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► **To cite this version:**

| Gerrit Coddens. Normalization errors in CHSH Bell inequalities. 2022. hal-03804940v3

HAL Id: hal-03804940

<https://hal.science/hal-03804940v3>

Preprint submitted on 30 Oct 2022

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Normalization problems in CHSH Bell inequalities

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25th October 2022

Abstract. We consider two different definitions of determinism, the traditional one and a new alternative one. The alternative definition introduces a new description of what happens during a photon correlation measurement. The CHSH Bell-type inequality is derived according to the traditional definition of determinism. It can then not be applied in the usual way to the experiments it has been designed for, when in the analysis of the experiments we follow the alternative type of definition. In fact, the usual procedure introduces then a normalization error with a confusion between absolute and conditional probabilities. Within the frame work of the alternative definition the inequality is no longer violated and determinism is not defeated.

PACS. 03.65.Ta, 03.65.Ud, 03.67.-a

1 Introduction

The subject matter of the Bell inequalities culminating in the experiments of Aspect *et al.* [1,2] can be supposed to be very well-known. For an introduction we refer to [3]. Further experimental developments have been the recent loophole-free experiments reported in [4,5].

Let us inspect the derivation of the CHSH Bell inequality given in [3]. We consider 4 variables $a_1 \in S$, $a_2 \in S$, $b_1 \in S$, $b_2 \in S$, where $S = \{0, 1\}$. The idea is that 0 corresponds to absorption in a polarizer, and 1 to transmission. There are other types of polarizers which deliver answers +1 and -1. Both types have been used by the teams led by Aspect. We focus here on the type that yields answers 0 or 1. We assume that the set-ups of these experiments are well-known. The values a_j are obtained with polarizer settings in one arm of the set-up, and b_k with polarizer settings in the other arm. There are thus 16 possible combinations for the values of (a_1, a_2, b_1, b_2) . By making a table of these 16 combinations it is easy to verify that we always have:

$$\forall (a_1, a_2, b_1, b_2) \in S^4 : Q = a_1 b_1 - a_1 b_2 - a_2 b_1 - a_2 b_2 + a_2 + b_2 \in S. \quad (1)$$

Throughout the paper we will use the notation $F(W_1, W_2)$ for the set of functions whose domain is the set W_1 and whose values belong to the set W_2 . Let us consider now functions $a_j \in F(V, S)$ and $b_k \in F(V, S)$. Here V is a set of relevant variables for the experiment. We can call the set V the set of hidden variables. One can imagine that V could be a subset of a vector space \mathbb{R}^n or of a manifold, e.g. a non-abelian Lie group like $SO(3)$ or $SU(3)$. We have then:

$$\forall \lambda \in V : 0 \leq Q(\lambda) \leq 1, \quad \text{where:} \\ Q(\lambda) = a_1(\lambda)b_1(\lambda) - a_1(\lambda)b_2(\lambda) - a_2(\lambda)b_1(\lambda) - a_2(\lambda)b_2(\lambda) + a_2(\lambda) + b_2(\lambda). \quad (2)$$

We can now consider a probability density p over V . The function p belongs then to the set of functions $F(V, [0, \infty[)$ with domain V and values in $[0, \infty[$. We further require that $\int_V p(\lambda)d\lambda = 1$. We can now integrate Equation 2 with p over V . Introducing the notations:

$$\begin{aligned}
p(a_j \wedge b_k) &= \int_V a_j(\lambda) b_k(\lambda) p(\lambda) d\lambda, \\
p(a_j) &= \int_V a_j(\lambda) p(\lambda) d\lambda, \quad p(b_k) = \int_V b_k(\lambda) p(\lambda) d\lambda,
\end{aligned} \tag{3}$$

we obtain then:

$$0 \leq p(a_1 \wedge b_1) - p(a_1 \wedge b_2) - p(a_2 \wedge b_1) - p(a_2 \wedge b_2) + p(a_2) + p(b_2) \leq 1. \tag{4}$$

Here \wedge is the logical “and” operator. This is the CHSH Bell inequality applied to the photon correlation experiments described in [1] and in its precursor [6]. It is a purely mathematical identity and does not depend on any physical considerations. It is also free of any considerations about statistical correlations and statistical independence, which must be dealt with in the definition of the probability density $p \in F(V, [0, +\infty[)$, i.e. its definition domain V , its values and its normalization to 1. In [1,6], the quantity $p(a_j \wedge b_k)$ becomes the probability for the event that photon 1 is transmitted by the filter A oriented along a direction defined by the angle α_j while its buddy photon 2 is transmitted by the filter B oriented along β_k .

