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Introduction to the Thematic Working Group 17 on Theoretical Perspectives and Approaches in Mathematics Education Research of CERME 12. Horizontal and vertical theorizing.

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Starting from previous lines of thought

Theories are essential parts of each scientific discipline; they can be used to solve problems, to answer research questions, to capture phenomena, to predict what can be expected or prescribe what can be done in practice (Prediger, 2019). However, in the field of mathematics education, there is no consensus of the notion of ‘theory’ (Assude et al., 2008). What is agreed upon in most communities in our field is that theories are “... individual or social constructions which serve to understand and describe a part of reality in a consistent manner” (see. Maier & Beck, 2001, p. 45, own translation). They provide a language and a lens “... to understand what are taken to be the things that can be questioned and what counts as an answer to that questioning.” (Mason & Waywood, 1996, p. 1056). Theories afford the coherence of a research framework and provide the space for consistent argumentation. As Bishop (1992) summarizes, a theory “is the way in which we represent the knowledge and understanding that comes from any particular research study. Theory is the essential product of the research activities, and theorizing, therefore, its essential goal.” (p. 711)

Radford (2008, 2012) has conceptualized theory as a way of understanding based on a collection of principles (P), methods connected to those principles (M), and paradigmatic research questions (Q). When researchers draw on a theory for research to produce results (R), those results can contribute to further theory development. Hence theories are dynamic, rather than static, and they continue to evolve via researchers’ engagement with them.

Previous TWG17s have addressed the question of how researchers and designers work with theories in the field of mathematics education (Kidron et al., 2018), taking tasks and tools into consideration. Two key, interrelated issues have been (1) how to grasp the complexity of teaching and learning of mathematics and (2) how to deal with the diversity of theories in the field. Working with theories is embedded in the culture of educational systems, and we witness a broad diversity of educational systems worldwide. Accordingly, our working group acknowledges diversity of theories as a kind of

richness in mathematics education, through which learning from and with each other provides the potential of advancing the field of mathematics education as a whole.

Coherence of a research framework and consistency of argumentation are guiding principles in the use of theories. This allows researchers to advance knowledge, which may be relevant even beyond mathematics education contexts. In its final discussion at CERME11, the TWG17 has agreed upon the dynamic, evolving nature of theories in research, as Bikner-Ahsbabs et al. (2019) summarize:

... scholars should neither demand that theories be used with absolute rigor nor allow arbitrarily applications of theory. To form coherent research frameworks, scholars engage in reconsidering, reinterpreting and reusing theories to investigate new phenomena, solve new problems and serve new purposes. Thus, theories develop and evolve through research. Working on coherence and consistency is an ongoing research task, particularly necessary for the Networking of Theories, in which reconsidering the compatibility of the theories or theoretical approaches is an additional epistemological necessity. Achieving generativity, generalizability, and generality affords the potential of the research results to be useful for answering new questions. (p. 3026)

The networking of theories approach (Bikner-Ahsbabs & Prediger, 2014) has been intensively discussed in previous TWG17s (Kidron et al., 2018). It has supported researchers' navigation of a variety of epistemological stances that may underlie the different theories involved in research and their exploration of how various theories can be used to investigate complex situations of teaching and learning. Yet, further elaboration of the networking of theories is needed to expand its research potential. A landscape of networking strategies has proven fruitful for guiding such efforts (Prediger et al., 2008). Specifically, the networking strategies of coordinating and locally integrating theoretical approaches have led to advancing our knowledge (Bikner-Ahsbabs & Prediger, 2014).

The TWG17 of CERME 11 identified the important role of certain sensitivities in theoretical work (e.g., the theory's ecology, its grain size, the nature of the mathematical content it considers, the research objects, etc.), and distinguished between generalizability, generality, and generativity in research methodologies: generalizability in empirical, generality in theoretical and generativity in design research. Chan and Clarke (2019) elaborated how our choices in research are based on mutual affordances between these theoretical sensitivities and the methodologies and methods to be used. Thus, researchers should be aware that theoretical, methodological, and methodical work are intermingled and related to the cultural context in which research is conducted.

Issues addressed in the call: connection to previous TWG17s

Design research has been prevalent in discussions of previous TWG17s. This pointed to various ways that researchers' decision-making could influence their theoretical approaches, specifically with respect to transforming versus depicting in mathematics education research processes. In this TWG17, we addressed this distinction by asking for contributions on theories related to design research, technology use and conditions for a productive dialogue between theorists.

