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# Introduction to the papers of TWG6: Applications and modelling

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## Introduction

The thematic working group TWG6 takes on the importance of exploring the role of mathematics when it is used and applied in the study of real-world phenomena in educational contexts. TWG6 considers the interplay between empirical research results and theoretical approaches to the teaching and learning of applications and mathematical modelling, in mathematics and other subjects, at primary, secondary and tertiary school levels, as well as in teacher education. It started at the fourth ERME Congress (CERME4) in 2005 and has since then continued to be an active thematic working group in the following nine congresses. In total, more than 200 papers and posters have been presented in the TWG6. In CERME12 the TWG6 received 36 submissions, resulting in a total of 25 papers and 5 posters presented. See Figure 1 for a summary of the TWG6 contributions in the different CERMEs. At CERME12, we have discussed theoretical, methodological, and empirical research contributions aiming to address a variety of topics, such as application, modelling and simulations in connection to other subjects, the use and impact of technology and other resources to support modelling, teacher education for applications and modelling, and assessment practices for mathematical modelling, among others. The contributions discussed at the congress revealed a strong and fruitful diversity in the research questions considered, the school levels addressed, the theoretical approaches chosen and their methodological development.

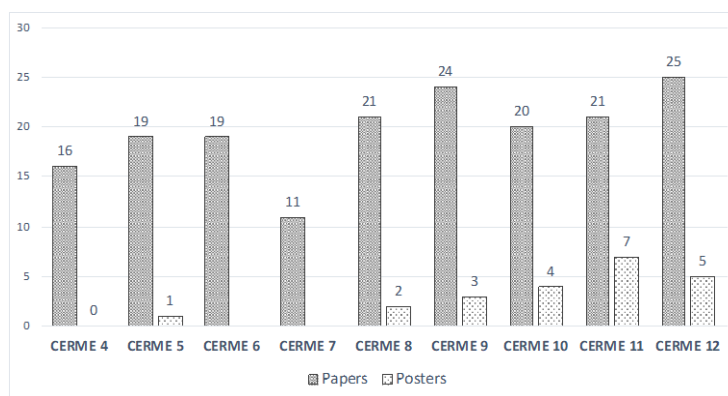


Figure 1: Evolution of the papers and posters presented in TWG6

A total of 45 participants participated in the online CERME12, representing 14 countries –most of them from Europe, but also from South and Central America, Israel, South Africa and Japan. The papers and posters were divided into six leading themes (see Table 1), so that in each session papers and posters were discussed on one of the themes.

**Table 1: Leading themes (LT) defined and number of contributions in the proceedings**

Leading themes	Papers	Posters
LT1. Theoretical and methodological developments in modelling	3	
LT2. Modelling in STEM topics and extra-mathematical topics	3	2
LT3. Design and analysis of modelling tasks and processes – Learning modelling at primary or secondary school education	6	2
LT4. Design and analysis of modelling tasks and processes – Teaching and learning of modelling at tertiary education	6	
LT5. Teacher education for mathematical modelling and application	4	1
LT6. Technology and other resources in mathematical modelling	3	

In the first leading theme, the reader can find papers focusing on theoretical and methodological developments of tools for the analysis of the modelling tasks and practices. The second theme focuses on the interplay and connections between mathematical modelling and other subjects, referring to the STEM topics and/or engineering context. The third and fourth themes address different strategies to support the design and analysis of modelling tasks and of the strategies to foster students' work on modelling tasks. These contributions are organised into two sub-topics according to the educational level the research contributed to: primary or secondary school education (LT3), or university education (LT4), the latter also including university students who are being prepared to become teachers. This last sub-topic links coherently with the fifth one, which covered teacher education for modelling and its applications, presenting several instructional proposals for preservice and in-service teacher education for application and modelling. Finally, the sixth theme focuses on the use of technology and other resources in mathematical modelling, mainly about how to combine different resources with technology in concept development by means of real word contexts. Table 2 summarizes the papers and posters included in the proceedings of CERME12, which we elaborate on in this introductory report discussing the leading themes.

