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Observer design for range-only localization in three dimensions with three measurements and asynchronous and aperiodic measurements

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Abstract—In this paper, a range based localization method is proposed for position estimation in three dimensions with only three measurements. The proposed scheme is based on a modification of the system to make it globally observable and a change of coordinates which allows to construct a continuous time observer. The obtained continuous time observer is further modified to be able to deal with aperiodic and asynchronous measurements. The performances of the proposed approach are illustrated with simulations. It is shown that the proposed observer performs well even in presence of noise.

Index Terms—High gain observer, multi-rate observers, nonlinear systems, sampled-data observers, range-only localization

I. INTRODUCTION

The problem of positioning, that is reconstructing the location of an object, has attracted many researchers since it finds applications in many areas [27], [33], [36]. The need for position estimation is quite universal, then the different methods are scattered across many fields. For example localization using radar has a long standing history and many researches have been devoted to this problem [3], [17]. Aerospace engineering is also very demanding, solutions based on GPS for example can be found in the literature [9]. More recently many schemes have been developed for indoor positioning based on ultra wide band [8], [29] or ultrasonic sound waves [21] for example. Oceanic engineering is another field where specific solutions have to be found since GPS cannot be used, in particular for underwater localization [6], [23]. The development of communications tools and network, especially wireless communication has also made the interest for localization grow [19], [24].

Many localization schemes rely on the use of range-measurements, since they can be obtained indirectly from Time Of Arrival or Received Signal Strength for example. The trilateration problem can be solved explicitly from algebraic manipulations [18], [30] and exact closed forms formulas can be obtained [28]. Nevertheless, problems arise when there are noise and uncertainties. Indeed, in this case a

solution might not exist. Therefore other formulations have been proposed to find an approximate solution. A way to formulate this problem is to minimize a criteria, leading to a nonlinear least square problem or as a maximum likelihood problem. It has been shown that under some assumptions these two problems are equivalent for a particular choice of the least square weights. Since the nonlinear least square problem cannot be solved explicitly in general, two directions have been followed. A first way, called direct method, consists in rewriting the least square problem in an approximate way so that it can be explicitly solved. The approximative least square problem can be either a single objective one [5], [11], [16] or a multi objective one [25], [32]. A second way is to approximate the solution of the nonlinear least square problem with iterative methods. A possible way is to transform the problem so that it can be solved through an eigenvalue decomposition, whose solution is found by determining the roots of a high-order polynomial [7], [15].

A limitation of the previously mentioned methods is the fact that they are designed for static targets. When a target is moving, better performances can be obtained if some knowledge about the dynamics of the target is used. The difficulties for applying filtering or estimation techniques mainly come from the fact that the measurements are nonlinear functions of the state. This prevents direct applications of classic linear filtering and estimation techniques. Nevertheless several solutions have been provided in the literature. Some approaches are based on Particle filters [20], [26]. Many declinations of Kalman filters have also been used in the literature. For example, an extended Kalman filter has been used in [2] for the position reconstruction of an underwater vehicle. A linear Kalman filter fed by the estimates provided by a maximum likelihood approach has been considered in [31]. A generalized Kalman filter able to deal with multiplicative measurement noises has been used in [14] and with a generalized iterated Kalman filter in [13]. The time varying properties of

measurements noise are taken into account in [35] with a sequential cubature filter. An unscented Kalman filter has been developed in [22] and a fusion based kalman filter that allows to deal with measurement uncertainties in [34].

Though these methods may be effective, the solutions based on Kalman filters suffer several drawbacks. Indeed, an estimation of the variances of the different noises are required to tune the algorithm, but they may be difficult to estimate. The kalman filter needs to be correctly initialized, indeed, the estimated trajectory may not converge to the true trajectory if this is not the case. The stability is difficult to obtain and may be only local, which is mainly due to the fact that approximations on the nonlinearities are made up to a certain order depending on the method. Finally, since the model used is usually discrete, it may be difficult to deal with aperiodic and asynchronous measurements, especially if they don't have a common divisor. For all these reasons, a deterministic approach based on a continuous time model may be interesting. But seldom approach exist mainly because of the high nonlinearities. For example, an observer has been proposed in [1], where the measurements are transformed to be linear, but it requires at least 4 range measurements, and noise cannot be assumed to be additive anymore. Although more sources lead to a better accuracy in general, it may not be possible to have too much sources due to cost or technical reasons, an efficient localization algorithm for the case where the minimum number of sources is available is then interesting.

A new scheme is proposed in this paper, which allows to reconstruct the position of the target in 3 dimensions with 3 range measurements from 3 non aligned sources. It is only assumed that the side on which the target lies compared to the plan passing through the three sources is known. The approach is based on a nonlinear transformation and an high-gain design. Several contributions have to be emphasized:

- i) there is only one tuning parameter with a physical meaning, rendering the tuning quite simple,
- ii) the proposed observer is locally stable,
- iii) aperiodic and asynchronous measurements can be naturally considered,
- iv) the estimates are in original coordinates and the measurements are not squared, nor transformed before use.

The paper is organized as follows. In section II, the considered problem is stated. The proposed observer is described in section III. Section IV contains some simulation results in order to illustrate the behavior of the proposed approach. Finally section V concludes this paper.

The following notations will be used throughout the paper. The zero matrix of dimension $n \times m$ is denoted $0_{n \times m}$, I_n is the identity matrix of dimension n . For a symmetric real matrix S , its largest and lowest eigenvalues are respectively

denoted $\lambda_M(S)$ and $\lambda_m(S)$. The euclidean ball of center $x \in \mathbb{R}^n$ and radius $r \geq 0$ is denoted $\mathcal{B}(x, r) = \{x \in \mathbb{R}^n \mid \|x\| \leq r\}$.

II. PROBLEM STATEMENT

The problem considered here consists in reconstructing the position of a target from three range measurements. More precisely, one considers here a target located at a position $x = (x_1, x_2, x_3)^T \in \mathbb{R}^3$ and one assumes that three range measurements are available:

$$\begin{aligned} r_i &= \|x - x^{r_i}\|, \quad i = 1, 2, 3 \\ &= \sqrt{(x_1 - x_1^{r_i})^2 + (x_2 - x_2^{r_i})^2 + (x_3 - x_3^{r_i})^2} \end{aligned} \quad (1)$$

where $x^{r_i} \in \mathbb{R}^3$ is the known position of the i -th source (a source can be a radar, a beacon or a buoy for example), and $\|\cdot\|$ is the Euclidean norm.

