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# Introduction to Thematic Working Group 1: Argumentation and Proof

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## Introduction

Argumentation and proof continue to attract the growing and wide interest of the mathematics education research community, which was also evident in CERME 12. In Thematic Working Group 1 (TWG01; “Argumentation and proof”), 39 participants from 15 different countries from 5 continents actively engaged in the discussion of the 30 papers and 3 posters. CERME 12 was the first to be organized online; a challenge and an opportunity to investigate ways of practicing and promoting communication, cooperation, and collaboration. For this purpose, the work of TWG01 was organized in both parallel split-group sessions and whole-group sessions, aimed to maximize active participation and to ensure the coherence and unity of the TWG01 identity in the subgroups. Moreover, the online format of the conference proved to be a great opportunity to organize a joint session with Thematic Working Group 9 (TWG09; “Mathematics and Language”). The session allowed the discussion of topics of the common interest of both TWGs, allowing for our practicing the ERME’s spirit of communicating, cooperating and collaborating beyond the conceptual boundaries of our TWG.

The papers of TWG01 were organized in seven themes, which were presented and discussed in split-group or whole group sessions: 1) Theoretical and epistemological perspectives about argumentation and proof, 2) National and international perspectives about argumentation and proof, 3) Argumentation and proof in primary mathematics education, 4) Argumentation and proof in school and university mathematics, 5) Argumentation and proof in teacher education, 6) Argumentation and proof beyond mathematics text and context, and 7) Argumentation, Language and Proof (this was the theme of the joint session with TWG09).

In this introduction chapter, the papers are presented and discussed in three broader topics, in line with the main themes elicited in the discussions of our group: a) Argumentation and proof in school and university, b) Theoretical, epistemological and sociocultural perspectives about argumentation and proof, and c) Argumentation and proof in different texts and contexts.

## **Discussion of Papers**

### **Argumentation and proof in school and university**

In the TWG01 Introduction section of the CERME 11 proceedings, it was noted that one of the areas that the TWG01 participants “would like, and hope, to see more research in future CERMEs was: *The teaching of proof and argumentation* in both school and university settings, including in teacher education with particular emphasis on argumentation and proof at the elementary school level.” In CERME 12, the topic of several papers appeared to address this, with a particular focus on primary education. The presented papers investigated diverse aspects of the primary school students’ argumentation and proof, including the students’ conceptions and understandings about proof (as presented in the papers of Sigrid Iversen and of Merve Dilberoğlu, Erdinç Çakıroğlu and Çiğdem Haser), designs to support the students’ proving skills and in highlighting the empirical-deductive gap (as discussed in the papers of Melanie Platz, of Jo Knox and Igor’ Kontorovich). Moreover, we had the opportunity to discuss different aspects of reasoning that occur at these educational levels: data-based argumentation as investigated in the paper of Jens Krummenauer and Sebastian Kuntze, and Simone Jablonski’s paper about mathematical reasoning outside of the classroom. Furthermore, we discussed topics specific to higher educational levels, including a paper about proofs without words at the secondary education level by Nadav Marco, Alik Palatnik and Baruch Schwarz, as well as Katharina Kirsten’s paper about the proving strategies employed by first year university students.

Considering teacher education, a central issue concerned the importance of working with future and in-service teachers to explore ways of efficiently incorporating research findings about argumentation and proving in teacher training and professional development programs. Such efforts were evident in the papers of Orly Buchbinder and Sharon McCrone and of Gabriel Chun-Yeung Lee. At the same time, the papers of Thomas Bauer and Eva Müller-Hill and of Fiene Bredow and Christine Knipping allowed us to gain deeper insight on the teachers’ practices through the lenses of different theoretical perspectives. Moreover, Lakatosian ideas were at the crux of three papers concentrating on teachers (the papers of Mei Yang, Andreas Stylianides and Mateja Jamnik, and of Dimitrios Deslis, Andreas Stylianides and Mateja Jamnik) and on teacher educators (as discussed in the paper of Magdalini Lada and Tore Alexander Forbregd).

Overall, the presented papers offered the opportunity for rich discussions about the commonalities and specificities of teaching and learning argumentation and proving at the different educational levels (with a special focus on primary education), as well as about the appropriateness of the respective research approaches. At the same time, the discussion about the implementation of Lakatosian ideas in teacher education raised fruitful deliberations about the way that ideas that have been developed in a specific sociocultural context may (or may not) be applied to educational research, which leads to the second broader topic of the papers discussed in TWG01.

