\mathcal{N} - 1), \tag{4}$$

such that considering the equality between both equilibria given in (3) and (4), we derive the following relationship between $\mu_{A,2}$ and K, that is

$$\mu_{A,2} = \frac{\mathcal{N}(\gamma + \mu_{A,1})}{K} = \frac{r\gamma\phi}{\mu_F K}.$$

Thus, taking into account the dependency of the parameters to Temperature and Rainfall, we deduce that

$$\mu_{A,2}(Temp, Rain) = \frac{r\gamma(Temp)\phi(Temp)}{\mu_F(Temp)K(Temp, Rain)}. \tag{5}$$

As it is well known, eggs are deposited above the waterline. They hatch once they are flooded by rainfall. However, the hatching rate can be seasonal. From *A. albopictus* eggs, picked up at Duparc, the mean hatching proportion is around 90% (G. Legoff, personal communication, TIS2B project).

For the other parameters, we will consider the data recalled in Table 11, page 19, as used in [24] to obtain the parameters estimates given in Table 12, page 19.

Then, we can use simple interpolation polynomials (with cubic spline, like in [25]) to estimate these parameters for any given temperature $Temp$, within the range $[15^o, 35^o]$. We assume that r, the sex-ratio, is independent of the weather data and stays fixed, along the year, to 0.5.

We will assume constant or periodic releases of sterile males, at rate $u_S(t)$ (either constant or variable), and μ_S, the sterile male mortality rate, which is supposed to be similar to the wild males mortality rate [26]. The dynamic of the sterile males is modeled by

$$\frac{dM_S}{dt} = u_S(t) - \mu_S(Temp)M_S. \tag{6}$$

Then following [4], we will consider the residual fertility, ε, in our SIT model. As recalled in the introduction, for constant value parameters, we need to verify $\varepsilon < \frac{1}{\mathcal{N}}$ in order to have an efficient SIT control, i.e. such that the wild population can be lowered under a given value, using appropriate (massive) releases. In [4], using a minimalistic model with constant parameters, the authors showed that, for instance, for *A. albopictus* mosquito, the RF should be lower than 2.5%. In [27], in a SIT program conducted on Mauritius island, which is close to Réunion island, the residual fertility was experimentally estimated at around 3.05% ($2.29\% - 3.92\%$), which could explain why mitigated results were obtained, before a cyclone occurred and broke the experiment. We believe

that 3% of RF is too large such that the reduction of the wild population with SIT is limited. Indeed, in [4], the authors showed that if RF is too large, then, whatever the size of the sterile males releases, the population cannot be lowered under a given threshold.

We derive the following temperature and rain dependent SIT-model

$$\left\{\begin{array}{rcl} \dfrac{dA}{dt} &=& \phi(Temp)F - (\gamma(T) + \mu_{1,A}(T) + \mu_{A,2}(T,R)A)\,A, \\ \dfrac{dM}{dt} &=& (1-r(T))\gamma(T)A - \mu_M(T)M, \\ \dfrac{dF}{dt} &=& r(T)\gamma(T)\dfrac{M+\varepsilon\beta M_S}{M+\beta M_S}A - \mu_F(T)F, \end{array}\right. \tag{7}$$

where β is the competition parameter. Since the variables T (Temperature) and R (Rainfall) change over time, our model is considered as a non-autonomous model.

2.1 Impulsive SIT massive-small releases strategy

We want to find the best period to start SIT control (without or with Mechanical Control) using a massive-small releases strategy as in [3] to minimize the number of massive periodic impulsive releases, and thus the amount of sterile males to release.
Thanks to [3], system (7) has the following long term behavior

- There exists a release rate threshold, u^*, such that when $u_S(t) > u^*$ then (A, M, F) converges to $\mathbf{0} = (\mathbf{0}, \mathbf{0}, \mathbf{0})$.
- When $0 < u_S(t) < u^*$, then there exist two positive equilibria, $\mathbf{E_1}$ and $\mathbf{E_2}$ such that $\mathbf{E_1} < \mathbf{E_2}$. Moreover $\mathbf{E_1}$ is unstable while $\mathbf{E_2}$ is stable and $[\mathbf{0}, \mathbf{E_1})$ lies in the basin of attraction of $\mathbf{0}$.

We will consider two different levels of massive releases (6000 or 12000 sterile males per ha) to decrease the wild population below a threshold set, $[\mathbf{0}, \mathbf{E_1})$, defined by (inexpensive) small releases (say 100 sterile males per ha), in a minimum time. We want to know how the duration of massive releases is influenced by temperate and rainfall conditions during the year, the residual fertility and the level of mechanical control. In other words, we search a control with the form

$$u(t) = \tau\Lambda_{\text{massive}} \sum_{i=1}^{N} \delta_{t_0+(i-1)\tau}(t), \tag{8}$$

where $\delta(t)$ is the Dirac function, t_0 is the starting time of the massive releases, τ the periodicity of the releases (here, $\tau = 7$), and N the number of weakly massive releases. The massive release Λ_{massive} and the small releases Λ_{small} are fixed. For a given t_0 there exists a $t_1(t_0) = t_0 + N_1(t_0)\tau$ (according to [3]) such that, after this time, the wild population (A, M, F) remains in the box $[\mathbf{0}, \mathbf{E_{1,min}}(\tau\mathbf{\Lambda_{small}}))$ where $\mathbf{E_{1,min}}(\tau\mathbf{\Lambda_{small}})$ is defined as follow: For a given Λ_{small}, for each time t, we compute the equilibrium $\mathbf{E_1(t)}$ of the system associated to the parameters at time t, and we define $\mathbf{E_{1,min}}(\tau\mathbf{\Lambda_{small}}) = \min_{\mathbf{t}} \mathbf{E_1(t)}$, where the minimum is taken between the beginning and the end of the time interval considered in the simulation.

The main goal is to find the (best) starting time of the massive releases, t_0, in order to minimize the duration of the releases and thus, the number of massive releases.

System (6)-(8) can be rewritten as an impulsive differential system with fixed moments of impulse effect, that is

$$\left\{\begin{array}{ll} \dfrac{dM_S}{dt} = -\mu_S(T)M_S, & t_0 + i\tau < t \leq t_0 + (i+1)\tau \\ M(t^+) = M(t) + \tau\Lambda_{massive}, & t = t_0 + i\tau, \end{array}\right. \tag{9}$$

for $i = 0, ..., N-1$. Since, the right-hand side of (7)-$(9)_1$ is locally Lipschitz-continuous on $\mathbb{R}^4$, we can use a classical existence Theorem (for instance Theorem 1.1 in [29], or Theorem 2.1 in [28]), to deduce that there exists $T_e > 0$ and a unique solution of system (7)-(9), defined from $(t_0, T_e) \longrightarrow \mathbb{R}^4$.

3 Numerical simulations and discussion

System (7)-(9) is solved thanks to `odeint` of the python library `scipy.integrate`. The codes and the data corresponding to the temperature and the rainfall in Réunion island are available on

`https://github.com/michelduprez/Impact-of-Rainfall-and-Temperature-on-IT-control-strategies.git`

We consider temperature, rainfall and humidity data based on (noised) data recorded in weather stations close to the site of Duparc (a 20 ha place), a neighborhood of Sainte Marie, located in the North of Réunion island from the 1st of January 2009 to the 14th of July 2021.

In La Réunion, since the sterile males are produced on-site, everything from the eggs to the sterilization is controlled (qualitatively), such that since the production started, the residual fertility is, in general, less than 1%, with an average value of around 0.6%, a very good results compared to other SIT projects, like [27]. That is why, in the forthcoming simulations, we will consider three cases of RF, namely 0%, 0.6% and 1.2%.

3.1 Reducing mosquito nuisance

As studied in [3], we consider the massive-small strategy, which consists first of periodic massive releases, such that $\tau\Lambda_{massive} = 20\times 6000$ or 20×12000 individuals, until the wild population has become lower than $\mathbf{E_{1,min}}(\mathbf{\Lambda_{small}})$, where here $\tau\Lambda_{small} = 20\times 100$ individuals, such that

$$\mathbf{E_{1,min}}(\mathbf{\Lambda_{small}}) \approx \begin{cases} (37.0, 29.8, 0.511) & \text{if } \varepsilon = 0.0, \\ (17.97, 14.47, 0.24) & \text{if } \varepsilon = 0.006, \\ (1.38, 0.350, 0.0059) & \text{if } \varepsilon = 0.012. \end{cases}$$

It is interesting to notice that, when the residual fertility increases, then the size of the box $[0, \mathbf{E_{1,min}}(\mathbf{\Lambda_{small}})]$ becomes smaller: SIT introduces a strong Allee effect while the residual fertility weakens it. Thus, in the forthcoming simulations, it will not be surprising to find large duration time values when the residual fertility is large. Once $[0, \mathbf{E_{1,min}}(\mathbf{\Lambda_{small}})]$ is reached, the periodic releases continue at rate Λ_{small}. Using the numerical simulations, we are able to estimate the date where the system can switch from massive releases to small releases and thus evaluate the duration of the (very) massive releases.

In Fig. 1, we consider the following values: $K_{\max} = 20\times 10000$, $K_0 = 20\times 100$ to derive the carrying capacity, K, and thus $\mu_{A,2}$.

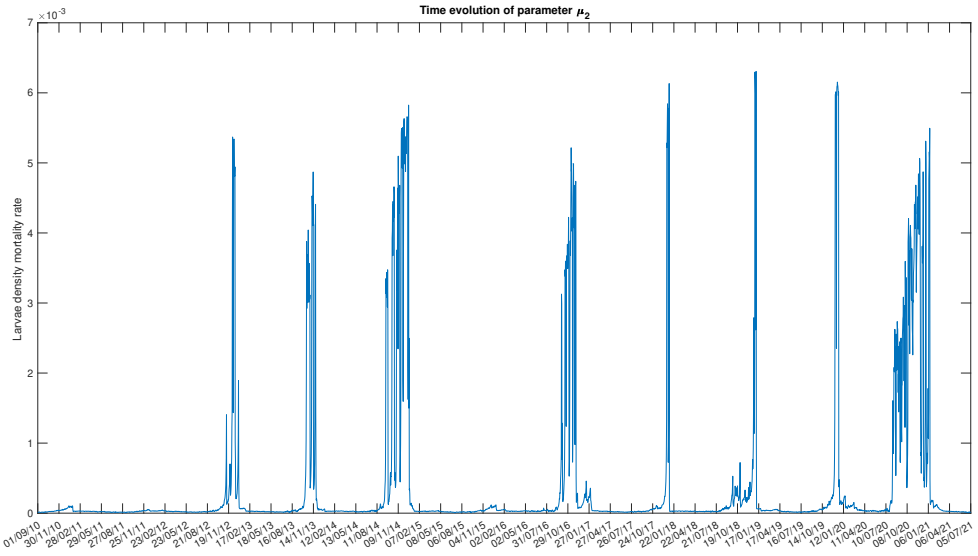

Figure 1: Time evolution of $\mu_{A,2}$ from the 1st of September 2010 until the 14th of July 2021

It is interesting to see the behavior of $\mu_{A,2}$ in the second-half of 2020, which was a particularly dry period compared to previous years in La Réunion. Note also that when mechanical control is considered, it will impact $\mu_{A,2}$: for instance, thanks to Formula (5), 40% of Mechanical control increases $\mu_{A,2}$ by 66.7%.

The choice of the initial condition for H and the initial conditions for the mosquitoes will impact respectively the initialization of the carrying capacity and the dynamic of the system for at most 12 months (see Fig. 2, page 7, where we have considered extreme initial values for $H(0)$ and for the population). That is why, in order to have a reliable estimate of the wild mosquito population at the beginning of the control, i.e. the 1st of Sept. 2010, we start the simulations at the beginning of January 2009.