Rather than using some experimental results with a limited accuracy, we can inject theoretical expressions for the outcomes of these experiments into the inequality and check if they violate it. By adopting theoretical expressions we can avoid the burden of discussing the experimental and statistical uncertainties. Many such uncertainties may occur and lead to the objection that a violation reported could in reality not be genuine and be due to experimental errors. This is then called an experimental loophole. There have therefore been elaborate attempts to make the experiments very precise such as to close all possible experimental loopholes [4,5]. These experiments and those described in [1,2,6] are all extremely meticulous, representing the state of the art of their time. While this entails admittedly an act of faith, the exact theoretical expressions can be considered to just express what the result of an infinitely precise loophole-free experiment would be. By plugging the exact theoretical expressions rather than experimental results into the inequality we free the argument from all experimental considerations. It becomes immune to them. The probabilities are thus identified with the mathematical expressions for the outcomes of the photon polarization experiments that are considered to be exact:

$$p(a_j \wedge b_k) = \frac{1}{2} \cos^2(\alpha_j - \beta_k), \quad p(a_j) = \frac{1}{2}, \quad p(b_k) = \frac{1}{2}, \tag{5}$$

where α_j and β_k are the angles of the polarizer settings in the two arms of the experiment. These expressions are known to violate the Bell inequality for certain choices of angles $\alpha_1, \alpha_2, \beta_1$ and β_2 . If the theoretical expressions in Equation 5 were not rigorously exact, then we could still test the inequality with the improved rigorous expressions. But the logic of the quest for rigorously exact data to test the Bell inequality falls apart if the Bell inequality in Equation 4 itself is wrong.

2 The model used to derive the inequality is not an adequate description of the experiments

We will now expose a fundamental problem with this derivation of the inequality. Note that in Equation 1 the number a_1 in $a_1 b_1$ is the same as in $a_1 b_2$, implying that $a_1 b_1$ and $a_1 b_2$ must be attributed to a same photon pair. The equation implies that all numbers a_1, a_2, b_1, b_2 must be attributed simultaneously to a same photon pair. As the two photons are identical, all photons are therefore assumed to carry the four numbers a_1, a_2, b_1, b_2 simultaneously. As we can choose α_j and β_k at will, an uncountable infinity of numbers a_j and b_k would have to be written inside each photon. A further serious problem is that in real-life experiments the polarizer A cannot be in the two positions α_1 and α_2 , at the same time. The same is true *mutatis mutandis* for the polarizer B . In the real world we can therefore not attribute simultaneously all four numbers a_1, a_2, b_1, b_2 to a same photon pair if we accept that we must base our attributions on hard data obtained by measurements. Everything will depend on the issue if one deems these objections as important or otherwise. Can one consider that it suffices to know the quantities a_1, a_2, b_1, b_2 in principle according to the idea that we can obtain them by deterministic calculations from some initial knowledge, such that we do not need to measure them? Or do we consider that it is necessary to work with measured quantities? In the latter case, we will not be able to attribute the four quantities a_1, a_2, b_1, b_2 simultaneously to a single photon as is assumed in the derivation of the Bell inequality, let alone an infinite amount of quantities a_j and b_k . The Bell inequality is then based on wrong modelling of the physical reality. When the quantities a_1, a_2, b_1, b_2 cannot be attributed to a single photon, the correlations between the numbers will be weaker [7] and the boundaries of the inequality that applies to them will be less stringent than in Eq. 2. However, Larsson [8] has dealt with this objection and argued that one can then nevertheless apply the Bell inequality to the measured probabilities, because they are averages.

3 Two types of determinism

The assumptions underlying the derivation of the Bell inequality are presumably faithfully reflecting Einstein's viewpoints about determinism. He may have had in mind two things:

- (E1) All four numbers a_j and b_k are already defined at the time when the photons are leaving the source.
- (E2) Strict determinism for whatever happens.

Bohr had a different vision:

- (B1) The values a_j and b_k are not known right from the start. They are produced by the experiment.
- (B2) There is no determinism and the outcomes of experiments are intrinsically probabilistic.

