Normalization errors in CHSH Bell inequalities

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Abstract. We show why a type of CHSH Bell-type inequality cannot be applied to the experiments it has been designed for due to a normalization error. The inequality is based on a confusion between two different definitions of probability, with the effect of erroneously assuming that they are identical.

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We want to point out a normalization error in the application of CHSH Bell inequalities to the experiments for which they have been designed, like e.g. the famous experiments carried out by Aspect *et al.* [1,2]. We will not dwell here on further experiments with improved set-ups that have been carried out after these first experiments.

Probabilities are defined as the number of desired outcomes divided by the total number of all possible outcomes. Let us call the set of all possible events or outcomes the event space. To teach probabilities one often draws the event space in the form of a probability tree, consisting of points and branches connecting these points. The two end points of a branch PQ on such a tree correspond to probabilities $p(P)$ and $p(Q)$. We assume that P is here the lower point of the branch where it sprouts from the tree. The branch itself corresponds to the conditional probability $p(Q|P)$ and we calculate $p(Q) = p(Q|P)p(P)$ in travelling upwards along the tree. In Fig. 1 we show the tree for the part:

$$p(a_1 \wedge b_1) - p(a_1 \wedge b_2) - p(a_2 \wedge b_1) - p(a_2 \wedge b_2), \tag{1}$$

which occurs within the CHSH Bell inequality:

$$0 \leq p(a_1 \wedge b_1) - p(a_1 \wedge b_2) - p(a_2 \wedge b_1) - p(a_2 \wedge b_2) + p(a_2) + p(b_2) \leq 1, \tag{2}$$

for the case that the polarizers deliver answers 1 for the transmission and 0 for the absorption of photons. There are other types of polarizers which deliver answers $+1$ and -1. Both types have been used by the team led by Aspect. We focus here on the type that yields answers 0 or 1. We assume that the set-ups of these experiments are well-known.

The polarizer A in the left arm of the set-up can be in the settings A_1 (angle α_1) or A_2 (angle α_2). The polarizer B in the right arm of the set-up can be in the settings B_1 (angle β_1) or B_2 (angle β_2). In Eqs. 1-2 a_j stands for the event that polarizer A in setting A_j yields the answer 1 for a photon. The same holds *mutatitis mutandis* for b_k. We have in total four possible configurations for the combined settings of the two polarizers. We can select the angles $\alpha_j, j \in \{1,2\}$ and $\beta_k, k \in \{1,2\}$ with random generators in such a way that both settings A_j occur with equal probability $1/2$, with the same *mutatis mutandis* for the settings B_k. And if the two random generators are statistically entirely independent the four combined settings (A_j, B_k) will each occur with probability $1/4$.

The probabilities $p(a_j)$ for transmission by filter A in settings A_j are both $1/2$. The probabilities $p(b_k)$ for absorption by filter B in settings B_k are also both $1/2$. The conditional probabilities $p(b_k|a_j)$ for transmission by filter B provided there is transmission by filter A is $p(b_k|a_j) = \cos^2(\alpha_j - \beta_k)$. We have further also $p(\neg b_k|\neg a_j) = \cos^2(\alpha_j - \beta_k)$, $p(\neg b_k|a_j) = \sin^2(\alpha_j - \beta_k)$, $p(b_k|\neg a_j) = \sin^2(\alpha_j - \beta_k)$. The quantities $p(a_j \wedge b_k)$ in Eqs. 1-2 are the probabilities for joint transmission $p(b_k|a_j)p(a_j) = \frac{1}{2}\cos^2(\alpha_j - \beta_k)$. Hence, in the setting (A_j, B_k) the probability for simultaneous transmission is $\frac{1}{2}\cos^2(\alpha_j - \beta_k)$. The event space for such a single setting (A_j, B_k) is illustrated in Fig. 1 for the case (A_1, B_2). It is symbolized by the rectangle in dashed lines. This rectangle corresponds to the event space for the real experiment in the setting (A_1, B_2). The total event space for the Bell inequality must contain all four set-ups (A_j, B_k) and corresponds therefore to the whole figure. In fact, this is compulsory because the inequality contains all