Using the parameters values and the estimates of $\mu_{A,2}$ given in Fig. 1, we derive the dynamic of the mosquito population in Duparc without release: see Fig. 3, page 7. As expected, periods, where the rainfall is low, imply

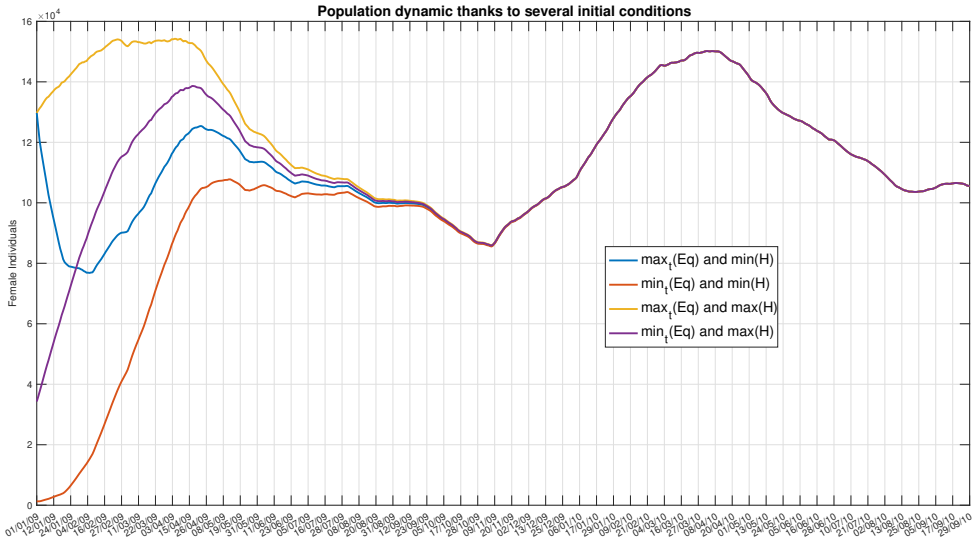

Figure 2: Temperature and rainfall dependent model - Simulations of the mosquito dynamics with several initial conditions, thanks to the initial rainfall data and the initial size of the mosquito population

a rapid decay of the population size, leading to an "almost" oscillatory behavior. This result is confirmed by Mark-Release-Recapture experiments derived in Duparc [21], where the ratio between the mosquito density/ha within the dry period and the mosquito density/ha within the wet period is a factor 10, which highlights the importance to consider a temporal dynamic in the parameters.

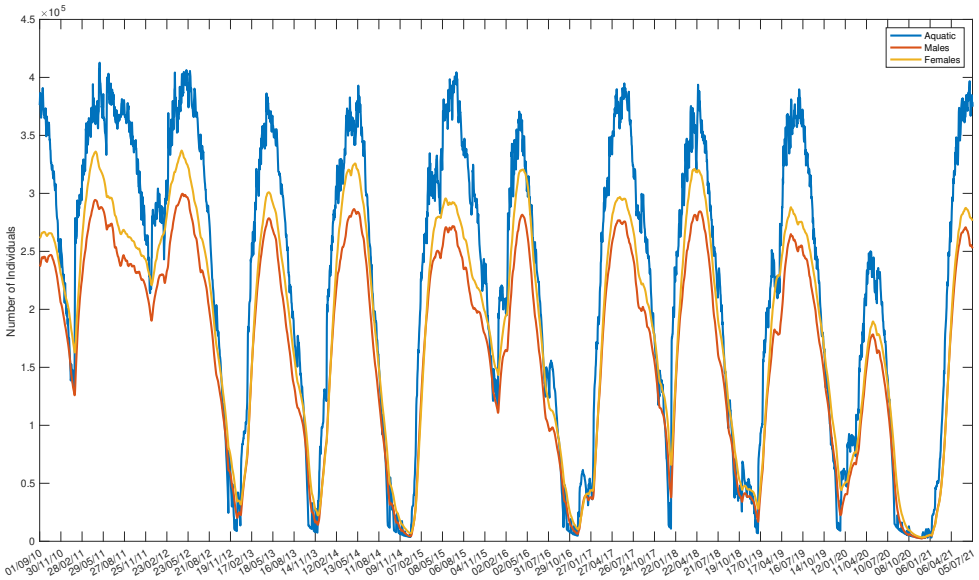

Figure 3: Temperature and rain dependent model - Mosquito population dynamics from the 1st of September 2010 until the 14th of July 2021

According to the time variation of the parameters, the basic offspring number $\mathcal{N}$ will vary between 29.7 and 85.7 with a mean of 48.41 over the considered period. Thus even if Formula (1) is not verified at each time, it is

satisfied in mean, i.e.

$$\varepsilon < \frac{1}{T}\int_0^T \frac{1}{\mathcal{N}(t)}dt = 0.0222. \tag{10}$$

An important issue in SIT control is to estimate the duration of the massive releases. Thanks to contrasted environmental conditions, within the period $[2010-2020]$, we show that the minimal time to decay the wild population under a certain threshold can greatly vary: see Figs. 4-5, from page 23 to page 24, for two weakly release rates (6000 Ind/ha and 12000 Ind/ha), for different levels of Mechanical Control (0%, 20% and 40%) and three levels of residual fertility (0%, 0.6% and 1.2%).

In general, in the literature, many SIT simulations are done with constant parameters, except, for instance, in [8, 30]. If we consider the mean value for each parameter over the whole period, we derive $\mathcal{N} \approx 49.3$ which is a quite large basic offspring number, but not surprising within a tropical context.

We will consider four cases to show the importance (for real field applications) to consider realistic environmental conditions: we consider model (7)-(9) with

1. temperature and rainfall dependent parameters.
2. average parameters values estimated from September 2010 to mid-July 2021.
3. temperature-dependent parameters only
4. constant average temperature and rainfall-dependent parameters.

We discuss and compare these four cases:

1. For the temperature and rainfall-dependent model, we derive several interesting facts: see Fig. 4, page 23, and Fig. 5, page 24.

   When mechanical control is considered, there is a gain in the duration of massive releases. However, the impact of mechanical control is more important in periods where the mosquito population is large, whatever the residual fertility: compare the years 2010-2011 with the years 2016, 2019, and 2020, in Fig. 4, page 23.

   As also showed in Fig. 4, page 23, the residual fertility is a very important parameter: if RF increases from 0% to 0.6%, this leads to an increase of the duration (of the massive releases) by 30%. Then, increasing RF from 0% to 1.2% increases the duration (of massive releases) by 250% (and even 350% when a minimal time is reached).

   Thus, for a given number of sterile males (here 6000 Ind/ha) to release, the impact of RF is important, such that it seems natural to increase the size of the releases. This is done in Fig. 5, page 24 with $\Lambda_M = 12000$ Ind/ha. Surprisingly, the benefit in duration is quite low. It is only when $RF = 1.2\%$ (see Fig. 5(c), page 24), that the gain (in time) is interesting, but only in periods when the mosquito population is large. Thus, increasing the release rate reduces a bit the duration of the massive releases, but the gain is very little compared to the production effort and thus to the cost increase. In fact, above a certain amount, it seems that a saturation effect occurs, such that very massive releases over a long time are not necessarily (always) an appropriate strategy.

   The death rate of mosquitoes is mainly linked to the temperature, we can intuitively think that the best period to act is during the Austral winter, i.e. from the end of June to September. According to Figs. 4 and 5, page 23 and 24, the best intervention period, i.e. the SIT starting-time, lasts, in general, from July to December. Thus, the window to start the SIT control is larger than expected, and is not necessarily reduced to the Austral winter period (from June to September) but can also include the Spring period (from October to December).

   Finally, in Table 1, page 9, we summarize the average total duration and the average total amount of sterile males needed to reach elimination: These values have to be compared with the average values obtained for the case of the constant parameters.

2. In Table 2, page 10, we provide the results obtained with the average values for the biological parameters, computed thanks to their mean values over the period $[2010, 2020]$: we obtain a mean estimate of the duration of the massive releases according to the levels of mechanical control, the levels of residual fertility and for the two weekly release rates. This case is interesting as it requires only **one** computation and thus can help to derive a first (rough) approximation in terms of releases strategy as well as the minimal amount of sterile males to produce over the massive releases period.

   While for small residual fertility, i.e. 0% and 0.6%, we obtain comparable average duration estimate (compare Table 2 to Table 1), for a large residual fertility, the result is clearly different, and certainly far from the reality.

   Increasing the sterile males release rate, i.e. switching from 6000 Ind/ha to 12000 Ind/ha, is not necessarily interesting: it is very costly in terms of sterile males with a little gain with respect to the duration of the massive releases, especially when mechanical control is strong, i.e. at 40%.

Table 1: **Simulations with temperature and rainfall dependent parameters: massive releases duration and the total mean amount of sterile males to release over the 20 hectares: (a) 0% of Residual Fertility; (b) 0.6% of Residual Fertility; (b) 1.2% of Residual Fertility**

(a)

	6000 Ind/ha		12000 Ind/ha	
$\varepsilon = 0$	Mean number of releases	Total mean amount of sterile males released	Mean number of releases	Total mean amount of sterile males released
0% of MC	56	6 720 000	49	11 760 000
20% of MC	52	6 240 000	46	11 040 000
40% of MC	48	5 760 000	44	10 560 000

(b)

	6000 Ind/ha		12000 Ind/ha	
$\varepsilon = 0.006$	Mean number of releases	Total mean amount of sterile males released	Mean number of releases	Total mean amount of sterile males released
0% of MC	68	8 160 000	60	14 400 000
20% of MC	63	7 560 000	57	13 680 000
40% of MC	58	6 960 000	54	12 960 000

(c)

	6000 Ind/ha		12000 Ind/ha	
$\varepsilon = 0.012$	Mean number of releases	Total mean amount of sterile males released	Mean number of releases	Total mean amount of sterile males released
0% of MC	164	19 680 000	152	36 480 000
20% of MC	157	18 840 000	148	35 520 000
40% of MC	150	18 000 000	143	34 320 000

Of course, as expected, mechanical control has a positive effect with, in mean, a gain of time between 5 and 10 weeks, depending on the level of residual fertility. Again, this is to balance between the cost of mechanical control and the effective gain in time, and thus the gain in terms of sterile males.

This "mean constant" simulation provides reasonable values, at least for residual fertility less than or equal to 0.6%, that can help to evaluate the maximal duration of the massive releases with and without mechanical control. However, this approach does not provide any information on the best period to start the SIT treatment to reduce the duration of the massive releases and, thus, optimize the sterile males production.

3. In the third case, we consider, like in many other so-called "realistic" models, temperature-dependent parameters. We use the approach developed in [8], where the carrying capacity, K, depends on the temperature only. It was defined as follows: the capacity is at its maximum, i.e. $K = K_{max}$, at 27^oC, which is the mean temperature at the end of the rainy season in Réunion Island. It is assumed that 15^oC corresponds to austral winter, when the precipitation is low (dry season), and the capacity is at the lowest, $K = K_{\min}$. We assumed that $K_{\min} = 0.1 \times K_{\max}$. Therefore, we assume that K increases when the temperature is goes from 15^oC to 27^oC. Then, when the temperature is above 27^oC, it is assumed that K decreases, either due to evaporation and to the fact that, in the period where high temperature occurs, heavy rains occur and can be detrimental to breeding sites, such that $K = 0.75 \times K_{\max}$ at 35^oC. Thus, a continuous relation between K and the temperature is obtained using linear interpolation.

   This leads to Fig. 6, page 25 and Fig. 7, page 26. Compared to the simulations obtained for the temperature and rainfall dependent model, the dynamics is rather different with a smoothing effect such that the amplitudes between the dry and the rainy seasons are strongly reduced: compare with Fig 4, page 23. This does not allow us to determine the best period to start the releases.

   In Table 3, page 11, we derive the average values for the duration and the total amount of sterile males to release. It is interesting to notice that the temperature-dependent model provides almost similar results whatever the level of residual fertility: compare with Table 2 to Table 1. The over-estimate of the duration might come from the way we estimate the carrying-capacity according to temperature only.

   Like in the previous cases, increasing the sterile males release rate does not provide, on average, a great benefit: compare the values in Table 3, page 11. However, on the contrary, mechanical control might have an interesting impact.

Table 2: **Simulations with constant (mean) parameters: massive releases duration and total amount of sterile males to release: : (a) 0% of Residual Fertility; (b) 0.6% of Residual Fertility; (c) 1.2% of Residual Fertility**

(a)

$\varepsilon = 0$	6000 Ind/ha		12000 Ind/ha	
	Number of releases	Total amount of sterile males released	Number of releases	Total amount of sterile males released
0% of MC	58	6 960 000	48	11 520 000
20% of MC	53	6 360 000	46	11 040 000
40% of MC	48	5 760 000	43	10 320 000

(b)

$\varepsilon = 0.006$	6000 Ind/ha		12000 Ind/ha	
	Number of releases	Total amount of sterile males released	Number of releases	Total amount of sterile males released
0% of MC	68	8 160 000	58	13 920 000
20% of MC	62	7 440 000	54	12 960 000
40% of MC	57	6 840 000	51	12 240 000

(c)

$\varepsilon = 0.012$	6000 Ind/ha		12000 Ind/ha	
	Number of releases	Total amount of sterile males released	Number of releases	Total amount of sterile males released
0% of MC	93	11 116 000	79	18 960 000
20% of MC	84	10 080 000	74	17 760 000
40% of MC	76	9 120 000	69	16 560 000

The main disappointment with this temperature-dependent only model comes from the fact that the inter-annual periodic behavior, for which the best period to start SIT would (roughly) be between May and September. This is exactly the same conclusion reached by entomologists.

4. In the fourth and last case, we consider the average values for the parameters related to temperature, but rainfall is taken into account following the description given in the previous section. The simulations are provided in Figs 8, page 27, and 9, page 28.

