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# The aRithmetic Discourse Profile as a tool for evaluating students' discourse according to the ritual to explorative continuum

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*We propose a new tool – the aRithmetic Discourse Profile (RDP) for analyzing the arithmetic discourse of students, based on the commognitive theory. The tool maps students' discourse on a continuum between ritual and explorative participation. We apply the tool to the discourse of 12 7<sup>th</sup> grade students and exemplify the analysis on three tasks: one in addition of whole numbers, one in multiplication and one with fractions. Tasks were taken from regular curricular materials of elementary school mathematics. We discuss the affordances of the tool in relation to existing diagnostic tools.*

*Keywords: Arithmetic, discourse, assessment, commognition.*

## Introduction

Assessing students' mastery of arithmetic has multiple potential benefits. It can assist teachers in evaluating students' performance in relation to curricular expectations and provide researchers with measures of pre- and post- interventions. Two major forms of assessment currently exist for assessing students' arithmetic discourse. One is the common school-based exams (or standardized tests). The other are “diagnostical tools” usually used by educational psychologists to assess students' “arithmetic abilities” (or disabilities) (e.g. Chinn, 2020). School-based exams are effective for efficiently assessing students' outcomes. Diagnostic psychological tools are effective for identifying certain common deficits (such as problems with fact retrieval) (e.g. Dowker, 2012). However, both these types of tools do not provide a fine-grained profile of students' arithmetic knowledge and skills.

Previous commognitive studies (Ben-Yehuda, 2003; Ben-Yehuda et al., 2005) have started offering an alternative to the above two types of assessment, by proposing a form of “profiling” a students' arithmetic discourse, based on transcriptions of think-aloud interviews. However, these studies were quite preliminary in their conceptualization. Their categories were difficult to replicate and not sufficiently connected to a broad theory of learning. In the past two decades, the commognitive theory has evolved and sharpened its tools of analysis. One useful conceptualization that has come out of commognitive studies is that of the “ritual-exploration” dyad, which qualifies participation in discourses both according to development (learners starting from ritual and moving to explorations) (Lavie & Sfard, 2019) and according to achievements (learners experiencing difficulties performing more ritually than successful learners) (Heyd-Metzuyanin, 2015). Armed with these new conceptualizations our goal is to develop and test a tool for mapping students arithmetic discourse based on the continuum of ritual to explorative participation.

## Theoretical Background

Commognition theorizes the learning of mathematics as a process whereby learners gradually become participants in the mathematical discourse. This process is characterized by movement from

performance of ritual routines to explorative routines. Routines are pairs of *task* (what it is that the performer feels obliged to do) and *procedure* (the steps to achieve the goal). Ritual routines are aimed primarily at pleasing the masters of the discourse, usually teachers or parents. As such, they are mostly characterized by thoughtful imitation (Lavie et al., 2019). The learner attempts to repeat actions that she saw the expert perform in situations that she perceives as similar to the present task. As the learner gains experience with similar tasks, her routines gradually become more explorative (Sfard & Lavie, 2005). Explorative participation is characterized by flexible usage of different procedures to achieve a single task, and by the learner initiating these procedures to achieve her own goals. That is, the learner performs the routines to produce narratives about the world, not to please an external authority. Explorative participation is thus characterized by agentivity. In addition, explorative routines are *bonded*, that is, the output of each step serves as an input to the next step in the procedure. In contrast, ritual routines may often be unbonded since they are made up of steps imitated of others, without the performer being aware of how or why each step is bonded with the former ones.