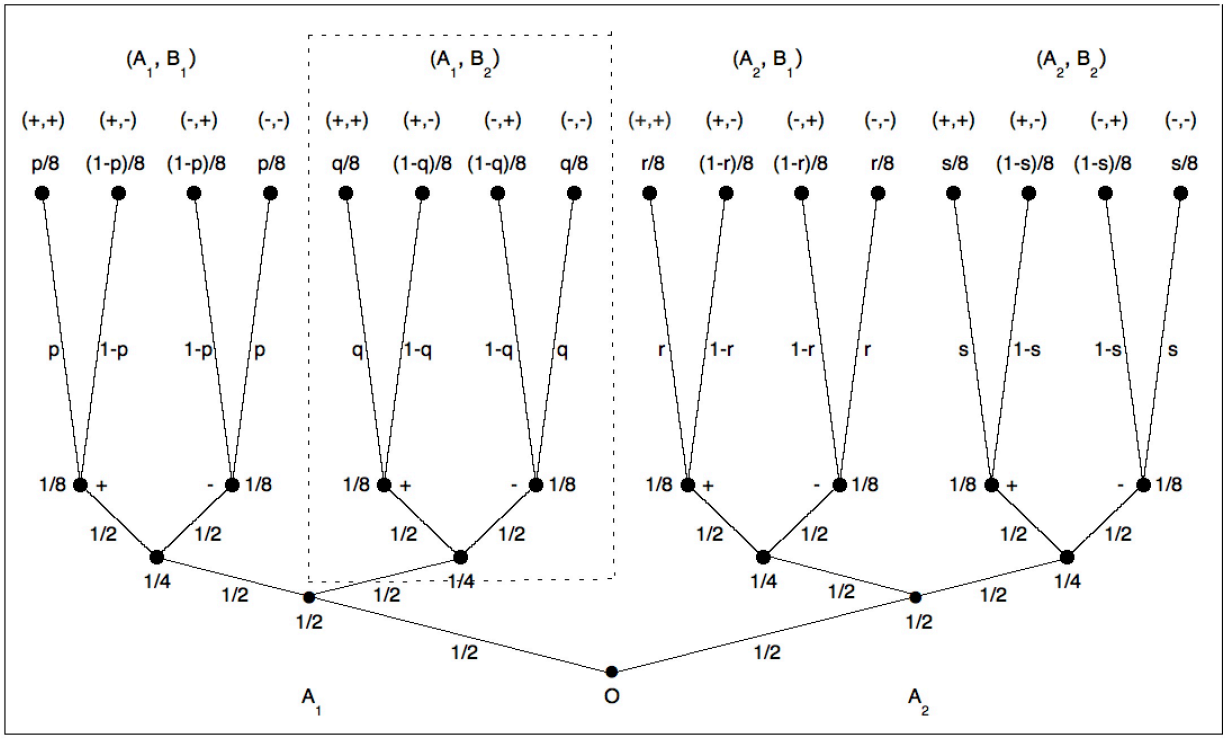

Fig. 1: Probability tree in the event space for the quantity in Eq. 1 which occurs in the CHSH Bell inequality in Eq. 2. The point O is the root of the tree. Two branches are starting from O. They represent the possibilities A_1 and A_2 for the setting of the filter A (in the left arm of the set-up). The end points of these branches are both the starting point for branches corresponding to the possibilities B_1 and B_2 for the setting of the filter B (in the right arm of the set-up). The four combined settings (A_j, B_k) occur each with the frequency 1/4. The higher parts of the tree represent the possible outcomes for the photon correlation experiments in the four settings. In order not to burden the figure we have introduced the notations $p = \cos^2(\alpha_1 - \beta_1)$, $q = \cos^2(\alpha_1 - \beta_2)$, $r = \cos^2(\alpha_2 - \beta_1)$, $s = \cos^2(\alpha_2 - \beta_2)$. The rectangle drawn in dashed lines represents the event space for the setting (A_1, B_2). In this setting the other points (A_j, B_k) and their related events are not elements of physical reality, because a polarizer cannot assume two orientations at the same time. The four points only exist simultaneously in our mind and in the event space for the Bell inequality. The latter corresponds to the complete figure. It shows that the experimental probabilities are normalized with respect to a smaller event space than the probabilities that occur in the Bell inequality, such that they must be normalized differently. The experimental probability is $q/2$, while probability to be inserted into the Bell inequality is $q/8$.

four probabilities $\frac{1}{2}\cos^2(\alpha_j - \beta_k)$ simultaneously and therefore the event space for the Bell inequality must allow for defining all these probabilities simultaneously. In real life the four settings (A_j, B_k) cannot exist simultaneously. The Bell inequality is therefore based on a wrong modelling of the physical reality.

We see thus that the event space needed to derive the inequality is larger than the individual event spaces in the four individual real-life experiments. This implies that the total number of outcomes in the event space we use to normalize the probabilities in real-life single set-ups is different from the total number of outcomes in the theoretical event space on which the Bell inequality is being proved.