One further assumes that the target is moving. The dynamical model of the target trajectory is supposed to be as follows:

$$\begin{aligned} \dot{x}(t) &= y(t), \\ \dot{y}(t) &= z(t), \\ \dot{z}(t) &= \varepsilon(t), \end{aligned} \quad (2)$$

where $y \in \mathbb{R}^3$ and $z \in \mathbb{R}^3$ are respectively the speed and the acceleration of the target. The following assumption is made on the jerk $\varepsilon \in \mathbb{R}^3$.

Assumption 1: The real function $\varepsilon(t)$ is uniformly bounded, that is there exists $\delta_\varepsilon \geq 0$ such that:

$$\|\varepsilon(t)\| \leq \delta_\varepsilon, \quad \forall t \geq 0. \quad (3)$$

Remark 1: The value of the jerk ε is supposed to be unknown for the observer design, one only assumes that it is bounded.

In order for the problem to be feasible, one assumes that the three sources are not on the same line. Since there are only three sources, they are coplanar and one assumes, without loss of generality, in the rest of the paper that $x_3^{r_1} = x_3^{r_2} = x_3^{r_3} = 0$ which means that their height is zero.

Since the measurements are symmetrical with respect to the sources plan, one cannot reconstruct the position from only three sources in general. But if one assumes that the target belongs to a known side of the sources plan, then the problem is feasible (see [18] for more details). The height of the target is then supposed to be positive and the target is supposed to be not too close to the sources and to lie in a bounded space. More precisely, the following assumption is made on the considered trajectory of the target.

Assumption 2: There exists some positive constants $\alpha, \beta, \Gamma \geq 0$ such that the trajectory $x(t) = (x_1(t), x_2(t), x_3(t))^T$, $t \geq 0$ of the target satisfies

- (i) $x_3(t) \geq \alpha$, for all $t \geq 0$,
- (ii) $\|x(t) - x^{r_i}\| \geq \beta$, for all $t \geq 0$,

$$(iii) \left\| \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} \right\| \leq \Gamma, \text{ for all } t \geq 0.$$

III. MAIN RESULTS

The construction of the proposed observer is split into different parts. Firstly the system is modified so that it is globally C^2 and observable. Secondly, new coordinates are considered and the dynamics of the system in these new coordinates is explicited. Thirdly, a continuous observer is proposed in the new coordinates and transformed back in the original coordinates. Finally, the observer is modified to deal with sampled aperiodic and asynchronous measurements.

A. Modified system

Given assumption 2, it is clear that system (1)-(2) is observable and C^2 on the subspace where the target lies. But the observer state can go outside of this subspace, during the transient response or because of noise. The system is then modified so that it is globally observable and C^2 on the whole state space. Two modifications on the system are then done.

Firstly, one considers the following additional output

$$r_4 = \kappa(x_3) \quad (4)$$

where $\kappa : \mathbb{R} \rightarrow \mathbb{R}$ is a C^2 function strictly increasing on $(-\infty, \alpha)$ and equal to zero on $[\alpha, +\infty)$.

Remark 2: Note that given assumption 2-(i), the target trajectory satisfy $x_3 \geq \alpha$, then the value of the output r_4 is known and always equal to zero for the system. But it allows to know if the observer height is positive or negative. This makes the system globally observable.

Secondly, the first three outputs r_i , $i = 1, 2, 3$ are modified. Indeed, due to the square root, the output function is not C^1 at the points $x = x^{r_i}$. The modified output functions are given by

$$r_i = \mu(\|x - x^{r_i}\|^2), \quad i = 1, 2, 3 \quad (5)$$

where $\mu : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is an increasing C^2 function such that $\mu(s) = \sqrt{s}$ for $s \geq \beta^2$ and $\mu'(s) > 0$ for all $s \in \mathbb{R}^+$.

Remark 3: Given assumption 2-(ii), it is clear that the new output given by equation (5) coincides with the original output given by equation (1) for the target trajectory.

Remark 4: An explicit possible construction for the functions κ and μ will be given in section IV.

B. Change of coordinates

The modified system used for the observer design is given by

$$\begin{aligned} \dot{x}(t) &= y(t) \\ \dot{y}(t) &= z(t) \\ \dot{z}(t) &= \varepsilon(t) \\ r(t) &= \phi(x(t)) \end{aligned} \quad (6)$$

which is globally observable, where $\phi : \mathbb{R}^3 \rightarrow \mathbb{R}^4$ is the function defined as

$$\phi(x) = \begin{pmatrix} \mu(\|x - x^{r_1}\|^2) \\ \mu(\|x - x^{r_2}\|^2) \\ \mu(\|x - x^{r_3}\|^2) \\ \kappa(x_3) \end{pmatrix}, \quad x \in \mathbb{R}^3. \quad (7)$$

Lemma 1: The function $\phi : \mathbb{R}^3 \rightarrow \mathbb{R}^4$ is one to one and for all $x \in \mathbb{R}^3$, its Jacobian matrix $F(x) = (\partial\phi/\partial x)(x)$ is full rank.

Proof 1 (of Lemma 1): The proof is direct and then not reported here.

The new system of coordinates, $r \in \mathbb{R}^4$, $u \in \mathbb{R}^4$, $v \in \mathbb{R}^4$ is defined as

$$\begin{pmatrix} r \\ u \\ v \end{pmatrix} = \begin{pmatrix} \phi(x) \\ F(x)y \\ F(x)z \end{pmatrix} \quad (8)$$

Then, in the new coordinates, system (6) can be rewritten as

$$\begin{aligned} \dot{r} &= u \\ \dot{u} &= v + \omega_1(x, y)y \\ \dot{v} &= \omega_2(x, z)y + F(x)\varepsilon \end{aligned} \quad (9)$$

with $\omega_1(x, y) = \partial(F(x)y)/\partial x$ and $\omega_2(x, z) = \partial(F(x)z)/\partial x$. Since $\phi : \mathbb{R}^3 \rightarrow \mathbb{R}^4$ is injective, one can construct a left inverse $\phi^c : \mathbb{R}^4 \rightarrow \mathbb{R}^3$, that is, such that

$$\phi^c(\phi(x)) = x, \quad \forall x \in \mathbb{R}^3 \quad (10)$$

Remark 5: One can explicitly construct a left inverse ϕ^c by using the formulas developed in [18]. Nevertheless, it will not be explicited here since it is only used for the design of the observer. Indeed, at the end, the observer is implemented in the original coordinates x, y, z without the need for determining ϕ^c .