In mathematics, routines are mostly concerned with producing narratives about mathematics objects. For example, the routine of multiplying  $5 \times 3$  leads, eventually (after, for example, adding three to itself five times), to the narrative “5 times 3 is 15”. Yet for such routines to be enacted with ease, learners need to *objectify* numbers. The process of objectification, according to commognition, is the main process that underlies what is often termed as “mathematical understanding”. Within this process, learners come to view processes (such as counting or adding) as products. For example, the process of counting “one, two, three, four” eventually gets reified into an object (“four”) that can be treated as existing on its own, regardless of the process of counting that led to it. The process of objectification, however, takes time, and ritual performers often treat the signifiers of mathematical objects (such as the digits of a number in the realm of tens) only *syntactically*, that is, without relation to the object that they are supposed to signify. The processes of objectification are never-ending in the learning of mathematics since mathematical discourses are hierarchical. Thus, for example, once the learner masters (or becomes explorative in) the discourse of natural numbers, they are expected to enter the discourse of fractions (or rationale numbers). Once this has been mastered, the expectations for participation are raised to the discourse of real numbers, algebra, etc.

School exams are ill-suited to examine the extent to which students perform explorative routines in a certain discourse, as they mostly give a picture of the final answer produced by the students’ routines, not the procedure by which the answer was produced. Therefore, students may be identified as “successful” (or achieve highly on exams) in school mathematics, while many of their mathematical routines are still ritual (Heyd-Metzuyanim, 2015). Another problem with standard school assessment is that they can miss certain explorative routines that students who err in their final answers do perform. This is especially relevant to low achieving students (Ben-Yehuda et al., 2005; Heyd-Metzuyanim, 2013). To overcome these limitations, early commognitive research came up with a tool called “Arithmetic Discourse Profiling” (Ben-Yehuda, 2003). This tool, based on a form of cognitive interviewing, attempts to map the student’s discourse in ways that highlight not just what the student does “wrong” (or non-canonically) but also (and more importantly) the actions the student does perform when attempting to solve mathematical tasks. In this work, we aim to extend the early attempts at profiling arithmetic discourse by using the more recent definitions and operationalizations

of ritual vs. explorative routines. Our research question is: Based on tasks taken from elementary school curriculum, how can students' arithmetic discourse be characterized on the continuum between ritual and explorative participation?

## Method

Our analytical tool was developed as a secondary data analysis, on data collected by the first author (Heyd-Metzuyanin, 2011). The data consists of arithmetic interviews conducted with 12 7<sup>th</sup> graders during 2008. The interviews were conducted as part of a larger study where the author taught the 12 students in three groups over a period of 5 months, in an out of school "course". The course was broadly defined as "mathematics enrichment" and the original study was aimed at examining interactions between emotional and cognitive aspects of learning. The students were placed in three groups (each of 4): Low achieving, middle-high achieving, and excelling. The excelling group consisted of 4 boys who came from an accelerated mathematics and science classroom (to which entrance was based on very high grades in elementary school mathematics and other subjects). Placement in the low and middle-high achieving group was based on reports coming from the school and parents of the students regarding their grades in elementary school mathematics and their achievements in the beginning of 7<sup>th</sup> grade.

All the interviews were conducted by the first author, in a one-on-one setting. Students were asked to think out loud as they solved the problems (Young, 2005). When they were not successful and asked for help, the interviewer attempted to scaffold the task, yet mostly she refrained from giving feedback on the correctness of the results. Interviews were videotaped with two cameras, one pointing at the student's face and one on his writing. Written artifacts were collected too.

The interview protocol was based on Ben-Yehuda's (2003) ADP protocol and consisted of 24 tasks (some very short) that had potential to illuminate various aspects of students' arithmetic discourse. For the present study, which goal was mainly to create analytical tools, we chose three tasks for analysis (the full protocol appears in (Heyd-Metzuyanin, 2011)). These tasks were (1) Add  $96+7936$  (if possible, "in your head") (2) Multiply  $25 \times 99$  and (3)  $\frac{2}{3} \times 9$ . We chose these tasks as they covered a relatively large domain of arithmetic skills, including manipulating whole numbers and fractions.