## **Leading themes and overarching questions**

### **LT1: Theoretical and methodological developments in mathematical modelling**

This first theme focuses on proposing research tools and methodologies to analyse modelling practices beyond the cognitive aspects and to discuss the rationale and specificities of mathematical modelling in comparison to other domains or disciplinary approach to modelling. Firstly, Vos and

*Frejd* present a suggestion for extending the modelling cycle by the dimensions of metacognitive strategies, tool use and social norms. Secondly, the paper of *Ärleback* and *Frejd* considers a dual integrated modelling approach to the teaching and learning of mathematics and the teaching and learning of mathematical modelling at the same time. Thirdly, *Kawakami* and *Ärleback* present a review on the characterization and comparison among the rationales of statistical modelling and mathematical modelling. In all three contributions it became clear that the teaching of modelling and the analysis of modelling processes should not be limited to modelling competencies but should also include other aspects such as mathematical concepts, metacognition, affect, social norms, group work and tool use. Some questions for discussion emerge for future developments on this topic:

- What might a modelling cycle as an analysis tool look like, that includes aspects such as metacognition, affect, tools, social norms or group work, in addition to the analysis of sub-competencies?
- What would a concept for teaching look like, in which modelling competencies and mathematical concepts or knowledge about the context can be taught at the same time?
- To what extent is it necessary to distinguish between mathematical modelling and, for example, statistical modelling? Does statistical modelling represent a sub-form of mathematical modelling?

## **LT2: Modelling in STEM topics and extra-mathematical topics**

The LT2-papers study the role of mathematical modelling in extra-mathematical, engineering, and STEM contexts. The paper by *Kacerja* and colleagues presents an analysis of teachers' discussions on the modelling involved in the body mass index (BMI). *Vásquez* and colleagues discuss a study and research path (SRP) for the teaching of modelling in secondary school in relation to the evolution of COVID-19, which connects to the poster from *Donner* and *Bauer* on a modelling project about the pandemic for grades 9 and 10. The poster by *Fleischmann* and colleagues presents the design of an SRP about modelling climate change. Finally, *Pablo-Díaz* and *Romo-Vazquez* present the design of a didactic activity for engineering education based on the Hazen-Williams model. The main areas and driving questions of the discussion with respect to LT2 can be summarized and highlighted as:

- The complexity of realistic and real-world (ill-defined) problems. How to deal with and conceptualize the extra-mathematical domain? How to transpose complex real-world modelling problems into different grade classrooms without losing authenticity, relevance and richness? How to support or motivate students to validate their results with the real context?
- Critical perspective, ethical considerations, and sensitivity towards students. How to support a critical perspective on the use of models in society (e.g. when addressing issues related to Covid-19, BMI)? How to consider in classrooms the potential implications of using the models for decision-making?
- Implications of using real data in modelling activities. How to foster a critical stance towards, and thinking about, data in students? Starting from the given data: what questions can be answered? Starting from questions: what data is needed in order to provide an answer?
- The role and importance of various aspects of posing questions. What is the role of problem posing in the context of applications and mathematical modelling? What can students learn

from posing questions with respect to modelling and extra-mathematical questions? How to support students in posing productive and answerable extra-mathematical questions?

- The nature of the models used and developed: To what extent are STEM contexts and activities providing new and different types of models to explore?

STEM and other extra-mathematical contexts provide rich sources for modelling problems but introduce new types of challenges and demands for students, teachers and researchers. We need to conceptualize and theorize about how to design activities and learning environments (including technology) that fundamentally connect to, and use, knowledge from other disciplines and extra-mathematical domains.

### **LT3: Design and analysis of modelling tasks and processes – Learning modelling at primary or secondary school education**

The papers related to this topic refer to different strategies to support students when solving modelling problems, teachers when guiding their implementation and strategies for the design and analysis of modelling activities. As one of the themes that are grouping more contributions, we can distinguish different focuses and issues raised.

Firstly, *Elias* and colleagues focus on examining the notion of the rate of change in modelling problems. The authors examine which aspects of the notion of rate of change are prone to subjective reasoning by learners, due to their structure or due to missing information, and which aspects are objective. Secondly, some contributions focus more on some particular steps of the modelling process and analyse, for instance, the influence of students' beliefs about modelling or the interest in the context of modelling tasks. *Geisler* focuses on the steps of validation and investigates how students validate their models within modelling tasks with experiments. Additionally, *Kanefke* and *Schukajlow* focus on the difficulties of students noticing when data are missing from some modelling problems. They aim to analyze the extent to which students noticed missing data while processing modelling problems with missing data. Furthermore, the poster from *Kaemmerer* presents a comparison between students' work on modelling tasks when they have an interest (or do not) in the real-world context where the task is proposed. The poster from *Surel* discusses strategies for a better comprehension of modelling tasks related to students' engagement. Some common questions for discussion linked to these contributions are about:

- What impact do students' beliefs about the nature of mathematics (e.g., exactness of mathematics) have on students' approaches to working with (missing) data? How can we influence students' beliefs, so they are able to be more flexible with their models (adaptations/reformulation/validation)?
- Are students' affective reactions dependent more on the type of modelling task or the phase in the modelling process? Are there important differences in students' affective reactions?