In addition, according to Lemma 1, $F(x)$ is full rank for each $x \in \mathbb{R}^3$, then it admits a left inverse. Denoting F^+ its pseudo inverse, one can write

$$y = F^+(\phi^c(r))u \quad (11)$$

$$z = F^+(\phi^c(r))v \quad (12)$$

Then, system (9) can be written in the following uniformly observable form

$$\begin{aligned} \dot{r} &= u \\ \dot{u} &= v + \eta_1(r, u) \\ \dot{v} &= \eta_2(r, u, v) + \eta_3(r)\varepsilon \end{aligned} \quad (13)$$

with $\eta_1(r, u) = \omega_1(\phi^c(r), F^+(\phi^c(r))u)F^+(\phi^c(r))u$, $\eta_2(r, u, v) = \omega_2(\phi^c(r), F^+(\phi^c(r))v)F^+(\phi^c(r))u$, $\eta_3(r) = F(\phi^c(r))$, for which an observer can be designed.

The system (13) can be written in a more compact form as follow

$$\dot{Z} = AZ + \eta(Z) + B\varrho(Z)\varepsilon \quad (14)$$

where $Z = (r^T u^T v^T)^T$, $\eta(Z) = (0_{1 \times 4} \eta_2(r, u) \eta_3(r, u, v)^T)^T$, $\varrho(Z) = \eta_3(r)$ and

$$A = \begin{pmatrix} 0_4 & I_4 & 0_4 \\ 0_4 & 0_4 & I_4 \\ 0_4 & 0_4 & 0_4 \end{pmatrix}, B = \begin{pmatrix} 0_4 \\ 0_4 \\ I_4 \end{pmatrix}$$

C. Observer Design

The proposed observer is based on a high-gain design, inspired by [10], and is given in the new coordinates by

$$\begin{aligned} \dot{\hat{Z}} &= A\hat{Z} + \eta(\hat{Z}) - \theta\Delta_\theta^{-1}K(\hat{r} - r) \\ &\quad - \theta M(\hat{Z})\Delta_\theta^{-1}K(\hat{r} - r) \end{aligned} \quad (15)$$

where $\hat{Z} = (\hat{r}^T \hat{u}^T \hat{v}^T)^T$, $K = [3I_4, 3I_4, I_4]^T$, $\Delta_\theta^{-1} = \text{diag}(I_4, \theta I_4, \theta^2 I_4)$, $\theta \geq 1$,

$$M(\hat{Z}) = \begin{pmatrix} 0_{4 \times 3} & 0_{4 \times 3} & 0_{4 \times 3} \\ \bar{\omega}_1(\hat{r}, \hat{u})F^+(\phi^c(\hat{r})) & 0_{4 \times 3} & 0_{4 \times 3} \\ \bar{\omega}_2(\hat{r}, \hat{v})F^+(\phi^c(\hat{r})) & 0_{4 \times 3} & 0_{4 \times 3} \end{pmatrix} \in \mathbb{R}^{12 \times 9}$$

with $\bar{\omega}_1(\hat{r}, \hat{u}) = \omega_1(\phi^c(\hat{r}), F^+(\phi^c(\hat{r}))\hat{u})$, $\bar{\omega}_2(\hat{r}, \hat{v}) = \omega_2(\phi^c(\hat{r}), F^+(\phi^c(\hat{r}))\hat{v})$.

While the expression of the observer may seem complicated in the new coordinates, it simplifies greatly in the original coordinates. Indeed, using (8) it can be shown that observer (15) can be expressed as follows in the original coordinates

$$\begin{aligned} \dot{\hat{x}} &= \hat{y} - 3\theta F^+(\hat{x})(\phi(\hat{x}) - r) \\ \dot{\hat{y}} &= \hat{z} - 3\theta^2 F^+(\hat{x})(\phi(\hat{x}) - r) \\ \dot{\hat{z}} &= -\theta^3 F^+(\hat{x})(\phi(\hat{x}) - r) \end{aligned} \quad (16)$$

This observer has only one tuning parameter, namely θ . The tuning parameter is directly related to the convergence speed of the observer. The higher θ is taken, the faster the observer will converge.

D. Adaptation of the observer for aperiodic discrete-time measurements

The continuous observer (16) is now adapted in order to deal with sampled aperiodic measurements. The measurements r_1, r_2, r_3 are supposed to be sampled and aperiodic, that is, each measurement r_i , $i = 1, 2, 3$, is received at some discrete instant t_k^i verifying

- i) $t_k^i < t_{k+1}^i$, $k \in \mathbb{N}$,
- ii) $\lim_{k \rightarrow +\infty} t_k^i = +\infty$,
- iii) $\tau_{\min} < |t_{k+1}^i - t_k^i| < \tau_{\max}$ for all $k \in \mathbb{N}$, with τ_{\min} and τ_{\max} some constants representing the minimum and maximum sampling time for the outputs.

The continuous-discrete time observer is inspired by [4] and given by

$$\begin{aligned} \dot{\hat{x}}(t) &= \hat{y}(t) - 3\theta F^+(\hat{x}(t))\tilde{\pi}(t) \\ \dot{\hat{y}}(t) &= \hat{z}(t) - 3\theta^2 F^+(\hat{x}(t))\tilde{\pi}(t) \\ \dot{\hat{z}}(t) &= -\theta^3 F^+(\hat{x}(t))\tilde{\pi}(t) \\ \dot{\tilde{\pi}}_1(t) &= -3\theta\tilde{\pi}_1(t), \quad t \in [t_k^1, t_{k+1}^1) \\ \dot{\tilde{\pi}}_2(t) &= -3\theta\tilde{\pi}_2(t), \quad t \in [t_k^2, t_{k+1}^2) \\ \dot{\tilde{\pi}}_3(t) &= -3\theta\tilde{\pi}_3(t), \quad t \in [t_k^3, t_{k+1}^3) \\ \tilde{\pi}_4(t) &= \phi_4(\hat{x}(t)) - r_4(t) \end{aligned} \quad (17)$$

and

$$\tilde{\pi}_i(t_k^i) = \phi_i(\hat{x}(t_k^i)) - r_i(t_k^i), \quad \forall k \in \mathbb{N}, \quad (18)$$

when a measurement is received.

Remark 6: Note that the output r_4 is a virtual output and then not sampled since its continuous measurements are available.

Remark 7: An explicit expression of the left-inverse matrix F^+ can be computed off-line since F only depends only on known parameters. The value of $F^+(x)$ then can be computed based on this expression and no on-line matrix inversion is required when the observer is running.

Theorem 1: There exists $\theta^* \geq 1$ such that for $\theta \geq \theta^*$, there exists $\delta > 0$ such that for $\tau_{\max} \leq \delta$, there exist $\epsilon > 0$ such that

$$\left\| \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} - \begin{pmatrix} \hat{x}(t) \\ \hat{y}(t) \\ \hat{z}(t) \end{pmatrix} \right\| \leq C_1 e^{-C_2 t} + \frac{C_3 \delta \epsilon}{\theta} \quad (19)$$

if

$$\left\| \begin{pmatrix} x(0) \\ y(0) \\ z(0) \end{pmatrix} - \begin{pmatrix} \hat{x}(0) \\ \hat{y}(0) \\ \hat{z}(0) \end{pmatrix} \right\| \leq \epsilon, \quad (20)$$

where $C_1, C_2, C_3 > 0$ are some constants.