**Analysis:** Analysis was done on full verbatim transcriptions of the interviews and proceeded in two steps. First, we divided each routine (task + procedure) into sub-routines (for example, adding  $96+7935$  often consisted of sub-routines such as adding  $5+6$ , adding 11 to 8020, etc). Next, we determined whether the routine (or sub-routines) were ritual or explorative according to eight categories, taken from the literature on ritual and explorations. These are explained in Table 1.

**Table 1 - Criteria of analysis for ritual-explorative routines**

	Criterion	Analytical actions	Characteristics of an explorative routine	Characteristics of a ritual routine
1	Objectified /syntactic mediation.	Searching for evidence that the nouns signify numbers/quantities and	In whole numbers, relating to the place value of the numeral. In fractions: relating to different realizations of the fraction as the same, including fraction as	In whole numbers: relating to operations as signaling procedures on digits rather than on the whole number. In

		not just the signifier of the number	operator, part of whole, part of quantity, etc.	fractions, relating separately to the numerator and denominator.
2	Flexibility	Look for multiple procedures that are associated with the same task.	More than one procedure is associated with the main task OR a non-standard procedure is applied to the task.	Relying on only one procedure while showing rigidity and reluctance to use any other procedure.
3	Agency/External authority	Look for subjectifying discourse; examine verbs/pronouns and non-verbal signals that indicate the confidence.	Mathematizing with high confidence (no hesitations, question marks, no looking for approval). Spontaneously articulating mathematical narratives	Talking with question marks; Verbally or Non-verbally seeking approval from the interviewer; Relating to external authority for justification (e.g. “that’s what I learned in school”)
4	Focus on goal or on procedure	Look for verbs indicating doing (e.g. “I add”) vs. being verbs indicating the result (“it is...”);	Talking about the result, checking it, or explaining it spontaneously	Talking about the actions of the procedure. Ending the procedure without relating to the reasonableness of the result.
5	Bondedness	Examine the procedure; sub-procedures and the bonds between them.	Each sub-procedure feeds the next sub-procedure. The narrative of the result of sub-procedure N serves as the input of sub-procedure N+1.	There is a disconnect between a certain sub-procedure and its following one OR sub-procedures using different realizations are not treated as the same.

The final three characteristics were *Canonical procedures* that were coded as explorative if all the steps in the procedure aligned with standard mathematical procedures; *Canonical narratives* were coded as explorative if the end result of a procedure (e.g. the end result for the task “two thirds of 9” was “six”) was canonical; and *Mediation* was coded as explorative if the procedure was fully initiated and enacted by the student, and ritual if some parts of it were mediated by the interviewer.

The first stages of constructing the coding criteria (exemplified in the findings section) were created by the first and third authors, through mutual agreement. After that, the second author was taught the coding scheme, blindly replicated it on 50% of the data, and created a coding manual.

After coding was complete, a “ritual/exploration” ratio was calculated for each student, on each of the tasks. The highest explorative ratio could be 0/8 (0 ritual, 8 explorative characteristics), whereas the most ritual performance could be 8/0. Notably some of the characteristics (agentivity, bondedness, canonical and non-canonical procedure/narrative) could be coded *both* as ritual and as explorative. This was necessary since we wanted to account for explorative elements of sub-routines. Thus ratios such as 7/3 or 5/6 were also possible. In general, ratios close to 1 showed “mixed” performance.

## Findings

**Table 2 - Ritual-Exploration ratios and relative placement of 12 students**

Student	Achievement group	96+7935	25×99	$\frac{2}{3} \times 9$
Dana	Low	6/4	8/2	7/3
Hili	Low	7/2	Not attempted	Not attempted

Hila	Low	7/3	7/1	Not attempted
Naor	Low	2/6	8/3	4/5
Edna	Middle-high	8/3	6/7	Not attempted
Idit	Middle-high	0/7	6/5	7/4
Dan	Middle-high	1/7	7/6	2/6
Ziv	Middle-high	3/6	3/8	2/6
Ram	Excelling	0/6	0/8	1/7
Gabby	Excelling	0/7	0/8	1/7
Yoram	Excelling	0/7	0/8	0/7
Amir	Excelling	0/7	1/8	0/8

Table 2 summarizes the ritual/explorative ratios of the 12 students' (all pseudonymed) performance on the three tasks. Before we delve into the exemplification of how these ratios were determined, there are a few observations worth mentioning regarding this table.

