



HAL
open science

Modeling of magnetoelectric effects in composite structures by FEM-BEM coupling

A Urdaneta-Calzadilla, Olivier Chadebec, Nicolas Galopin, Innocent Niyonzima,
Gerard Meunier, Bertrand Bannwarth

► **To cite this version:**

A Urdaneta-Calzadilla, Olivier Chadebec, Nicolas Galopin, Innocent Niyonzima, Gerard Meunier, et al.. Modeling of magnetoelectric effects in composite structures by FEM-BEM coupling. 20th Biennial IEEE Conference on Electromagnetic Field Computation (IEEE-CEFC'2022), Oct 2022, Denver, United States. <hal-03808722>

HAL Id: hal-03808722

<https://hal.science/hal-03808722v1>

Submitted on 10 Oct 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

Modeling of magnetoelectric effects in composite structures by FEM-BEM coupling

Alberto Urdaneta-Calzadilla

Univ. Grenoble Alpes, CNRS, Grenoble INP, G2Elab
alberto.urdaneta-calzadilla@g2elab.grenoble-inp.fr

Nicolas Galopin

Univ. Grenoble Alpes, CNRS, Grenoble INP, G2Elab
nicolas.galopin@g2elab.grenoble-inp.fr

Gerard Meunier

Univ. Grenoble Alpes, CNRS, Grenoble INP, G2Elab
gerard.meunier@g2elab.grenoble-inp.fr

Olivier Chadebec

Univ. Grenoble Alpes, CNRS, Grenoble INP, G2Elab
olivier.chadebec@g2elab.grenoble-inp.fr

Innocent Niyonzima

Univ. Grenoble Alpes, CNRS, Grenoble INP, G2Elab
innocent.niyonzima@g2elab.grenoble-inp.fr

Bertrand Bannwarth

Univ. Grenoble Alpes, CNRS, Grenoble INP, G2Elab
bertrand.bannwarth@g2elab.grenoble-inp.fr

Abstract—In this paper, a coupling of the Finite Element Method (FEM) and the Boundary Element Method (BEM) is used to model the behaviour of magnetoelectric effects in composite structures. This coupling of numerical methods makes it possible not to have to consider a free domain, and thus to use a single mesh for the magnetic, mechanical and electrical problems. This results in a consequent reduction of the number of unknowns which is accompanied by shorter computation times compared to a classical FEM approach. A mixed magnetic vector potential-reduced magnetic scalar potential formulation is used for the magnetic problem, and classical FEM formulations are used for electrical and mechanical problems. The resulting global algebraic system is solved by a block Gauss-Seidel solver.

Index Terms—FEM-BEM coupling, magnetoelectric, piezoelectricity, piezomagnetism, multiphysics, Gauss-Seidel method

I. INTRODUCTION

Without relying on induction, electromagnetic coupling can be obtained through the mechanical association of piezoelectric and magnetostrictive materials. The resulting composite structure displays the magnetoelectric effect [1] and can allow for magnetic to electric energy conversion or vice-versa. For the modelling of magnetoelectric composites, the FEM allows to solve for the behaviour of materials with non trivial structures. Nevertheless, a classical FEM approach has several disadvantages. An air-box has to be considered to properly model the decay of magnetic fields at infinity. A FEM-BEM coupling has previously been successfully used to overcome this problem and thus to only mesh the active material [2]. The same approach can be applied to the modelling of the multiphysics phenomena arising in magnetoelectric composite structures. The magnetoelectric problem will be decomposed into three distinct sub-problems: a magnetic problem solved by the FEM-BEM approach, and electrical and mechanical problems solved by classical FEM formulations.

II. BEHAVIOURAL LAWS

The behavior of smart materials (piezoelectric and magnetostrictive materials) is characterized by strong

electro(magneto)-mechanical coupling. Piezoelectric materials can be described by a set of linear relationships between the electrical and mechanical quantities [3]. This is not the case for magnetostrictive materials which generally exhibit non-linear behavior [4]. However, it is possible to induce a polarization state by applying a magnetic field and/or a mechanical prestress and thus define a set of linear relationships [4]. In this framework, the electrical, magnetic and mechanical behavior relations are written :