Remark 8: Theorem 1 states that the proposed observer is locally convergent. Though only local stability is obtained, it will be shown in the simulation section that the area of convergence is very large.

Remark 9: As it has been noted before, the tuning parameter θ is directly related to the convergence speed of the observer. It therefore has to respect some restrictions. First, since the design is of high-gain type, θ has to be taken large enough to dominate the nonlinear terms. Since the output is discretized, for a given value of θ the maximum sampling periods may not be taken as small as desired. Finally, since the convergence is local, the observer may have to be initialized close enough to the true value of the state to obtain the convergence, depending on the value of θ (but as it will be shown in the simulation section this point is not very restrictive in practice).

The following technical lemma will be used in the proof of the Theorem.

Lemma 2: Let $f : [-\tau, +\infty) \rightarrow \mathbb{R}^+$ be a C^1 function such that $\dot{f}(t) \leq -a_1 f(t) + a_2 \sup_{t-\tau \leq \sigma \leq t} f(\sigma) + a_3$ for $t \geq 0$ and if $a_1 > a_2 > 0$, then one has $f(t) \leq b_1 e^{-b_2 t} + \frac{a_3}{a_1 - a_2}$, with b_2 the solution of $b_2 - a_1 + a_2 e^{b_2 \tau} = 0$ and $b_1 > \sup_{t-\tau \leq t \leq 0} f(t)$.

Proof 2 (Lemma 2): Lemma 2 is a direct extension of the Lemma p. 378 in [12] and the proof is then not detailed here.

Proof 3 (Theorem 1): The stability analysis is done in the coordinates (r, u, v) and can be transposed in a straightforward way in the original coordinates since the convergence is established on a compact set and the change of coordinates is locally Lipschitz.

The proof is split into three parts. In the first part, the dynamics of the error is given and new coordinates are considered which are more adapted to the stability analysis. In the second part, a candidate Lyapunov function is given and an over-valuation of its derivative is obtained. Finally, in the last part the obtained over-valuation is used to show the stability of the error system.

Part 1

Let us denote $\tilde{r} = \hat{r} - r$ and $e = \hat{Z} - Z$, then the error system is given by

$$\begin{aligned} \dot{e} = & Ae - \theta \Delta_\theta^{-1} K \tilde{r} + \theta \Delta_\theta^{-1} K (\tilde{r} - \tilde{\pi}) \\ & - \theta M(\hat{Z}) \Delta_\theta^{-1} K \tilde{r} + \theta M(\hat{Z}) \Delta_\theta^{-1} K (\tilde{r} - \tilde{\pi}) \\ & + (\eta(\hat{Z}) - \eta(Z)) - B \varrho(Z) \varepsilon \end{aligned} \quad (21)$$

One now considers the classical high-gain coordinates $\bar{e} = \Delta_\theta e$, which gives

$$\begin{aligned} \dot{\bar{e}} = & \theta(A - KC)\bar{e} + \theta K(\tilde{r} - \tilde{\pi}) \\ & - \theta \Delta_\theta M(\hat{Z}) \Delta_\theta^{-1} K(C\bar{e} - (\tilde{r} - \tilde{\pi})) \\ & + \Delta_\theta(\eta(\hat{Z}) - \eta(Z)) - \frac{1}{\theta^2} B \varrho(\hat{Z}) \varepsilon \end{aligned} \quad (22)$$

by using the equalities $\Delta_\theta A \Delta_\theta^{-1} = \theta A$, $C \Delta_\theta^{-1} = C$ and $\Delta_\theta B = \frac{1}{\theta^2} B$.

Part 2

Let us define the candidate Lyapunov function as $V(\bar{e}) = \bar{e}^T S \bar{e}$, with S the positive definite matrix solution of the equation $S + A^T S + SA - C^T C = 0$. The derivative of $V(\bar{e})$ is given by

$$\begin{aligned} \dot{V}(\bar{e}) = & \theta \bar{e}^T ((A - KC)^T S + S(A - KC)) e \\ & + 2 \bar{e}^T S \Delta_\theta (\eta(\hat{Z}) - \eta(Z)) + 2 \theta \bar{e}^T S K (\tilde{r} - \tilde{\pi}) \\ & - 2 \theta \bar{e}^T S \Delta_\theta M(Z) \Delta_\theta^{-1} K (C\bar{e} - (\tilde{r} - \tilde{\pi})) \\ & + 2 \theta \bar{e}^T S \Delta_\theta (M(Z) - M(\hat{Z})) \Delta_\theta^{-1} K (C\bar{e} - (\tilde{r} - \tilde{\pi})) \\ & - 2 \bar{e}^T S \left(\frac{1}{\theta^2} B \varrho(r) \varepsilon \right) \end{aligned} \quad (23)$$

Noticing that $K = S^{-1} C^T$, using the Cauchy-Schwarz

inequality and the Rayleigh quotient yields

$$\begin{aligned} \dot{V}(\bar{e}) \leq & -\theta V(\bar{e}) + 2\sqrt{\lambda_M(S)} \sqrt{V(\bar{e})} \|\Delta_\theta(\eta(\hat{Z}) - \eta(Z))\| \\ & + 2\theta \sqrt{\lambda_M(S)} \|K\| \sqrt{V(\bar{e})} \|\tilde{r} - \tilde{\pi}\| \\ & + 2\sqrt{\lambda_M(S)} \sqrt{V(\bar{e})} \|\theta \Delta_\theta M(Z) \Delta_\theta^{-1}\| \|K\| \\ & \quad \times (\|C\bar{e}\| + \|\tilde{r} - \tilde{\pi}\|) \\ & + 2\sqrt{\lambda_M(S)} \sqrt{V(\bar{e})} \|\theta \Delta_\theta (M(Z) - M(\hat{Z})) \Delta_\theta^{-1}\| \\ & \quad \times \|K\| (\|C\bar{e}\| + \|\tilde{r} - \tilde{\pi}\|) \\ & + \frac{2}{\theta^2} \sqrt{\lambda_M(S)} \sqrt{V(\bar{e})} \|B\| \|\varrho(Z)\| \|\varepsilon\| \end{aligned} \quad (24)$$

