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Strategy combination in human cognition: a behavioral and ERP study in arithmetic

Thomas Hinault · Stéphane Dufau · Patrick Lemaire

Abstract It is well known that people use several strategies to accomplish most cognitive tasks. Unknown is whether they can combine two strategies. The present study found that such strategy combination can occur and improves participants' performance. Participants verified complex multiplication problems that violated the five rule ($5 \times 32 = 164$), parity rule ($5 \times 12 = 65$), both parity and five rules ($5 \times 31 = 158$), or no rule ($5 \times 26 = 140$). Participants obtained better performance on problems violating both five and parity rules than on problems violating either (or no) rule. Moreover, we found event-related potential (ERP) differences between two-rule and one-rule violation problems between 550 ms and 850 ms post-stimulus presentation, and ERP differences between parity-rule and five-rule violation problems between 850 ms and 1,400 ms. These findings have important implications to further our understanding of strategic variations in human cognition and suggest that strategy combination may occur in a wide variety of cognitive domains.

Keywords Arithmetic · Strategy combination · Five rule · Parity rule · Electroencephalography

Introduction

Three decades of research have established that people use several strategies to accomplish most cognitive tasks, select and execute strategies on a problem-by-problem basis, and that strategy choices and execution are influenced by problem, people, strategy, and situation characteristics (see Siegler, 2007, for a review). A strategy can be defined as “a procedure

or a set of procedures for achieving a higher level goal or task” (Lemaire & Reder, 1999, p 365). Several computational models of strategies have formalized strategy selection and execution processes (e.g., Lovett & Anderson's, 1996 ACT-R model; Lovett & Schunn's, 1999 RCCL model, Payne et al.'s, 1993 Adaptive Decision Maker model, Rieskamp & Otto's, 2006 SSL model, or Siegler & Araya's, 2005 SCADS* model). Although these models differ in some details, they share the core assumption that participants select the best strategy on each problem (i.e., the strategy that requires the least mental resources to execute and that yields the best performance). However, none of previous theoretical and empirical works have envisaged the possibility that participants can combine two (or more) strategies on a given item. The present experiment was run to test strategy combination that would occur if participants combine two different strategies into a single one, more efficient, strategy. We also determined whether strategy combination results in better performance. We pursued these goals in the context of arithmetic problem solving verification tasks.

Previous works found that participants obtain better performance when they verify parity-rule violation problems (to be true, a product must be even if either or both of its multipliers is even; otherwise, it must be odd; e.g., $4 \times 38 = 149$) and five-rule violation problems ($N \times 5$ product that ends in 0 or 5; $5 \times 32 = 164$) relative to no-rule violation problems (e.g., $4 \times 38 = 154$), and better performance on five-rule violation problems than on parity-rule violation problems (Krueger, 1986; Lemaire & Fayol, 1995; Lemaire & Reder, 1999; Lemaire & Siegler, 1995; Masse & Lemaire, 2001; Siegler, 1988). Better performance on five-rule violation problems or parity-rule violation problems relative to no-rule violation problems is usually accounted for by assuming that participants use a quick rule-violation checking strategy on rule-violation problems and a slower exhaustive verification strategy on no-rule violation problems. For example, when verifying five-rule

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violation problems, participants check only whether unit digits of proposed answers equal 0 or 5 or not. If not, they can quickly reject the equations without calculating the correct product, which they must do while checking no-rule violation problems.

Previous works in arithmetic using ERPs showed a late positive complex peaking between 500 and 1000 ms, which has been interpreted as resulting from mental calculation involved in problem solving task (e.g., Galfano et al., 2004; Kiefer & Dehaene, 1997). Also, authors found an N400 component which was interpreted as reflecting activation of arithmetic facts (e.g., Niedeggen et al., 1999). Finally, ERPs were left-lateralized, possibly as a result of language involvement (e.g., Dehaene et al., 1999; Stanescu-Cosson et al., 2000).

Here, we compared participants' performance for problems that violated only five rule (e.g., $5 \times 32 = 164$), only parity rule (e.g., $5 \times 12 = 65$), both parity and five rules (e.g., $5 \times 31 = 158$), or no rule ($5 \times 26 = 140$). Behaviorally, we predicted that strategy combination would result in better performance if participants combine both parity and five rules on two-rule violation problems compared to either parity-rule violation or five-rule violation problems. A combination of both rules occurs when people check whether (a) an even operand \times five = product that ends with 0 and (b) an odd operand \times five = product that ends with 5.

We expected electrophysiological signatures of such strategy combination with larger cerebral activities and/or differences in time course of event-related potentials (ERPs) for two-rule violation problems compared to one-rule violation problems. Moreover, we expected this modulation to be lateralized in left hemisphere of the brain. We also tested ERP differences between parity-rule and five-rule violation problems. Given that the five rule is used more consciously than parity rule (Krueger, 1986; Lemaire & Fayol, 1995; Lemaire & Reder, 1999; Masse & Lemaire, 2001), we expected larger amplitudes for five-rule than for parity-rule violation problems.

Methods

Participants

Nineteen young adults (four men; 18–25 years of age; mean age: 21.5 years) participated in this experiment. All reported normal or corrected-to-normal vision. Participants were paid 15 Euros for their participation. An informed consent was obtained prior to participation.

Stimuli

The stimuli were multiplication problems presented in a standard form ($a \times b = c$) with "a" as a single digit and "b" as a double digit, or reversed (see Tables 1, 2, 3 and 4 in appendix). Single-digit operands ranged from 3 to 8 while two-digit operands

ranged from 12 to 98. Each participant solved 640 problems. There were 320 five problems (e.g., $5 \times 89 = 445$) and 320 non-five problems (e.g., $3 \times 17 = 51$). Half of five and non-five problems were true problems (e.g., $4 \times 26 = 104$) and half were false problems (e.g., $5 \times 41 = 201$). Thus, there were 160 problems of each type (i.e., five true, five false, non-five true, and non-five false problems). Half of the five problems had an even second operand (e.g., $5 \times 64 = 320$), whereas the other half had an odd second operand (e.g., $5 \times 93 = 465$). For the non-five problems, there were problems with two even operands (even \times even), two odd operands (odd \times odd), and one even operand (even \times odd or odd \times even). Following previous works in arithmetic (e.g., Ashcraft & Battaglia, 1978), false problems were created by varying the difference between correct and proposed products. This made it possible to create problems either consistent or inconsistent with five and parity rules. For non-five problems, the false problems had products with splits of $\pm 1, 2, 3, 4, 7, 9, 14,$ or 20 between proposed and correct products. This resulted in half of the false non-five problems being parity-rule violation problems (e.g., $6 \times 17 = 103$) and half being parity-rule non violation problems (e.g., $6 \times 23 = 134$). Four types of false five problems were tested: (1) parity-rule only violation problems, with splits of ± 5 or ± 15 from correct products (e.g., $5 \times 12 = 65$), (2) five-rule only violation problems, with splits of ± 2 or ± 4 from correct products (e.g., $5 \times 32 = 164$), (3) parity- and five-rule violation problems, with splits of ± 1 or ± 3 from correct products (e.g., $5 \times 31 = 158$), and (4) no parity- or five-rule violation problems, with splits of ± 10 or ± 20 from correct products (e.g., $5 \times 26 = 140$).