$$\begin{cases} \underline{T} = \underline{c}^{E,B} : \underline{S} - \underline{t}_e \cdot \underline{E} - \underline{t}_h \cdot \underline{B}, & (1) \\ \underline{D} = \underline{\epsilon} : \underline{S} + \underline{\epsilon}^S \cdot \underline{E}, & (2) \\ \underline{H} = -\underline{h} : \underline{S} + \underline{\nu}^S \cdot \underline{B}, & (3) \end{cases}$$

where \underline{T} , \underline{S} , \underline{D} , \underline{E} , \underline{B} and \underline{H} are, respectively, the stress tensor, the strain tensor, the electric flux density, the electric field, the magnetic flux density and the magnetic field. \underline{c} is the stiffness tensor at constant electric field and magnetic flux density, $\underline{\epsilon}$ is the electrical permittivity tensor at constant strain, $\underline{\nu}$ is the magnetic reluctivity tensor at constant strain, \underline{e} is the piezoelectric tensor and \underline{h} the piezomagnetic tensor.

III. MAGNETIC PROBLEM

The magnetoelectric problem consists in solving Maxwell's equations in the framework of electrostatics and magnetostatics as well as the mechanical equilibrium equation with small strains assumptions and by taking into account the relations (1), (2) and (3). In order to use a single mesh containing no air, a mixed formulation [5] is used for the magnetic problem, with a magnetic vector potential \underline{a} inside the active materials domain Ω_m and a reduced magnetic scalar potential ϕ_{red} in the air domain. From magnetostatic equations and (3), the weak formulation (4) inside Ω_m is developed considering the reduced magnetic scalar potential ϕ_{red} for the boundary term and the field created by external currents H_0 , calculated by

the Biot & Savart law. The discretization is based on edge elements in the H_{curl} function space for the magnetic vector potential and 0-order surface elements for the reduced scalar potential ϕ_{red} . Then the weak form reads: find $(\underline{a}, \phi_{red})$ such that

$$\begin{aligned} & \int_{\Omega_m} \nabla \times \underline{\delta a} \cdot \underline{\nu}^S \cdot \nabla \times \underline{a} \, d\Omega_m - \int_{\Omega_m} \nabla \times \underline{\delta a} \cdot \underline{h} : \underline{S} \, d\Omega_m \\ & + \int_{\Gamma_m} (\nabla \times \underline{\delta a}) \cdot \underline{n} \, \phi_{red} \, d\Gamma_m = \int_{\Gamma_m} (\underline{\delta a} \times \underline{n}) \cdot \underline{H}_0 \, d\Gamma_m, \quad \forall \underline{\delta a}, \end{aligned} \quad (4)$$

where Γ_m is the exterior surface of Ω_m . The behavior of the magnetic field in the air, especially at infinity, is taken into account by adding an equation to the system. Since $\Delta \phi_{red} = 0$, Green's third identity applied to the external surface of Ω_m leads to the relation (5),

$$\frac{1}{2} \phi_{red} + \int_{\Gamma_m} G \frac{B_n}{\mu_0} \, d\Gamma_m - \int_{\Gamma_m} \phi_{red} \frac{\partial G}{\partial n} \, d\Gamma_m = \int_{\Gamma_m} G H_{0n} \, d\Gamma_m, \quad (5)$$

where G is the Green function of the 3D Laplacian operator. This equation is discretized using a Galerkin approach with 0-order surface elements. As $\iint B_n \, dS = \oint a^{surf} \, dl$, after discretization of (5), the magnetic vector potential in edge elements is introduced in the discretized equation from the flux of the magnetic induction through each facet element. The resulting matrix system relates the magnetic vector potential discretized using edge elements to the reduced scalar magnetic potential discretized using 0-order surface elements, both in the boundary of the active material.

IV. ELECTRIC AND MECHANICAL PROBLEMS

The electric and mechanical problems are limited to the active material domain Ω_m . Indeed even if in all generality an electric FEM-BEM coupling can be formulated, the presence of electrodes and the strong permittivity of some piezoelectric materials makes the leaks of the displacement field negligible. The FEM is then used to solve the electric and mechanical problems, considering, respectively, (2) and (1) and electric scalar potential and displacement vector formulations [6] without magnetic forces.