Using the fact that $\|B\| = \|C\| = 1$, the following inequality (which can be obtained by following the same steps as in [4])

$$\|\tilde{r}(t) - \tilde{\pi}(t)\| \leq \frac{\theta}{\sqrt{\lambda_m(S)}} \int_{t-\tau_{\max}}^t \sqrt{V(\bar{e}(s))} ds \quad (25)$$

and denoting $\sigma(S) = \lambda_M(S)/\lambda_m(S)$ leads to

$$\begin{aligned} \dot{V}(\bar{e}) \leq & -\theta V(\bar{e}) + 2\sqrt{\lambda_M(S)} \|\Delta_\theta(\eta(\hat{Z}) - \eta(Z))\| \sqrt{V(\bar{e})} \\ & + 2\theta^2 \sqrt{\sigma(S)} \|K\| \sqrt{V(\bar{e})} \int_{t-\tau_{\max}}^t \sqrt{V(\bar{e}(s))} ds \\ & + 2\sqrt{\sigma(S)} \|K\| \|\theta \Delta_\theta M(Z) \Delta_\theta^{-1}\| V(\bar{e}) \\ & + 2\theta \sqrt{\sigma(S)} \|K\| \|\theta \Delta_\theta M(Z) \Delta_\theta^{-1}\| \\ & \quad \times \sqrt{V(\bar{e})} \int_{t-\tau_{\max}}^t \sqrt{V(\bar{e}(s))} ds \\ & + 2\sqrt{\sigma(S)} \|K\| \|\theta \Delta_\theta (M(Z) - M(\hat{Z})) \Delta_\theta^{-1}\| V(\bar{e}) \\ & + 2\theta \sqrt{\sigma(S)} \|K\| \|\theta \Delta_\theta (M(Z) - M(\hat{Z})) \Delta_\theta^{-1}\| \\ & \quad \times \sqrt{V(\bar{e})} \int_{t-\tau_{\max}}^t \sqrt{V(\bar{e}(s))} ds \\ & + \frac{2}{\theta^2} \sqrt{\lambda_M(S)} \sqrt{V(\bar{e})} \|\varrho(Z)\| \delta_\varepsilon \end{aligned} \quad (26)$$

Then, there exists constants $k_1, \dots, k_7 \geq 0$ independent of θ such that

$$\begin{aligned} \frac{d}{dt}(\sqrt{V(\bar{e})}) & \leq \frac{\dot{V}(\bar{e})}{2\sqrt{V(\bar{e})}} \\ & \leq -\frac{\theta}{2} \sqrt{V(\bar{e})} + k_1 \|\Delta_\theta(\eta(\hat{Z}) - \eta(Z))\| \\ & \quad + \theta^2 k_2 \int_{t-\tau_{\max}}^t \sqrt{V(\bar{e}(s))} ds \\ & \quad + k_3 \|\theta \Delta_\theta M(Z) \Delta_\theta^{-1}\| \sqrt{V(\bar{e})} \\ & \quad + \theta k_4 \|\theta \Delta_\theta M(Z) \Delta_\theta^{-1}\| \int_{t-\tau_{\max}}^t \sqrt{V(\bar{e}(s))} ds \\ & \quad + k_5 \|\theta \Delta_\theta (M(Z) - M(\hat{Z})) \Delta_\theta^{-1}\| \sqrt{V(\bar{e})} \\ & \quad + \theta k_6 \|\theta \Delta_\theta (M(Z) - M(\hat{Z})) \Delta_\theta^{-1}\| \\ & \quad \quad \times \int_{t-\tau_{\max}}^t \sqrt{V(\bar{e}(s))} ds \\ & \quad + \frac{k_7}{\theta^2} \|\varrho(Z)\| \delta_\varepsilon \end{aligned} \quad (27)$$

One can prove that the following inequalities holds, with L_1, L_2, L_3, L_4 constants independant of θ ,

- (i) $\|\Delta_\theta(\eta(\hat{Z}) - \eta(Z))\| \leq L_1\|e\|, \forall Z, \hat{Z} \in \mathcal{B}(0, 2\Gamma),$
- (ii) $\|\theta\Delta_\theta M(Z)\Delta_\theta^{-1}\| \leq L_2, \forall Z \in \mathcal{B}(0, \Gamma),$
- (iii) $\|\theta\Delta_\theta(M(Z) - M(\hat{Z}))\Delta_\theta^{-1}\| \leq L_3\|e\|, \forall Z, \hat{Z} \in \mathcal{B}(0, 2\Gamma),$
- (iv) $\|\varrho(Z)\| \leq L_4, \forall Z \in \mathcal{B}(0, 2\Gamma),$

with classical high gain techniques (as in [10] for example), since η is lower triangular and M strictly lower triangular and both functions are locally Lipschitz.

Then, using inequalities (i), (ii), (iii) and (iv) with inequality (28) gives the existence of $\bar{k}_1, \dots, \bar{k}_7$, independent of θ , such that

$$\begin{aligned} \frac{d}{dt}(\sqrt{V(\bar{e})}) &\leq -\frac{\theta}{2}\sqrt{V(\bar{e})} + (\bar{k}_1 + (\bar{k}_3 + \bar{k}_5)\|e\|)\sqrt{V(\bar{e})} \\ &\quad + \theta(\theta\bar{k}_2 + (\bar{k}_4 + \bar{k}_6)\|e\|) \int_{t-\tau_{\max}}^t \sqrt{V(\bar{e}(s))} ds \\ &\quad + \frac{\bar{k}_7}{\theta^2} \delta_\varepsilon \end{aligned} \quad (29)$$

Taking $\theta \geq 1$ such that $\theta/8 \geq \bar{k}_1$, $e \in \mathcal{B}(0, 1/(8(\bar{k}_3 + \bar{k}_5)))$, there exists K_1 and K_2 , independent of θ , such that

$$\begin{aligned} \frac{d}{dt}(\sqrt{V(\bar{e})}) &\leq -\frac{\theta}{4}\sqrt{V(\bar{e})} + \theta^2 K_1 \tau_{\max} \sup_{t-\tau_{\max} \leq s \leq t} \sqrt{V(\bar{e}(s))} \\ &\quad + \frac{K_2}{\theta^2} \delta_\varepsilon \end{aligned} \quad (30)$$

Note that inequality (30) is valid for $\hat{Z}, Z \in \mathcal{B}(0, 2\Gamma)$ and for $e \in \mathcal{B}(0, 1/(8(\bar{k}_3 + \bar{k}_5)))$. In the next part, we show that if the observer error e is initialized such that $\|e\|$ is sufficiently small, then \hat{Z} stay in $\mathcal{B}(0, 2\Gamma)$ and $\|e\|$ decreases.

Without loss of generality, one assumes that $\Gamma \geq 1/(8(\bar{k}_3 + \bar{k}_5))$, then if one ensures that e stays in $\mathcal{B}(0, 1/(8(\bar{k}_3 + \bar{k}_5)))$, this automatically guarantees that $\hat{Z} \in \mathcal{B}(0, 2\Gamma)$, since $Z \in \mathcal{B}(0, \Gamma)$ by assumption.