The probabilities in the Bell inequality must therefore be normalized differently in the event space for the inequality than the probabilities in the event spaces for isolated experiments. Consequently, the probabilities to be fed into the Bell inequality must be attributed the values $\frac{1}{8}\cos^2(\alpha_j - \beta_k)$ rather than $\frac{1}{2}\cos^2(\alpha_j - \beta_k) = \frac{1}{8}\cos^2(\alpha_j - \beta_k)/(1/4)$, where 1/4 is the probability for selecting the configuration (A_j, B_k). In the Bell inequality, the real-world probability becomes a conditional probability, whereby the condition is the selection of the configuration (A_j, B_k). The violation

of the Bell inequality in Eq. 2 is therefore due to a normalization error in the definitions of the quantities which occur in Eq. 1. With the correct normalization the inequality is not violated. We could rephrase this by stating that there has been a confusion between two different definitions of probability, by tacitly assuming that they were identical.

Everybody knows that probability calculus is full of pitfalls, as illustrated by the ferocious polemics that have historically surrounded the correct solution of the Monty Hall problem by Marilyn von Savant. It is therefore not recommended practice to question special relativity on the basis of some results obtained by probability calculus. That might be considered as a rather heedless attack on special relativity. The weak element in such a confrontation is *a priori* not the theory of relativity but the probability calculus. Furthermore, if there were really a flaw in special relativity then it must be replaced by something else and it is then useless to search for a theory of quantum gravity before the problem with special relativity has been settled. Entanglement implies that the connectivity of space-time is different from what has been assumed in special relativity. Not many persons seem to bother about such issues of all-out importance. Perhaps they consider blatant contradictions in the foundations of physics as not very much of a problem. For the rebuttal of other experiments which claim that the existence of entanglement has been proved, other approaches are needed. A much extended version of this paper [3] was submitted to Universe where it received three vicious referee reports full of underhanded violence. When I rebutted this the editor Nathen Zhang used double standards refusing to see the beam in his own eye and those of the referees.

References

1. Aspect, A.; Dalibard, J.; and Roger, G.; Experimental Test of Bell's Inequalities Using Time-Varying Analyzers. Phys. Rev. Lett. **49**, 1804 (1982).
2. Aspect, A.; Grangier, P.; Roger, G.; Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment: A New Violation of Bell's Inequalities, Phys. Rev. Lett. **49**, 91 (1982).
3. Coddens, G.; Inadequacy of the CHSH Bell inequality for an application to its associated photon correlation experiments, https://hal.archives-ouvertes.fr/hal-03482574v6.