   It is interesting to notice that, like in Figs. (4) and (5), we recover larger amplitude in the oscillations between rainy and dry periods. This allows to capture years where rainfall was more abundant than usual (like in 2010 and 2011, for instance), and, also to capture, like for the temperature and rainfall-dependent model, the best periods to start SIT.

   Mechanical control already has an impact on massive duration treatment

   However, while for small residual fertility, less than or equal to 0.6%, the rainfall-model seems to provide an equivalent result to the full model, for large residual fertility, namely 1.2%, the results seem to be worst, and, even worse than those obtained for the average parameters model (case 2).

To summarize: from the four cases, we derive contrasted results. On average, and for small residual fertility, we obtain almost similar results: see Tables 2, Table 3, Table 4, and Table 1. In particular, the average (constant) model is not so bad. This is very interesting, because, in general, biologists often criticize the use of constant parameters models for practical applications. However, since we are not yet capable of predicting weather data accurately, it is important to have an initial estimate for the duration of the massive treatment and then refine this estimate while the process is in progress.

Based on the average values given in the previous Tables, we can estimate, roughly at least, the duration and the amount production of sterile males needed for the massive releases to reduce the nuisance. In general, the amount of sterile males to produce is considerable. However, all along the treatment, the size of the massive releases can be adapted to the wild male population, using Mark-Release-Recapture experiments, as well as the periodicity of the releases, like in [4, 31].

Last, if the residual fertility is too large, say to 2%, then it is impossible to reduce (in a reasonable amount of time) the wild population under the given threshold, such that the wild population can be controlled with small releases of $\tau\Lambda_M = 20 \times 100$ individuals. The recommendation is thus to improve the sterilization process to have a residual fertility as small as possible, at least, less than 0.6%, for instance.

Our simulations took place in the context of nuisance reduction, i.e. to reduce the wild mosquito population in order to reduce the number of bites. It is not always necessary to reach this objective. In particular, in a tropical context, where people are used to mosquitoes. There, the most important goal is to reduce the epidemiological

Table 3: **Simulations with temperature-dependent parameters only: massive releases duration and total mean amount of sterile males to release over the 20 hectares: (a) 0% of Residual Fertility; (b) 0.6% of Residual Fertility; (b) 1.2% of Residual Fertility**

(a)

$\varepsilon = 0$	6000 Ind/ha		12000 Ind/ha	
	Mean Number of releases	Mean Total amount of sterile males released	Mean Number of releases	Mean Total amount of sterile males released
0% of MC	68	8 160 000	54	12 960 000
20% of MC	61	7 320 000	51	12 240 000
40% of MC	55	6 600 000	48	11 576 000

(b)

$\varepsilon = 0.006$	6000 Ind/ha		12000 Ind/ha	
	Mean Number of releases	Total amount of sterile males released	Mean Number of releases	Mean Total amount of sterile males released
0% of MC	84	10 080000	68	16 320000
20% of MC	76	9 120000	64	15 360000
40% of MC	68	8 160000	60	14 400000

(c)

$\varepsilon = 0.012$	6000 Ind/ha		12000 Ind/ha	
	Mean Number of releases	Total amount of sterile males released	Mean Number of releases	Mean Total amount of sterile males released
0% of MC	174	10080000	153	16320000
20% of MC	162	9120000	147	15360000
40% of MC	151	8160000	140	14400000

risk.

3.2 Reducing the epidemiological risk

In the previous simulations, we derive numerics to lower the mosquito population under a given release threshold for sterile males, for instance, 100 Ind/ha/week. As explained, in a tropical context, another option is to reduce the population in order to reduce the epidemiological risk, to prevent the risk of an epidemic. Since Dengue is often circulating in La Réunion, we can couple our entomological model with a Dengue model, like the one developed in [32]. According to the epidemiological model developed in [32], and recalled in Appendix B, we derive the following formula for the SIT basic reproduction number

$$\mathcal{R}^2_{0,SIT} = \frac{\nu_m}{\nu_m + \mu_F} \frac{B\beta_{mh}}{\mu_F} \frac{B\beta_{hm}}{\eta_h + \mu_h} \frac{F^*_S}{N_h}, \tag{11}$$

where F^*_S is the amount of susceptible adult females at the Disease Free Equilibrium (DFE) equilibrium, estimated according to the value taken by $\tau\Lambda_{Massive}$. In [32], the authors showed that F_S is either strictly positive or equal to zero, depending on the amount of sterile males released. The positive parameters μ_h and $1/\nu_h$ represent respectively the average human mortality rate and the average viremic period. Since no disease-induced mortality is considered, the total human population is supposed to be constant and equal to N_h. The average rate of mosquito bites per individual is denoted $B > 0$, and $\beta_{mh} > 0$ ($\beta_{hm} > 0$) is the probability of dengue transmission from an infected female mosquito (human) to a susceptible human (mosquito) during such an event. The positive parameter $\nu_m > 0$ is the extrinsic incubation rate (EIR). However, it is well known that the previous parameters, ν_m, β_{hm}, and β_{mh}, are Temperature-dependent. Using results from [33], we consider the following transmission probability for an infected *Aedes albopictus* to transmit DENV-2, using a Lactin-1 function

$$\beta_{mh}(T) = \exp(\alpha \times T) - \exp\left(\alpha \times T_{max} - \frac{(T_{max} - T)}{\delta_T})\right),$$

with $\alpha = 0.20404$, $T_{max} = 37.354$, and $\delta_T = 4.89694$. The probability of transmission from humans to mosquitoes is negligible for "low" temperatures, increases linearly to one at a maximum temperature and remains at one for

Table 4: **Simulations with constant (average) temperature and rainfall-dependent parameters: massive releases duration and total mean amount of sterile males to release over the 20 hectares: (a) 0% of Residual Fertility; (b) 0.6% of Residual Fertility; (b) 1.2% of Residual Fertility**

	6000 Ind/ha		12000 Ind/ha	
$\varepsilon = 0$	Mean number of releases	Total mean amount of sterile males released	Mean number of releases	Total mean amount of sterile males released
0% of MC	52	6 240 000	45	10 800 000
20% of MC	48	5 760 000	42	10 080 000
40% of MC	44	5 280 000	40	9 600 000

(b)

	6000 Ind/ha		12000 Ind/ha	
$\varepsilon = 0$	Mean number of releases	Total mean amount of sterile males released	Mean number of releases	Total mean amount of sterile males released
0% of MC	60	7 200 000	52	12 480 000
20% of MC	55	6 600 000	49	11 760 000
40% of MC	51	6 120 000	46	11 040 000

(c)

	6000 Ind/ha		12000 Ind/ha	
$\varepsilon = 0.012$	Mean number of releases	Total mean amount of sterile males released	Mean number of releases	Total mean amount of sterile males released
0% of MC	75	9 000 000	65	15 600 000
20% of MC	69	8 280 000	61	14 640 000
40% of MC	63	7 560 000	57	13 680 000

higher temperatures. Thus, following [34], we consider

$$\beta_{hm}(T) = \frac{T^7}{T^7 + \beta_h^7},$$

with $\beta_h = 18.9871$. Last but not least, the EIP (Extrinsic incubation Period) decreases according to the temperature. Using again [33], we derive the following interpolation:

$$\nu_m(T) = aT^2 + bT + c,$$

with $a = -0.001$, $b = 0.0670$, and $c = -0.866$. All other (epidemiological) parameters are supposed to be constant.

The threshold $\mathcal{R}^2_{0,SIT_c}$ is related to the long-time behavior of the system. For practical purpose, we will consider $\mathcal{R}_{eff}$, the effective reproduction number, that is defined as follow

$$\mathcal{R}_{eff}(t) = \frac{\nu_m(t)}{\nu_m(t) + \mu_F(t)} \frac{B^2 \beta_{mh}(t)\beta_{hm}(t)}{\mu_F(t)\,(\eta_h + \mu_h)} \frac{F_S(t)}{N_h}. \tag{12}$$

In fact, assuming that t_{DENV} is the time where a DENV virus starts circulating, we will estimate $\mathcal{R}_{eff}$ at time t_{DENV}. Clearly, if $\mathcal{R}_{eff}(t_{DENV}) < 1$ and $\mathcal{R}^2_{0,SIT_c} < 1$, then no epidemics will occur. In contrary, even if $\mathcal{R}^2_{0,SIT_c} < 1$ but $\mathcal{R}_{eff}(t_{DENV}) > 1$ then an outbreak may occur.

In the forthcoming simulations, we will estimate the time needed to lower $\mathcal{R}_{eff}$ below 0.5 for different sizes of massive releases, i.e. 6000 or 12000 sterile males per ha, and for different residual fertility, with and without mechanical control. Thus, it suffices to find t^* such that the wild female population verifies

$$F(t^*) < \frac{\nu_m(t^*) + \mu_F(t^*)}{\nu_m(t^*)} \frac{\mu_F\,(\eta_h + \mu_h)}{B^2 \beta_{mh}(t^*)\beta_{hm}(t^*)} \frac{N_h}{2}, \tag{13}$$

for a given N_h. For the numerical simulations, we will consider the parameters values given in Table 5, page 13.

In Fig. 10, page 29, and Fig. 11, page 30, for a given SIT starting date, we compute the number of releases necessary to reach $R_{eff}(t^*) < 0.5$ for the full model. Of course, the duration of the SIT control. When $N_h = 2000$, it is very interesting to see that the results differ from the previous objective of reducing the nuisance. Indeed,

Table 5: ***DENV* epidemiological constant parameter values [32]**

Symbol	B	μ_h	η_h	N_h
Value	0.2	$\frac{1}{365 \times 78}$	1/7	2000
Unit	day^{-1}	day^{-1}	day^{-1}	Ind

depending on the year and the period within the year, the number of releases varies between 2 and 34 (22), when 6000 (12000) sterile males are released every week, whatever the residual fertility. This shows that the objective of reducing the epidemiological risk is easier to reach, even when residual fertility occurs than reducing the nuisance. In addition, releasing more sterile males is only beneficial in years where the wild population is very large (2010, 2011, for instance), but globally there is no important gain. Last but not least, residual fertility is not really an issue here, and, again, mechanical control is only useful when the wild population is large. Otherwise, when the population is small (from September to December), SIT alone could work.

Simulations with the temperature-only and the rainfall-only dependent models are also given in Fig. 12, page 31 and Fig. 13, page 32, for a weakly release rate of 6 000 $Indiv/ha$. We recover the same results as with the full model, i.e. a very low impact of the residual fertility; in some periods, reaching $R_{eff}(t^*) < 0.5$ can be fast

Like in the nuisance reduction section, it is to compare the results obtained with the full model with results obtained with the temperature-dependent or rainfall-dependent parameters SIT model. As expected from the previous computations the amplitudes of oscillations with this temperature-dependent model are small compared to the rainfall-temperature model, while they are almost similar with the rainfall-dependent model. Thus, the number of releases varies between 20 (14) and 34 (23) at most, when 0% (40%) mechanical control occur: see Fig. 12_1. Also, the simulations in Fig. 12, page 31, confirm that the residual fertility has less negative effect to reach $\mathcal{R}_{eff} < 0.5$ than to reach the objective of nuisance reduction. Still, with 1.2% residual fertility, the amount of releases varies between 21 (14) and 36 (24), almost the same values as those obtained with 0% residual fertility.

In fact, the results provided by the temperature-only model are quite acceptable for periods where the mosquito population is large but seems not so accurate to derive the appropriate amount of releases when the population is (strongly) regulated by rainfall, i.e. in periods where there is a rain deficit.

We also derive Table 6, page 14, for the average values model in order to compare with the estimates obtained for the temperature-dependent model (Table 7, page 15), the rainfall-dependent model (Table 8, page 16), and the temperature and rainfall dependent model (Table 9, page 17). The conclusions are almost the same: the impact of residual fertility is (very) low, as well as the impact of mechanical control and very massive releases (12 000 MS/ha). However, of course, this approach can not provide the best period to start in order to minimize the amount of the release.

Of course, once $\mathcal{R}(t^*) < 0.5$ is reached, it will be necessary to continue to release enough sterile males to maintain $\mathcal{R}(t^*)$ below 0.5, as long as needed. This can be evaluated through numerical simulations.