What has been implicit in previous TWG17s were basic commitments underlying theoretical work. For this TWG17 we called for more explicit discussion of ethical commitments in theorizing and theory networking. We also called for attention to ontological, epistemological and axiological

presumptions of theories. Adapted from Patterson and Williams (1998), Daene (2018, n.p.) distinguished and described four such commitments. These were “the nature of reality and what really exists (ontology); the relationship between the knower and what is known (epistemology), what we value and how we determine that value (axiology), the strategy and justifications in constructing a specific type of knowledge (methodology), as linked to individual techniques (method/s).” In theories on teaching and learning we—sometimes implicitly—could make ontological assumptions about the piece of reality addressed (e.g., whether teaching-learning is an irreducible entity or consists of two different processes). In epistemology we could build on this assumption to ask how we can know something about the ontological entity and how this knowledge must be so designed. In methodology we could address the question of how knowledge can be produced (and by what means). Ethics, being part of axiology, could enable us “to rethink and re-evaluate some of the taken-for-granted commonplaces of our practices” (Ernest, 2012, as cited in Stinson, 2017, p. 2), “which opens up new possibilities for theorizing and researching mathematics teaching and learning.” (Stinson, 2017, p. 2)

Main contributions achieved

Fifteen papers and four posters were presented in this working group. In sum, 52 authors from 17 countries worldwide were involved. This indicated that doing research happens in communities rather than individually. Fifteen out of 19 contributions followed a networking of theories approach, indicating that networking of theories is being normalized in research although it still needs to be further developed. Design research became more prominent, with many papers involving this approach. In line with previous TWG17s many theory elements were addressed. There were 22 theories and 27 theoretical concepts or ideas addressed across the contributions. The awareness of our own achievements in our home field of mathematics education seemed to be growing. This was not so obvious in the papers but more so in our ways of talking about theories. Scholars became more often named as originators of the theories together with the theories, for example when talking about Duval’s cognitive theory of representation or Schoenfeld’s Resources-Orientations-Goals theory.

A key outcome of our discussion is a new characterization of theorizing, in terms of two dimensions: *horizontal* and *vertical* theorizing, which we illustrate in Figure 1. Horizontal theorizing happens when researchers draw on a theory (or theories) to make sense of problems or phenomena. With horizontal theorizing, researchers focus on the *how* of theory use, for the purpose of illuminating new aspects of complex phenomena. For example, researchers may network different theories to further the investigation of empirical phenomena, such as students’ reasoning in engineering education. In contrast, vertical theorizing happens in the semiosphere, the cultural semiotic space of theory cultures (Radford, 2008; Lotman, 1990). With vertical theorizing, researchers focus on meta-issues of theory use for the purpose of *understanding theories as entities in and of themselves*. For example, researchers may weigh the epistemological ramifications of networking different theories, and argue for the viability of doing so. Researchers’ theorizing may address horizontal or vertical theorizing, or both. As suggested by our examples, the networking of theories entails both. The rule of *keeping our feet on the empirical ground* (Figure 1) takes seriously the purpose of a theory to act as a tool for understanding the empirical world. It means that vertical theorizing can only increase our understanding of theories when it is grounded in horizontal theorizing.