V. MATRIX SYSTEM AND RESOLUTION METHOD

The resulting global matrix system is composed of sparse FEM matrices and full BEM matrices, and has a big scaling difference between coefficients: coefficients of $\underline{c}^{\underline{E}, \underline{B}}$ are of the GPa magnitude, $\underline{\nu}^S$ around 10^5 m/H, and $\underline{\epsilon}^S$ between 10^{-7} and 10^{-11} F/m. This system can be solved iteratively using a block Gauss-Seidel scheme (6). Contrary to a direct resolution, the Gauss-Seidel method allows for solving well conditioned sub-systems as extra-diagonal sub-matrices are scaled by the current solutions and introduced as second hand terms (6).

$$[A_{ii}]\{x_i^{n+1}\} = \{b_i\} - \sum_j^{i-1} [A_{ij}]\{x_j^{n+1}\} - \sum_{i+1}^n [A_{ij}]\{x_j^n\} \quad (6)$$

The Gauss-Seidel method also allows to use an optimized solver for each sub-system: preconditioned GMRES iterative

solver for the magnetic sub-system and MUMPS direct solver for the electrical and mechanical sub-system.

VI. RESULTS

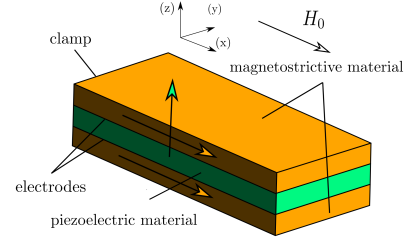


Fig. 1. Diagram of the composite structure, the colored arrows represent the poling direction of the material

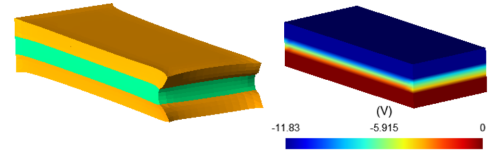


Fig. 2. Amplified displacement field ($\times 6.5 \cdot 10^3$) and nodal voltage (V)

The proposed model was tested on a composite structure whose geometry is presented in Figure 1. It is composed of a piezoelectric layer in between two magnetostrictive layers and measures 3x6x14 mm. It functions as follows: the source field drives the deformation of the magnetostrictive layer. As both phases are mechanically bonded together, the piezoelectric layer will deform and an electric voltage will appear between the electrodes as shown in Figure 2. For all linear coupling tensors, the magneto-mechanical formulation was successfully validated against a pure FEM approach. A good convergence of the resolution algorithm is observed, around 15 iterations for a convergence of 10^{-7} , and also, we observe convergence of the output voltage as we refine the mesh. In these conditions, as expected, the magnetoelectric coefficient, i.e. the output voltage divided by the source field of the magnetoelectric, is constant vs the source field.

REFERENCES

- [1] Y. Cheng, B. Peng, Z. Hu, Z. Zhou, M. Liu, "Recent development and status of magnetoelectric materials and devices", *Physics Letters A* 382 (2018) 3018–3025. doi:https://doi.org/10.1016/j.physleta.2018.07.014.
- [2] G. Meunier, J. Coulomb, S. Salon and L. Krahenbul, "Hybrid finite element boundary element solutions for three dimensional scalar potential problems," in *IEEE Transactions on Magnetics*, vol. 22, no. 5, pp. 1040-1042, September 1986, doi: 10.1109/TMAG.1986.1064625.
- [3] Piefort, V. (2001). *Finite element modelling of piezoelectric active structures*, Ph. D. thesis. Bruxelles, Belgium: Université Libre de Bruxelles, Department for Mechanical Engineering and Robotics.
- [4] G. Engdhal *Handbook of Giant Magnetostrictive Materials*. Academic Press, 2000. doi: 10.1016/B978-0-12-238640-4.X5014-1.
- [5] Q. Phan, G. Meunier, O. Chadebec, J. Guichon and B. Bannwarth, "BEM-FEM formulation based on magnetic vector and scalar potentials for eddy current problems," 2019 19th International Symposium on Electromagnetic Fields in Mechatronics, Electrical and Electronic Engineering (ISEF), 2019, pp. 1-2, doi: 10.1109/ISEF45929.2019.9096899.
- [6] N. Galopin, X. Mininger, F. Bouillault, L. Daniel, "Finite Element Modeling of Magnetoelectric Sensors", *IEEE Transactions on Magnetics*, vol. 44, no. 6, pp. 834-837, June 2008. doi:10.1109/TMAG.2008.915781