Thirdly, other papers are more focused on the proposal of analytic tools for the ideal, individual, or collective modelling routes. The paper from *Schneider* and colleagues investigates whether and to what extent knowledge about ideal-typical modelling processes have an influence on phase transitions in individual modelling. *Göksen-Zayim* and colleagues present an observation instrument to study collaborative modelling. The authors use three main components: collaborative learning,

mathematical modelling and the language that students use while working together. Moreover, *Bassi* and *Brunetto* are interested in the affective factors, cognitive and motivational, that can be associated with the different modelling activities. Some common questions about these contributions refer to:

- To what extent does group work condition the modelling process? How to take into consideration the aspects related to group work conditions?
- Which tools are more useful for the systematic analysis of the influence of interactions on the resolution process?

#### **LT4: Design and analysis of modelling tasks and processes – Teaching and learning of modelling at university**

This fourth leading theme, closely related to the previous and the next ones, is now focused on the design and analysis of mathematical modelling at university level. Most of papers on this topic choose the university students who are in fact preservice teachers.

*Zhou* and *Hansen* investigate how the introduction of the pedagogical method “mathematics in three acts” to preservice teachers influenced their mathematical modelling. *Hartmann* and colleagues investigate the modelling activities that take place when posing problems that are based on given real-world situations and the extent to which modelling activities occur with different problem-posing activities. *Segura* and *Ferrando* aim to categorize the different types of errors when university students solve some modelling tasks involving estimations and to inquire into the efficient use and learning from errors to improve initial teacher training. *Sevinc* and *Ferrando* analyze the commonalities and divergences in the resolution of a group of Turkish and Spanish preservice mathematics teachers’ ways of modeling approaching a Fermi-based modeling problem. The contribution from *Andresen* aims to inquire about learning trajectories in the context of modelling with differential equations. Textual analysis of the reports from teachers in a masters’ program implied the marking of notions and terms related to progressive, horizontal and vertical mathematizing,

We can divide the main topics into three areas. They are school-university transfer, modelling processes and modelling tasks. In the *school-university transfer*, the importance of the transfer of studies from school to university, was discussed. Here, the question arises, how to compare the conditions under which the implementation runs and the results. Another question discussed is for example: How to take into account the university context and the institutions to which some modelling tasks are to be transferred?

In the *context of modelling processes*, the question of understanding reality and mathematics in the modelling process as “part of the world” was discussed. Modelling is seen, especially at university, as a way to learn new advanced mathematics rather than just using familiar university-level content. The connection between mathematical modelling and problem posing and the importance of their relationship was also pointed out here.

The discussion of *modelling tasks* for university students includes criteria for evaluating the “quality of the problem” (for example openness and complexity). Fermi tasks also have great potential as modelling tasks at the university. They provide an opportunity to analyse possible errors in more

detail. Modelling tasks examples at university, if used in teacher education, should also be considered in terms of their pedagogical significance. Looking to the future with respect to this fourth topic, questions arise about:

- What differences between engineering students and preservice teachers are relevant to the construction of modelling opportunities?
- What are the main differences between mathematical modelling in university and mathematical modelling in the school?

#### **LT5: Teacher education for mathematical modelling and application (professional content knowledge and competencies)**

This fifth theme is focused on teacher education, based on the acknowledged need for preparing preservice and practicing teachers for the teaching of applications and modelling. Papers on this leading theme are closely related to the previous ones. As mentioned before, papers in LT4 present the analysis of preservice teacher activity (considered as university students) and, in the current one, the focus is on professional content knowledge for mathematical modelling, pedagogical knowledge for modelling and simulation, and/or on teachers modelling skills.

On the one hand, *Greefrath* and colleagues consider the professional content knowledge of mathematical modelling as a competence facet of teachers to present and discuss the development and use of a test instrument. In this same direction, but referring more to pedagogical knowledge, *Gerber* and colleagues propose a theory-based model and subsequently present items of an associated test instrument to measure the preservice mathematics teachers' professional knowledge for teaching simulations and mathematical modelling with digital tools. On the other hand, *Montejo-Gómez* and colleagues present the characterization and the analysis of the kinds of assumptions made by a group of preservice teachers on the models considered and the impact on modelling outcomes and teachers' skills. The poster presented by *Ulbrich* and colleagues focuses on developing mathematical modelling skills for mathematics teachers through 3D Modelling and 3D Printing. Some main issues and questions related to this leading theme are about:

*Content knowledge (CK) and Pedagogical content knowledge for mathematical modelling (PCK):* Considering the specificities of the knowledge domains (mathematics, stochastics, grade level, ...), how can we describe mathematical, technological and modelling knowledge involved in PCK in order to design assessment tools for measuring PCK?

*Noticing mathematical modelling processes:* How is teachers' noticing competencies and knowledge (regarding metacognition) in modelling and its development related to one another?