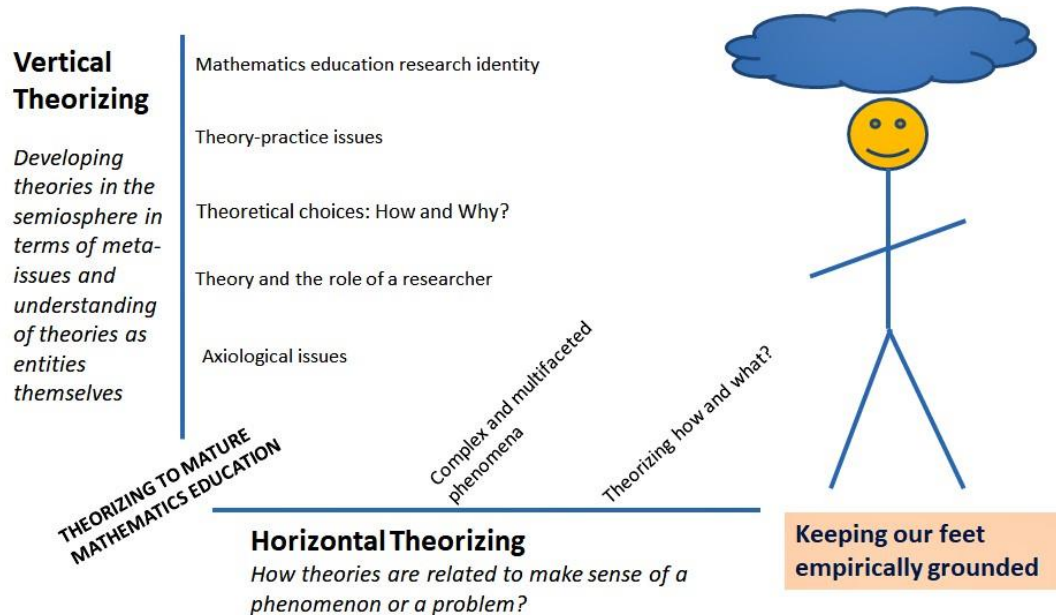


Figure 1: two dimensions of theorizing

Contributions of horizontal theorizing

Some papers addressed complex and multifaceted phenomena as a unity, a balance or tension between theoretical parts. Fosse et al. used Cultural Historical Activity Theory (CHAT) to investigate expansive learning when students crossed institutional cultures in their career. They explored masters students' transitions from university to school to identify if contradictions spark their learning and how and what teacher educators can learn from it for their own teaching. Herbst et al. proposed to use networking of theories to investigate teacher decision making and the role of structure and agency in these processes. Kuzniak and Nechache presented the Mathematical Working Space as a framework to be linked with other theories (e.g., Abstraction in Context) to describe how cognitive actions start from the epistemological level and how it develops with respect to semiotic, discursive and instrumental genesis. They posited that actions may link mathematical work with other theories.

Several contributions theorized the use of artifacts or tools and considered their role for progressing. Shvarts et al. analysed Freudenthal's and Davydov's work disclosing their common understanding of mathematical perception in learning to advance the embodied design framework. Starting with concrete actions, *ascending from the abstract to the concrete* entailed viewing the concrete in a completely new way through the lens of acquired artifacts. Thus, one's perception of the concrete would change as learning proceeds. Salinas-Hernandez et al. linked a cultural-historical approach with a semiotic view. They explored the "Imaging Teaching Scheme" of a physics teacher who used various artifact representations for the production of signs to guide learning. Santi et al. networked the theory of objectification with a differentiation approach for design research. Open Activity Theory Lesson Plan is an artifact they developed to guide designing stations to foster inclusive instruction by processes of objectification and subjectification of all learners. Kanwal networked a CHAT frame with the concept of creative and imitative reasoning to investigate teaching mathematics in engineering education at university. Computer software provided the artifact that shaped the conditions for the students' operating that constrained their reasoning actions.

In some contributions, horizontal theorizing pursued specific purposes. This way, researchers could clarify why networking was needed and helpful. Bach et al. networked Duval's (2017) cognitive theory of representation and the Instrumental Approach in a design research approach for learning functions to foster the representation competency defined in the Danish KOM framework. In their paper, they showed how the networking strategy of coordinating helped them to structure developing design principles. Petersen networked concept image and concept definition also with the cognitive theory of representation to enlighten mathematical thinking related to the KOM framework with a focus on differentiability. Kanwal's theoretical approach allowed for the consideration of students within the activity system as a wider entity. Researchers also could use theories to make new aspects visible. Tuktamyshov offered a "picture of the world." Mali et al. drew on the concept of "perezhivanie" to expose learners' life-changing experiences, and Zagorianakos linked perezhivanie with phenomenology to explore what this might add to previous insights.

Another purpose of horizontal theorizing was to take up and explore new challenges. Bikner-Ahsbabs et al. met the challenge of the pandemic and shifted the summer school YESS10 to an online conference format. This new situation allowed them to conceptualize the rhythmic orchestration of the research pentagon (Bikner-Ahsbabs, 2019). Gardesten coordinated the Knowledge Quartet and the Pedagogical Relational Teachership to build a methodological tool to explore teachers' pedagogical content knowledge and relational abilities when teaching mathematics in inclusive settings. Barquero et al. identified points of contact between the Anthropological Theory of the Didactic and the Theory of Didactic Situations to explore constraints of a new paradigm of instruction, the Study and Research Path.

Contributions to vertical theorizing

Vertical theorizing can happen in many ways. Lensing has provided an example of vertical theorizing as he looked at the three essential systems in mathematics education, the social, the individual and the body, and the problem of the impossibility to theorize them jointly. His solution of this complexity problem was to look at regularities the systems share and, thus, build theories on a more formal level. Other ways of vertical theorizing ask for how we understand and practice theorizing, and what kind of ethical, ontological, epistemological, and methodological commitments are present in theorizing. Critical for the ethical dimension are the aspects to which a theory attends while leaving others left aside, which means that some aspects are valued over others. For example, the theory of objectification (TO) addresses the dialectic between objectification and subjectification in teaching and learning mediated by tools (Radford, 2021), but it does not tell Santi et al. how to differentiate tasks for an inclusive setting. Thus, differentiation is invisible in the TO. Santi et al.'s networking theories approach is a way to display differentiation and simultaneously adhering to the TO.