Based on previous findings in arithmetic (see Geary, 1994 or Campbell, 2005, for reviews), we controlled the following factors: (1) no double-digit operand had 0 or 5 as unit digits, (2) no double-digit operand had 5 as decade digit (e.g., 53), (3) no double-digit operand had the same unit digit as decade digit (e.g., 44), (4) the size and side of double-digit operands was controlled, (5) size and direction of splits were matched across problems. The mean splits did not differ across five problems (mean = 7.5) and non-five problems (mean = 7.5), $t(159) < 1$. Moreover, the correct products were larger than the proposed products in half the problems and smaller in the other problems, for both five and non-five problems, (6) the magnitude of the proposed products did not differ significantly between five problems (mean = 275) and non-five problems (mean = 275), $t(159) < 1$. Furthermore, the magnitude of the proposed products did not differ significantly between parity-rule violation problems, five-rule violation problems, no-rule violation problems, and two-rule violation problems (means 275; $F < 1$). Finally, no false problem had proposed products equal to 100.

Procedure

Participants were seated comfortably in a quiet room. Stimuli were presented on a 800×600 resolution computer screen in a

18-point Courier New font. Problems were displayed horizontally in the center of the screen in the form of ' $a \times b = c$ '. When presented the problems, participants had to decide whether it was true or false. Participants were instructed to press the "F" key on an AZERTY keyboard if the problem was true and the "J" key if the problem was false, both with their index fingers. Response keys were counterbalanced across participants who had to solve the problems as quickly and accurately as possible.

Each trial started with a 200-ms blank screen (see Fig. 1). A warning fixation point (!) was then displayed at the center of the screen for 300 ms, followed by the equation. The equation remained on the screen until participants responded. During the inter-trial interval, participants saw 'XXXX' for 2,000 ms on the screen. Participants were instructed to blink only during inter-trial intervals. Before the experiment, participants solved 16 training problems that were similar but different from the experimental problems. After the practice trials, participants saw 10 blocks of 64 trials each (about 10 trials by condition, number of trial of each condition was counterbalanced between blocs), with a brief pause (<5 min) between each blocks. Participants were individually tested in one session that lasted approximately 60–90 min, with a pause between blocks.

Data processing

Latencies larger than the mean of the condition + 2 SDs were removed as well as all erroneously solved problems. We first analyzed true versus false problems as well as five versus non-five problems. Then, to test our predictions, we analyzed participants' performance on five false problems with ANOVAs and planned comparisons to test significance of differences in participant's performance between different problem types (no-rule violation, five-rule only violation, parity-rule only violation, and two-rule violation problems).

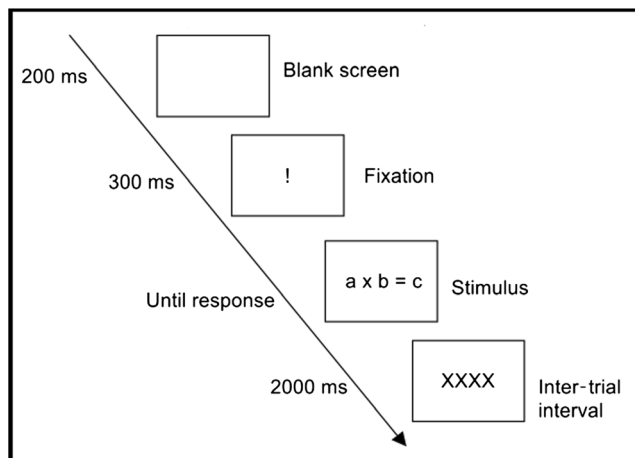


Fig. 1 Sequence of events within a trial

Electrophysiological activity was continuously recorded by a "Biosemi Active 2" system with 64 active electrodes positioned on an elastic cap (Electro-cap-Inc) following the 10–10 international system (FP1/2, AF7/8, AF3/4, F7/8, F5/6, F3/4, F1/2, FT7/8, FC5/6, FC3/4, FC1/2, T7/8, C5/6, C3/4, C1/2, TP7/8, CP5/6, CP3/4, CP1/2, P9/10, P7/8, P5/6, P3/4, P1/2, PO7/8, PO3/4, CMS/DRL, O1/2, FPz, AFz, Fz, FCz, Cz, CPz, Pz, POz, Oz, Iz). Six additional external electrodes were used: two on the left and right mastoids for the offline reference, and four electrodes placed on the left and right temples and below the eyes, to identify eye blinks and horizontal eye movements. The signal was continuously recorded at a frequency of 256 Hz and processed off line using EEGLAB software (Delorme & Makeig, 2004). The signal was filtered ([1–30] Hz bandpass filter), epoched ([–200, 3.200] ms stimulus based), and baselined ([–200, 0] ms). All electrodes were re-referenced off line using the mean activity of the left and right mastoid electrodes. Event-related potentials containing blinks or activity exceeding $\pm 100 \mu\text{V}$ were rejected. Also, any epoch with a channel containing amplitudes of more than five standard deviations from the epoch mean was rejected. Only ERPs corresponding to correct answers were analyzed.

We calculated mean amplitudes of latency windows that were determined by a combination of visual inspection and previous analyses of ERPs in strategy selection (e.g., El Yagoubi et al., 2003; Uittenhove et al., 2013). We defined seven windows of interest: 0–200; 200–550; 550–850; 850–1400; 1400–1700; 1700–2300; and 2300–3200-ms windows. We conducted ANOVAs in each window, on the data from 46 lateral electrodes, and 16 midline electrodes.

Results

Behavioral data

Three participants were removed from the analyses: One participant did not finish the experiment, and two other participants had accuracy at chance (average accuracy: 54 %). Preliminary analyses with the block factor (First/Second half of the experiment) revealed no main effect or interaction involving this factor. Unless otherwise noted, differences are significant at least to $P \leq 0.05$. Effect sizes are reported for each significant analysis.