These two scenarios may look mutually exclusive. As a matter of fact, according to Einstein's viewpoint, (E2) would presumably already imply (E1). The four numbers exist then already from the start and all we have to do is register them. We can call this Einstein's determinism. But when (E2) and (E1) collapse into one single assumption, then Einstein's viewpoint can only be right or wrong: *tertium non datur!*

The ultimate consequences of sticking to Einstein's viewpoint are all the objections we formulated in Section 2. But the problem when one denies determinism as Bohr did is that one cannot explain the 100 percent correlations observed between the results in the two arms of the set-up when the two polarizers are parallel. And when we are allowed to position the polarizers far enough apart, entanglement enters the scene.

As the collapse of the two conditions (E1) and (E2) into a single one shows, there is actually no reason for presenting Einstein's vision as consisting of two points like we did. It looks outlandish. The reason why we did it anyway is that we wanted to break away from the spell of the *tertium non datur* by thinking out of the box in a wholesome act of rebellion. This permits us to introduce a different form of determinism, whereby we combine (E2) from Einstein and (B1) from Bohr. We can cite Remark 1 on p.3 of [7] with some small modifications to explain how this ought to roll out:

"We assume that the photon has only a few polarization parameters when it leaves the source and that the rest is determined by the interaction of the photon with the set-up, i.e. the two polarizers and their orientations. In other words, the outcome of the experiment is produced by the interactions. In fact, when we change the polarizer settings, e.g. from α_1 to α_2 we do not modify the hidden variables of the photons, but the probabilities change from $p(a_1 \wedge b_k)$ to $p(a_2 \wedge b_k)$, such that there must be hidden variables in the polarizers. However, this raises the question how we can consistently obtain the same response in both arms of the set-up when the two polarizers are parallel. The systematic agreement shows that the interactions that come into play cannot be the interaction of a photon with an individual electron, atom or molecule of a polarizer. That would give rise to fluctuations with the result that the responses in both arms cannot be consistently the same. The mechanism must therefore be more like the interaction of a photon with a macroscopic field generated by a very large amount of electrons, atoms or molecules of the polarizers. It could e.g. be an electric field. That means that the hidden variables within the polarizer can be replaced by macroscopic quantities that do not allow for fluctuations. We will call these macroscopic quantities, which describe the polarizers, "the fields". We have then identical photons interacting with identical fields in both arms, which explains the 100 percent correlation. All statistical fluctuations must then be due to those in the distribution of the completely correlated pairs of photons which interact deterministically with the fields. We can understand the introduction of the concept that there are some fields by drawing an analogy with a similar correlation experiment for electrons (instead of photons) and two Stern-Gerlach filters (instead of two polarizers). In the Stern-Gerlach filters, the fields at stake are magnetic fields. The analogy is not perfect because for electrons the probabilities will rather be of the form $p(a_j \wedge b_k) = \frac{1}{2} \cos^2((\alpha_j - \beta_k)/2)$ and in the Stern-Gerlach filters there are field gradients, which are needed to separate the beams, but the idea that there must be some macroscopic fields at work is then clarified."

In summary, we assume that some properties of the photons are determined right from the start. The photons behave deterministically such that the outcome of the experiment is determined by the deterministic interactions of the photons with the set-up. However, the information is no longer all concentrated at the source within the photons at the time of the emission by the source. Some information is also stored in the set-up. And to explain the 100 percent correlations when the polarizers are parallel, one must then accept that the set-ups define macroscopic fields. This is our version of determinism. It is different from Einstein's but the whole scenario it is still totally deterministic.

We have no logical, *deductive* means to decide which one of the two versions of determinism is right. One important difference is that in our alternative definition of determinism the probabilities must be actualized by experiments, while in Einstein's definition they exist independently of a measurement. But it would be arbitrary to weaponize such issues in order to promote one definition at the expense of the other one, by claiming that it would be more logical, even if it is true that the Moon is also there when nobody looks. Proving by pure deductive reasoning which one of the two versions is right could be a tall order. However, the derivation of the Bell inequality used in the experiment of Aspect *et al.* is based on Einstein's definition of determinism. In order to defeat determinism on the basis of their results Aspect *et al.* become therefore endowed with the charge of proving that also the alternative definition of determinism we are proposing here is wrong. We on the other hand do not have such an obligation to prove that Einstein's version is wrong. With our objections we went a long way in that direction, but they do not constitute a conclusive watertight proof. However, we will see that the data violate the Bell inequality following Einstein's philosophy while they do not violate the inequality following ours. This amounts to an experimental *inductive* proof of our version of determinism.