Part 3

It can be seen that since inequality (30) is valid for $e \in \mathcal{B}(0, 1/(8(\bar{k}_3 + \bar{k}_5)))$, then inequality (30) is valid on the whole set

$$\left\{ e \in \mathbb{R}^{12} \mid \sqrt{V(\bar{e})} \leq \left(\frac{\sqrt{\lambda_m(S)}}{8(\bar{k}_3 + \bar{k}_5)} \frac{1}{\theta^2} \right) \right\} \quad (31)$$

since $\|e\| \leq \frac{\theta^2}{\sqrt{\lambda_m(S)}} \sqrt{V(\bar{e})}$.

Take $\tau_{\max} \leq 1/(8\theta K_1)$, $\theta \geq \left(\frac{128(\bar{k}_3 + \bar{k}_5)K_2}{\sqrt{\lambda_m(S)}} \right) \delta_\varepsilon$ and assume that e is initialized such that $\sqrt{V(\bar{e}(s))} < \left(\frac{\sqrt{\lambda_m(S)}}{16(\bar{k}_3 + \bar{k}_5)} \frac{1}{\theta^2} \right)$ for $s \in [-\tau_{\max}, 0]$, then applying lemma 2 with $a_1 = \theta/4$, $a_2 = \theta K_1 \tau_{\max}$ and $a_3 = (K_2 \delta_\varepsilon)/\theta^2$ gives

$$\sqrt{V(\bar{e}(t))} \leq b_1 e^{-b_2 t} + \frac{a_3}{a_1 - a_2} \quad (32)$$

with $b_1 = \left(\frac{\sqrt{\lambda_m(S)}}{16(\bar{k}_3 + \bar{k}_5)} \frac{1}{\theta^2} \right)$. It can be directly seen that e always belong to the set defined by (31) since $b_1 + \frac{a_3}{a_1 - a_2} \leq$

$$\left(\frac{\sqrt{\lambda_m(S)}}{8(\bar{k}_3 + \bar{k}_5)} \frac{1}{\theta^2} \right).$$

Finally, noticing that $\|e\| \leq \frac{\theta^2}{\sqrt{\lambda_m(S)}} \sqrt{V(\bar{e})}$ ends the proof.

IV. SIMULATION RESULTS

The performances of the proposed observer are now illustrated with simulations. One assumes that three sources positions are known and located at

$$\begin{aligned} x^{r1} &= (500, 500, 0)^T \\ x^{r2} &= (500, -500, 0)^T \\ x^{r3} &= (-500, 500, 0)^T \end{aligned}$$

The target is supposed to follow a circular trajectory in the (x_1, x_2) plan with a height varying between 400m and 600m (see figure 3).

The minimum value for the height is considered to be equal to $\alpha = 10m$ and the trajectory is at least at a distance of $\beta = 10m$ of the sources.

The function $\mu(s)$ is equal to the square root for $s \geq \alpha^2$ and equal to a polyomial of order 5 on $[0, \alpha^2]$, such that $\mu(0) = 0$, $\mu'(0) = 1$, $\mu(\alpha^-) = \mu(\alpha^+)$, $\mu'(\alpha^-) = \mu'(\alpha^+)$, $\mu''(\alpha^-) = \mu''(\alpha^+)$. The computed function μ is depicted on figure 1.

The function $\kappa(s)$ is equal to zero for $s \geq \beta$, it is linear, with a slope S_κ , for low values and a polynomial of order 5 between the linear part and the zero value, making a C^2 connection. The function κ is then linear with a slope S_κ for $s \leq \beta - 10S_\kappa$, equal to $-50S_\kappa$ at the point $\beta - 10S_\kappa$, then equal to a polynomial of order 5 making the function κ C^2 and then equal to zero for $s \geq \beta$. The corresponding function with a slope of $S_\kappa = 10$ is depicted on figure 2.

The matrix $F(x)$ is equal to

$$F(x) = \begin{pmatrix} 2\mu_{p1}(x_1 - 500) & 2\mu_{p1}(x_2 - 500) & 2\mu_{p1}(x_3) \\ 2\mu_{p2}(x_1 - 500) & 2\mu_{p2}(x_2 + 500) & 2\mu_{p2}(x_3) \\ 2\mu_{p3}(x_1 + 500) & 2\mu_{p3}(x_2 - 500) & 2\mu_{p3}(x_3) \\ 0 & 0 & \kappa_p \end{pmatrix}$$

with $\mu_{pi} = \mu'(\|x - x^{ri}\|^2)$, $i = 1, 2, 3$ and $\kappa_p = \kappa'(x_3)$. The Moore-Penrose inverse F^+ of matrix F has been computed with the help of the symbolic Matlab toolbox.

For all the simulations, the observer is initialized at 0 for the position, speed and acceleration, except for \hat{x}_3 which is initialized at -100 to prove the effectiveness of the proposed approach.

A first set of simulations has been done with $\theta = 1.5$, $\tau_{\min} = 50ms$ and $\tau_{\max} = 60ms$ and are depicted on figures 3 and figure 4. It can be seen that despite the fact that the initialization of the observer is far away from the correct one, it converges to the true values. Furthermore, the residual error due to the unknown jerk is very small.

A second set of simulations has been done, but this time with noise on the measured outputs, with the same parameters as in the first set of simulations. The measured outputs and

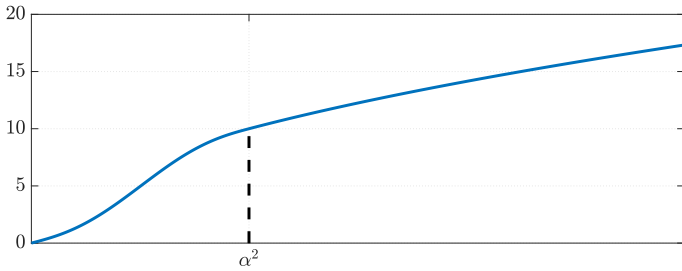


Fig. 1: Function μ

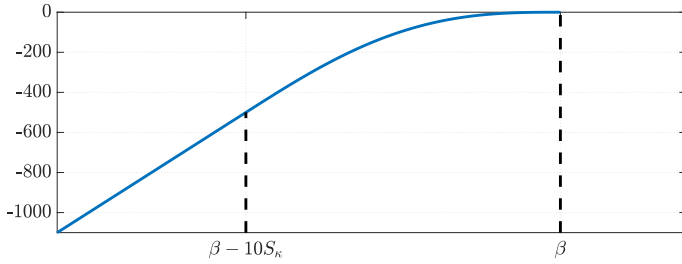


Fig. 2: Function κ

their estimated values are depicted on figure 6. The true position of the target and its estimation are reported on figure 5. It can be seen that despite an important noise, the estimation remains quite accurate.