Our first observation is that the ritual/exploration tool seems to capture a wide range of routine enactments, from those very high in exploration (0/8) to those almost only featuring ritual characteristics (8/2). Secondly, we see the low achieving group very high in ritual characteristics (or not attempting tasks at all); the moderate-high group is fluctuating widely, between 0 and 8; and the excelling group is quite consistent around the 0 – meaning high exploration. This lends validity to the tool as capturing features that reflect students' success in school mathematics. A third observation is that students can be inconsistent with respect to different tasks. For example, Dan's performance is explorative in the addition task, ritual in the multiplication task, and relatively explorative again in the fractions task. We do see, however, that in the excelling group these fluctuations do not exist.

Next, we demonstrate our analysis on two episodes from students' interviews, showing different types of routine enactment, associated with different ritual/explorative ratios.

### Episode I: Hila, 96+7935 – Dominantly ritual performance

	What is said (what is done)	Writing
1	Hila: Ah, ninety-six plus seven, seven... seven thousand, seven thousand ninety and thirty, um, can I calculate?	96 + 7935 = 8021
2	Interviewer: Is there a way that you can do it in your head?	
3	Hila: I, um, it's so difficult for me, I-	
4	Interviewer: Try it	
5	Hila: OK. Ah, wow, three plus (..) nine, twelve. Um twelve, um twelve and five plus six is eleven. Eleven plus twelve, equals thirty-three (..), yes, ah no, twenty-three. So, it's eight thousand ninety and thirty, Ah no. Twenty three. OK.	
6	Interviewer: OK, look at the answer and tell me if it looks alright to you?	$\begin{array}{r} 111 \\ 7935 \\ + 96 \\ \hline 8021 \end{array}$
7	Hila: Oops.	
8	Interviewer: Wait, oops. Why oops? What's not good about it?	
9	Hila: 'cause I had to do plus	
10	Interviewer: And what did you do?	
11	Hila: Aah... (points at the paper) I didn't combine like... here the...	
12	Interviewer: Now do you want to do it vertically?	

13	Hila: (Writes). Eighty, three. Five – six, nine, yeah twelve, ten, yeah aah (erases) eighty, eighty (erases).	
14	Interviewer: And that's a better way?	
15	Hila: Yes.	

We see in Hila's performance 7 characteristics of ritual and 3 or explorative participation, thus her ritual/explorative ratio is 7/3. Following is the characteristics analysis: A. Non-objectified discourse. The digits in Hila's discourse are treated as independent entities, to be combined and manipulated in some form, but not as indicative of a whole number or as place value (see line 5, where the digits are first added horizontally, and then line 13). B. Rigid performance: Hila sticks only to procedures of adding the digits separately. Even when asked to do it "out loud", she tries (albeit unsuccessfully) to reconstruct some sort of procedure for adding the numbers digit by digit. C. External authority: Hila's discourse is hesitant, and she relies on the interviewer to encourage her to try out the task to begin with [4], to monitor her answer [6, 8] and to suggest an alternative procedure [12]. D. Focus on procedure: Hila focuses only on the procedure ("I had to do plus" [9] "I didn't combine" [11]). She does not check her answer showing no interest in the result. E. Canonical procedures: Some of the procedures Hila enacts are non-canonical (e.g. adding the sums of the unit digits and tens digits: 11+12). Other procedures are canonical (e.g. writing the "carry on" digits in the appropriate places in the vertical solution). Therefore, she got 1 on both "canonical" and "non-canonical" sides of the ritual/explorative table. F. Canonical narratives: Similar to E. some of Hila's narratives are canonical (e.g. 5+6=11) while others are not (the overall sum is non-canonical). G. Mediation: Hila's performance is mediated by several comments of the interviewer, especially the suggestion to "do it vertically" [12]. H. Bondedness: all of Hila's sub-procedures feed one into the other. Therefore, she received a '1' on the bonded criteria (explorative).