We first conducted 2 Problem Type (five; non-five) \times 2 Correctness (true; false) repeated-measures ANOVAs on mean solution latencies and percentages of errors. Participants were faster for five problems than for non-five problem, $F(1,15) = 16.55$, $MSe = 621,779.42$, $np^2 = 0.53$, and on false problems compared to true problems, $F(1,15) = 14.96$, $MSe = 684651.94$, $np^2 = 0.50$. The Problem Type \times Correctness interaction was significant and revealed larger difference between five and non-five problems on true

problems compared to false problems, $F(1,15) = 8.07$, $MSe = 45943.71$, $np^2 = 0.35$.

ANOVAs of false five problems revealed a main effect of problem type (see Fig. 2), $F(1,15) = 13.32$, $MSe = 137025.19$, $np^2 = .47$. Planned comparisons showed that participants were slower at rejecting no-rule violation problems than parity-rule violation problems, $F(1,15) = 42.21$, $MSe = 3390735.37$, $np^2 = 0.74$, than five-rule violation problems, $F(1,15) = 84.37$, $MSe = 6051294.31$, $np^2 = 0.85$, or than two-rule violation problems, $F(1,15) = 109.89$, $MSe = 7240805.34$, $np^2 = 0.88$. Moreover, participants took more time to reject parity-rule violation problems than five-rule violation problems, $F(1,15) = 12.94$, $MSe = 382595.34$, $np^2 = 0.46$ or than two-rule violation problems, $F(1,15) = 14.98$, $MSe = 721615.43$, $np^2 = 0.50$. Most importantly, participants were faster to reject two-rule violation problems than five-rule violation problems, $F(1,15) = 5.27$, $MSe = 53331.70$, $np^2 = 0.26$.

Analyses of percentages of errors revealed a main effect of rule violation, $F(3,45) = 25.87$, $MSe = 505.43$, $np^2 = 0.63$. Participants rejected no-rule violation problems less accurately than parity-rule violation problems, $F(1,15) = 24.63$, $MSe = 1614.61$, $np^2 = 0.62$, than five-rule violation problems, $F(1,15) = 28.57$, $MSe = 2190.10$, $np^2 = 0.66$, or than two-rule violation problems, $F(1,15) = 27.20$, $MSe = 2175.03$, $np^2 = 0.65$. Furthermore, participants made more errors to reject parity-rule violation problems than five-rule violation problems, $F(1,15) = 7.44$, $MSe = 43.78$, $np^2 = 0.33$, or than two-rule violation problems, $F(1,15) = 6.88$, $MSe = 41.67$, $np^2 = 0.31$. Two-rule violation problems and five-rule violation problems were rejected equally accurately, $F < 1$ (see Fig. 2). We arc-sine corrected the error rates to normalize the data. Analyses yielded the same results as analyses on untransformed data.

Finally, we compared performance for two-rule violation problems and found no differences between $5 \times$ even operand

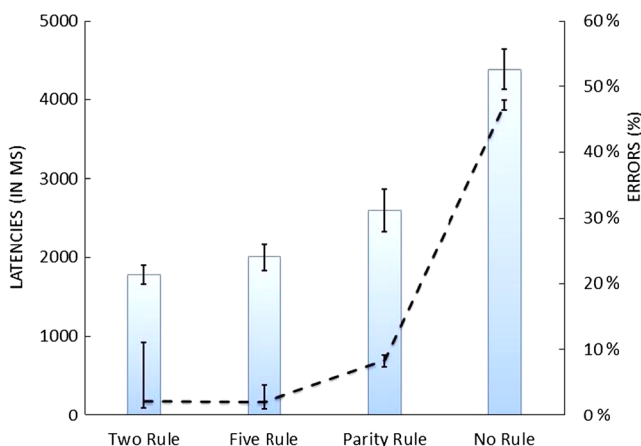


Fig. 2 Mean solution times (columns) and percentages of errors (dashed lines) for each problem type. Error bars represent standard error of the mean

(e.g., 5×82) and $5 \times$ odd (e.g., 5×71) in either solution times or mean percent errors, $F_s < 2$ s.24.

Electrophysiological data

Data were analyzed to examine ERP signatures of strategy combinations and of differences between rule-violation checking strategies. To test strategy combination, we conducted repeated-measures ANOVA, with the number of violated rules (one, two), anteroposterior (anterior, posterior), and hemisphere (left, right) of electrodes.

ANOVAs revealed no significant effects ($F_s < 3.5$, $P_s > 0.08$) of the number of violated rules in the 0–200 ms, 200–550 ms, 850–1,400 ms, 1,400–1,700 ms, or 2,300–3,200 ms windows. In the 550–850 ms window, there was a larger negativity (see Fig. 3a) for two-rule violation problems than for one-rule violation problems, $F(1,15) = 5.89$; $MSe = 0.52$, $np^2 = 0.28$. Also, the number of violated rules interacted with hemisphere ($F(1,15) = 7.70$, $MSe = 0.13$, $np^2 = 0.34$). The Number of violated rules \times Location \times Hemisphere ($F(1,15) = 4.50$, $MSe = 0.03$, $P = 0.05$, $np^2 = 0.23$) interaction was marginally significant. Two-rule violation problems elicited larger negative amplitudes than one-rule violation problems in the left anterior and posterior regions of the brain (see Fig. 3b, Fig. 4).

In the 1,700–2,300 ms window, the Number of violated rules \times Hemisphere interaction ($F(1,15) = 4.55$, $MSe = 0.04$, $np^2 = 0.23$) revealed a larger negative amplitude for two-rule violation problems than for one-rule violation problems in the left hemisphere of the brain.

Next, to determine the electrophysiological basis of the difference between rule-checking strategies, we conducted repeated-measures ANOVAs with Anteroposterior \times Hemisphere \times Problem type (no-rule; parity-rule; five-rule; two-rule violation problems).

ANOVAs revealed no significant effects ($F_s < 2.48$) of problem Type in the 0–200 ms, 200–550 ms, and 1,400–1,700 ms windows either at midline or at lateral electrodes. In the 550–850 ms window, overall activities were larger at posterior regions of the brain, $F(1,15) = 13.46$, $MSe = 1.66$, $np^2 = 0.47$. There was a main effect of problem type, $F(3,45) = 3.26$, $MSe = 0.35$, $np^2 = 0.18$. Contrasts revealed that two-rule violation problems led to a larger negativity than no-rule violation problems, $F(1,15) = 7.70$, $MSe = 1.79$, $np^2 = 0.34$, than parity-rule violation problems, $F(1,15) = 5.07$, $MSe = 0.92$, $np^2 = 0.25$, and most importantly, than five-rule violation problems, $F(1,15) = 5.18$, $MSe = 1.30$, $np^2 = 0.26$.

