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# Magnetic Field Upscaling and B-Conforming Magnetodynamic Multiscale Formulation

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**Abstract**—This paper proposes two novel methods for upscaling the macroscopic magnetic field from the local solutions in magnetodynamic multiscale problems. Unlike the volume average method classically used, these methods yield accurate values of the macroscale magnetic field for problems with strong locally-confined eddy currents which enables B-conforming multiscale formulations of eddy current problems at higher frequencies.

**Index Terms**—Eddy currents, Heterogeneous Multiscale Method, Homogenization, Nonhomogeneous media

## I. INTRODUCTION

Multiscale methods have been extensively studied in the electromagnetic community thanks to their easy parallelization and ability to speed up numerical simulations of problems involving heterogeneous materials such as laminated cores, stranded inductors and soft magnetic composites [1]–[4]. These methods make possible the simulation of 3D nonlinear eddy current problems with composite materials, unlike classical methods such as the finite element method (FEM) which are still too expensive for these problems.

Multiscale methods generally necessitate the upscaling of some quantity like the homogenized magnetic permeability or the homogenized dual field such as the magnetic field  $\mathbf{h}$  for B-conforming formulations or the magnetic flux density  $\mathbf{b}$  for H-conforming formulations. In this paper, we consider a multiscale formulation that uses the upscale of the magnetic field. The upscaling of the magnetic and the electric fields can unfortunately be non trivial for problems with local eddy currents. Indeed, a simple average of the local magnetic field can fail to filter out the contribution of the locally confined eddy currents into the macroscale magnetic field [3]. This paper proposes two novel methods for upscaling the magnetic field  $\mathbf{h}$  that are valid for problems with a lot of locally confined eddy currents. These methods only depend on the solutions of the mesoscale problem and the topology of its conducting region. They have been tested and validated on a 2D eddy current problem solved with a B-conforming formulation.

## II. MULTISCALE MODELING

We are interested in solving the following Maxwell’s equations on a domain  $\Omega \in \mathbb{R}^d$  in the magnetodynamic regime:

$$\text{curl } \mathbf{h} = \mathbf{j}, \quad \text{div } \mathbf{b} = 0, \quad \text{curl } \mathbf{e} = -\partial_t \mathbf{b}, \quad \mathbf{j} = \boldsymbol{\sigma} \mathbf{e}, \quad \mathbf{h} = \boldsymbol{\nu} \mathbf{b}. \quad (1)$$

In (1),  $\mathbf{h}$ ,  $\mathbf{b}$ ,  $\mathbf{j}$  and  $\mathbf{e}$  are the magnetic field, magnetic flux density, electric current density and electric field, respectively, and  $\boldsymbol{\sigma}$  and  $\boldsymbol{\nu}$  are the electrical conductivity and magnetic reluctivity. The dimension  $d$  is 2 or 3.

### A. The two scale model

In our multiscale approach, the reference problem (1) is replaced by the macroscale problem and many mesoscale problems both governed by equations similar to (1) and solved in an iterative scheme. Mesoscale fields are defined on a periodic unit cell denoted by  $Y$  and admit the following splitting [3]:

$$\begin{aligned} \mathbf{h} &= \mathbf{H} + \mathbf{J} \wedge \frac{\mathbf{y}}{d-1} + \mathbf{h}_c, & \mathbf{b} &= \mathbf{B} + \mathbf{b}_c \\ \mathbf{e} &= \mathbf{E} - \partial_t \mathbf{B} \wedge \frac{\mathbf{y}}{d-1} + \mathbf{e}_c, & \mathbf{j} &= \mathbf{J} + \mathbf{j}_c \end{aligned} \quad (2)$$

where capital letters denote the macroscale fields assumed constant on the periodic cell  $Y$ , the index  $c$  is used to denote correction fields assumed periodic on the cell  $Y$  and  $\mathbf{y}$  represents local coordinates in the cell. As  $\mathbf{b}$  and  $\mathbf{j}$  are divergence free,  $\mathbf{b}_c$  and  $\mathbf{j}_c$  are constrained to have a zero volume average on the cell  $Y$  (see [3]), thus leading to the following upscaling equations:

$$\mathbf{B} = \langle \mathbf{b} \rangle_Y, \quad \mathbf{J} = \langle \mathbf{j} \rangle_Y \quad (3)$$

where  $\langle \cdot \rangle_Y$  denotes the volume average on the cell  $Y$ .