### Episode II: Idit solving 25×99 - Example of mixed performance

- 1 Idit: (Reads the task, sighs) I usually get messed up with such exercises
- 2 interviewer: Is there something that you can do with the... ninety-nine? (..) That is very close to another number?
- 3 Idit: A hundred (Interviewer: OK) You can round it (up) to like... (looks at Interviewer), no. First, I'll round it to 25 times 100. Can I do (that)? (Interviewer: OK) Which turns out two thousand and five hundred (Interviewer: Umhmm) and then take off one. But (comments on failing on such exercises. Interviewer encourages her to try, nevertheless). So, it turns out a thousand... No (erases)... (writes 1049) wait, no (erases, mumbles, writes 2499).
- 4 interviewer: Two thousand four hundred and ninety-nine
- 5 Idit: that's what seems most reasonable
- 6 Interviewer: But do you think it's correct? (Idit shrugs "I guess so" and smiles). If I tell you that this is 99. I tell you that this is 99 times (intrusion outside). If I tell you this as an exercise of 99 times 25 would it help?
- 7 Idit: (giggles) No

Idit's ritual/explorative ratio was determined as 6/5 for the following reasons: A. Objectified discourse: we only find indications of objectified discourse in Idit's solution. She treats 25 times 100 as objects ("which turns out two thousand and five hundred" and "I'll round *it* to 25 times 100" [3]). B. Rigid performance: Idit does not have an alternative procedure for multiplying 25×100 (e.g. by long multiplication), even though she is unhappy with the result [7] C. Both Agentivity and External

authority: in some points, Idit makes independent choices and seeks no guidance (e.g. decides 1049 is an error and instead writes 2499). In other parts, she seeks approval from the interviewer (e.g. “Can I do that?” [3]) D. Focus on product. There are no indications that Idit is focused on the procedure and she does comment on the final product (“That’s what seems most reasonable” [5]) E. The overall procedure is non-canonical since she solves  $25 \times 99$  with  $25 \times 100 - 1$  F. Canonical and non-canonical narratives: all the sub-narratives ( $25 \times 100 = 2500$ ;  $2500 - 1 = 2499$ ) are canonical however the overall narrative  $25 \times 99 = 2499$  is non-canonical G. Mediation: Idit’s performance is mediated by the interviewer suggesting the similarity between 99 and 100 [2] H. Bondedness and non-bondedness: all of Idit’s sub-procedures are bonded, yet the overall procedure is not bonded to the overall task.

## Discussion

In this study we asked how can students’ arithmetic discourse be characterized on the continuum between ritual and explorative participation? Our results indicate that the eight characteristics of ritual/explorative participation are useful for locating students on such a continuum. These characteristics are: Objectification, Flexibility, Agentivity, Focus on procedure/product, Canonical procedures, Canonical narratives, Mediation and Bondedness. We used these characteristics to construct a ritual/explorative ratio, where ratios close to 0 indicated high explorative performance, whereas ratios nearing 8/0 indicate ritual performance. Ratios around 1 indicate mixed performance, which mostly show parts of the sub-procedures are ritual while others are explorative.

The validity of this tool is stemming from three sources. First, we were able to achieve blind inter-rater reliability showing that our criteria of analysis were operational. Second, we found general coherence between the ratios and students school achievements (as indicated by placement into the achievement groups in the study, see table 1). Third, the ratios generally cohered with the first author’s experience with the students, gained through teaching them for five months.