In the 850–1400 ms window, mean amplitude was larger in the anterior regions, $F(1,15) = 6.39$, $MSe = 0.97$, $np^2 = 0.30$. Furthermore, the Anteroposterior \times Hemisphere \times Problem Type interaction was marginally significant, $F(3,45) = 2.77$, $MSe = 0.01$, $P = 0.05$, $np^2 = 0.16$. Contrasts revealed a

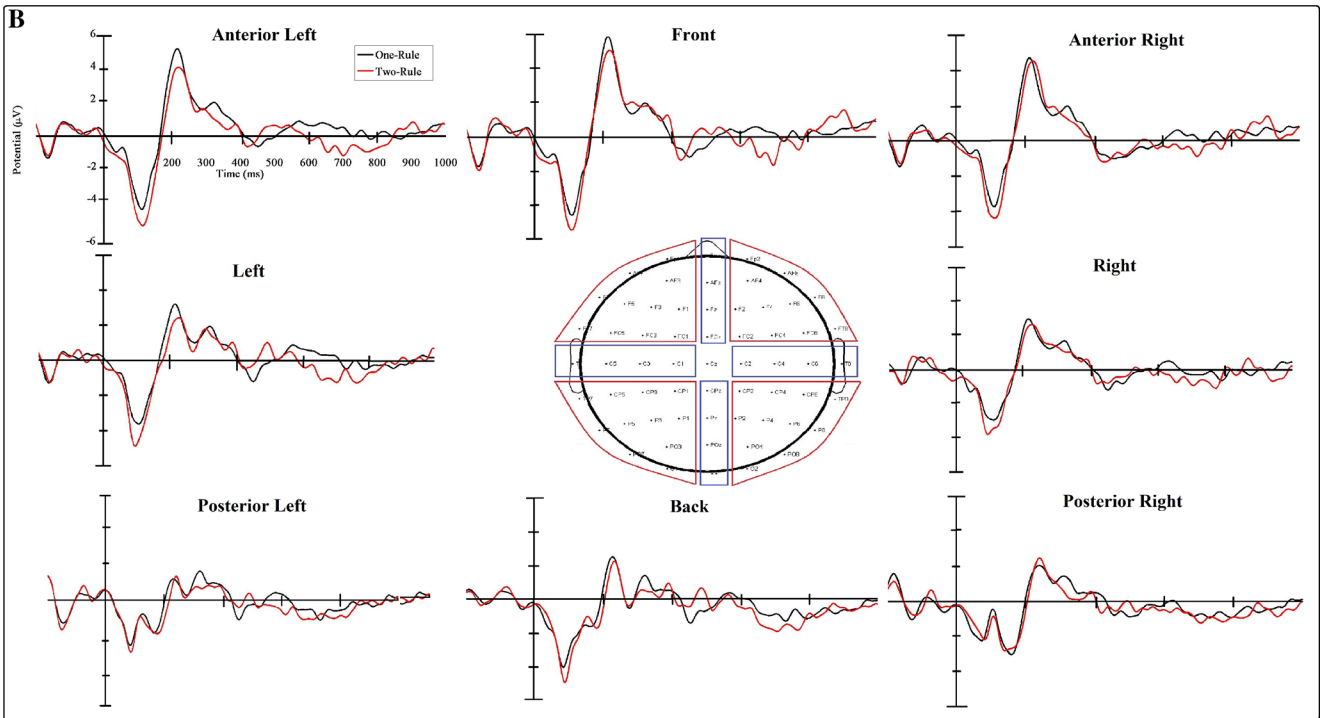
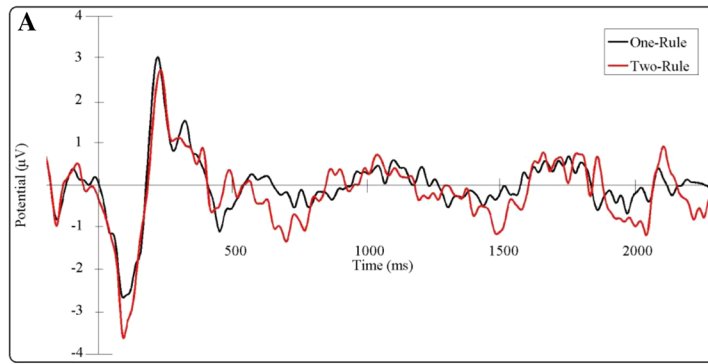


Fig. 3 Event-related potentials (ERPs) for one-rule violation problems (*black*) and two-rule violations problems (*red*). **a** Overall average of electrodes, **b** right and left anterior and posterior electrodes, and midline electrodes

larger positivity for five-rule violation problems than parity-rule violation problems in the anterior left regions, $F(1,15) = 5.95$, $MSe = 0.23$, $np^2 = 0.28$. Moreover, in the posterior right region (see Fig. 5a and 5b), five-rule violation problems yielded a larger positivity than parity-rule violation problems ($F(1,15) = 6.08$, $MSe = 0.07$, $np^2 = 0.29$); and parity-rule violation problems yielded larger

positivity than no-rule violation problems, $F(1,15) = 5.44$, $MSe = 0.05$, $np^2 = 0.27$.

In the 1700–2300 ms window, mean amplitudes of electrodes were more negative in posterior regions ($F(1,15) = 8.52$, $MSe = 1.19$, $np^2 = 0.36$). Also, the significant Hemisphere \times Problem Type interaction ($F(3,45) = 2.87$, $MSe = 0.04$, $np^2 = 0.16$) revealed a larger

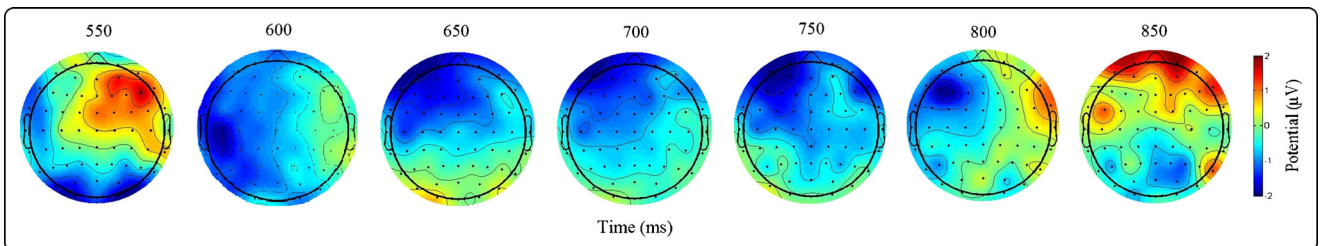


Fig. 4 Localization of ERP differences between one-rule violation problems and two-rule violation problems, from 550 to 850 ms after stimulus presentation (every 50 ms)

