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# Isogeometric FEM-BEM coupling for magnetostatic problems using magnetic scalar potential

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**Abstract**—An original magnetic scalar potential FEM-BEM coupling based on isogeometric analysis is presented to address magnetostatic problems. The efficiency of the method is demonstrated on an academic problem and on a more complex multipatch geometry.

**Index Terms**—FEM-BEM, isogeometric analysis, scalar potential, magnetostatics

## I. INTRODUCTION

The finite element method (FEM) is widely used to study low frequency electromagnetic devices. However, it is well known that this method has a number of limitations due to its dependence on the mesh. When modelling electrical machines for instance, the accuracy of the solution in the airgap is a key point and is extremely sensitive to the discretization used for its description. In 2005, a method called Isogeometric Analysis (IGA) has been introduced. It provides interesting solutions to the previous difficulties. Coming from the mechanical simulation community, it has increasing interest since its implementation in the two last decades [2].

This method uses generalized Bézier curves, generated by a basis of spline functions, to exactly model complex geometries. The mathematical properties of the spline basis makes it a suitable choice for standard FEM projection functions, and allows to naturally embed the geometry it describes, without the need to mesh the physical domain. The support of the elements of the spline basis provides parametric elements that can be used as elements during the assembly process, making it very similar to standard FEM assembly. In addition, having a spline basis as shape functions offers great flexibility as regularity can be locally imposed, and Spline orders can be elevated at will. Isogeometric analysis has proven great efficiency in standard FEM for a wide variety of problems (contact mechanics, fluid mechanics, wave propagation...) as presented in the review [1].

However, the conventional IGA and FEM based models need a full domain representation including active (magnetic and/or conductive) regions and inactive (air) ones. As a result, the number of degrees of freedom (DoFs) can be huge. Moreover, IGA does not provide an efficient way to represent inactive regions since in magnetostatics, the air region often has a complex topology.

In order to overcome this drawback, and inspired by the FEM/BEM coupling already developed in [5], a coupling between the IGA-FEM and the boundary element method (IGA-BEM) can be developed. The BEM is used to model only the linear unbounded regions (air domain) and the FEM is used to model the active regions, allowing to benefit from both methods. In this context, as the geometry is represented exactly by the NURBS curve, a better precision per degree of freedom is expected as it will only depend on the interpolation of the fields by the shape functions. IGA-FEM-BEM coupling started developing in 2014, in mechanics [3], with only few contributions to magnetostatics except very recently in [4].

In this paper, we introduce an original isogeometric FEM-BEM coupling, based on the magnetic scalar potential formulation for 2D and 3D magnetostatic problems, and discuss its advantages and drawbacks. Numerical considerations regarding the implementation will also be discussed (assembly, singular integration, ... ).

## II. NURBS

Non-Uniform Rational B-Spline (NURBS) curves are generated as a rational sum of weighted elements of a B-Spline, themselves weighted by control points, allowing geometries to be modeled. To generate a NURBS curve, two structures must be given:

A knot vector  $\Xi = [\xi_1, \xi_2, \dots, \xi_{n+p+1}]$ , used to generate the base spline functions (defining their support, degree, and regularity). Given such knot vector, the  $n$  elements of the spline basis functions of degree  $p$ ,  $(N_j^p)_{j=1, \dots, n}$  are given by the recurrence formula:

$$N_j^0(\xi) = \begin{cases} 1, & \text{if } \xi_j \leq \xi < \xi_{j+1} \\ 0, & \text{otherwise.} \end{cases}$$

$$N_j^p(\xi) = \frac{\xi - \xi_j}{\xi_{j+p} - \xi_j} N_j^{p-1}(\xi) + \frac{\xi_{j+p+1} - \xi}{\xi_{j+p+1} - \xi_{j+1}} N_{j+1}^{p-1}(\xi). \quad (1)$$

Control points  $(P_i)_{i=1, \dots, n}$  and weights  $(\omega_i)_{i=1, \dots, n}$ , determining the shape of the NURBS curve:

$$C(\xi) = \frac{\sum_{j=1}^{m-p-1} \omega_j P_j N_j^p(\xi)}{\sum_{j=1}^{m-p-1} \omega_j N_j^p(\xi)}, \quad (2)$$

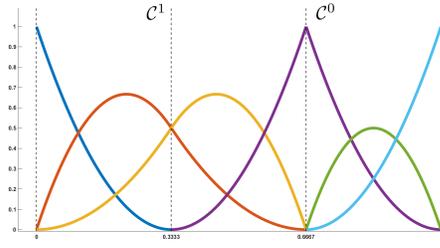


Fig. 1. Example of the Spline basis  $(N_j^2)_{j=1,\dots,6}$  generated by the knot vector  $\Xi = [0, 0, 0, \frac{1}{3}, \frac{2}{3}, \frac{2}{3}, 1, 1, 1]$ .

where  $m$  is the number of knots in the knot vector. NURBS surfaces and volumes can then be generated using tensor product of NURBS curve/surface.