Our findings generally support the commognitive theory of the development of students’ mathematical discourse in several respects. First, they show that students generally achieving higher (and thus, presumably more fluent in the arithmetic discourse) indeed perform more exploratively, while low achieving students (who have not mastered the discourse) perform ritually (Heyd-Metzuyanim, 2015). Second, we see from table 1 that students generally performed more exploratively in discourses that are primary (e.g. natural numbers) while in a newer discourse (fractions) the performance was more ritual. This is consonant with the theory that states higher level discourses are built upon primary discourses that are subsumed by them (Sfard, 2008). However, we also found some anomalies, such as a student whose performance on the multiplication task was ritual while his discourse on fractions was explorative. Future studies should look more into the question of whether these are anomalies that are characteristic of “transition” phases (such as at the level of beginning middle school) or whether they can be seen also in later stages of mathematical learning.

The study of course has several limitations, the main one of which is the relatively small number of participants. This is a limitation related to the high work-intensiveness of commognitive analysis and may be overcome in future studies by finding ways to make the analysis more efficient. Nevertheless, the method suggested in this study is the first form of quantifying mathematical performance according to the ritual-explorative continuum. As such, it opens up multiple avenues for further



inquiry. For example, future studies can examine whether the pattern of “being around 1 ratio” in students who are identified as middle-high achieving repeats itself, while the pattern of “close to 0 ratio” is common in students identified as excelling. The latter question is especially important since we note that the curriculum mostly advances according to the assumption that students have mastered primary discourses. For example, middle-school curriculum is based on the assumption that students are explorative participants in the arithmetic discourse (Karsenty & Arcavi, 2003). Whether and how students “fill in” formerly learned discourses is an open question, which the tool proposed in this study may help to answer.

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## References

- Ben-Yehuda, M. (2003). *Using discourse analysis to investigate arithmetical thinking processes of students with learning difficulties (Unpublished Dissertation)*. Haifa: Haifa University.
- Ben-Yehuda, M., Lavy, I., Linchevsky, L., & Sfard, A. (2005). Doing wrong with words: what bars students' access to arithmetical discourses. *Journal for Research in Mathematics Education*, 36(3), 176–247.
- Chinn, S. (2020). *More trouble with maths* (3rd ed.). Routledge.
- Dowker, A. (2012). Numeracy recovery: A pilot scheme for early intervention with young children with numeracy difficulties. *Supporting Mathematical Thinking*, 16(1), 28–38. <https://doi.org/10.4324/9780203963234-10>
- Heyd-Metzuyanim, E. (2011). *Emotional aspects of learning mathematics - how the interaction between identifying and mathematizing influences the effectiveness of learning (Unpublished Dissertation)*. Haifa: University of Haifa.
- Heyd-Metzuyanim, E. (2013). The co-construction of learning difficulties in mathematics—teacher–student interactions and their role in the development of a disabled mathematical identity. *Educational Studies in Mathematics*, 83(3), 341–368.
- Heyd-Metzuyanim, E. (2015). Vicious cycles of identifying and mathematizing: A case study of the development of mathematical failure. *Journal of the Learning Sciences*, 24(4), 504–549. <https://doi.org/10.1080/10508406.2014.999270>
- Karsenty, R., & Arcavi, A. (2003). *Characteristics of learning and thinking of low achievers in mathematics (Hebrew)*.
- Lavie, I., & Sfard, A. (2019). How Children Individualize Numerical Routines: Elements of a Discursive Theory in Making. *Journal of the Learning Sciences*, 0(00), 1–43. <https://doi.org/10.1080/10508406.2019.1646650>
- Lavie, I., Steiner, A., & Sfard, A. (2019). Routines we live by: from ritual to exploration. *Educational Studies in Mathematics*, 101(2), 153–176. <https://doi.org/10.1007/s10649-018-9817-4>
- Sfard, A. (2008). *Thinking as communicating*. Cambridge University Press.
- Young, K. A. (2005). Direct from the source: The value of “think-aloud” data in understanding learning. *Journal of Educational Enquiry*, 6(1), 19–33.