Remark 1. *In fact for this particular objective of reducing the epidemiological risk, and because in La Réunion we have some seasonality, the meaningful strategy, as already proposed in [8, Fig.4], would be to consider massive releases only over the wet period to maintain the mosquito population at a level corresponding to the population size during the dry period, the Austral winter, where, in general, the epidemiological risk is low because the vector population is low, except, of course, when rainfalls occur during this period, like in 2010 and 2011. Thus, we would have a "massive and stop" SIT strategy, contrary to the nuisance reduction where a "massive and small" releases strategy seems more appropriate. According to the temperature and rainfall dependent model, the duration of the massive releases could occur from October-November to May-June, that is over* 8 *months, with, eventually, a reduction of the size of the massive releases, using, for instance, a closed-loop control strategy*

4 Conclusion

We build a minimalistic rainfall-temperature entomological model to derive the dynamics of *Aedes albopictus* population in the place of Duparc (Sainte-Marie), La Réunion island. Since we are in a sub-tropical environment, our simulation shows that rainfall definitively plays a great role in the dynamics of the mosquito population, with rapid decay or growth. In fact, it seems that they are periods where the mosquito dynamics are mainly driven by temperature, while in other periods the dynamic is more driven by rainfall. Other approaches, based either on constant, temperature-dependent or rainfall-dependent parameter values, provide similar results on average. However, they are not satisfactory enough to be an "accurate" alternative.

Quality control within SIT is an important issue: if it fails, then release cannot occur. Within the quality control, we focus on residual fertility. We show that it may have an important impact on SIT duration and,

Table 6: **Reduction of the epidemiological risk. Simulations with the mean values parameters: massive releases duration and total mean amount of sterile males to release over the 20 hectares: (a) 0% of Residual Fertility; (b) 0.6% of Residual Fertility; (b) 1.2% of Residual Fertility**

	6000 Ind/ha		12000 Ind/ha	
$\varepsilon = 0$	Mean number of releases	Total mean amount of sterile males released	Mean number of releases	Total mean amount of sterile males released
0% of MC	23	2 760 000	18	4 320 000
20% of MC	19	2 280 000	16	3 840 000
40% of MC	16	1 920 000	14	3 360 000

(b)

	6000 Ind/ha		12000 Ind/ha	
$\varepsilon = 0.006$	Mean number of releases	Total mean amount of sterile males released	Mean number of releases	Total mean amount of sterile males released
0% of MC	23	2 760 000	18	4 320 000
20% of MC	20	2 240 000	16	3 840 000
40% of MC	17	2 040 000	14	3 360 000

(c)

	6000 Ind/ha		12000 Ind/ha	
$\varepsilon = 0.012$	Mean number of releases	Total mean amount of sterile males released	Mean number of releases	Total mean amount of sterile males released
0% of MC	24	2 880 000	19	4 560 000
20% of MC	20	2 400 000	17	4 080 000
40% of MC	17	2 040 000	15	3 600 000

eventually on its efficacy. In nuisance reduction, the lower the residual fertility, the lower the number of massive releases needed to switch from a massive releases strategy to a small releases strategy. However our results show that the massive releases duration for "high" residual fertility can greatly vary depending if the model takes into account the temperature, the rainfall or both. Also, residual fertility seems to be less detrimental when it comes to consider a SIT strategy to reduce the epidemiological risk.

When SIT is considered, coupled or not with Mechanical control, we recommend starting the releases of sterile males within a period that last from July to December, when the mosquito population is, in general, at its lowest. However, the duration of the SIT treatment or the number of periodic releases, constant in size, of sterile males may vary thanks to the environmental parameters.

Clearly, combining Mechanical control with SIT is strongly recommended, in particular when the SIT treatment starts within a humid year: see for instance the years 2010 and 2011 where the dry period was more rainy than usual.

Last, the massive-small releases strategy is only useful to reduce substantially the nuisance due to mosquitoes, i.e. to reach nearly elimination and to maintain the population under a certain level, related, here, with the number of sterile male individuals considered in the small releases. If the objective of SIT control is only to reduce the epidemiological risk, i.e. $\mathcal{R}_{eff} < 0.5$, then the duration of the SIT treatment, with massive releases, will be short compared to elimination. In addition, having non-zero residual fertility, as long as $\varepsilon < 1/\mathcal{N}$, seems to be less problematic to reach $\mathcal{R}_{eff} < 0.5$, than to reach elimination. From a practical point of view, this can be very convenient. However, once $\mathcal{R}_{eff} < 0.5$ or elimination is reached, it will be necessary to continue to release a sufficient amount of sterile males in order to keep the wild population under a certain threshold. At this stage, a closed-loop control (taking into account feedback from the system, like the size of the mosquito population through Mark Release and Recapture experiments) can be used from time to time, in order to reduce the overall cost, as described in [4, 31].

Our model, while minimalistic from the variables and parameters point of view, captures relatively well the dynamics of the *Aedes albopictus* population along the year, and this without too many details. However, as with all models, improvements could be made. For instance, by taking into account the migration of males and females from neighboring places. Indeed, we have considered our area closed, in the sense that no external mosquitoes can invade the treated area. Migration is another big concern in SIT treatment. We recently showed that if

Table 7: **Reduction of the epidemiological risk. Simulations with temperature-dependent parameters: massive releases duration and total mean amount of sterile males to release over the 20 hectares: (a) 0% of Residual Fertility; (b) 0.6% of Residual Fertility; (b) 1.2% of Residual Fertility**

	6000 Ind/ha		12000 Ind/ha	
$\varepsilon = 0$	Mean number of releases	Total mean amount of sterile males released	Mean number of releases	Total mean amount of sterile males released
0% of MC	29	3 480 000	21	5 040 000
20% of MC	24	2 880 000	18	4 320 000
40% of MC	20	2 400 000	16	3 840 000

(b)

	6000 Ind/ha		12000 Ind/ha	
$\varepsilon = 0.006$	Mean number of releases	Total mean amount of sterile males released	Mean number of releases	Total mean amount of sterile males released
0% of MC	29	3 480 000	21	5 040 000
20% of MC	24	2 880 000	19	4 560 000
40% of MC	20	2 400 000	16	3 840 000

(c)

	6000 Ind/ha		12000 Ind/ha	
$\varepsilon = 0.012$	Mean number of releases	Total mean amount of sterile males released	Mean number of releases	Total mean amount of sterile males released
0% of MC	30	3 600 000	22	5 280 000
20% of MC	25	3 000 000	19	4 560 000
40% of MC	20	2 400 000	17	4 080 000

migration is little, then SIT treatment can work [5]. Otherwise, it is mandatory to isolate the targeted domain. This is not so easy with mosquitoes because, so far, no (killing) attractant is efficient.

Last, we assume that no sterile females are released. In fact, we know that this is not the case: there is always a (small) percentage of sterile females that are released. If, in the past, only less than 5% of sterile females was acceptable [35], this is not the case now. IAEA recommends not to release more than 1% of sterile females. This is part of the control quality process. Thus, following [32], accidental releases of females could be taken into account, in order to derive how they could impact the SIT releases strategy along a year, from the nuisance reduction or epidemiological point of view.

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Table 8: **Reduction of the epidemiological risk. Simulations with rainfall-dependent parameters: massive releases duration and total mean amount of sterile males to release over the** 20 **hectares: (a)** 0% **of Residual Fertility; (b)** 0.6% **of Residual Fertility; (b)** 1.2% **of Residual Fertility**

	6000 Ind/ha		12000 Ind/ha	
$\varepsilon = 0$	Mean number of releases	Total mean amount of sterile males released	Mean number of releases	Total mean amount of sterile males released
0% of MC	21	2 520 000	17	4 080 000
20% of MC	18	2 160 000	15	3 600 000
40% of MC	15	1 800 000	13	3 120 000

(b)

	6000 Ind/ha		12000 Ind/ha	
$\varepsilon = 0.006$	Mean number of releases	Total mean amount of sterile males released	Mean number of releases	Total mean amount of sterile males released
0% of MC	22	3 480 000	17	4 080 000
20% of MC	19	2 880 000	15	3 600 000
40% of MC	16	2 400 000	13	3 120 000

(c)

	6000 Ind/ha		12000 Ind/ha	
$\varepsilon = 0.012$	Mean number of releases	Total mean amount of sterile males released	Mean number of releases	Total mean amount of sterile males released
0% of MC	22	3 600 000	17	4 080 000
20% of MC	19	3 000 000	16	3 840 000
40% of MC	16	2 400 000	14	3 360 000

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Table 9: **Reduction of the epidemiological risk. Simulations with temperature and rainfall dependent parameters: massive releases duration and total mean amount of sterile males to release over the** 20 **hectares: (a)** 0% **of Residual Fertility; (b)** 0.6% **of Residual Fertility; (b)** 1.2% **of Residual Fertility**

	6000 Ind/ha		12000 Ind/ha	
$\varepsilon = 0$	Mean number of releases	Total mean amount of sterile males released	Mean number of releases	Total mean amount of sterile males released
0% of MC	20	2 400 000	16	3 840 000
20% of MC	17	2 040 000	14	3 360 000
40% of MC	14	1 680 000	12	2 880 000

(b)

	6000 Ind/ha		12000 Ind/ha	
$\varepsilon = 0.006$	Mean number of releases	Total mean amount of sterile males released	Mean number of releases	Total mean amount of sterile males released
0% of MC	20	2 400 000	16	3 840 000
20% of MC	17	2 040 000	14	3 360 000
40% of MC	15	1 800 000	13	3 120 000

(c)

	6000 Ind/ha		12000 Ind/ha	
$\varepsilon = 0.012$	Mean number of releases	Total mean amount of sterile males released	Mean number of releases	Total mean amount of sterile males released
0% of MC	21	2 520 000	16	3 840 000
20% of MC	18	2 160 000	14	3 360 000
40% of MC	15	1 800 000	13	3 120 000

Table 10: **Description of the epidemiological and entomological parameters**

Symbol	Description	Unit
$1/\mu_h$	Average lifespan of human	Day
$1/\eta_h$	Average viremic period	Day
B	Daily average mosquito bites	Day^{-1}
β_{mh}	Transmission probability from infected mosquito	-
β_{hm}	Transmission probability from infected human	-
ϕ	Number of eggs at each deposit per capita	Day^{-1}
γ	Maturation rate from larvae to adult	Day^{-1}
$\mu_{A,1}$	Density independent mortality rate of the aquatic stage	Day^{-1}
$\mu_{A,2}$	Density dependent mortality rate of the aquatic stage	Day^{-1} Individuals $^{-1}$
r	Sex ratio	-
$1/\nu_m$	Average extrinsic incubation period (EIP)	Day
$1/\mu_F$	Average lifespan of fertilized and eggs-laying females	Day
$1/\mu_M$	Average lifespan of males	Day
Λ_M	Sterile male release rate	Individuals $\times$ Day^{-1}

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Table 11: **Entomological parameters of *Aedes albopictus* at different temperatures (from [7])**

Symbol	Name	T=15^o	T=20^o	T=25^o	T=30^o	T=35^o
r_{viable}	Proportion of viable eggs (E-L1)	8.2	66.9	49.2	51.4	10
N_{eggs}	Number of eggs deposited	0	50.8	65.3	74.2	48.7
τ_{gono}	Duration of the gonotrophic cycle	NA	8.1	3.1	3.9	1.3
τ_A	Time from hatching to emergence	35	14.4	10.4	8.8	12.3
s_A	Survivorship from larva first instar to adult	50	77.5	76.3	67.5	2.5
τ_M	Adult male half-life	15.45	10.25	9.6	8.55	7.4
τ_F	Adult female half-life	19.65	15.15	15.3	16.9	10

Table 12: **Parameter values for system** (2) **deduced from Table 11, page 19**

Symbol Symbol	Name Name	Formula Formula at	mean value at 15^o	mean value at 20^o	mean value at 25^o	mean value at 30^o	mean value at 35^o
ϕ	Effective fecundity	$\frac{r_{viable}N_{eggs}}{\tau_{gono}}$	0	4.1957	10.3637	9.7792	3.7462
$\mu_{A,1}$	Aquatic death rate	$-\frac{log(s_A)}{\tau_A}$	0.0198	0.0177	0.0260	0.0447	0.2999
ν_A	Aquatic to adult transition rate	$\frac{1}{\tau_A}$	0.0286	0.0694	0.0962	0.1136	0.0813
μ_M	Adult male death rate	$\frac{\log(2)}{\tau_M}$	0.0449	0.0676	0.0722	0.0811	0.0937
μ_F	Adult female death rate	$\frac{log(2)}{\tau_F}$	0.0353	0.0458	0.0453	0.0413	0.0693

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5 Appendix: Parameters values

See Table 11, page 19, and Table 12, page 19.

6 Appendix A: Equilibria

- Without release

In the case of constant coefficients, the equilibria of System (2) are 0 and

$$\begin{cases} A^* = \dfrac{\gamma+\mu_{A,1}}{\mu_{A,2}}(\mathcal{N}-1), \\ M^* = \dfrac{(1-r)\gamma A^*}{\mu_M} = Q(\mathcal{N}-1), \\ F^* = \dfrac{r\gamma A^*}{\mu_F}, \end{cases} \tag{14}$$

where $Q = \dfrac{(1-r)\gamma\left(\gamma+\mu_{1,A}\right)}{\mu_{2,A}\mu_M}$.