### III. ISOGOMETRIC FEM-BEM COUPLING

Using the well known magnetic scalar potential formulation ( $\mathbf{H} = \mathbf{H}_s - \nabla\phi$ ), the general problem reads: find the magnetic scalar potential  $\phi$  on  $\Omega$  and the normal component of the flux density  $\mathbf{B}_n$  on  $\Gamma$  such that:

$$\text{div}(\mu(\mathbf{H}_s - \nabla\phi)) = 0 \text{ on } \Omega \quad (3)$$

$$\mathbf{n} \times \mathbf{H} = 0 \text{ on } \Gamma \quad (4)$$

$$\mathbf{n} \cdot \mathbf{B} = 0 \text{ on } \Gamma \quad (5)$$

where  $\mu$  is the magnetic permeability,  $\mathbf{H}_s$  the source magnetic field. The studied domain  $\Omega$  is decomposed into  $\Omega_{\text{active}}$  and  $\Omega_{\text{air}}$ . By imposing interface conditions for the magnetic flux density and the magnetic field, the weak formulations for the air and for the active domain can be derived:

In the active domain, it can be directly obtained with an integration by parts and the divergence theorem :

$$\begin{aligned} \int_{\Omega} \text{div}(w_i \cdot (\mu(\mathbf{H})(\mathbf{H}_s - \nabla\phi))) - \int_{\Omega} \nabla w_i \mu(\mathbf{H})(\mathbf{H}_s - \nabla\phi) &= 0 \\ \implies \int_{\Omega} \nabla w_i \mu(\mathbf{H}) \nabla\phi + \int_{\Gamma} w_i \mathbf{B}_n &= \int_{\Omega} \nabla w_i \mu(\mathbf{H}) \mathbf{H}_s. \end{aligned}$$

In  $\Omega_{\text{air}}$ ,  $\mu$  is linear and equation (3) becomes a Laplacian. Denoting  $G$  the associated Green kernel, and using Green's identity :

$$\frac{\phi}{2} = - \int_{\Gamma} \frac{dG}{dn} \phi + \int_{\Gamma} G(\mathbf{H}_{s_n} - \frac{\mathbf{B}_n}{\mu_0}),$$

leading to the coupled weak formulation :

$$\int_{\Omega} \nabla w_i \mu(\mathbf{H}) \nabla\phi + \int_{\Gamma} w_i \mathbf{B}_n = \int_{\Omega} \nabla w_i \mu(\mathbf{H}) \mathbf{H}_s \quad (6)$$

$$\int_{\Gamma} v_i \frac{\phi}{2} + \int_{\Gamma} v_i \int_{\Gamma} \frac{dG}{dn} \phi + \int_{\Gamma} v_i \int_{\Gamma} G \frac{\mathbf{B}_n}{\mu_0} = \int_{\Gamma} v_i \int_{\Gamma} G \mathbf{H}_{s_n} \quad (7)$$

Where  $(w_i, v_i) \in (H^1(\Omega) \times H^{(-\frac{1}{2})}(\Gamma))$ . Contrary to standard FEM-BEM assemblies, multiple specificities arise: The number of shape functions per parametric element may vary (as in Fig. 1), adjacent NURBS patches needs their DoFs to be linked, the trace of shape functions of dimension  $d-1$  needs to be recovered from the NURBS curves/surfaces that generated them through tensor product.

### IV. VALIDATION AND RESULTS

The validation of the developed model has been carried out first on a 2D simple academic case consisting in a disk in a uniform magnetic field  $\mathbf{H}_s$ , where the analytical solution is known. The Fig. 2 shows the relative error comparison of  $\nabla\phi$  and  $\mathbf{B}_n$ . It should be noticed that a good accuracy is obtained using only a few DoFs.

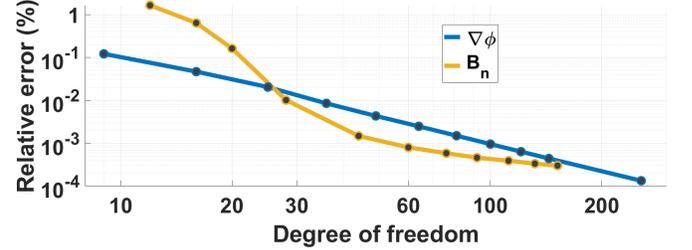


Fig. 2. Relative error comparison of  $\nabla\phi$  and  $B_n$  obtained using the presented IGA-FEM-BEM coupling to a reference solution.

The method has then been used to model an actuator: a magnetic circuit with a coil as shown in Fig.3(a). In order to handle the complex geometry, the active domain is decomposed into multiple NURBS patches. The figure 3(b) illustrates the magnetic field cartography. More results about the study of the convergence rate and the time computation will be detailed in the extended paper.

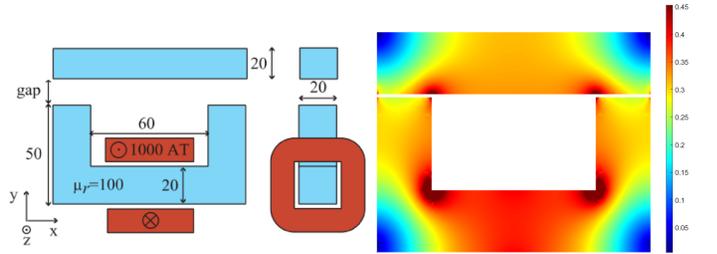


Fig. 3. a) scheme of the modeled Actuator. b) Norm of the Magnetic field  $B$  inside the actuator generated with multiple NURBS patches.

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