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## **Introduction to the papers of TWG24:**

### **Representation in mathematics teaching and learning**

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### **Introduction**

When TWG24, *Representations in Mathematics Teaching and Learning*, was initially opened at CERME10 (Robotti et al., 2017) it included 24 participants from 13 countries with 16 accepted papers and 2 posters; at CERME11 (Baccaglini-Frank et al, 2019) it grew to welcome 31 participants from 16 countries, with 18 accepted papers and 4 accepted posters; and at CERME12 TWG24 enjoyed the online participation of 28 researchers from 10 different countries with 19 papers and 2 posters presented. At CERME12 the structure of the working sessions allocated for discussion of each paper or poster was designed to stimulate interaction and collaboration among participants, even though the conference was held entirely online. Each paper or poster was allocated to one of six working sessions, which typically included three or four papers with theoretical, methodological or thematic similarities. All papers were shared with the participants in advance of the conference, and each presenting author was asked to concentrate especially on providing feedback or questions on the papers presented within his/her session. Presenting authors were asked to prepare a short presentation of their paper, including one slide explicitly addressing the theoretical perspective taken on “representations”. The allocated time (20 minutes for papers and 10 minutes for posters) was split equally between author presentation and working group discussion. There was also a workshop session dedicated entirely to working in smaller rotating subgroups on questions designed by the co-leaders, which had emerged from the previous four days’ discussions. The final session was devoted to a conclusive discussion chaired by TWG co-leaders with contributions from each of the morning’s subgroups.

In this short report of the main themes that were discussed, we divide the themes between more practice-based ones and more theoretical ones.

### **Practice-based emerging themes**

In thinking about representations in practice, important themes emerged related to sharing representational practices in three overlapping zones: 1) across educational systems, 2) within interpersonal learning activities and 3) inside encounters with various mathematical technologies and tools. At the largest scale, we found ourselves asking: *What can we observe about how representations are used across different educational systems? What issues arise when we ‘import’ something from one education system to another?* This macro-level question was inspired in large part by Palop del Río and Santaengracia’s paper exploring the introduction of a concrete approach to

the bar-model imported from Singapore educational system into a fifth-grade classroom in Spain. Interested in the bar-model's flexibility in a wide variety of problem scenarios, Palop del Río and Santaengracia sought to test the implicit assumption in Spanish curricula that this representation must be introduced to students in their earliest years of schooling to be an effective tool for thinking.

Although the potentials of (and contested approaches to) the “Singapore bar” remain a paradigmatic example of “importation”, the theme of working across educational systems also surfaced in papers which sought to better understand a variety of under-studied educational activities, asking to what extent these activities can be understood as “systems” in their own right. Angeloni, Wille and Hausch, for example, explicitly challenged the concept of “importation”, arguing that the invention and development of mathematical representations in Austrian Sign Language was a much more complicated affair than “importation” or “translation” might imply. While papers like this one did not directly address the complex politics of national educational systems, their ideas were expanded in the subsequent discussion, highlighting and exploring the emergence of representations in minority or marginalized linguistic, pedagogical, and digital spaces (some more of which are noted below). Challenging views of learning about or with mathematical representations as being static, normative or universalizable experiences, practice-based evidence inspires us to find new ways of drawing on marginalized mathematical experiences as sources of broader pedagogical insight in their own right.

Several papers focusing on students working collaboratively on mathematical tasks (either with peers or a teacher/researcher) also led our group to focus on the development of interpersonal/interactional representational strategies, thinking about the generation and sharing of representational systems within both individual learning support and whole class contexts. These papers led us to discuss: *How much and in which ways should learner-generated representational strategies be encouraged and incorporated into educational discourse by the teacher (in her classroom or beyond)?* Finesilver, for example, provided a vivid case study of one student, struggling with division problems, personalizing and modifying pictorial representations and metaphors as ways to engage with multiplicative structure, including eventually more abstract tasks. This case demonstrated how idiosyncratic representations can be harnessed to combat exclusion of marginalized learners, in this case neurodiverse experiences. Lisarelli and Poli also reported on a teaching sequence in a class which aimed at developing responsive representational strategies drawing on student-generated imagery as a tool to think with while problem solving. While on the first activity students used a collection of representations, the subsequent whole class discussion helped the teacher navigate towards a consensual representation in the class. Meanwhile Velez, Serrazina and da Ponte aimed to understand exactly how a teacher managed his pupils’ use and interpretation of representations during whole class discussion. Hence their focus was predominantly on the verbal interactions, such as the ways the teacher changed the question type as students’ representations varied.