The magnetic field  $\mathbf{h}$  can also be upscaled using  $\langle \mathbf{h} \rangle_Y$  for magnetostatic problems. Although some previous work could obtain good results using the volume average of  $\mathbf{h}$  for dynamical problems (see e.g., [1], [2] and [4]), the upscaling of  $\mathbf{h}$  cannot generally be done like this. Especially for problems

with strong locally-confined eddy currents as  $\langle \mathbf{h} \rangle_Y$  fails to filter out from  $\mathbf{h}$  their contribution to the macroscale Maxwell-Ampere law. In the next sections, two methods are proposed for recovering  $\mathbf{H}$  from  $\mathbf{h}$  and  $\mathbf{j}$ .

### B. Splitting global and local currents in the cell

A three-term Helmholtz–Hodge decomposition (HHD) [5] of the electric current density  $\mathbf{j}$  leads to a splitting into the local distribution of macro currents  $\mathbf{j}_M$  and the confined eddy currents  $\mathbf{j}_e = \text{curl } \mathbf{t}_0$ :

$$\mathbf{j} = \overline{\text{grad } \phi_0} + \text{curl } \mathbf{t}_0 + \mathbf{j}_M \quad \text{in } Y_c. \quad (4)$$

The potential  $\mathbf{t}_0$  is such that  $\mathbf{t}_0 \wedge \mathbf{n} = 0$  on  $\partial Y_c$ , is  $Y_c$ -periodic and with  $\text{div } \mathbf{t}_0 = 0$ . The field  $\mathbf{j}_M$  is the harmonic component of the HHD with  $\text{curl } \mathbf{j}_M = 0$  and  $\text{div } \mathbf{j}_M = 0$ . Thus, the gradient term disappears as  $\text{div } \mathbf{j} = 0$ . The integral  $\int_Y \mathbf{j}_e = \int_{Y_c} \text{curl } \mathbf{t}_0 = \int_{\partial Y_c} \mathbf{t}_0 \wedge \mathbf{n} = 0$ . Therefore  $\langle \mathbf{j}_M \rangle_Y = \mathbf{J}$ , so  $\mathbf{j}_M$  contains all the macro current, and flow lines of  $\mathbf{j}_M$  don't curl on themselves in  $Y$  as  $\text{curl } \mathbf{j}_M = 0$ .

The potential  $\mathbf{t}_0$  can be extracted from  $\mathbf{j}$  by solving the FEM projection  $\text{curl}(\mathbf{j} - \text{curl } \mathbf{t}_0) = 0$  in  $Y_c$ . The local currents  $\mathbf{j}_e$  have no effect on the macro current  $\mathbf{J}$ . Therefore, the potential  $\mathbf{t}_0$  from which it is derived should have no influence on the macroscopic magnetic field  $\mathbf{H}$  in order to respect the macro Maxwell-Ampere's law, that is:

$$\mathbf{H} = \langle \mathbf{h} - \mathbf{t}_0 \rangle_Y. \quad (5)$$

Finally (5) allows to upscale  $\mathbf{H}$  from  $\mathbf{h}$  and  $\mathbf{j}$ .

### C. Alternative on an insulated cell

When  $Y_c$  is insulated, i.e.  $\partial Y \cap Y_c = \emptyset$ , it is possible to compute  $\mathbf{H}$  without  $\mathbf{t}_0$ . Introducing the tangential average of a field  $\mathbf{u}$  over a closed surface  $S$ :

$$\langle \mathbf{u} \rangle_{S_{\parallel}} = \left( \int_S \mathbf{1}_t \right)^{-1} \cdot \int_S \mathbf{u}_t \quad (6)$$

where  $\mathbf{1} = (1, 1, 1)$ ,  $\mathbf{u}_t = (\mathbf{n} \wedge \mathbf{u}) \wedge \mathbf{n}$  with  $\mathbf{n}$  the normal to  $S$ . The inverse and dot product are to be taken component wise, i.e.  $v^{-1} \cdot w = (w_x/v_x, w_y/v_y, w_z/v_z)$ .

