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An Air Gap Model for Magnetostatic Volume Integral Formulation

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Abstract—This paper presents an air gap model for thin air gaps using the magnetostatic volume integral method. The thin air gap is considered as a surface region with a numerical treatment instead of a volume region to avoid having a thin mesh that could lead to long computations. An application to a current transformer is reported to show its accuracy.

Index Terms—Volume integral method, air gap, magnetostatics.

I. INTRODUCTION

The volume integral method (VIM) is a powerful approach to solve magnetic problems. Contrary to the finite element method (FEM), only the active regions have to be discretised, thus avoiding to mesh the air. VIM is known since 1970s, but it wasn't until the last decade that it became more popular as a result of matrix compression algorithms such as the fast multipole method (FMM) [2]. A common feature of magnetic problems is the presence of air gaps, that can be very thin, arising for example when two regions are welded. Modelling air gaps is not obvious, if the mesh elements have an elongated shape the numerical treatment is difficult therefore we need a thin mesh that leads to costly calculations. A model of thin air gaps considered as a surface region instead of a volume region for VIM is presented in this paper. It allows to avoid a thin mesh of the air gap volume region that is computationally expensive, keeping the precision of computations. The model is applied to a current transformer and it is compared with a FEM solution to show its accuracy.

II. VOLUME INTEGRAL FORMULATION

We consider Maxwell's equations under the magnetostatics assumptions in a domain composed of a ferromagnetic region

and coils. A volume integral method is used to solve the equations choosing induction \mathbf{B} as a state variable and interpolating it with face elements (Whitney 2-form) as in [1]. The resulting discretized integral equation for magnetostatics is:

$$[\mathbf{Z}]\{\Phi\} = \{\Delta\varphi\} + \{S\} \quad (1)$$

where Φ is the flux of each face of the mesh, $\Delta\varphi$ is the difference of magnetic potential of two mesh elements, S is a term coming from the external sources and $[\mathbf{Z}]$ is a matrix composed of two parts: a first finite element matrix $[\mathbf{R}]$ with

$$R_{ij} = \int_{\Omega} \mathbf{w}_i \nu \mathbf{w}_j d\Omega \quad (2)$$

where \mathbf{w}_i is the face shape function and ν is the inverse of the permeability of the ferromagnetic region and a second full integral matrix $[\mathbf{L}]$ with

$$L_{ij} = \frac{1}{4\pi} \int_{\Gamma_{\text{ext}}} \frac{1}{s_i} \int_{\Gamma_{\text{tot}}} \frac{1}{s_j} \delta\nu_j \frac{1}{r} d\Gamma_{\text{ext}} d\Gamma_{\text{tot}} \quad (3)$$

where s_i the surface of the mesh element i , $\delta\nu$ is the reluctivity jump between two elements and r is the distance between the integration point and the observation point. The region Γ_{tot} considers all the faces of the mesh and Γ_{ext} refers to the surface elements of the ferromagnetic/air boundary.

To ensure $\text{div}\mathbf{B} = 0$ we consider a change of basis on the system (1) with an incidence matrix. The latter strategy can also be seen as an equivalent circuit approach that verifies either the Kirchhoff's nodal rule or the Kirchhoff's mesh rule, then \mathbf{R} matrix can be seen as the internal reluctance matrix for the magnetic material and \mathbf{L} matrix as the one taking into account flux leakage in air region.

III. AIR GAP MODEL

The aim is to create a method to deal with thin regions of air gap type as a surface region of a given thickness and not as a volume region. Let us consider the following assumptions:

- Thickness of the air gap is small compared to the dimension of the other elements (ratio inferior to 5E-4).
- Flux leakage from the external faces of the air gap is null.
- Flux inside the air gap is perpendicular to the air gap. We can assume this because the permeability of the material is much higher than the permeability of the vacuum and the air gap is thin.

The equivalent magnetic circuit can be built adding the air gap contribution to the matrices [R] and [L]. For the finite element matrix [R], we consider an additional reluctance between the faces of the air gap, given by $\frac{\mu_0 e}{s}$, where e is the thickness of the air gap, s is the surface of the element and μ_0 is the vacuum permeability. For matrix [L], since the air gap is characterized by having two regions that are near each other, we will evaluate two parallel faces Γ_1 and Γ_2 that are in front of each other and separated by a distance equal to the thickness of the air gap. Now, using a first order Taylor expansion $\frac{1}{r_1} + \frac{1}{r_2} \approx \frac{2}{r}$, the approximation $\frac{1}{r_1} - \frac{1}{r_2} \approx \mathbf{egrad}(\frac{1}{r}) \cdot \mathbf{n}$ and writing $\nu_1 = \nu_{\text{avg}} + \frac{\delta\nu}{2}$, $\nu_2 = \nu_{\text{avg}} - \frac{\delta\nu}{2}$, where ν_{avg} is the average of ν , [L] matrix reads:

$$L_{ij} = \frac{1}{4\pi} \int_{\Gamma_{\text{ext}}} \frac{1}{s_i} \int_{\Gamma_{\text{tot}}} \frac{1}{s_j} \delta\nu_j \frac{1}{r} d\Gamma_{\text{ext}} d\Gamma_{\text{tot}} + \frac{1}{4\pi} \int_{\Gamma_{\text{ext}}} \frac{1}{s_i} \int_{\Gamma_{\text{tot}}} (\nu_{\text{avg}} - \nu_0) \frac{e}{s_j} \mathbf{egrad}(\frac{1}{r}) \cdot \mathbf{n} d\Gamma_{\text{ext}} d\Gamma_{\text{tot}}. \quad (4)$$

We observe that the first term of [L] matrix corresponds to the term we had when there is no air gap, eq. (3), being the second term the additional contribution given by the air gap. When the thickness of the air gap is small or the two materials divided by the air gap have similar reluctivity, the correction of finite element matrix [R] is enough to model the air gap behaviour. Otherwise, the additional correction to [L] matrix must be considered.

IV. APPLICATION

The proposed air gap model is applied to a current transformer composed of a ferromagnetic region with a linear material $\mu_r = 1,000$ and two coils as Fig. 1 shows. The primary coil has 1 turn and the secondary coil has 980 turns. The calculation of the magnetic flux through the secondary coil following [3] is compared with the reference values obtained using the FEM performed by the commercial software Altair FLUX to verify the accuracy of the model. The VIM model has 52,581 tetrahedral mesh elements in the magnetic region and 208 on the surface air gap region, FEM model has 412,523 in the air region, 96,588 in the ferromagnetic region and 400 face elements on the surface air gap region. Results for two different air gap thicknesses of 5e-6 meters and 50e-6 meters imposing 1 ampere on the primary coil and 0 amperes on the secondary coil are given in Table I. VIM R refers to VIM taking into account the air gap addition of matrix [R], whereas

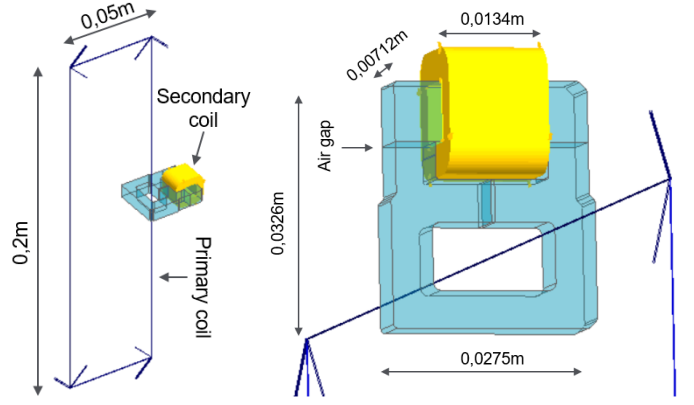


Fig. 1: Geometry of magnetic region (light blue), primary coil (dark blue) and secondary coil (yellow).

VIM R-L2 considers VIM with the air gap contribution of matrix [R] and the second term of matrix [L].

	FEM	VIM R	VIM R-L2	Diff R	Diff R-L2
Air gap 5e-6	-5.47e-4	-5.51e-4	-5.48e-4	0.73%	0.18%
Air gap 50e-6	-3.57e-4	-3.65e-4	-3.58e-4	2.24%	0.28%

TABLE I: Magnetic flux through secondary coil.

Last two columns show the relative difference between FEM and VIM R and between FEM and VIM R-L2 respectively. Considering the two contributions R-L2 the difference between FEM and VIM is 0.18% and 0.28% for 5e-6m and 50e-6m respectively. For the R contribution an air gap of 5e-6m has a difference of 0.73% that increases to 2.24% when the air gap is wider. We then validate the VIM air gap model, taking into account that for thin air gaps the R contribution is enough, but for thicker air gaps the R-L2 contribution will be needed.

V. CONCLUSION

A model of thin air gaps using the VIM is proposed. It considers the air gap as a surface region with a given thickness, avoiding an air gap volume region that leads to a thin mesh. It is made for a B-facet volume integral formulation for linear and non linear magnetostatic problems. An application to a current transformer is shown, comparing the results with FEM and keeping the accuracy of the model.

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