In thinking about the empirical evidence presented within our group, we also sought to engage with the fundamental question: *What representational practices are demonstrably effective (and less effective) in promoting meaningful mathematical learning? How might this vary in different educational environments?* In our general discussion of the questions highlighted above, many participants talked about expanding from or avoiding overreliance on conventional and ubiquitous representations. They discussed encouraging a mix of speech, gesture, and tool-use as powerful

multimodal representational activities in the classroom and beyond. In encouraging and incorporating learner-generated representations, the group discussed balancing freedom and creativity with learning the necessary conventions for participating in the wider mathematical community. The power dynamics of classrooms were also addressed in our efforts to differentiate between representations that are an integral part of thinking and problem solving, as opposed to those produced on request just for pleasing the teacher, or retroactively after having already solved the problem without observable external representational strategy.

### **Theory-based emerging themes**

As for the representations in theory, various themes emerged. We decided to focus the group discussions around the three questions, introduced below:

- a) Of the theoretical and analytical frameworks presented, which have synergy? Where are conflicts? What might be fruitful combinations?
- b) What hidden assumptions might we have about representations that will be made by or offered to different kinds of learners? How does this affect our choices regarding research participants, methods, and theory? What assumptions need to be uncovered and changed, and how to do this?
- c) What are the links between representations in our research data and how we (re)present it to others? What representations serve us well in our professional practices as researchers and how can we develop them further?

Regarding question a), identifying synergy in representational frameworks proved quite challenging, and is deserving of more sustained consideration. However, the variety of analytical systems presented gave rise to many lively discussions considering how complementary aspects might be adapted, combined or developed for research in other contexts or with different types of datasets. Conflicts included some based on familiar divisions, such as between more platonic perspectives, according to which mathematical objects are pure and abstract, accessible only via representations, as opposed to others, in which mathematical objects do not reside in some hyper-reality, but in the discourse itself; there were also competing systems of terminology and classification to navigate. Nevertheless, we consider this diverse and multifaceted – yet interconnected – form of ‘rhizomatic’ theory-building to be a strength of the field (Deleuze & Guattari, 1980/1987).

In thinking through question b), we recognised that researchers in our group have worked with a wide variety of participants in terms of age, stage of education and level of expertise – from primary education up to PhD students – but also diverse learner groups, not only in terms of the different national educational systems from which we hail, but including e.g. sign language users, one-to-one intervention work with struggling students, and mathematics clubs or communities outside of formal schooling. One example of an assumption that came up was that students perceived as high achieving can easily work with multiple representations, together with the assumption that those perceived as low achieving have trouble transitioning between one representation and another. The result of such thinking is that in many educational settings, the latter students are offered narrower representational experiences, which limits learning opportunities and so disadvantages them even further. This was noted by many of our participants.

An interesting consideration, which emerged from discussion of question c), was about the difficulty of selecting and pairing appropriate and creative analytical tools with representations for communicating data interpretation and results. Choices and decisions about analytical tools, frameworks and procedures may highlight certain aspects of the data, but result in the loss of other aspects, particularly when we then communicate these to others. It was proposed to look to other fields for representational inspiration (e.g. computer science, media studies, dance); however, our group's papers already included interesting examples. Ott and Wille explored patterns of communication in one-on-one teacher-student support, seeking to understand individual learning support through the analysis of how and when communications between students and their instructors moved between two diagrammatic representations of number: the natural numbers and the field of twenty. To do this, the authors developed a visual system for coding the flow of diagrammatic conversation, which allowed them to make general observations about pedagogical patterns in their empirical study. Miragliotta and Lisarelli drew on Sfard's (2008) *realisation trees* in their research. This allowed them to make predictions about the ways in which classroom discourse might take hold of the geometric concept of "the height of a triangle". After analysing the lesson, the realisation tree helped them to map missing strategies and connections, as well as highlight novel ways of thinking about a triangle's height that occurred inside the classroom discussion.

## Looking forward

As these questions and examples highlight, issues related to representations – both at the practical and theoretical levels, and in the relationships between practice and theory – seem to be of continued (or perhaps increasing) interest to the educational research community. This includes exploring underlying tensions between the multiple theoretical lenses through which representations can be conceived and studied (Baccaglini-Frank et al., 2022), a theme that was also addressed in the plenary panel at CERME12. Hopefully, both an exciting new fusion of theoretical and practice-based observations will be further discussed in the near future. We welcome more researchers to join our group over the next CERME in Budapest!

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