To date it is not so clear what, specifically, theorizing entails when the aim is not to apply a theory to an empirical situation but rather to create new theoretical steps. Valdés-Zorrilla et al. have proposed to consider theorizing as a kind of *metaphorizing*, starting from the source domain of a metaphor to theorize the target domain with the help of the metaphor. In such a process, the role of the researcher as a user, borrower, adaptor, developer, or creator of theories comes into play. This necessitates being explicit about how, why, and for what theoretical steps are made. In this context, two purposes of

theory use become relevant: using a theory to depict versus using a theory to transform a teaching-learning situation. The former requires quite different kinds of reasoning than the latter. This has consequences for methodological choices as Chan and Clarke (2019) have pointed out. They have explored the reciprocity of theory and methodology showing the mutual affordance between theoretical and methodical choices. It requires vertical theorizing to describe particularities of how mutual affordance is involved in research. Following this path of vertical theorizing, Johnson et al. have elaborated what mutual affordance might mean for the networking of theories. They have proposed to use the metaphor of a multifocal lens to guide mutual affordances between theory networking and methodical choices.

Vertical theorizing is also present in the researchers' sensitivity to decide how, why, and for what purpose a theoretical step is needed and where to start with theoretical choices. Researchers are not free in their choices, as these are constrained by theory traditions. Theory development is only possible within certain limitations of a theory culture. When a transformative step goes beyond the realm of the theory culture, it can create epistemological obstacles. However, some theories such as CHAT may inform quite different research directions. Fosse et al. offer contradiction as a starting point to explore expansive learning. Kanwal also draws on CHAT to relate students' reasoning to the software the students used and to embed both into a wider activity system. When a new challenge such as the pandemic situation emerges, it may constrain research. However, it also may provide opportunities to see calls for new theorizing steps, such as Bikner-Ahsbabs et al.'s description of rhythm to extend the concept of instrumental orchestration.

Vertical theorizing also may address the relation between theory and practice and contribute to shape a disciplinary identity. In the practice of research, a theory needs an appropriate methodology to be put to work and lead to relevant insights. However, theory and methodology do not transform teaching and learning by itself. The practice of teaching and learning can be transformed by design research, which may in turn also transform research and gain new insights for theories. Design research may even be considered a research genre that is specific for mathematics education, thus contributing to form a mathematics education research identity. This can allow researchers to line up and navigate the power-filled landscape of disciplines and institutions. Theoretical work, including further developing the networking of theories approach and design-based research, may help the field of mathematics education to become aware of its strengths and of what it has to offer to other disciplines. In turn, this can strengthen mathematics education's status as a research discipline.

Lessons learned and future directions

Did we proceed with respect to consider the dimensions of ontology, epistemology, and axiology in processes of theorizing more explicitly? Three contributions point to progress in this area. Gardesten and Santi et al. are aiming at establishing inclusive mathematics education research. The dimension of ethics, and hence, axiology, is very explicit in their contributions. Santi et al. appeal to Radford's (2021) theory of objectification (TO) to position students and teachers as "reflective and ethical subjects" who engage in "joint labour" to develop new knowledge. In the analysis of Shvarts et al., we may consider the relationship between the works from Freudenthal and Davydov as being based on the common epistemological assumption that abstraction leads to a new vision of reality.

Ontologically, this means that reality is structured in a new way when it is perceived through using a mathematical artifact. In future TWG17s, there needs to be continued and more explicit attention to ontological, epistemological and axiological dimensions grounding our research.

The distinction between horizontal and vertical theorizing is an insightful step this TWG17 has made because it accredits us as researchers with the sensitivity of what theorizing means and in what kind of theorizing we are involved. Horizontal theorizing on concrete phenomena or problems is a necessary step in research, and we also need to strengthen our practice of vertical theorizing in order to be able to clarify basic assumptions in our research. Through vertical theorizing we can decide whether and how networking of theories is a consistent approach to reveal a coherent body of results. Whereas the objects of horizontal theorizing are phenomena in mathematics education, the objects of vertical theorizing are the theories or theory elements themselves (i.e., abstract entities in the semiosphere). Vertical theorizing is therefore more difficult to communicate, much more difficult than communicating how theories frame our research. Reflecting on concrete work may strengthen our theoretical understanding of a piece of reality allowing us to advance vertical theorizing.

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