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Rayleigh limit of generalized Lorenz-Mie: axicon terms revisited

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A systematic study of optical forces in the Rayleigh limit of the generalized Lorenz-Mie theory, GLMT in short, has been recently investigated [1]. Although performed in the case of circularly symmetric Bessel beams, a general formulation for arbitrary shaped beams has been simultaneously obtained, both for longitudinal in the z -direction [2] and for transverse forces in the x - and y -directions, see [3], with corrections for the y -component of forces provided in [4]. In this framework, optical forces are expressed in terms of beam shape coefficients $g_{n, TM}^m$ and $g_{n, TE}^m$ (n from 1 to ∞ , m from $(-n)$ to $(+n)$, TM standing for "Transverse Magnetic" and TE standing for "Transverse Electric"). A remarkable result is that optical forces are then expressed in terms of BSCs associated with only ($n = 1$)- and ($n = 2$)-partial waves. This is deep contrast with the dipole theory of forces, e.g. [5], [6] in which optical forces are *explicitly* expressed in terms of the *total* electric field of the illuminating beam \mathbf{E} , i.e. encompassing all partial waves from $n = 1$ to ∞ of this beam, and asks the question to know what is the relationship between the Rayleigh limit of the GLMT and the dipole theory of forces (let us note what we here comply with the definition of *total* field in the dipole theory of forces denoting the illumination beam, in contrast with the point of view in the GLMT framework where *total* field instead denotes the summation of the incident and scattered fields). Relying on numerical [4], [7] and on formal analyses [8], [9], it has however been demonstrated that the

Rayleigh limit of GLMT perfectly identifies with the dipole theory of forces. This implies that the dipole theory of forces actually contains mainly superfluous contributions, specifically all contributions from partial waves of order $n > 2$. These results furthermore imply that it is necessary to revisit a paper devoted to the existence of axicon terms in the framework of the Rayleigh limit of GLMT [10]. To this purpose, it will be sufficient to limit ourselves to a discussion of longitudinal optical forces.

We start with scattering forces that we define as forces proportional to the Poynting vector at the position of the scattering particle, denoted by the subscript P (this is here the most convenient definition although alternative points of view may be taken). In the Rayleigh limit of GLMT, the longitudinal component of the Poynting vector taken at P is found to read as, e.g. Eq.(66) in [2]:

$$(\widetilde{S}_z)_P = 2 \operatorname{Re}[i(g_{1, TM}^{-1} g_{1, TE}^{-1*} - g_{1, TM}^1 g_{1, TE}^{1*})] \quad (1)$$

in which the normalization condition $E_0 H_0^*/2 = 1$ has been used and which shows that scattering forces only depend on $(n = 1)$ -partial waves. This implies that, when expressing the corresponding scattering forces in the dipole theory of forces, all partial waves of order $n > 1$ are superfluous and produce a null contribution as formally checked (after lengthy formal computations) in [8].

Let us now consider gradient forces that we define as being proportional to the derivative of $|\mathbf{E}|^2$ taken at P . In the case of circularly symmetric Bessel beams, this gradient is 0 in agreement with the fact that longitudinal gradient forces are found to be 0 as well in the Rayleigh limit of GLMT [2]. However, it has been recognized soon after that this result was unsatisfactory and somehow *inconsistent* insofar as, according to the understanding gained by working in the Rayleigh limit of GLMT, it was clear that most of the partial waves involved in \mathbf{E} were irrelevant to the computation of the gradient of $|\mathbf{E}|^2$. Inspired by Eq.1 above, it was then decided to evaluate this gradient by working with a restricted electric field $|\widetilde{\mathbf{E}}|$ in which only $(n = 1)$ -partial wave contributions would be retained rather than working with the total field \mathbf{E} . In the case of on-axis circularly symmetric Bessel beams, it has been found that the gradient of $|\widetilde{\mathbf{E}}|^2$ was indeed zero as expected [11]. However, when applied to off-axis beams, the gradient of $|\widetilde{\mathbf{E}}|^2$ generated axicon terms which were not zero, as it was expected at this time [10]. The solution to this issue is that, in contrast with the case of scattering forces which are correctly predicted by using only $(n = 1)$ -contributions, the evaluation of gradient forces and of the corresponding gradient of $|\mathbf{E}|^2$ taken at P require to account both for $(n = 1)$ - and for $(n = 2)$ -contributions. This may be demonstrated as follows. When establishing the identification between the Rayleigh limit of GLMT and the dipole theory of forces, it was established that the longitudinal gradient of $|\mathbf{E}|^2$ taken at P reads as [8]:

$$(\partial_z |\mathbf{E}|^2)_P = -2k |E_0|^2 \text{Im}(G) \quad (2)$$

in which, see Eq.(24) in [2]:

$$G = \frac{1}{3} g_{1, TM}^0 g_{2, TM}^{0*} + g_{1, TM}^1 (g_{2, TM}^{1*} - i g_{1, TE}^{1*}) + g_{1, TM}^{-1} (g_{2, TM}^{-1*} + i g_{1, TE}^{-1*}) \quad (3)$$

But, it has been demonstrated that, in the case of circularly symmetric Bessel beams, see Eqs.(28), (37) and (41) in [2]:

$$\text{Im}(G) = 0 \quad (4)$$

which is to be compared with the fact that the gradient forces are found to be proportional to $\text{Im}(G)$, e.g. Eq.(27) in [2]. Therefore, taking into account both ($n = 1$)- and for ($n = 2$)-contributions when evaluating $(\partial_z |\mathbf{E}|^2)_P$ does not produce any axicon term, but a perfectly coherent formulation.

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