- With release

  In the case of constant coefficients, some calculations are needed to find the release threshold, $M_{T_1,\varepsilon}$ and also to derive the equilibria. We have to solve

$$\begin{cases} \phi F = \left(\gamma+\mu_{1,A}+\mu_{2,A}A\right)A, \\ (1-r)\gamma A = \mu_M M, \\ \dfrac{M+\varepsilon\beta M_T}{M+\beta M_T} r\gamma A = \mu_F F. \end{cases}$$

  Using the first and the third equalities leads to

$$\frac{M+\varepsilon\beta M_T}{M+\beta M_T} r\gamma = \frac{\mu_F}{\phi}\left(\gamma+\mu_{1,A}+\mu_{2,A}A\right),$$

  and using the fact that

$$M = \frac{(1-r)\gamma}{\mu_M}A,$$

  we have

$$\frac{(1-r)\gamma A+\varepsilon\mu_M\beta M_T}{(1-r)\gamma A+\mu_M\beta M_T} r\gamma = \frac{\mu_F}{\phi}\left(\gamma+\mu_{1,A}+\mu_{2,A}A\right),$$

  that is

$$r\gamma\phi\left((1-r)\gamma A+\varepsilon\mu_M\beta M_T\right) = \mu_F\left((1-r)\gamma A+\mu_M\beta M_T\right)\left(\gamma+\mu_{1,A}+\mu_{2,A}A\right),$$

  leading to second order polynomial

$$\mu_F(1-r)\gamma\mu_{2,A}A^2 + \left[\mu_F(1-r)\gamma\left(\gamma+\mu_{1,A}\right)+\mu_F\mu_M\beta M_T\mu_{2,A}-r\gamma\phi(1-r)\gamma\right]A + \mu_F\mu_M\beta M_T\left(\gamma+\mu_{1,A}\right)-r\gamma\phi\varepsilon\mu_M\beta M_T = 0$$

$$(1-r)\gamma\mu_{2,A}A^2 + \left[(1-r)\gamma\left(\gamma+\mu_{1,A}\right)+\mu_M\beta M_T\mu_{2,A}-\frac{r\gamma\phi}{\mu_F}(1-r)\gamma\right]A + \mu_M\beta M_T\left(\gamma+\mu_{1,A}\right)-\frac{r\gamma\phi}{\mu_F}\varepsilon\mu_M\beta M_T = 0$$

$$(1-r)\gamma\mu_{2,A}A^2 + \left[(1-r)\gamma\left(\gamma+\mu_{1,A}\right)(1-\mathcal{N})+\mu_M\beta M_T\mu_{2,A}\right]A + \mu_M\beta M_T\left(\gamma+\mu_{1,A}\right)(1-\varepsilon\mathcal{N}) = 0,$$

  or equivalently

$$\frac{(1-r)\gamma}{\mu_M}A^2 - \left[Q\left(\mathcal{N}-1\right)-\beta M_T\right]A + \beta M_T\frac{\left(\gamma+\mu_{1,A}\right)}{\mu_{2,A}}\left(1-\varepsilon\mathcal{N}\right) = 0,$$

  that is, using (14),

$$\frac{(1-r)\gamma}{\mu_M}A^2 - \left[M^*-\beta M_T\right]A + \beta M_T\frac{\left(\gamma+\mu_{1,A}\right)}{\mu_{2,A}}\left(1-\varepsilon\mathcal{N}\right) = 0. \tag{15}$$

  We compute the discriminant of the last equation

$$\Delta(\varepsilon) = \left[M^*-\beta M_T\right]^2 - 4Q\beta M_T\left(1-\varepsilon\mathcal{N}\right).$$

  We will distinguish three cases:

– Assume $\varepsilon > 1/\mathcal{N}$, then $\Delta(\varepsilon) > 0$, such that there always exists one positive equilibrium

$$A^*_\varepsilon = \mu_M \frac{M^* - \beta M_T + \sqrt{\Delta(\varepsilon)}}{2(1-r)\gamma}.$$

After straightforward computations, we can show that even with very massive releases, the aquatic equilibrium is bounded from below by

$$2\frac{\varepsilon\mathcal{N}-1}{\mathcal{N}-1}A^*$$

– Assume $\varepsilon = 1/\mathcal{N}$. Then $\Delta(\varepsilon) = 0$ iff $\beta M_T = M^*$. In fact if $\beta M_T > M^*$, then $A^*_\varepsilon = 0$.

– Assume $\varepsilon < 1/\mathcal{N}$. Setting $y = \beta M_T$, we derive

$$\begin{aligned}\Delta(\varepsilon) &= [Q(\mathcal{N}-1) - y]^2 - 4Q(1-\varepsilon\mathcal{N})y \\ &= y^2 - 2Q((\mathcal{N}-1) + 2(1-\varepsilon\mathcal{N}))y + (Q(\mathcal{N}-1))^2\end{aligned}$$

The discriminant of the equation $\Delta(\varepsilon) = 0$ following the variable y is given by

$$\begin{aligned}\delta(\varepsilon) &= (2Q)^2((\mathcal{N}-1) + 2(1-\varepsilon\mathcal{N}))^2 - (2Q(\mathcal{N}-1))^2 \\ &= 16(1-\varepsilon\mathcal{N})(1-\varepsilon)Q^2\mathcal{N}.\end{aligned}$$

Since $\varepsilon < 1/\mathcal{N}$, then $\delta(\varepsilon) > 0$, and we obtain two roots

$$\beta M_{T_1,\varepsilon} = Q\left(\mathcal{N} + 1 - 2\varepsilon\mathcal{N} - 2\sqrt{(1-\varepsilon\mathcal{N})(1-\varepsilon)\mathcal{N}}\right)$$

and

$$\beta M_{T_2,\varepsilon} = Q\left(\mathcal{N} + 1 - 2\varepsilon\mathcal{N} + 2\sqrt{(1-\varepsilon\mathcal{N})(1-\varepsilon)\mathcal{N}}\right).$$

When $\varepsilon = 0$, we recover the result obtained in [3]

$$\beta M_{T_1,0} = Q\left(\sqrt{\mathcal{N}} - 1\right)^2.$$

Assume $0 < \varepsilon < \dfrac{1}{\mathcal{N}}$. Then, when $0 < M_T < M_{T_1,\varepsilon}$, we have $\Delta(\varepsilon) > 0$, and thus two equilibria

$$A_{1,\varepsilon} = \mu_M \frac{Q(\mathcal{N}-1) - \beta M_T - \sqrt{[Q(\mathcal{N}-1) - \beta M_T]^2 - 4Q\beta M_T(1-\varepsilon\mathcal{N})}}{2(1-r)\gamma},$$

and

$$A_{2,\varepsilon} = \mu_M \frac{Q(\mathcal{N}-1) - \beta M_T + \sqrt{[Q(\mathcal{N}-1) - \beta M_T]^2 - 4Q\beta M_T(1-\varepsilon\mathcal{N})}}{2(1-r)\gamma}.$$

Since

$$\begin{aligned}A_{1,\varepsilon} + A_{2,\varepsilon} &= \tfrac{\mu_M}{2(1-r)\gamma}(Q(\mathcal{N}-1) - \beta M_T) \\ &> \tfrac{\mu_M}{2(1-r)\gamma}(Q(\mathcal{N}-1) - \beta M_{T_1,\varepsilon}) \\ &= \tfrac{\mu_M}{2(1-r)\gamma}\left(Q\left(2\varepsilon\mathcal{N} + 2\left(\sqrt{(1-\varepsilon\mathcal{N})(1-\varepsilon)\mathcal{N}} - 1\right)\right)\right).\end{aligned}$$

We remark that

$$(1-\varepsilon\mathcal{N})(1-\varepsilon)\mathcal{N} - 1 = \mathcal{N} - 1 - (\mathcal{N}+1)\varepsilon + \mathcal{N}\varepsilon^2.$$

The discriminant of the last polynomial in ε is given by

$$\Delta_\varepsilon = (\mathcal{N}+1)^2 - 4\mathcal{N}(\mathcal{N}-1) = -3\mathcal{N}^2 + 6\mathcal{N} + 1$$

In general, $\mathcal{N}$ is large, i.e. $\mathcal{N} >> 1 + \frac{2}{\sqrt{3}}$, such that $\Delta_\varepsilon < 0$, such that the ε-polynomial is always positive. Then $A_{1,\varepsilon} + A_{2,\varepsilon} > 0$. Moreover, since the last term in (15), we have also $A_{1,\varepsilon}A_{2,\varepsilon} > 0$. Thus the two roots $A_{1,\varepsilon}$ and $A_{2,\varepsilon}$ are positive. Therefore the system has two equilibria $E_{1,2} = (A_{1,2}, M_{1,2}, F_{1,2})$ such that $\mathbf{0} << E_1 << E_2$.

In practice, we are, in general, in the case $\varepsilon < 1/\mathcal{N}$, see (10) and $N > 1 + \frac{2}{\sqrt{3}}$.

Appendix B: The epidemiological Model We briefly recall the SIR-SEI model of dengue transmission, studied in [32], without taking into account the accidental releases of sterile females. The evolution of the human population is given by the following SIR model, with Susceptible, Infected, and Recovered compartments:

$$\begin{cases} \dot{S}_h = \mu_h N_h - B\beta_{mh}(T) F_I \frac{S_h}{N_h} - \mu_h S_h, \\ \dot{I}_h = B\beta_{mh}(T) F_I \frac{S_h}{N_h} - (\eta_h + \mu_h) I_h, \\ \dot{R}_h = \eta_h I_h - \mu_h R_h, \end{cases} \tag{16}$$

Extending the model of evolution of the mosquito population described in the previous sections, we use here a SEI model for the wild female mosquitoes, with Susceptible, Exposed and Infected compartments, adapted from (7):

$$\begin{cases} \dfrac{dA}{dt} = \phi(T)(F_S + F_E + F_I) - (\gamma(T) + \mu_{1,A}(T) + \mu_{A,2}(T,R)A)\, A, \\ \dfrac{dM}{dt} = (1 - r(T))\gamma(T)A - \mu_M(T)M, \\ \dfrac{dF_S}{dt} = r(T)\gamma(T)\dfrac{M + \varepsilon\beta M_S}{M + \beta M_S} A - B\beta_{mh}(T) F_S \frac{I_h}{N_h} - \mu_F(T)F, \\ \dot{F}_E = B\beta_{mh}(T) F_S \frac{I_h}{N_h} - (\nu_m(T) + \mu_F(T))F_E \\ \dot{F}_I = \nu_m(T) F_E - \mu_F(T) F_I. \end{cases} \tag{17}$$

where M_S is driven by (6).

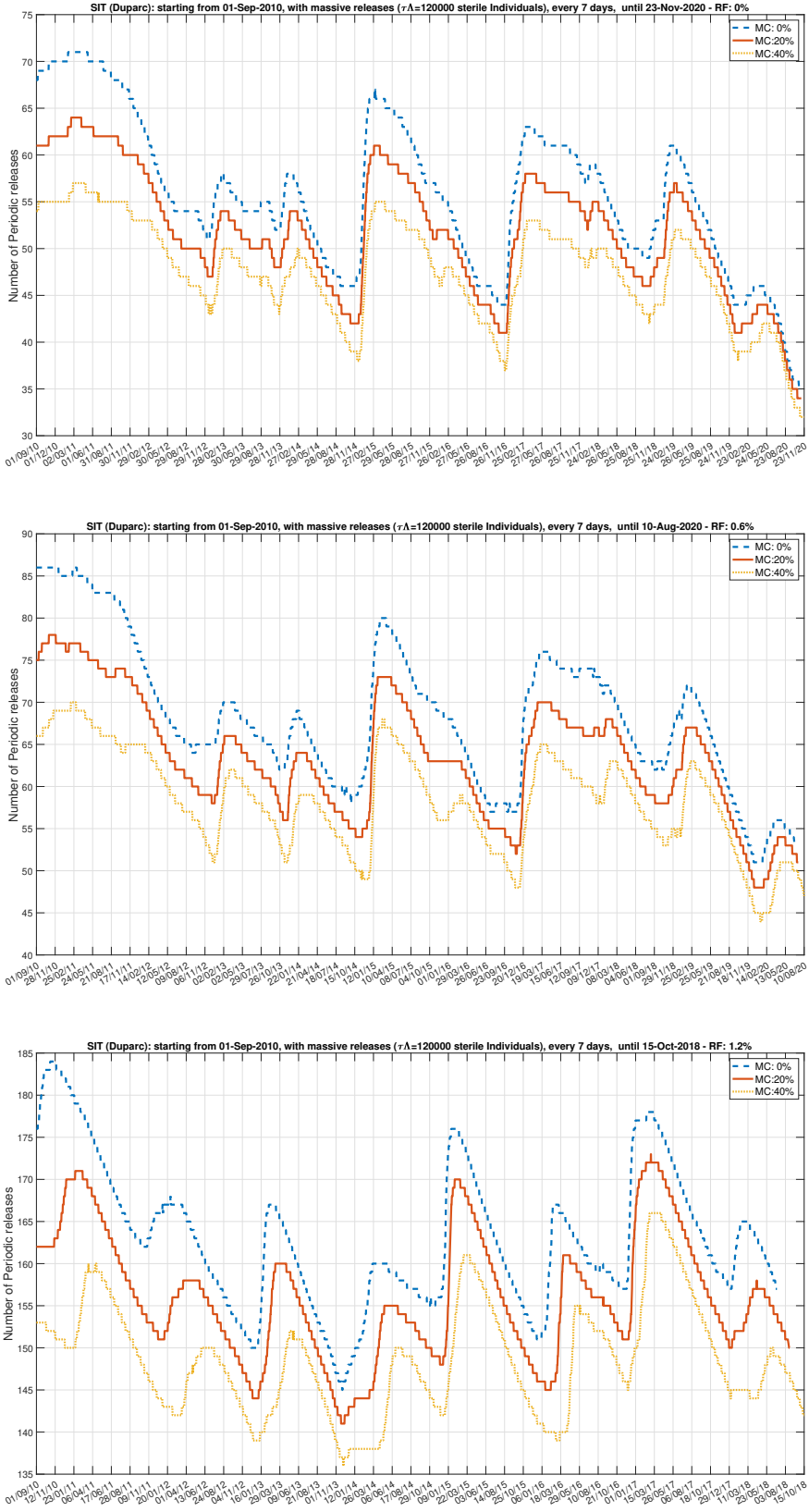

Figure 4: Temperature and rainfall dependent model - Weakly SIT control with 6000 sterile Ind/ha for various level of Mechanical control - Residual fertility: (a) 0%, (b) 0.6%, (c) 1.2%