V. CONCLUSION

In this paper a nonlinear observer has been proposed that allows the reconstruction of the position of a target in three dimensions from three range measurements. The range measurements are sampled and the sampling instants are asynchronous and each range measurement sampling is independent from the other range measurements. The proposed scheme is based on a modification of the system, which render the system globally observable, and a nonlinear transformation. Local convergence is further ensured theoret-

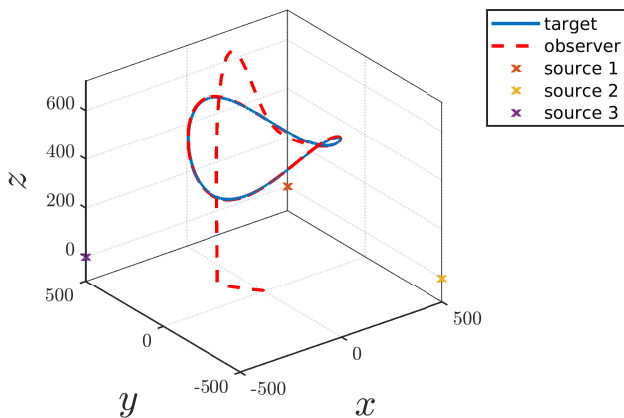
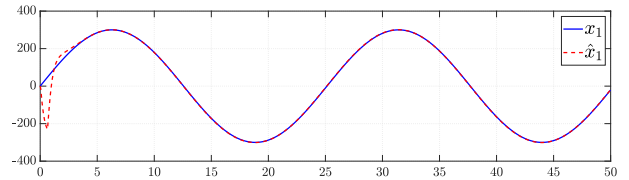
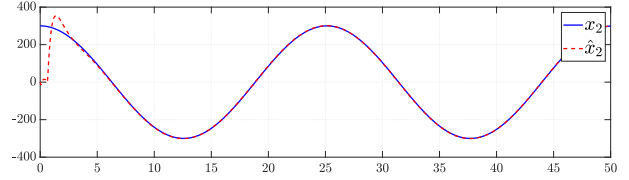


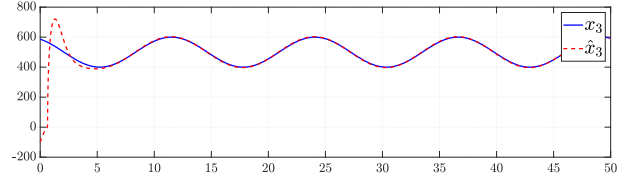
Fig. 3: Trajectory and estimation in 3 dimension without noise



(a) x_1

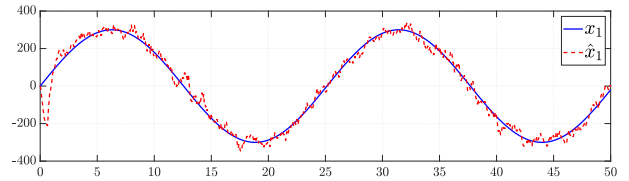


(b) x_2

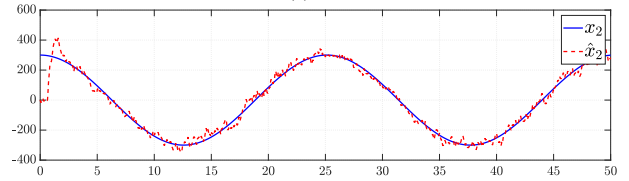


(c) x_3

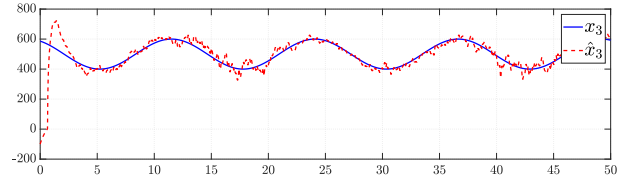
Fig. 4: Trajectory of the target and its estimates without noise



(a) x_1



(b) x_2



(c) x_3

Fig. 5: Trajectory of the target and its estimates with noise

ically.

The performances of the observer have been assessed with simulations. It is shown to behave well even in the presence of noise and it converges even with an initialization of the estimates that is far from the correct value.

Future works will focus on the case of more than three range measurements and the modification of the observer to obtain semi-global stability.