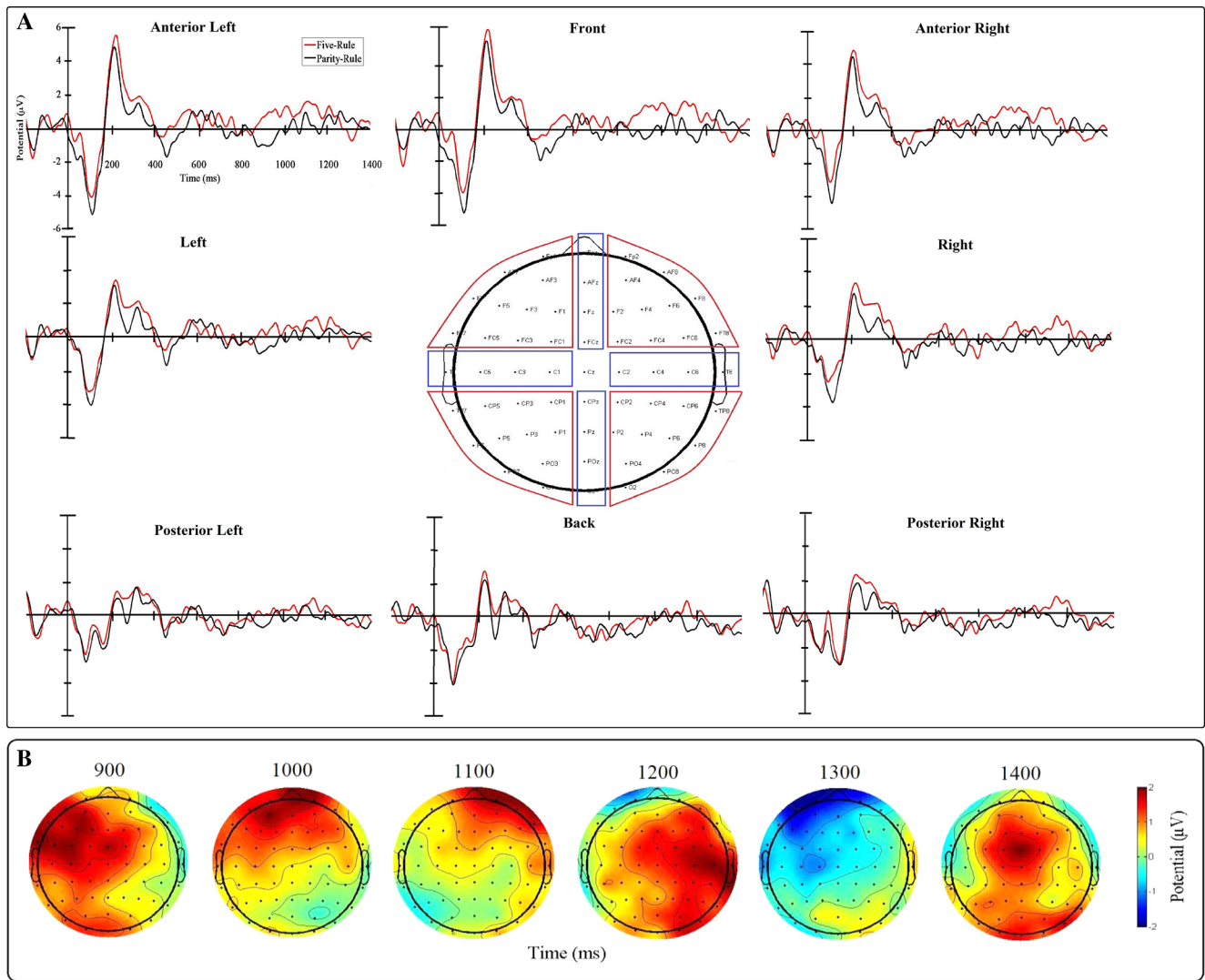


Fig. 5 ERPs for five-rule violation and parity-rule violation problems after stimulus presentation (a) in the right and left anterior and posterior regions and midline electrodes, and (b) ERP differences between five-rule

and parity-rule violation problem from 900 to 1,400 ms after stimulus presentation (every 100 ms)

positivity for five-rule violation problems than parity-rule violation problems in right hemisphere ($F(1,15) = 5.00$, $MSe = 0.08$, $np^2 = 0.25$). Furthermore, in the 2,300–3,200 ms window, larger positivity was observed in anterior regions, $F(1,15) = 5.53$, $MSe = 1.06$, $np^2 = 0.27$. The significant Anteroposterior \times Hemisphere \times Problem Type interaction ($F(3,45) = 2.92$, $MSe = 0.01$, $np^2 = 0.16$) revealed a larger positivity for five-rule violation problems than for parity-rule violation problems, $F(1,15) = 5.25$, $MSe = 0.29$, $np^2 = 0.26$, in the anterior left region of the brain.

Discussion

The present findings revealed both behavioral and electrophysiological signatures of strategy combination. First,

participants were faster to reject two-rule violation problems than no-rule or one-rule violation problems. Second, ERPs showed larger negativity while rejecting two-rule violation problems relative to one-rule violation problems in the 550–850 ms window. We also found larger positivity while rejecting five-rule violation problems relative to parity-rule violation problems in the 850–1,400 ms window.

Better performance for two-rule violation problems is consistent with the hypothesis that participants used a two-rule checking strategy on problems violating two rules rather than either a parity-rule or a five-rule checking strategy. There are at least two variants of the two-rule checking strategy. First, participants check first one of the two rules and then the other rule. Second, they combine two rule-violation checking strategies into a single one. The present data are inconsistent with a sequential checking of the two rules,

which predicts slower latencies for two-rule violation problems than for one-rule violation problems, in contrast to the present faster latencies to verify two-rule violation problems relative to one-rule violation problems. Furthermore, ERP data did not reveal any major temporal shift between the two strategies. We propose that participants verified the two-rule violation problems by combining both rule-checking strategies. This combination of both rules occurred when people checked whether (1) an even operand \times five = product that ends with 0, and (2) an odd operand \times five = product that ends with 5. Accumulation of evidence for both rules being violated while participants are processing arithmetic equations may be one of the mechanisms underlying strategy combination. This would be a quickly occurring mechanism, as suggested by larger negativity in ERPs for two-rule violation problems in the 550–850 window. This larger negativity is consistent with activation of arithmetic information (Niedeggen et al., 1999) for strategy combination. Significant ERP differences in the 850–1,400 ms window between five-rule violation and parity-rule violation problems suggest that rule-violation checking mechanisms were slower when problems violated only one rule, possibly because problem features automatically activated more useful features to combine both parity- and five-rule violation strategies.