With our scenario we save determinism in a modified form with the asset that we are no longer forced to accommodate conceptually for the existence of entanglement. As our approach is based on a different paradigm for determinism than the one used by Aspect *et al.* the experimental results can no longer be used to invalidate determinism.

4 Model according to the alternative definition of determinism

Let us now work this out. Probabilities are defined as the number of desired outcomes divided by the total number of all possible outcomes. Let us call the set of all possible events or outcomes the event space. To teach probabilities one often draws the event space in the form of a probability tree, consisting of points and branches connecting these points. The two end points of a branch PQ on such a tree correspond to probabilities $p(P)$ and $p(Q)$. We assume that P is here the lower point of the branch where it sprouts from the tree. The branch itself corresponds to the conditional probability $p(Q|P)$ and we calculate $p(Q) = p(Q|P)p(P)$ in travelling upwards along the tree. In Fig. 1 we show the tree for the part:

$$p(a_1 \wedge b_1) - p(a_1 \wedge b_2) - p(a_2 \wedge b_1) - p(a_2 \wedge b_2), \quad (6)$$

which occurs within the CHSH Bell inequality in Eq. 4. In Eqs. 4-6 a_j stands for the event that polarizer A in setting A_j yields the answer 1 for a photon. The same holds *mutatis mutandis* for b_k . We have in total four possible configurations for the combined settings of the two polarizers. We can select the angles $\alpha_j, j \in \{1, 2\}$ and $\beta_k, k \in \{1, 2\}$ with random generators in such a way that both settings A_j occur with equal probability $1/2$, with the same *mutatis mutandis* for the settings B_k . And if the two random generators are statistically entirely independent the four combined settings (A_j, B_k) will each occur with probability $1/4$.

The probabilities $p(a_j)$ for transmission by filter A in settings A_j are both $1/2$. The probabilities $p(b_k)$ for absorption by filter B in settings B_k are also both $1/2$. The conditional probabilities $p(b_k|a_j)$ for transmission by filter B provided there is transmission by filter A is $p(b_k|a_j) = \cos^2(\alpha_j - \beta_k)$. We have further also $p(\neg b_k|\neg a_j) = \cos^2(\alpha_j - \beta_k)$, $p(\neg b_k|a_j) = \sin^2(\alpha_j - \beta_k)$, $p(b_k|\neg a_j) = \sin^2(\alpha_j - \beta_k)$. The quantities $p(a_j \wedge b_k)$ in Eqs. 6-4 are the probabilities for joint transmission $p(b_k|a_j)p(a_j) = \frac{1}{2} \cos^2(\alpha_j - \beta_k)$ in the configuration (A_j, B_k) . Hence, in the setting (A_j, B_k) the probability for simultaneous transmission is $\frac{1}{2} \cos^2(\alpha_j - \beta_k)$. The event space for such a single setting (A_j, B_k) is illustrated in Fig. 1 for the case (A_1, B_2) . It is symbolized by the rectangle in dashed lines. This rectangle corresponds to the event space for the real experiment in the setting (A_1, B_2) . The total event space for the Bell inequality must contain all four set-ups (A_j, B_k) and corresponds therefore to the whole figure (according to our definition of determinism, see below). In fact, this is compulsory because the inequality contains all four probabilities $\frac{1}{2} \cos^2(\alpha_j - \beta_k)$ simultaneously and therefore the event space for the Bell inequality must allow for defining all these probabilities simultaneously. In real life the four settings (A_j, B_k) cannot exist simultaneously.

We see thus that according to Fig. 1, based on our definition of determinism, the event space needed to derive the inequality is larger than the individual event spaces in the four individual real-life experiments. This implies that the total number of outcomes in the event spaces we use to normalize the probabilities in real-life single set-ups is different from the total number of outcomes in the theoretical event space on which the Bell inequality is being proved.

The probabilities in the Bell inequality must therefore be normalized differently in the event space for the inequality than the probabilities in the event spaces for isolated experiments. Consequently, the probabilities to be fed into the Bell inequality must be attributed the values $\frac{1}{8} \cos^2(\alpha_j - \beta_k)$ rather than $\frac{1}{2} \cos^2(\alpha_j - \beta_k) = \frac{1}{8} \cos^2(\alpha_j - \beta_k)/(1/4)$, where $1/4$ is the probability for selecting the configuration (A_j, B_k) . In the Bell inequality, the real-world probability becomes a conditional probability, whereby the condition is the selection of the configuration (A_j, B_k) . The violation of the Bell inequality in Eq. 4 appears this way as due to a normalization error in the definitions of the quantities which occur in Eq. 6. With the correct normalization the inequality is not violated. We could rephrase this by stating that there has been a confusion between two different definitions of probability, by tacitly assuming that they were identical.