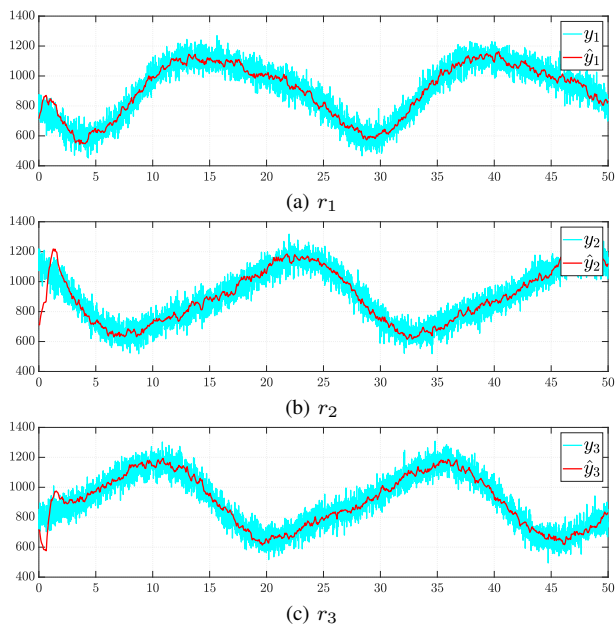


Fig. 6: Measured and reconstructed outputs with noise

REFERENCES

- [1] A. Alcocer. Positioning and navigation systems for robotic underwater vehicles. *Doctor thesis, Instituto Superior Tecnico*, 2009.
- [2] A. Alcocer, P. Oliveira, and A. Pascoal. Study and implementation of an ekf gib-based underwater positioning system. *Control engineering practice*, 15(6):689–701, 2007.
- [3] Y. Bar-Shalom, P.K. Willett, and X. Tian. *Tracking and data fusion*, volume 11. YBS publishing Storrs, CT, USA., 2011.
- [4] T. Bonargent, T. Menard, O. Gehan, and E. Pigeon. Adaptive observer design for a class of lipschitz nonlinear systems with multirate outputs and uncertainties: Application to attitude estimation with gyro bias. *International Journal of Robust and Nonlinear Control*, 31(8):3137–3162, 2021.
- [5] J.J. Caffery. A new approach to the geometry of toa location. In *Vehicular Technology Conference Fall 2000. IEEE VTS Fall VTC2000. 52nd Vehicular Technology Conference (Cat. No. 00CH37152)*, volume 4, pages 1943–1949. IEEE, 2000.
- [6] A. Caiti, A. Garulli, F. Livide, and D. Prattichizzo. Localization of autonomous underwater vehicles by floating acoustic buoys: a set-membership approach. *IEEE Journal of Oceanic Engineering*, 30(1):140–152, 2005.
- [7] K.W. Cheung, H.C. So, W.K. Ma, and Y.T. Chan. Least squares algorithms for time-of-arrival-based mobile location. *IEEE transactions on signal processing*, 52(4):1121–1130, 2004.
- [8] K. Cisek, A. Zolich, K. Klausen, and T.A. Johansen. Ultra-wide band real time location systems: Practical implementation and uav performance evaluation. In *2017 Workshop on Research, Education and Development of Unmanned Aerial Systems (RED-UAS)*, pages 204–209. IEEE, 2017.
- [9] J.J. Dougherty, H. El-Sherief, D.J. Simon, and G.A. Whitmer. Gps modeling for designing aerospace vehicle navigation systems. *IEEE transactions on aerospace and electronic systems*, 31(2):695–705, 1995.
- [10] M. Farza, M. M’Saad, and L. Rossignol. Observer design for a class of mimo nonlinear systems. *Automatica*, 40(1):135–143, 2004.
- [11] I. Guvenc, S. Gezici, F. Watanabe, and H. Inamura. Enhancements to linear least squares localization through reference selection and ml estimation. In *2008 IEEE Wireless Communications and Networking Conference*, pages 284–289. IEEE, 2008.
- [12] A. Halanay. *Differential equations: Stability, oscillations, time lags*, volume 6. Elsevier, 1966.
- [13] X. Hu, M. Bao, X.P. Zhang, L. Guan, and Y.H. Hu. Generalized iterated kalman filter and its performance evaluation. *IEEE Transactions on Signal Processing*, 63(12):3204–3217, 2015.
- [14] X. Hu, Y.H. Hu, and B. Xu. Generalised kalman filter tracking with multiplicative measurement noise in a wireless sensor network. *IET Signal Processing*, 8(5):467–474, 2014.
- [15] Y. Huang, J. Benesty, G.W. Elko, and R.M. Mersereati. Real-time passive source localization: A practical linear-correction least-squares approach. *IEEE transactions on Speech and Audio Processing*, 9(8):943–956, 2001.
- [16] Z. Li, W. Trappe, Y. Zhang, and B. Nath. Robust statistical methods for securing wireless localization in sensor networks. In *IPSN 2005. Fourth International Symposium on Information Processing in Sensor Networks, 2005.*, pages 91–98. IEEE, 2005.
- [17] B.R. Mahafza. *Introduction to radar analysis*. CRC press, 2017.
- [18] S.E. Manolakis. Efficient solution and performance analysis of 3-d position estimation by trilateration. *IEEE Transactions on Aerospace and Electronic systems*, 32(4):1239–1248, 1996.
- [19] G. Mao, B. Fidan, and B.D.O. Anderson. Wireless sensor network localization techniques. *Computer networks*, 51(10):2529–2553, 2007.
- [20] E.B. Mazomenos, J.S. Reeve, and N.M. White. A range-only tracking algorithm for wireless sensor networks. In *2009 International Conference on Advanced Information Networking and Applications Workshops*, pages 775–780. IEEE, 2009.
- [21] N.B. Priyantha. *The cricket indoor location system*. PhD thesis, Massachusetts Institute of Technology, 2005.
- [22] H. Qian, P. Fu, B. Li, J. Liu, and X. Yuan. A novel loss recovery and tracking scheme for maneuvering target in hybrid wsns. *Sensors*, 18(2):341, 2018.
- [23] M.J. Rendas and I.M.G. Lourtie. Hybrid navigation system for long range operation. In *Proceedings of IEEE Symposium on Autonomous Underwater Vehicle Technology (AUV’94)*, pages 353–359. IEEE, 1994.
- [24] A.H. Sayed, A. Tarighat, and N. Khajehnouri. Network-based wireless location: challenges faced in developing techniques for accurate wireless location information. *IEEE signal processing magazine*, 22(4):24–40, 2005.
- [25] H.C. Schau and A.Z. Robinson. Passive source localization employing intersecting spherical surfaces from time-of-arrival differences. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, 35(8):1223–1225, 1987.
- [26] M. Sepahvand, A. Naseri, M. Raeesdanaee, and M.H. Khanzadeh. Target tracking with unknown maneuvers using adaptive parameter estimation in wireless sensor networks. *Journal of Communication Engineering*, 7(2):50–66, 2018.
- [27] G. Shen, R. Zetik, and R.S. Thoma. Performance comparison of toa and tdoa based location estimation algorithms in los environment. In *2008 5th Workshop on Positioning, Navigation and Communication*, pages 71–78. IEEE, 2008.
- [28] P. Teunissen. Nonlinear least squares. *Manuscripta Geodaetica*, 15:137–150, 1990.
- [29] R.S. Thoma, O. Hirsch, J. Sachs, and R. Zetik. Uwb sensor networks for position location and imaging of objects and environments. In *The Second European Conference on Antennas and Propagation, EuCAP 2007*, pages 1–9. IET, 2007.
- [30] F. Thomas and L. Ros. Revisiting trilateration for robot localization. *IEEE Transactions on robotics*, 21(1):93–101, 2005.
- [31] X. Wang, M. Fu, and H. Zhang. Target tracking in wireless sensor networks based on the combination of kf and mle using distance measurements. *IEEE Transactions on mobile computing*, 11(4):567–576, 2011.
- [32] J. Yan, C.J.M. Tiberius, G. Bellusci, and G.J.M. Janssen. A novel non-iterative localization solution. In *GLOBECOM 2009-2009 IEEE Global Telecommunications Conference*, pages 1–6. IEEE, 2009.
- [33] J. Yan, C.J.M. Tiberius, G.J.M. Janssen, P.J.G. Teunissen, and G. Bellusci. Review of range-based positioning algorithms. *IEEE Aerospace and Electronic Systems Magazine*, 28(8):2–27, 2013.
- [34] X. Yang, W.A. Zhang, A. Liu, and L. Yu. Linear fusion estimation for range-only target tracking with nonlinear transformation. *IEEE Transactions on Industrial Informatics*, 16(10):6403–6412, 2019.
- [35] X. Yang, W.A. Zhang, L. Yu, and F. Yang. Sequential gaussian approximation filter for target tracking with nonsynchronous measurements. *IEEE Transactions on Aerospace and Electronic Systems*, 55(1):407–418, 2018.
- [36] R. Zekavat and R.M. Buehrer. *Handbook of position location: Theory, practice and advances*, volume 27. John Wiley & Sons, 2011.