Analyses of ERPs revealed that differences between one- and two-rule violation problems as well as between five-rule and parity-rule had left hemisphere origin. This result is consistent with previous work demonstrating language implication in mental calculation (Dehaene et al., 1999; Grabner & De Smedt, 2011). The larger activity in anterior sites of the scalp is interesting, given the link between frontal regions and executive functions (Kane & Engle, 2002). Likewise, studies have indicated that the prefrontal cortex is involved in arithmetic tasks (e.g., Menon et al., 2000; Stanesco-Cosson et al., 2000; Zago & Tzourio-Mazoyer, 2002).

Theoretically, our results have implications for how models of strategy selection (e.g., Lovett & Anderson's, 1996 ACT-R model; Lovett & Schunn's, 1999 RCCL model; Payne et al.'s, 1993 Adaptive Decision Maker model; Rieskamp & Otto's, 2006 SSL model; or Siegler & Araya's, 2005 SCADS* model) need to take into account the possible combination of several rule-checking strategies while accomplishing cognitive tasks. Such strategy combination may be used in many domains, other than arithmetic, where strategic variations have been documented like episodic memory (Dunlosky & Hertzog, 2001), verbal reasoning (Carpenter et al., 2000), and information search in decision making (e.g., Johnson, 1990). For example, in episodic memory, when participants encode paired associates, they could

use both interactive imagery and verbal encoding. In sentence-picture verification tasks, people could apply verbal reasoning by combining a linguistic and a pictorial strategy. In decision making, people could combine a compensatory attribute comparison and a noncompensatory alternative comparison strategy. Future empirical research will test the generality of such strategy combination, its benefits, as well as its conditions of occurrence; and future computational modeling will help specify the underlying mechanisms.

Appendix

List of stimuli

Table 1 True five problems. *SD* Single digit, *DD* double digit

Odd * Odd		Odd* Even	Even * Odd
SD * DD	DD*SD	SD * DD	DD*SD
5 × 21 = 105	17 × 5 = 85	5 × 14 = 70	12 × 5 = 60
5 × 23 = 115	19 × 5 = 95	5 × 18 = 90	14 × 5 = 70
5 × 27 = 135	21 × 5 = 105	5 × 24 = 120	16 × 5 = 80
5 × 29 = 145	23 × 5 = 115	5 × 28 = 140	18 × 5 = 90
5 × 43 = 215	31 × 5 = 155	5 × 32 = 160	32 × 5 = 160
5 × 47 = 235	37 × 5 = 185	5 × 36 = 180	34 × 5 = 170
5 × 61 = 305	41 × 5 = 205	5 × 64 = 320	36 × 5 = 180
5 × 63 = 315	43 × 5 = 215	5 × 68 = 340	38 × 5 = 190
5 × 67 = 335	47 × 5 = 235	5 × 72 = 360	46 × 5 = 230
5 × 69 = 345	49 × 5 = 245	5 × 74 = 370	48 × 5 = 240
5 × 81 = 405	61 × 5 = 305	5 × 82 = 410	72 × 5 = 360
5 × 83 = 415	63 × 5 = 315	5 × 84 = 420	74 × 5 = 370
5 × 87 = 435	73 × 5 = 365	5 × 92 = 460	76 × 5 = 380
5 × 89 = 445	79 × 5 = 395	5 × 94 = 470	78 × 5 = 390
5 × 93 = 465	87 × 5 = 435	5 × 96 = 480	92 × 5 = 460
5 × 97 = 485	89 × 5 = 445	5 × 98 = 490	96 × 5 = 480
5 × 13 = 65	13 × 5 = 65	5 × 12 = 60	24 × 5 = 120
5 × 17 = 85	27 × 5 = 135	5 × 16 = 80	26 × 5 = 130
5 × 19 = 95	29 × 5 = 145	5 × 26 = 130	28 × 5 = 140
5 × 31 = 155	39 × 5 = 195	5 × 34 = 170	42 × 5 = 210
5 × 37 = 185	67 × 5 = 335	5 × 38 = 190	62 × 5 = 310
5 × 39 = 195	69 × 5 = 345	5 × 42 = 210	64 × 5 = 320
5 × 41 = 205	71 × 5 = 355	5 × 46 = 230	68 × 5 = 340
5 × 49 = 245	81 × 5 = 405	5 × 48 = 240	82 × 5 = 410
5 × 71 = 355	83 × 5 = 415	5 × 62 = 310	84 × 5 = 420
5 × 73 = 365	91 × 5 = 455	5 × 76 = 380	86 × 5 = 430
5 × 79 = 395	93 × 5 = 465	5 × 78 = 390	94 × 5 = 470
5 × 91 = 455	97 × 5 = 485	5 × 86 = 430	98 × 5 = 490