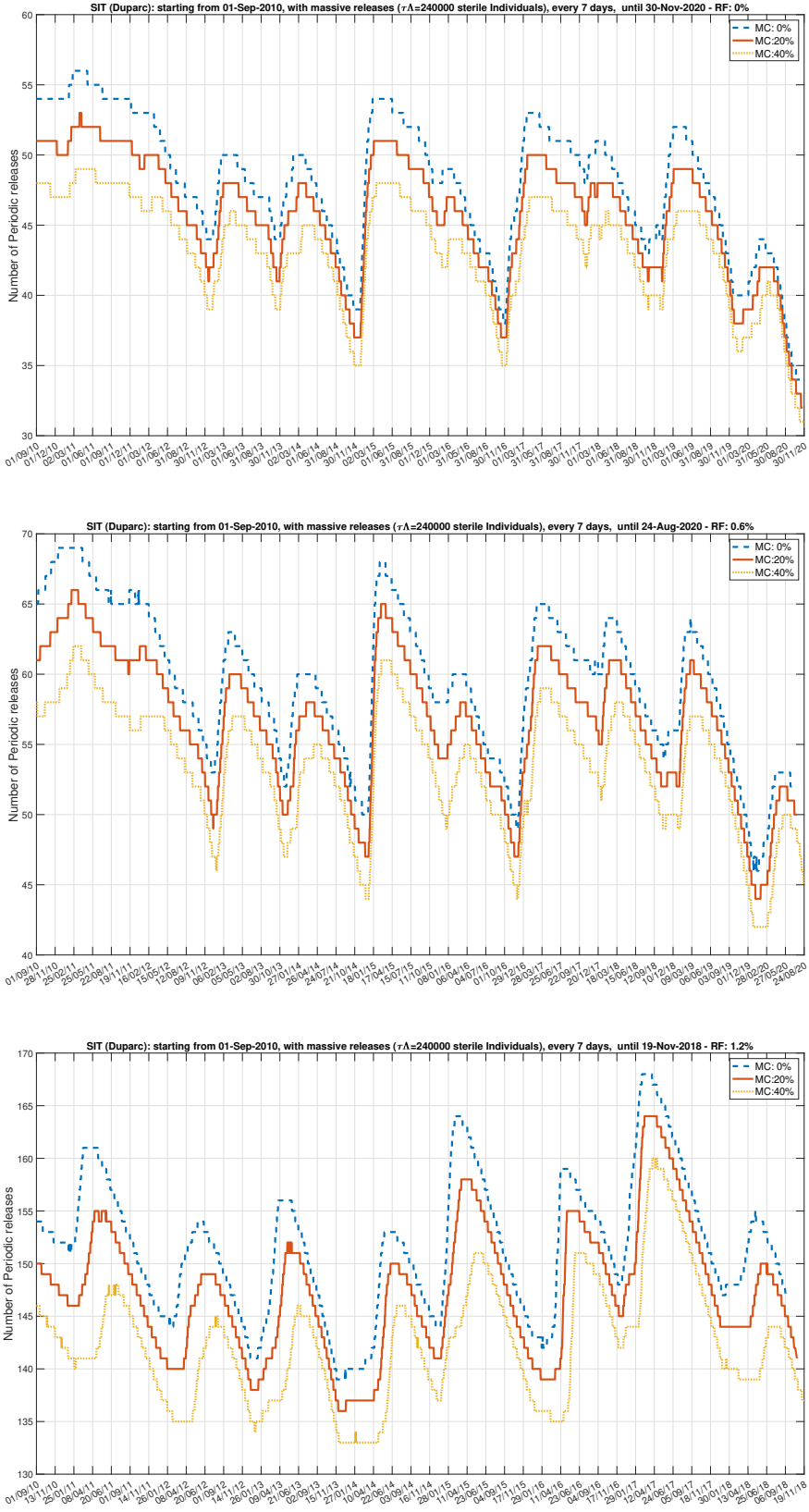

Figure 5: Temperature and rainfall dependent model - Weakly SIT control with 12000 sterile Ind/ha for various level of Mechanical control - Residual fertility: (a) 0%, (b) 0.6%, (c) 1.2%

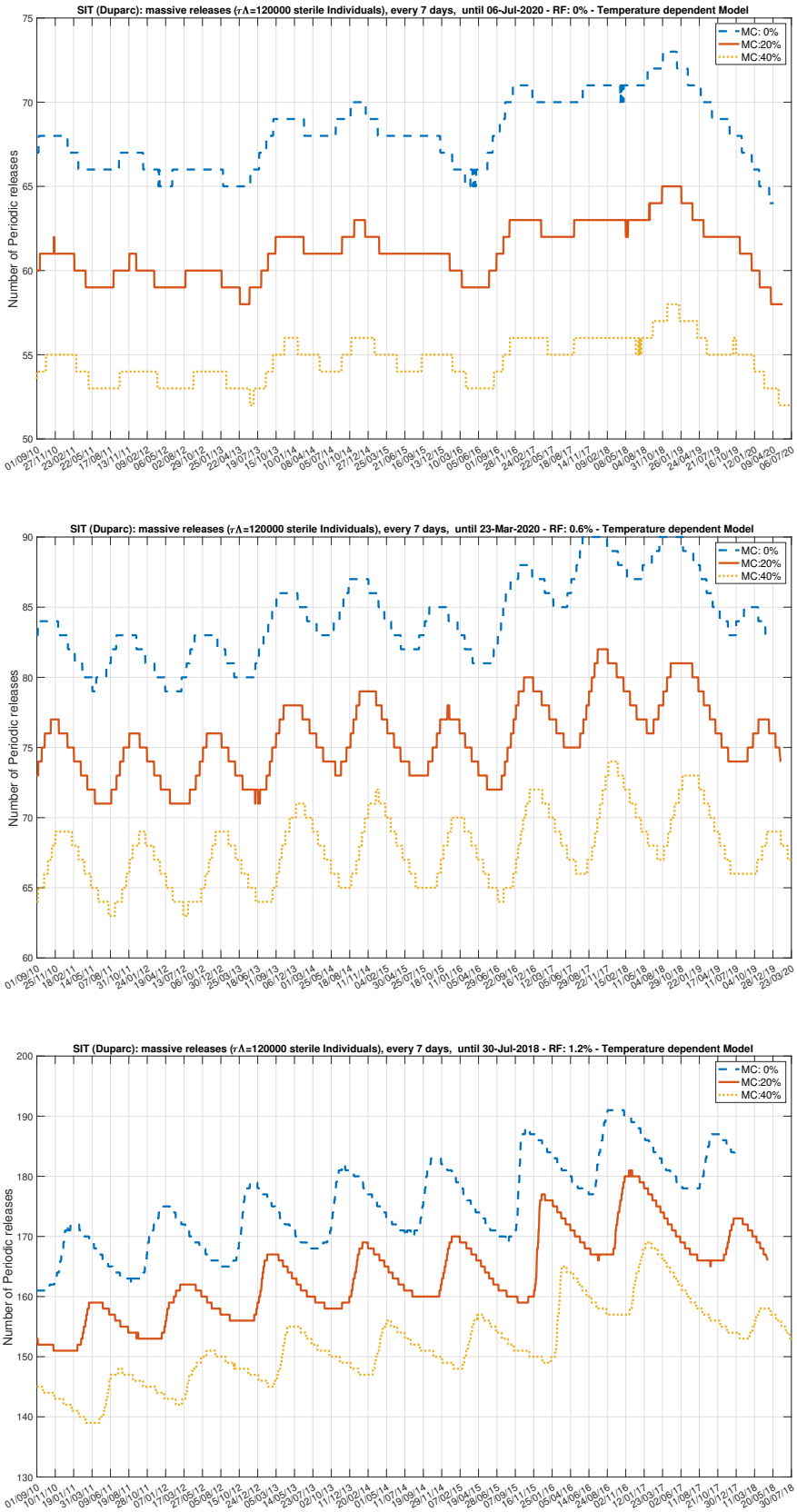

Figure 6: Temperature-dependent model - Weakly SIT control with 6000 sterile Ind/ha for various level of Mechanical control - Residual fertility: (a) 0%, (b) 0.6%, (c) 1.2%

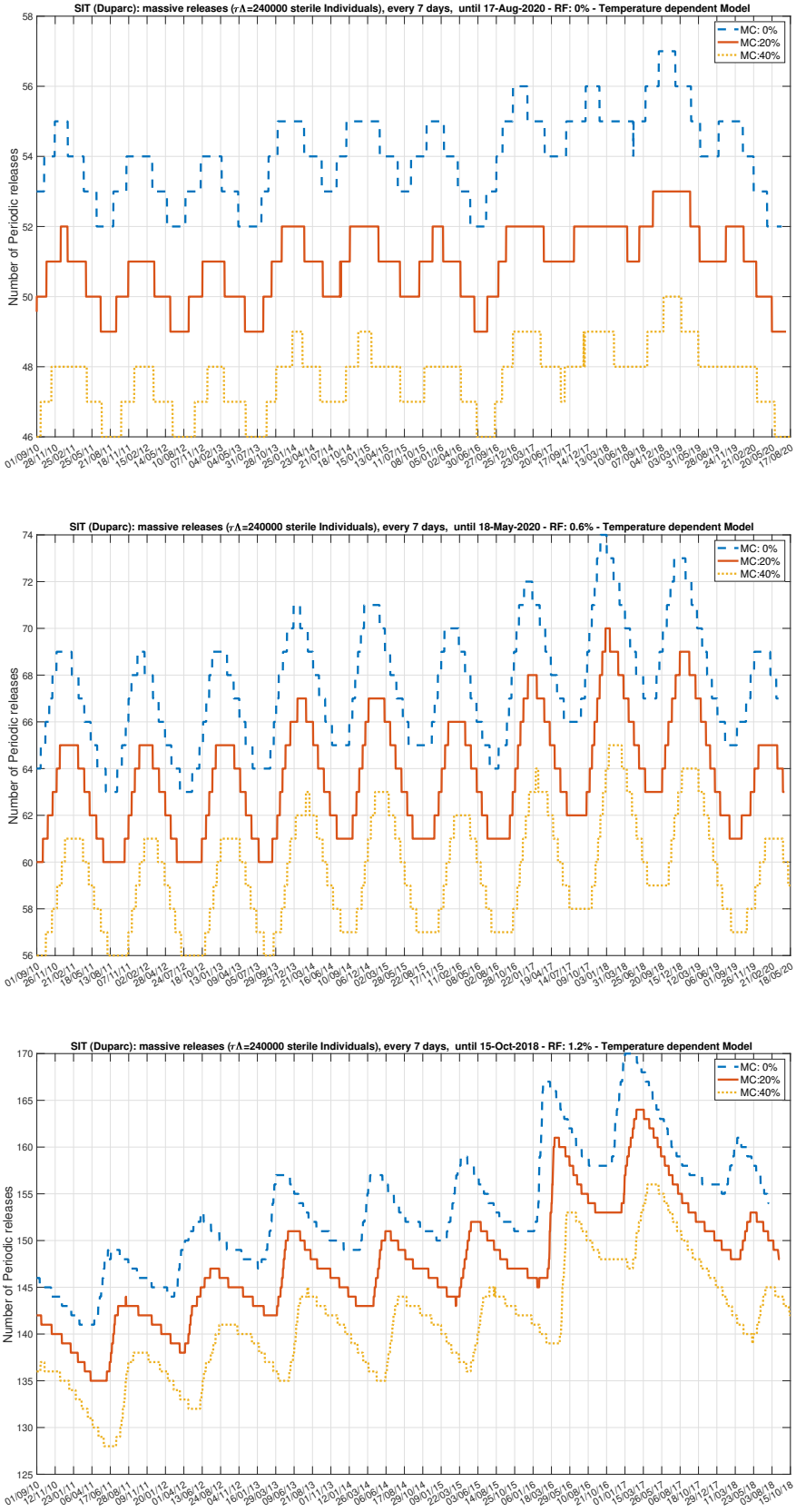

Figure 7: Temperature-dependent model - Weakly SIT control with 12000 sterile Ind/ha for various level of Mechanical control - Residual fertility: (a) 0%, (b) 0.6%, (c) 1.2%