*Validity and replicability of research in teacher education:* How do we deal with the question about validity and replicability in our field of research (in particular, in teacher education for mathematical modelling)? What might be the impact of teacher education for modelling on teachers' practice? How can we replicate studies in light of the diversity of conditions (and limitations) under which teachers can act? In the future development of our field of research, questions arise about what knowledge on technologies (which tools, when to use tools, black boxing, instrumental genesis, ...) might also be involved in the discussion of teacher knowledge for mathematical modelling and of teacher education.

## **LT6: Technology and other resources in mathematical modelling**

This theme focused on the use of technology and digital resources in mathematical modelling. *Touma* and *Olsher* explore the design principles of computer-based modelling activities. From design-based research, the results show that technology plays several relevant roles, namely simulation, investigating scenarios, and simplifying procedures. In her study, *Jessen* aims at characterising the roles of digital resources in mathematical modelling, by using the Anthropological Theory of the Didactic in terms of media-milieu dialectics. The digital tools are seen both as media and milieu. *Lieban* and *Bueno* examine students' use of 2D and 3D resources to find out how ideas from the Three Worlds of Mathematics come into play. They conclude that connections result from experimentation, exploration, understanding, and manipulation, in physical or digital environments. Some of the matters largely discussed were:

*Contributions of digital tools to the modelling activity:* What is the relationship between simulation and modelling? In considering the different roles that digital tools can play in the modelling process, to what extent does technology define the modelling activity? How does technology trigger students' creativity in the modelling process?

*Design of technology-based tasks and environments:* How can we find a balance between the teacher's and the students' intervention in a modelling task? How to design good tasks that foster the students' modelling competencies using digital tools? How to develop modelling tasks considering the diversity of students' knowledge of technology in a class? How to promote student's self-confidence on the use of technology for modelling?

*Teachers' knowledge on mathematical modelling with digital tools and STEM practices:* Is there a specialized PCK concerning the implementation of mathematical modelling with technology? How can technology be used as a communication tool and a collaboration tool in modelling activities? Is it possible to establish a relationship between a modelling task and the technology that is more effective to carry out some or all the steps of the modelling process?

In the future development of our field of research, we need more studies on technology-based modelling activities. New conceptualizations and theoretical approaches concerning mathematical modelling with technology are also necessary. The design of modelling activities with digital tools is important in feeding the research on this theme.

## **Concluding remarks and perspectives**

The leading themes addressed by the TWG6 show again the variety of research approaches and questions the papers dealt with (Carreira et al., 2019; Kaiser & Sriraman, 2006). Furthermore, the educational levels spanned from primary to tertiary education, also covering preservice and in-service teacher education. With the overview presented in the previous section, we now focus on each of the six leading themes central in CERME12 and summarize the main driving questions of the discussion, as well as some remarks and questions, looking ahead for future research with respect to each leading theme.

The first leading theme (LT1) discusses research tools and methodologies to analyse modelling practices beyond the cognitive aspects, suggesting extending the modelling cycle by considering



metacognitive strategies, social norms, affect, group work and tool use. Moreover, this discussion is complemented by the comparison between mathematical modelling with modelling in other domains, such as statistical modelling, to discuss their possible meeting point in terms, for instance, of their rationale or conceptualisation. The second theme (LT2) argues the role of mathematical modelling in extra-mathematical, engineering and STEM contexts (or more in general, interdisciplinary contexts). There is a long tradition in TGW6 of discussing examples under this theme. The discussions highlight the need to conceptualize and theorize about how to design activities and learning environments that connect disciplinary knowledge, in particular mathematical and mathematical modelling knowledge, to other disciplines and extra-mathematical domains.

The third and fourth themes are more associated with the design and analysis of modelling tasks and processes in primary and secondary education (LT3) and tertiary education (LT4). These two themes group about half of the contributions. LT3 groups the papers that refer to different strategies to support students when solving mathematical modelling problems and strategies to design and analyse modelling activities at primary and secondary school levels. Closely related to the previous one, LT4 is focused on the design and analysis of mathematical modelling at tertiary education. Most of the papers choose the university training of preservice teachers to analyse what future teachers do when they are being trained (as students) at university.

The fifth theme (LT5) focuses on teacher education, based on the need of preparing preservice and in-service teachers for the teaching of applications and modelling. LT5, with the previously mentioned LT4, included many papers, continuing with the dynamics initiated in the previous CERME11. Teachers and their initial and continuous professional development are crucial for the integration of mathematical modelling into mathematics education at all school levels. Some questions about the linkages and replicability of teacher education and its impact on teachers' practice remain open for future research. Finally, the sixth theme (LT6) is focused on the use of technology and digital resources in the teaching and learning of modelling. Questions about the need for theoretically-based designs of modelling activities with digital tools or the need for extending research on teacher education for the use of simulations and modelling remain highlighted for future development of our field of research.

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