### **Theoretical, epistemological and sociocultural perspectives about argumentation and proof**

The meaning(s) of proof, its relationships with the validity of mathematical knowledge and with the notion of truth, are important epistemological issues that are constantly being re-visited in the TWG01 meetings; question certainties helps to enrich, broaden and alter perspectives. Along these lines, Viviane Durand-Guerrier discussed the dialectical relationships between truth and proof, while the

discussion was enriched by Vergnauds' ideas as employed in Nadia Azrou's paper and by Habermas' rationality in the paper of Paolo Boero. Moreover, the role of logic and deductive reasoning is at the crux of argumentation and proof and, hence, in this CERME, logic was again a central theme of our discussions in TWG01. Miglena Asenova's paper, challenged the traditional perspective of classical logic and set-theoretical assumptions, while we had the opportunity to consider the role of unitizing predicates as presented in the paper of Paul Christian Dawkins and Kyeong Hah Roh, as well as the role of deductive reasoning in word problems as investigated in the paper of Rimas Norvaiša.

Considering the complexity of the argumentation and proof phenomena, the discussions about epistemological and theoretical perspectives were explicitly linked with the role of the sociocultural aspects in the teaching and learning of argumentation and proof. For this purpose, we focused on the diverse perspectives and realizations of assessment in different countries (for example, Chile, Hungary, Norway), drawing upon the papers of Kinga Szűcs and of Manuel Goizueta, Constanza Ledermann and Helena Montenegro. Furthermore, the paper of David Reid broadened the discussion to include 'reasoning' in the national curricula and standards in several countries. Moreover, the paper of Karolína Mottlová and Jana Slezáková offered an insight of implementing ideas of the curriculum of one country to another (respectively, from Singapore to Czech Republic), focusing on word problems. At the same time, language appears to be a crucial factor in mastering the logical structure of proofs, as discussed by Kerstin Hein. The latter issues about language are also explicitly linked to the third broad topic of the papers discussed in TWG01.

### **Argumentation and proof in different texts and contexts**

Argumentation and proof have been traditionally linked with language and verbal communication, but in CERME 12 the participants draw the attention to broader conceptualizations of text, including non-verbal and multimodal aspects. Within this context, we discussed the explanation norms expanded to include explanation videos and the explanation norms, as presented in the paper of Jessica Kunstler. Moreover, in the last CERME, the participants' discussions involved various aspects of the issues of language in argumentation and proof, while it was noted that it might be sensible to address this complexity in collaboration with colleagues of TWG09 and linguists. In this CERME, we addressed this issue by having a joint session with TWG09, where we had the opportunity to discuss language, argumentation and proof. For this purpose, we initiated the inter-TWG collaboration by critically focusing on Toulmin's scheme, which has been a staple tool for analyzing argumentation. The various implementations and extensions to the Toulmin's model were synthesized in the paper of Jenny Cramer and Leander Kempen, while the paper of Andreas Moutsios-Rentzos explicitly acknowledged multimodality in the discussion about argumentation and text. Two papers of colleagues of TWG09 (of Christoph Körner and Michael Meyer, and of Jorge Toro and Walter Castro, which may be found in the TWG09 part of the CERME 12 proceedings) allowed us to reflect upon the commonalities and differences of the two groups with respect to language issues in argumentation and proof.

Furthermore, mathematics is at the crux of the modern scientific, non-mathematical texts, such as physics. The participants of TWG01 investigated aspects of argumentation and proof in historical and

physics texts, as presented in the paper of Laura Branchetti, Alessia Cattabriga, Olivia Levrini and Sara Satanassi.

## **Conclusions and Future Directions for TWG01**

We argue that CERME12, the first to be conducted online, offered the participants the opportunity to be engaged in rich, broad and deep discussions about a variety of issues and perspectives. Importantly, we noted a valuable mix of a continuity of topics from previous CERMEs with novel ideas. A series of questions were posed that paint potential paths of future research projects.

Considering the *teaching of argumentation and proof*, we noted the tensions amongst research, intentions and actuality in everyday teaching and the importance of findings ways to bridge the potential divides. Furthermore, we ponder how can we make the teaching of argumentation and proof feasible for the different grades, curricular, educational and sociocultural settings? Should it be incorporated in everyday teaching practices across mathematical contents or should there also be a dedicated section to specific argumentation and proof practices (for example, about logic)?

Moreover, drawing upon the fact that in the modern curricula mathematics is present in non-mathematical courses, we identified a need for investigating *interdisciplinary perspectives about argumentation and proving* in texts, textbooks, and teaching practices.

The rapid technological advances appear to crucially affect the established communication norms and modalities, as they become part of the everyday teaching. Within this context, conceptualizing and investigating *language issues related to argumentation and proof* seems to need to be re-visited, and broadened to include non-verbal (multimodal, embodied, affective etc) and/or implicit aspects.

The *sociocultural aspects of argumentation and proof* seem to be another area of interest that transcends various perspectives, including the implementation of specific theoretical and/or epistemological perspectives in different context (to the one that the perspective originated), as they may entail both cultural and cognitive dimensions.

All these areas of interest intersect in complex ways and derive from this TWG01 meetings and should not be interpreted as prioritizing specific lines of research over others. We are aware that proof and argumentation are approached from different perspectives (and in other TWGs groups) and in TWG01 we are committed to voicing and exploring this diversity.