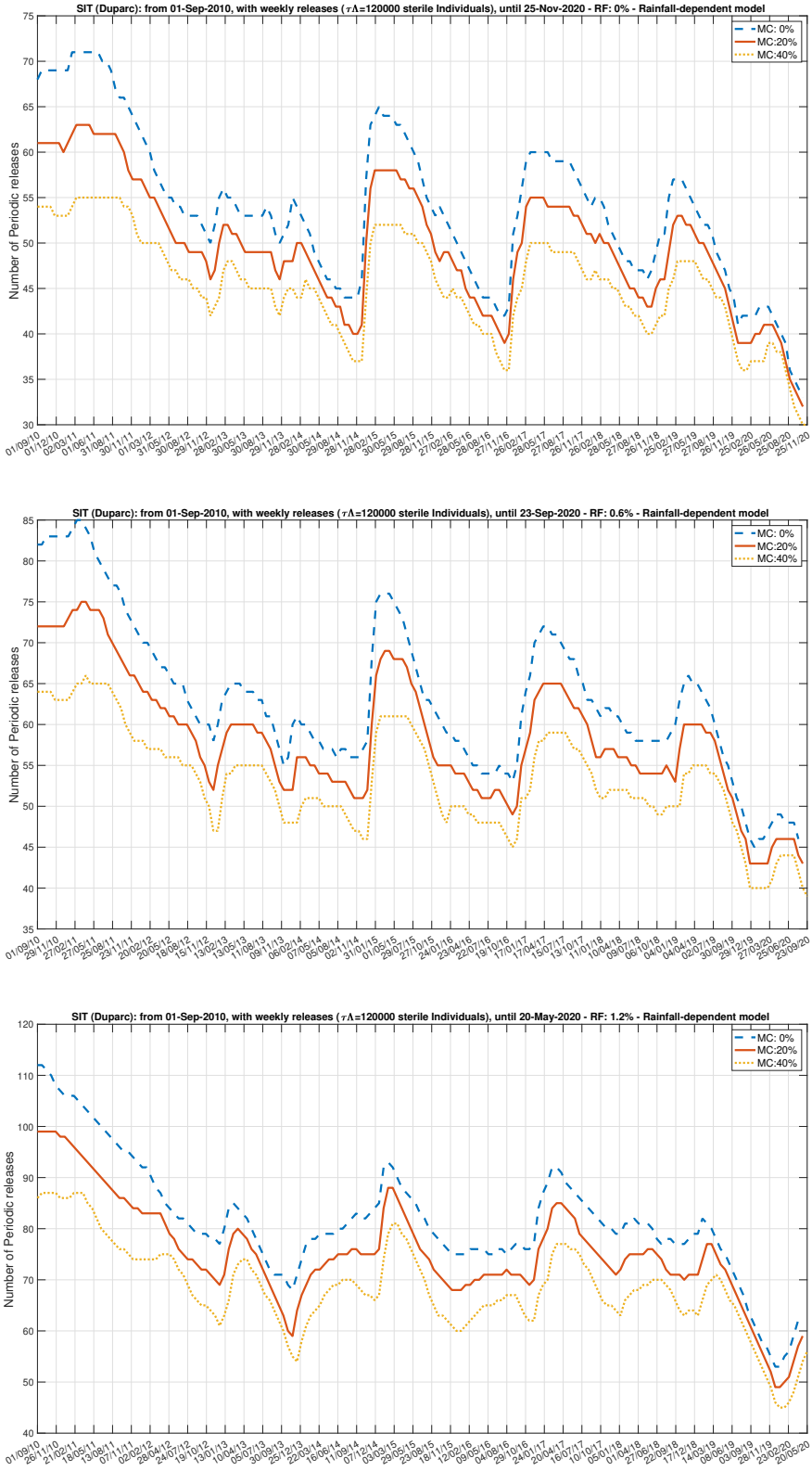

Figure 8: Rainfall-dependent model - Weakly SIT control with 6000 sterile Ind/ha for various level of Mechanical control - Residual fertility: (a) 0%, (b) 0.6%, (c) 1.2%

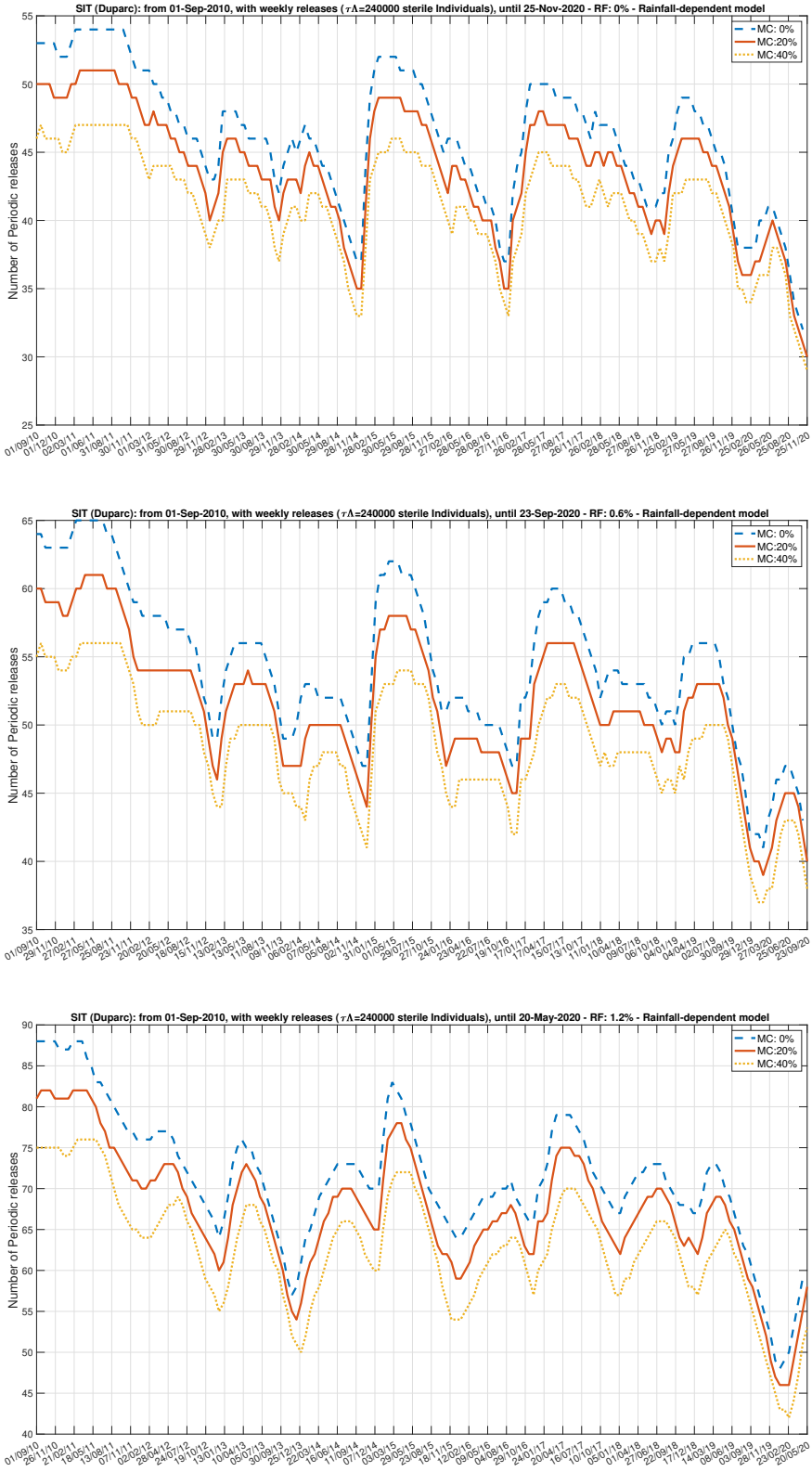

Figure 9: Rainfall-dependent model - Weakly SIT control with 12000 sterile Ind/ha for various level of Mechanical control - Residual fertility: (a) 0%, (b) 0.6%, (c) 1.2%

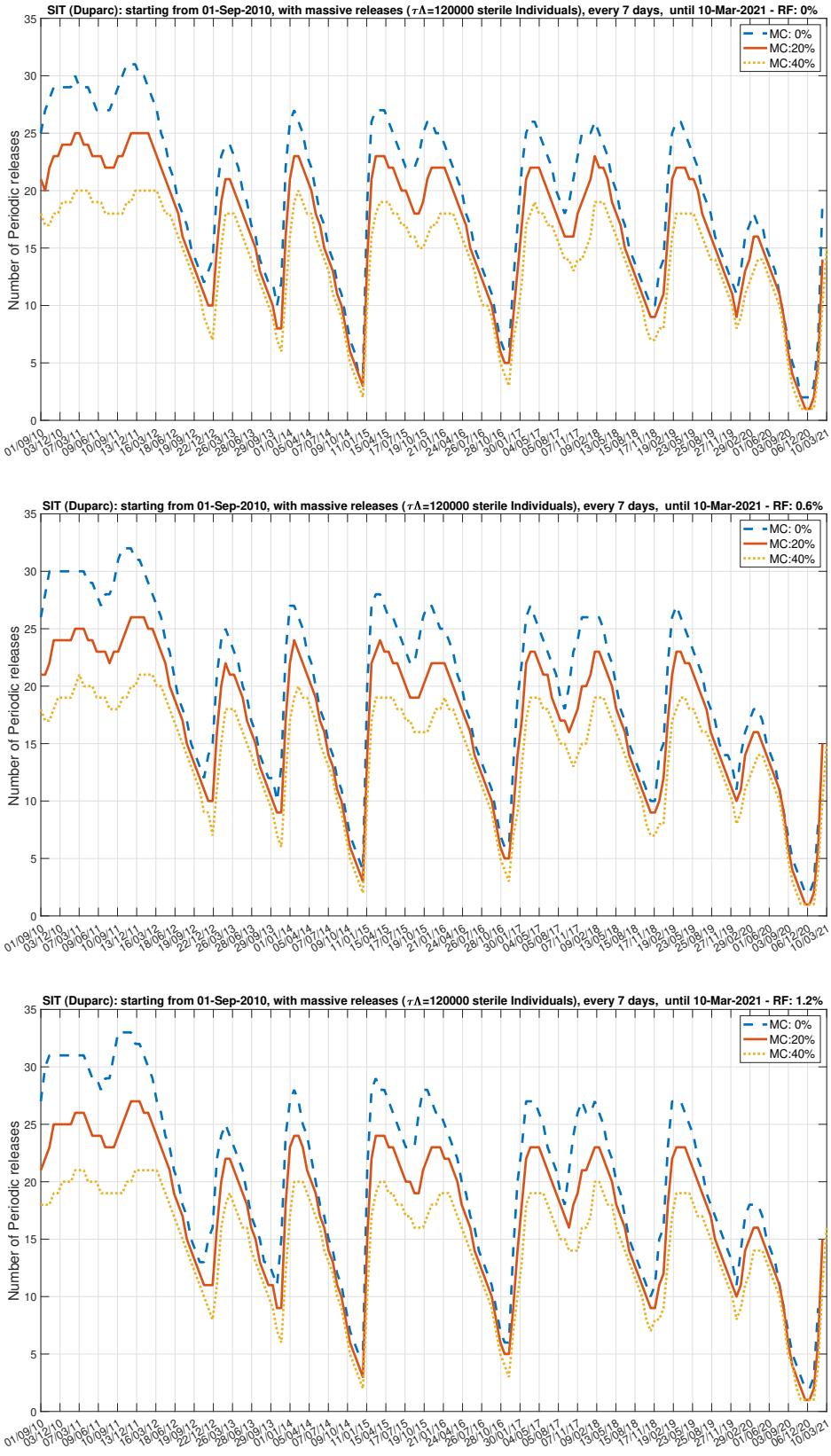

Figure 10: Temperature and rainfall dependent model - Weakly SIT control of 6000 sterile Ind/ha for various level of Mechanical control to reach $\mathcal{R}_{eff} < 0.5$ - Residual fertility variation: (a) 0%, (b) 0.6%, and (c) 1.2%