If  $Y$  is a rectangular parallelepiped, one can prove that  $\langle \mathbf{u} \rangle_{\partial Y_{\parallel}} = \langle \mathbf{u} \rangle_Y$  whenever  $\mathbf{u}$  is  $Y$ -periodic and  $\text{curl } \mathbf{u} = 0$  on  $Y$ . As  $\mathbf{J} = 0$  with an insulated cell,  $\text{curl}(\mathbf{h} - \mathbf{t}_0) = 0$  and the previous formula yields

$$\mathbf{H} = \langle \mathbf{h} - \mathbf{t}_0 \rangle_{\partial Y_{\parallel}} = \langle \mathbf{h} \rangle_{\partial Y_{\parallel}}. \quad (7)$$

The last equality is due to  $\mathbf{t}_0|_{\partial Y} = 0$ .

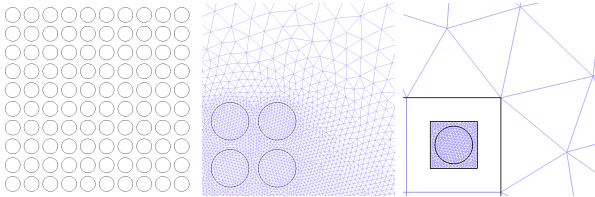


Fig. 1. Geometry of the core to homogenize (left), reference mesh in its corner (middle), corresponding macro and meso meshes (right).

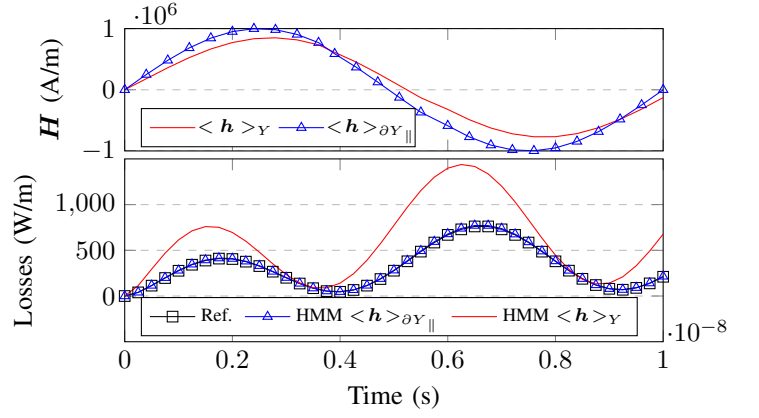


Fig. 2. Magnetic field upscaling strategies in a cell excited with sinusoidal  $\mathbf{B}$  at 100 MHz (top). Joule losses power in the full test case at 100 MHz (bottom). At 100 kHz,  $\mathbf{t}_0$  is negligible in (5) and  $\langle \mathbf{h} \rangle_Y$  works.

## III. VALIDATION

A full 2D multiscale problem is solved with a FEM B-conforming formulation of the macro and meso problems. The periodic geometry studied is a square core of 10 by 10 insulated conducting disks of radius  $100 \mu\text{m}$  (see Fig. 1). There is  $\mu = \mu_0$  everywhere,  $\sigma = 1.01 \times 10^7$  S/m in the disks and  $\sigma = 0$  outside. A source field is generated by an external coil surrounding the core and fed by a sinusoidal current.

HMM is used, i.e. a cell problem is associated to each Gauss point of the macro mesh, in which one element covers four disks of the real geometry (Fig. 1). The macro and meso problems are solved alternatively [4]. The reference mesh takes into account the real geometry and is solved with FEM. Upscaling using  $\langle \mathbf{h} \rangle_Y$  and (7) are compared. Our method yields correct evaluation of the Joule losses with 1.34% of error over the first period at 100 MHz (Fig. 2), versus 90.2% with  $\langle \mathbf{h} \rangle_Y$ . These methods have roughly the same cost, but (5) is more expensive because it requires computing  $\mathbf{t}_0$ .

We emphasize that the macro  $\mathbf{H}$ - $\mathbf{B}$  law is not linear here even if the meso  $\mathbf{h}$ - $\mathbf{b}$  law used is linear. Indeed it depends on the magnetic dipole created by  $\mathbf{j}_e$  and thus on  $\partial_t \mathbf{B}$  and  $\partial_t \mathbf{b}$ .

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