Table 2 False five problems

Odd*Odd		Odd*Even	Even*Odd
SD*DD	DD*SD	SD*DD	DD*SD
Parity-rule violation			
$5 \times 13 = 70$	$17 \times 5 = 90$	$5 \times 12 = 65$	$16 \times 5 = 95$
$5 \times 19 = 110$	$31 \times 5 = 160$	$5 \times 34 = 185$	$24 \times 5 = 105$
$5 \times 23 = 110$	$39 \times 5 = 210$	$5 \times 36 = 185$	$26 \times 5 = 115$
$5 \times 27 = 120$	$61 \times 5 = 320$	$5 \times 46 = 215$	$28 \times 5 = 125$
$5 \times 37 = 180$	$67 \times 5 = 350$	$5 \times 62 = 325$	$42 \times 5 = 195$
$5 \times 43 = 210$	$69 \times 5 = 340$	$5 \times 68 = 355$	$48 \times 5 = 225$
$5 \times 49 = 230$	$71 \times 5 = 350$	$5 \times 72 = 345$	$82 \times 5 = 415$
$5 \times 73 = 380$	$79 \times 5 = 390$	$5 \times 78 = 385$	$84 \times 5 = 425$
$5 \times 81 = 410$	$91 \times 5 = 450$	$5 \times 92 = 445$	$86 \times 5 = 435$
$5 \times 89 = 450$	$93 \times 5 = 480$	$5 \times 98 = 485$	$94 \times 5 = 465$
Five-rule violation			
$5 \times 13 = 69$	$37 \times 5 = 189$	$5 \times 16 = 82$	$12 \times 5 = 62$
$5 \times 17 = 87$	$39 \times 5 = 199$	$5 \times 32 = 162$	$18 \times 5 = 92$
$5 \times 21 = 101$	$63 \times 5 = 317$	$5 \times 38 = 194$	$24 \times 5 = 118$
$5 \times 29 = 143$	$67 \times 5 = 331$	$5 \times 48 = 236$	$26 \times 5 = 128$
$5 \times 31 = 153$	$69 \times 5 = 347$	$5 \times 62 = 312$	$28 \times 5 = 138$
$5 \times 41 = 201$	$71 \times 5 = 353$	$5 \times 64 = 322$	$42 \times 5 = 206$
$5 \times 47 = 233$	$73 \times 5 = 363$	$5 \times 74 = 366$	$46 \times 5 = 226$
$5 \times 79 = 399$	$91 \times 5 = 451$	$5 \times 76 = 378$	$82 \times 5 = 414$
$5 \times 83 = 419$	$93 \times 5 = 461$	$5 \times 94 = 466$	$84 \times 5 = 424$
$5 \times 87 = 439$	$97 \times 5 = 487$	$5 \times 96 = 478$	$86 \times 5 = 434$
Parity- and five-rule violation			
$5 \times 17 = 88$	$13 \times 5 = 68$	$5 \times 14 = 71$	$32 \times 5 = 161$
$5 \times 31 = 158$	$19 \times 5 = 98$	$5 \times 16 = 83$	$34 \times 5 = 171$
$5 \times 39 = 196$	$21 \times 5 = 102$	$5 \times 24 = 119$	$62 \times 5 = 313$
$5 \times 41 = 204$	$27 \times 5 = 132$	$5 \times 26 = 127$	$64 \times 5 = 319$
$5 \times 63 = 318$	$29 \times 5 = 142$	$5 \times 38 = 187$	$68 \times 5 = 343$
$5 \times 67 = 338$	$47 \times 5 = 234$	$5 \times 42 = 209$	$74 \times 5 = 367$
$5 \times 71 = 352$	$49 \times 5 = 244$	$5 \times 48 = 237$	$78 \times 5 = 387$
$5 \times 79 = 394$	$81 \times 5 = 406$	$5 \times 76 = 381$	$94 \times 5 = 473$
$5 \times 91 = 452$	$83 \times 5 = 416$	$5 \times 82 = 411$	$96 \times 5 = 479$
$5 \times 93 = 464$	$87 \times 5 = 436$	$5 \times 86 = 431$	$98 \times 5 = 489$
No-rule violation			
$5 \times 19 = 85$	$13 \times 5 = 45$	$5 \times 12 = 50$	$14 \times 5 = 60$
$5 \times 37 = 165$	$23 \times 5 = 135$	$5 \times 18 = 70$	$36 \times 5 = 160$
$5 \times 39 = 185$	$27 \times 5 = 155$	$5 \times 26 = 140$	$38 \times 5 = 170$
$5 \times 49 = 265$	$29 \times 5 = 165$	$5 \times 28 = 160$	$62 \times 5 = 290$
$5 \times 61 = 285$	$41 \times 5 = 225$	$5 \times 34 = 180$	$64 \times 5 = 310$
$5 \times 69 = 325$	$43 \times 5 = 235$	$5 \times 42 = 220$	$68 \times 5 = 350$
$5 \times 71 = 375$	$81 \times 5 = 395$	$5 \times 46 = 250$	$72 \times 5 = 370$
$5 \times 73 = 375$	$83 \times 5 = 405$	$5 \times 78 = 370$	$76 \times 5 = 390$
$5 \times 91 = 465$	$89 \times 5 = 435$	$5 \times 84 = 410$	$92 \times 5 = 470$
$5 \times 97 = 505$	$97 \times 5 = 495$	$5 \times 86 = 420$	$98 \times 5 = 470$

Table 3 True non-five problems

SD*DD	DD*SD	SD*DD	DD*SD
Odd*Odd		Even*Odd	
$3 \times 17 = 51$	$19 \times 3 = 57$	$4 \times 21 = 84$	$14 \times 7 = 98$
$3 \times 21 = 63$	$23 \times 7 = 161$	$4 \times 27 = 108$	$16 \times 3 = 48$
$3 \times 23 = 69$	$27 \times 3 = 81$	$4 \times 39 = 156$	$26 \times 7 = 182$
$3 \times 37 = 111$	$29 \times 7 = 203$	$4 \times 61 = 244$	$28 \times 3 = 84$
$3 \times 39 = 117$	$31 \times 7 = 217$	$4 \times 67 = 268$	$34 \times 3 = 102$
$3 \times 61 = 183$	$43 \times 7 = 301$	$4 \times 83 = 332$	$34 \times 7 = 238$
$3 \times 67 = 201$	$47 \times 3 = 141$	$4 \times 97 = 388$	$38 \times 7 = 266$
$3 \times 79 = 237$	$49 \times 7 = 343$	$6 \times 17 = 102$	$42 \times 7 = 294$
$3 \times 89 = 267$	$61 \times 7 = 427$	$6 \times 23 = 138$	$46 \times 3 = 138$
$3 \times 97 = 291$	$63 \times 3 = 189$	$6 \times 37 = 222$	$48 \times 7 = 336$
$7 \times 17 = 119$	$69 \times 3 = 207$	$6 \times 49 = 294$	$62 \times 3 = 186$
$7 \times 21 = 147$	$69 \times 7 = 483$	$6 \times 63 = 378$	$68 \times 7 = 476$
$7 \times 27 = 189$	$71 \times 7 = 497$	$6 \times 79 = 474$	$72 \times 3 = 216$
$7 \times 37 = 259$	$73 \times 3 = 219$	$6 \times 91 = 546$	$72 \times 7 = 504$
$7 \times 47 = 329$	$79 \times 7 = 553$	$8 \times 13 = 104$	$76 \times 3 = 228$
$7 \times 63 = 441$	$81 \times 3 = 243$	$8 \times 21 = 168$	$78 \times 3 = 234$
$7 \times 67 = 469$	$83 \times 3 = 249$	$8 \times 29 = 232$	$82 \times 7 = 574$
$7 \times 73 = 511$	$89 \times 7 = 623$	$8 \times 47 = 376$	$84 \times 7 = 588$
$7 \times 81 = 567$	$91 \times 3 = 273$	$8 \times 63 = 504$	$86 \times 3 = 258$
$7 \times 91 = 637$	$93 \times 3 = 279$	$8 \times 73 = 584$	$94 \times 3 = 282$
Odd*Even		Even*Even	
$3 \times 18 = 54$	$13 \times 4 = 52$	$4 \times 26 = 104$	$14 \times 6 = 84$
$3 \times 26 = 78$	$13 \times 6 = 78$	$4 \times 36 = 144$	$14 \times 8 = 112$
$3 \times 38 = 114$	$17 \times 8 = 136$	$4 \times 62 = 248$	$16 \times 6 = 96$
$3 \times 48 = 144$	$21 \times 6 = 126$	$4 \times 64 = 256$	$16 \times 8 = 128$
$3 \times 64 = 192$	$23 \times 8 = 184$	$4 \times 82 = 328$	$18 \times 8 = 144$
$3 \times 68 = 204$	$29 \times 4 = 116$	$4 \times 92 = 368$	$24 \times 4 = 96$
$3 \times 74 = 222$	$31 \times 8 = 248$	$6 \times 18 = 108$	$28 \times 4 = 112$
$3 \times 82 = 246$	$37 \times 4 = 148$	$6 \times 24 = 144$	$28 \times 8 = 224$
$3 \times 84 = 252$	$39 \times 6 = 234$	$6 \times 46 = 276$	$32 \times 6 = 192$
$3 \times 98 = 294$	$47 \times 6 = 282$	$6 \times 48 = 288$	$32 \times 8 = 256$
$7 \times 12 = 84$	$49 \times 4 = 196$	$6 \times 74 = 444$	$34 \times 6 = 204$
$7 \times 24 = 168$	$49 \times 8 = 392$	$6 \times 78 = 468$	$36 \times 6 = 216$
$7 \times 32 = 224$	$61 \times 6 = 366$	$6 \times 98 = 588$	$42 \times 8 = 336$
$7 \times 36 = 252$	$61 \times 8 = 488$	$8 \times 12 = 96$	$46 \times 4 = 184$
$7 \times 46 = 322$	$63 \times 4 = 252$	$8 \times 38 = 304$	$48 \times 4 = 192$
$7 \times 62 = 434$	$67 \times 8 = 536$	$8 \times 46 = 368$	$62 \times 6 = 372$
$7 \times 64 = 448$	$71 \times 6 = 426$	$8 \times 48 = 384$	$64 \times 8 = 512$
$7 \times 78 = 546$	$79 \times 4 = 316$	$8 \times 62 = 496$	$68 \times 4 = 272$
$7 \times 86 = 602$	$79 \times 8 = 632$	$8 \times 68 = 544$	$76 \times 4 = 304$
$7 \times 92 = 644$	$81 \times 4 = 324$	$8 \times 82 = 656$	$78 \times 4 = 312$

