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Sensitivity Analysis using the Virtual Work Principle For 2D Magnetostatic Problems

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***Abstract*—The rise of shape and topology optimization techniques has opened new possibilities to find innovative designs for electromagnetic devices such as actuators or rotating machines. This paper provides an analytical sensitivity analysis for the magnetic force and torque based on the virtual work principle and suitable for those kinds of optimization.**

***Index Terms*—Sensitivity analysis, Virtual work method, Design optimization, Inverse problems.**

I. Introduction

The optimization of electromagnetic devices is a popular topic. It allows to find innovative designs which are not intuitive. A crucial step when using those algorithms is the computation of the sensitivity with respect to the design variables of the optimization. When it comes to optimize the magnetic force or torque, a simple method to perform the sensitivity analysis is the finite difference method, which is simple but numerically unstable and unefficient. Other papers use the Maxwell tensor method to formally compute the force and its sensitivity. The virtual work method is known to have a better numerical behavior [2]. Another advantage is that it does not require the additional choice of a path surrounding the region where the force is computed. The originality of this paper is to provide an analytical method for the sensitivity analysis of the magnetic force and torque via the virtual work method using the adjoint method for shape and topology optimization. This paper details the formal computation of this sensitivity and shows an example of its use for topology optimization on a 2D actuator with a non-linear ferromagnetic material.

II. Problem description

A. Context

Let us consider a 2D domain with a current density $\boldsymbol{J} = J_z\mathbf{e_z}$ in a coil Ω_S and zero elsewhere, and two domains Ω_{F_1} and Ω_{F_2} which are both filled with a ferromagnetic material and surrounded with air. A typical illustration is shown in Fig. 1. The magnetostatic equations with 2D assumptions satisfied in Ω are:

$$\begin{aligned} \nabla \times \boldsymbol{H} = \boldsymbol{J} \quad in \quad \Omega \\ \boldsymbol{H} = \nu \boldsymbol{B} \ ; \ \boldsymbol{B} = \nabla \times \boldsymbol{A} \\ \boldsymbol{A} = 0 \quad on \quad \partial\Omega \end{aligned} \tag{1}$$

where $\boldsymbol{H}$ is the magnetic field $(A.m^{-1})$, $\boldsymbol{B}$ is the magnetic induction (T), $\boldsymbol{A} = A_z\mathbf{e_z}$ is the 2D potential vector and ν is the magnetic reluctivity. The reluctivity is described by the non-linear law $\nu = \nu(|\boldsymbol{B}|)$ of the ferromagnetic material in $\Omega_{F_1} \bigcup \Omega_{F_2}$ and $\nu = \nu_0$ elsewhere with ν_0 the vacuum reluctivity. This non-linear scalar equation is solved using the Newton-Raphson algorithm which iteratively solves the following matrix system corresponding to (1):

$$S(\boldsymbol{A_z})\boldsymbol{A_z} = \boldsymbol{b} \tag{2}$$

where the vector $\boldsymbol{A_z}$ is the values of the potential vector A_z on the nodes of the mesh of the domain Ω.

B. The virtual work method for magnetic force and torque

The virtual work principle allows to compute the magnetic force along a direction s, or the torque around an axis ω. Starting from the virtual work expression in [2], the force on Ω_{F_1} can be rewritten as:

$$F_s = \nu_0 \sum_e \int_{\Omega_e} \boldsymbol{B}^T M_e \boldsymbol{B} \quad d\Omega \tag{3}$$

$$\text{with } M_e = -G_e^{-1}\frac{\partial G_e}{\partial s} + \frac{1}{2|G_e|}\frac{\partial |G_e|}{\partial s} I_3.$$

where I_3 is the identity matrix, G_e is the Jacobian matrix of the transformation from the mesh element Ω_e to the reference element Δ_e [2] and $|G_e|$ its determinant. The terms $\frac{\partial G_e}{\partial s}$ and $\frac{\partial |G_e|}{\partial s}$ are the derivative of G_e and $|G_e|$ with respect to the virtual translation of the mesh nodes of Ω_{F_1} in the direction s. They are zero everywhere except on the air layer of finite elements around Ω_{F_1}. The exact same formula holds true for

the torque around ω, but in this case the mesh nodes are virtually moved with a rotating movement around ω.

III. Sensitivity analysis

A. Design variables for shape and topology optimization

A classical optimization problem for electromagnetic devices is to minimize the volume of ferromagnetic material used in Ω_{F_1} and Ω_{F_2} with a constraint on the force or torque value in the mobile region Ω_{F_1}. When using a gradient-based optimization, the sensitivity computation of the objective and constraints with respect to the design variables of the optimization is crucial.