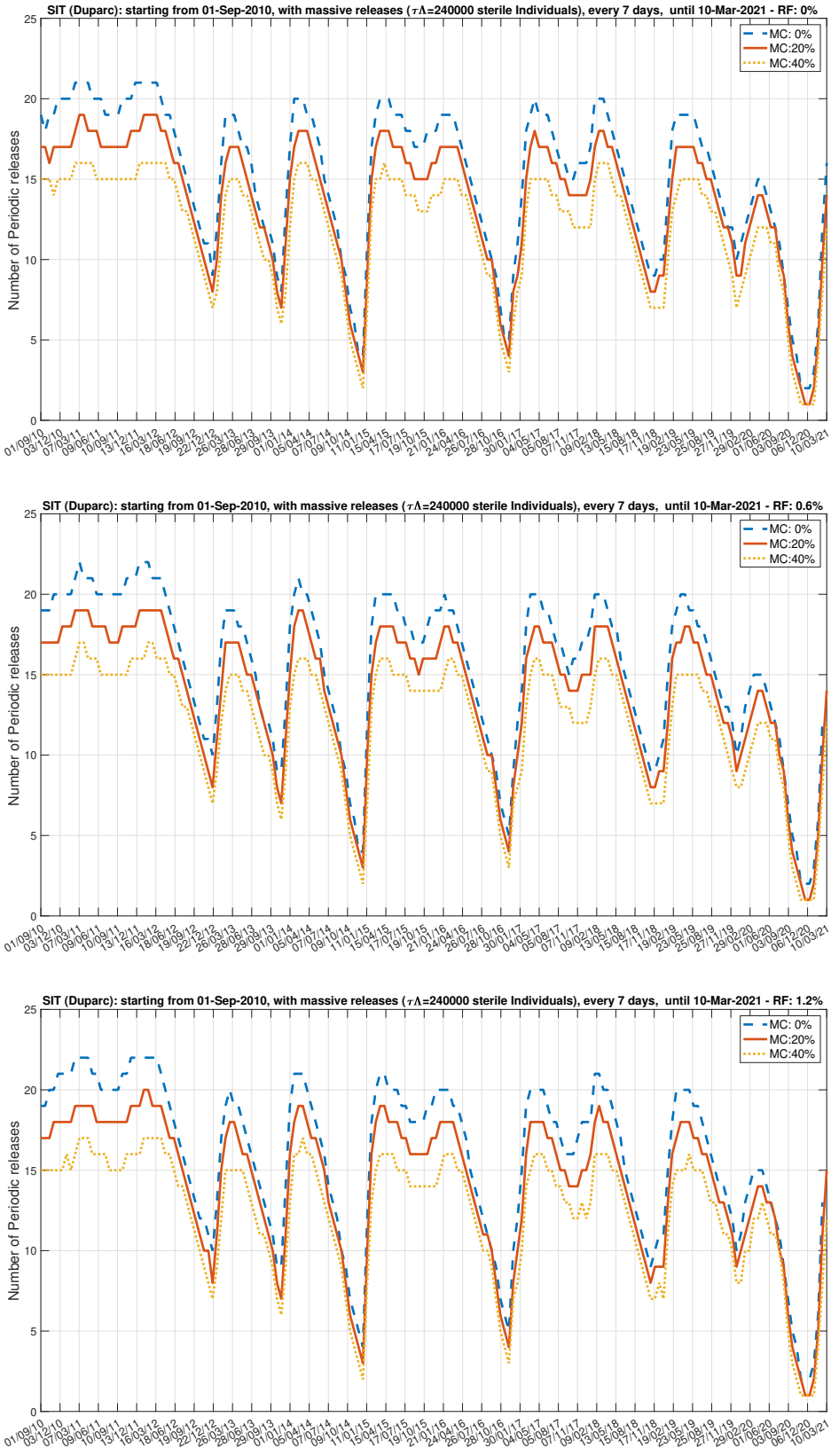

Figure 11: Temperature and rainfall dependent model - Weakly SIT control of 12000 sterile Ind/ha for various level of Mechanical control to reach $\mathcal{R}_{eff} < 0.5$ - Residual fertility variation: (a) 0%, (b) 0.6%, and (c) 1.2%

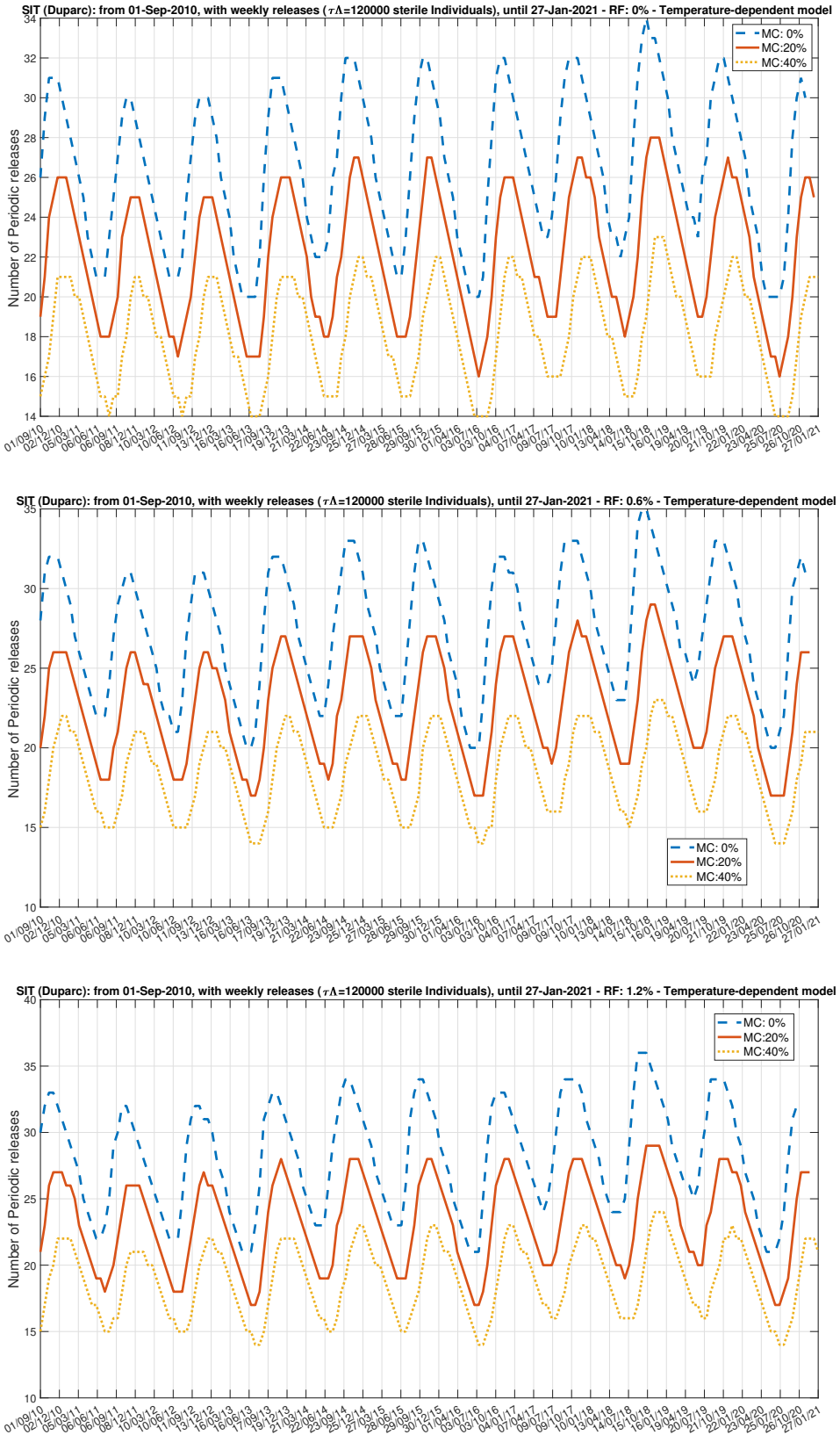

Figure 12: Temperature-dependent model. Weakly SIT control of 6000 sterile Ind/ha for various level of Mechanical control to reach $\mathcal{R}_{eff} < 0.5$ - Residual fertility variation: (a) 0%, (b) 0.6%, and (c) 1.2%

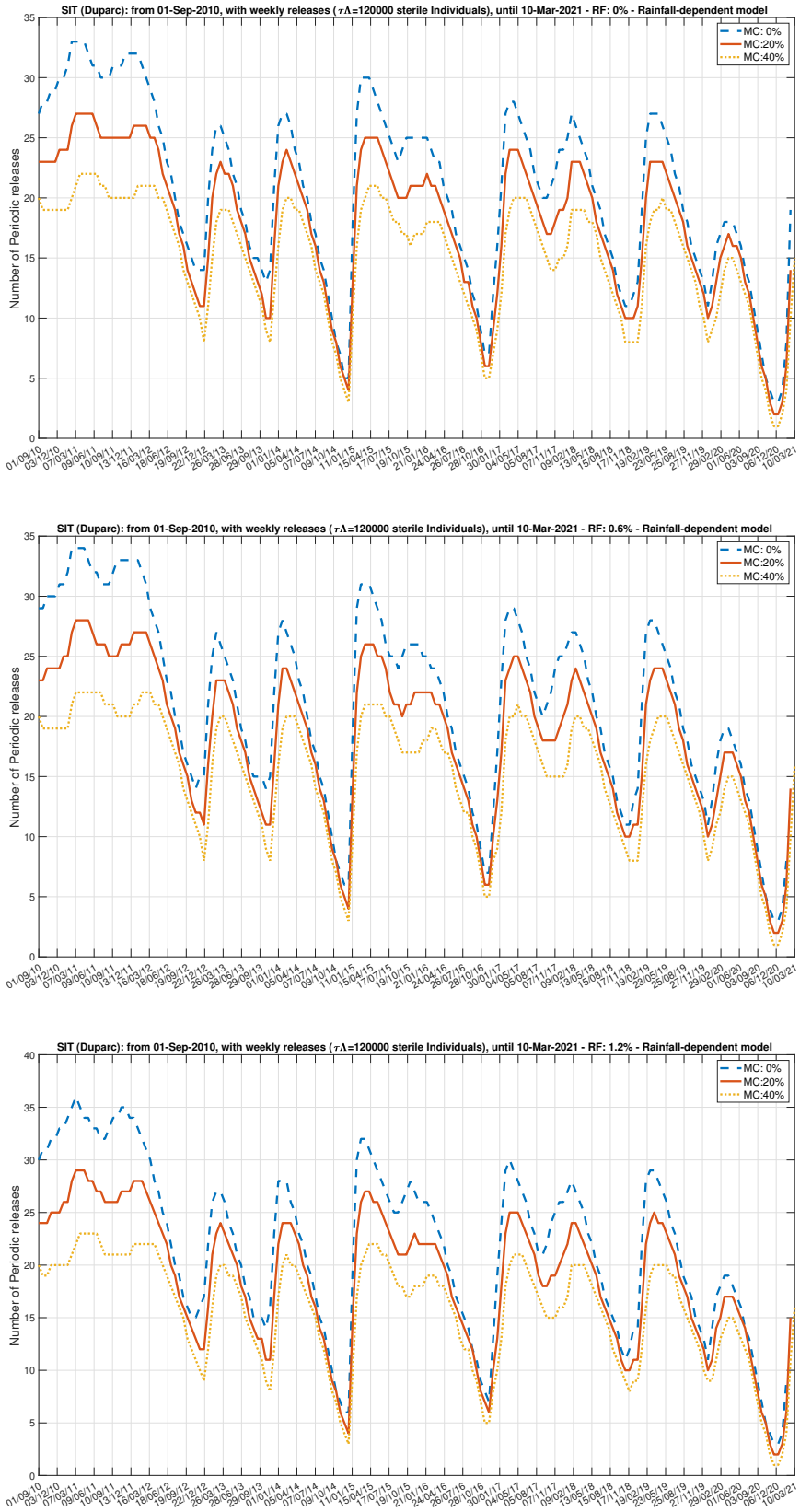

Figure 13: Rainfall-dependent model. Weakly SIT control of 6000 sterile Ind/ha for various level of Mechanical control to reach $\mathcal{R}_{eff} < 0.5$ - Residual fertility variation: (a) 0%, (b) 0.6%, and (c) 1.2%