Table 4 False non-five problems

Odd*Odd		Odd*Even	
SD*DD	DD*SD	SD*DD	DD*SD
Parity-rule violation			
7 × 17 = 116	23 × 7 = 162	7 × 12 = 93	13 × 6 = 71
7 × 21 = 150	29 × 7 = 202	7 × 24 = 159	29 × 4 = 123
7 × 27 = 190	47 × 3 = 150	7 × 32 = 221	21 × 6 = 123
7 × 37 = 258	43 × 7 = 292	7 × 36 = 245	47 × 6 = 285
7 × 47 = 338	49 × 7 = 336	7 × 46 = 329	49 × 4 = 197
7 × 63 = 434	71 × 7 = 504	7 × 62 = 437	49 × 8 = 391
7 × 67 = 460	61 × 7 = 424	7 × 64 = 449	71 × 6 = 435
7 × 73 = 518	69 × 7 = 486	7 × 78 = 545	79 × 4 = 307
7 × 81 = 564	79 × 7 = 554	7 × 86 = 611	61 × 8 = 481
7 × 91 = 640	89 × 7 = 622	7 × 92 = 635	79 × 8 = 639
No parity-rule violation			
3 × 17 = 37	19 × 3 = 77	3 × 18 = 50	13 × 4 = 50
3 × 21 = 77	27 × 3 = 61	3 × 26 = 82	17 × 8 = 138
3 × 23 = 89	31 × 7 = 213	3 × 38 = 112	23 × 8 = 170
3 × 37 = 91	63 × 3 = 193	3 × 48 = 146	37 × 4 = 162
3 × 39 = 113	69 × 3 = 205	3 × 64 = 178	31 × 8 = 268
3 × 61 = 187	73 × 3 = 221	3 × 68 = 218	39 × 6 = 214
3 × 67 = 199	83 × 3 = 235	3 × 74 = 242	63 × 4 = 248
3 × 79 = 239	81 × 3 = 257	3 × 82 = 226	67 × 8 = 540
3 × 89 = 253	91 × 3 = 293	3 × 84 = 248	61 × 6 = 364
3 × 97 = 305	93 × 3 = 259	3 × 98 = 298	81 × 4 = 326
Even*Odd		Even*Even	
SD*DD	DD*SD	SD*DD	DD*SD
Parity-rule violation			
6 × 17 = 103	14 × 7 = 107	8 × 12 = 89	16 × 6 = 93
4 × 27 = 107	26 × 7 = 173	6 × 24 = 151	46 × 4 = 187
8 × 21 = 177	34 × 3 = 95	4 × 36 = 141	34 × 6 = 205
6 × 37 = 213	42 × 7 = 301	8 × 46 = 371	14 × 6 = 83
4 × 61 = 237	48 × 7 = 333	6 × 46 = 277	28 × 4 = 121
8 × 47 = 383	34 × 7 = 241	4 × 62 = 247	42 × 8 = 327
6 × 63 = 375	68 × 7 = 477	8 × 48 = 393	36 × 6 = 209
4 × 83 = 335	72 × 7 = 503	6 × 78 = 459	78 × 4 = 319
8 × 73 = 585	82 × 7 = 583	4 × 92 = 361	16 × 8 = 125
6 × 91 = 545	84 × 7 = 579	8 × 82 = 663	32 × 8 = 259
No parity-rule violation			
4 × 21 = 104	16 × 3 = 44	6 × 18 = 106	24 × 4 = 82
8 × 13 = 84	28 × 3 = 88	4 × 26 = 106	18 × 8 = 158
6 × 23 = 134	38 × 7 = 264	8 × 38 = 290	28 × 8 = 244
4 × 39 = 160	46 × 3 = 140	6 × 48 = 302	48 × 4 = 172
8 × 29 = 230	62 × 3 = 172	4 × 64 = 276	14 × 8 = 108
6 × 49 = 296	78 × 3 = 248	8 × 68 = 524	32 × 6 = 196
4 × 67 = 254	72 × 3 = 236	6 × 74 = 440	76 × 4 = 302
8 × 63 = 518	76 × 3 = 208	4 × 82 = 332	64 × 8 = 514
6 × 79 = 494	86 × 3 = 254	8 × 62 = 494	62 × 6 = 358
4 × 97 = 368	94 × 3 = 286	6 × 98 = 590	68 × 4 = 286

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