For shape optimization, the goal is to optimize the initial boarders of Ω_{F_1} and Ω_{F_2}. Thus, the design variables are the mesh nodes of this boarder. For topology optimization, the SIMP method can be used [3], [4]. The goal is to distribute a given amount of ferromagnetic material in the regions Ω_{F_1} and Ω_{F_2} using a design variable $\rho \in [0, 1]$ which interpolates the reluctivity value $\hat{\nu}$ between air and material in every mesh element of Ω_{F_1} and Ω_{F_2} using the law $\hat{\nu} = \nu_0 + g(\rho)(\nu - \nu_0)$ where g is a penalization law used to avoid non discrete density values.

B. Expression of the sensitivity

The sensitivity of the force or torque (3) with respect to the design variables X_i can be expressed for shape and topology optimization using the adjoint method [3]:

$$\frac{dF_s}{dX_i} = \frac{\partial F_s}{\partial X_i} + \boldsymbol{\lambda}^T \mathbf{P_i} \tag{4}$$

where $\boldsymbol{\lambda}$ is the adjoint vector and $\mathbf{P_i} = \frac{\partial \boldsymbol{b}}{\partial X_i} - \frac{\partial S}{\partial X_i}(\boldsymbol{A_z})\mathbf{A_z}$ the pseudo-load vector [3] with $\mathbf{A_z}$ the physical solution of (1) obtained with the Newton-Raphson resolution (2).

The pseudo-load vector is independent of the specific application to the magnetic force and its implementation has been already treated [3]. The originality of this work lies in the analytical development of the term $\frac{\partial F_s}{\partial X_i}$ and of the adjoint vector λ derived from the force expression (3).

The term $\frac{\partial F_s}{\partial X_i}$ is zero in general but a difficulty arrises for the specific case of shape optimization when optimizing the boarders of Ω_{F_1}. In this case the optimization is performed at the same place where the force is computed, thus the term $\frac{\partial F_s}{\partial X_i}$ must be taken into account. Its development is tedious and will be detailed in the full paper.

C. Expression of the adjoint vector

The novelty of this work is the analytical expression of the adjoint vector $\boldsymbol{\lambda}$ as the solution of the following linear problem with the quantity $\boldsymbol{M_{adj}}$ obtained by taking the derivative of (3) with respect to $\boldsymbol{A_z}$ [1], [4]:

$$\begin{aligned} &\nabla \times (\nu(|\mathbf{B}|)\nabla \times \boldsymbol{\lambda} - \boldsymbol{M_{adj}}) = 0 \quad in \quad \Omega \\ &\boldsymbol{\lambda} = 0 \quad on \quad \partial\Omega \\ &\boldsymbol{M_{adj}} = \nu_0 \left[\sum_e (M_e + M_e^T) \right] \boldsymbol{B} \end{aligned} \tag{5}$$

where $\boldsymbol{B}$ and $\nu(|\boldsymbol{B}|)$ are the physical solutions of (2). The development of this formula will be detailed in the full paper.

IV. Application

The computation of the sensitivity (4) has been implemented in Altair Flux™ both for shape and topology optimization. The software is then coupled with the gradient-based optimizer Altair Optistruct™. Starting from a given design, the sensitivity of the force is calculated and sent to the optimizer which modify the current design variables such that it decreases the objective function. Then, the process iterates until the optimization converges or a maximum number of iteration is reached. The presented example is a topology optimization of an actuator made of non-linear magnetic material. We aim to find an optimal design for the magnetic circuit Ω_{F_2} such that it maximizes the vertical component of the force in the orange mobile zone Ω_{F_1} and it removes 50% of the initial circuit volume (See Fig. 1). In this example, the initial width of Ω_{F_2} has been voluntary exaggerated to see if the optimization correctly removes the material surplus on the sides that is obviously unefficient to improve the magnetic force in Ω_{F_1}.

The initial and optimized designs are shown in Fig.1. In the initial design, the whole domain Ω_{F_2} (red and grey areas) is filled with a constant density $\rho = 0.5$. The final material design is shown in red. The grey parts are filled with air. This example shows a reduction of half of the initial magnetic circuit volume while keeping 97% of the force.

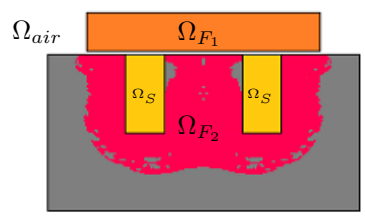

Fig. 1. Example of optimized design (in red) using topology optimization.

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