The error becomes most obvious by noticing that the sum of all probabilities in the top line in Fig. 1 is 1 as it should be, while following the values used by Aspect *et al.* it would be 4. One may object that our drawing constitutes a leading argument and that the probabilities might be conceived as corresponding to overlapping realities, such that they should not be juxtaposed and summed as suggested by Fig. 1. But that objection is itself also a leading argument. It opposes Einstein's vision of determinism to ours, because with our definition of determinism what it says is not true. Let S_{jk} be the set of hidden variables needed to describe the configuration (A_j, B_k) . The set S_{jk} contains then couples of numbers (λ_j, μ_k) where λ_j are hidden variables for filter A in its setting A_j and μ_k are hidden variables for filter B in its setting B_k . This shows that the sets S_{jk} are disjoint because their elements (λ_j, μ_k) just have different meanings (even if there might be numerical coincidences), such that the representation in Fig. 1 is correct [7]. Furthermore Fig. 1 also represents the correct measurement protocol. The photons and their parameters intervening in the four top branches of the tree can be considered to be different in each branch as in the real measurement protocol or identical in all four branches as assumed in the derivation of the Bell inequality [7]. The difference between the two choices,

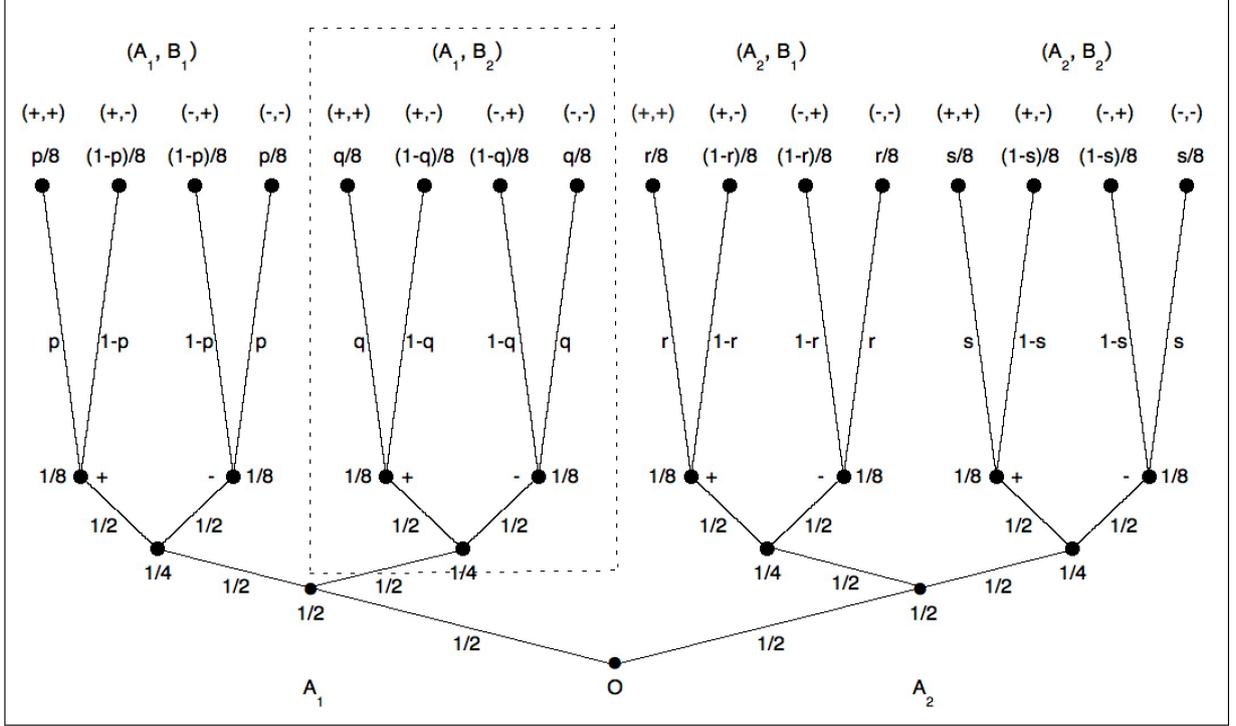


Fig. 1: Probability tree in the event space for the quantity in Eq. 6 which occurs in the CHSH Bell inequality in Eq. 4. The point O is the root of the tree. Two branches are starting from O . They represent the possibilities A_1 and A_2 for the setting of the filter A (in the left arm of the set-up). The end points of these branches are both the starting point for branches corresponding to the possibilities B_1 and B_2 for the setting of the filter B (in the right arm of the set-up). The four combined settings (A_j, B_k) occur each with the frequency $1/4$. The higher parts of the tree represent the possible outcomes for the photon correlation experiments in the four settings. In order not to burden the figure we have introduced the notations $p = \cos^2(\alpha_1 - \beta_1)$, $q = \cos^2(\alpha_1 - \beta_2)$, $r = \cos^2(\alpha_2 - \beta_1)$, $s = \cos^2(\alpha_2 - \beta_2)$. The rectangle drawn in dashed lines represents the event space for the setting (A_1, B_2) . In this setting the other points (A_j, B_k) and their related events are not elements of physical reality, because a polarizer cannot assume two orientations at the same time. The four points only exist simultaneously in our mind and in the event space for the Bell inequality. The complete figure describes exactly the event space for the Bell inequality as explained in the main text. It shows that the experimental probabilities are normalized with respect to a smaller event space than the probabilities that occur in the Bell inequality, such that they must be normalized differently. The experimental probability is $q/2$, while the probability to be inserted into the Bell inequality is $q/8$ (see main text).

which are two choices of definition of determinism, will not change the structure of the tree. It will also not change the accurately measured probabilities according to Larsson's argument¹.

¹ Indeed, we can then attribute the probability 1 to the four branches of the tree, because they all address the same photons. This argument saves the derivation of the Bell inequality, with its virtual unmeasured probabilities. However, the set of hidden variables needed to describe the photons in a real-life experiment and their accurately measured real probabilities is smaller than the set of hidden variables needed to describe the photons in the derivation of the Bell inequality and the corresponding unmeasured virtual probabilities. This leads to a difference in normalization between the real and the virtual probabilities (see [7], Eqs. 13-15) such that the argument is flawed. This is just the argument about the hidden variables in the sets S_{jk} which pops up in a different guise. It is the consequence of a wrong modelling of the physical reality and we are bound to run into this normalization problem whatever the presentation, because mathematics is self-consistent.

Finally, each argument stipulating that one should reason differently than we have done in the construction of the tree could be rebutted by invoking the violation of the inequality that might result from it as a proof that this alternative reasoning is wrong. An example of such an argument could be claiming that for all photons and all possible angles α_j and β_k the numbers a_j and b_k do not require a measurement to be real. That would be imposing Einstein's vision and supposedly invalidate the structure of the tree or alternatively replace the values 1/4 by 1 (see Footnote 1) on the tree in Fig. 1, in agreement with the objection formulated. But nobody is entitled to impose a straightjacket on the definition of determinism by some personal choice based on preconceived notions that with hindsight look caricatural. Informed consent and following the science implies that we must leave that choice to mother nature instead of a ministry of truth. Our approach differs from the traditional approach by considering also hidden variables in the set-up, which is unusual but still perfectly open to classical understanding.

5 Epilogue

In conclusion, our scenario to describe the photon correlation experiments does not violate the Bell inequality, preserves determinism and does not require introducing the concept of entanglement based on the results of Aspect *et al.* Everybody knows that probability calculus is full of pitfalls, as illustrated by the ferocious polemics that have historically surrounded the correct solution of the Monty Hall problem by Marilyn Vos Savant. Also Bertrand's paradox illustrates this. It is therefore not recommended practice to question special relativity on the basis of some results obtained by probability calculus even if the calculus is advertized to be fool-proof and trivial. That might be considered as a rather heedless attack on special relativity. The weak element in such a confrontation is *a priori* not the theory of relativity but the probability calculus. Furthermore, if there were really a flaw in special relativity then it must be replaced by something else and the existence of entanglement could be revealing such a flaw. It could e.g. be telltale of faster-than-light transfer of information or of a different connectivity for space-time than assumed hitherto. Not many physicists seem to bother about resolving the issues of all-out importance for the foundations of physics that are raised by accepting the existence of entanglement. They prefer to jump on the bandwagon. In order to try to rebut other experiments which claim that the existence of entanglement has been proved, other approaches are needed (see [7] and the references therein). It is thereby imperative to fully understand what is going on behind the scenes of the group representation theory of SU(2).

Data Availability Statement: No Data associated in the manuscript.

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