HAL
open science

High frequency variations of the earth's instantaneous angular velocity vector : determination from VLBI data analysis

S. Bolotin, Christian Bizouard, Sylvain Loyer, Nicole Capitaine

To cite this version:

S. Bolotin, Christian Bizouard, Sylvain Loyer, Nicole Capitaine. High frequency variations of the earth's instantaneous angular velocity vector : determination from VLBI data analysis. Astronomy and Astrophysics - A&A, 1997, 317, pp.601-609. hal-03801666

HAL Id: hal-03801666

https://hal.science/hal-03801666v1

Submitted on 8 Oct 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Astron. Astrophys. 317, 601–609 (1997)

ASTRONOMY AND ASTROPHYSICS

High frequency variations of the Earth's instantaneous angular velocity vector

Determination from VLBI data analysis

S. Bolotin[⋆]**, C. Bizouard, S. Loyer, and N. Capitaine**

Observatoire de Paris, avenue de l'Observatoire, 61, F-75014 Paris, France (e-mail: name@mesiob.obspm.fr.)

Received 31 July 1995 / Accepted 6 June 1996

Abstract. A new determination of the Earth's orientation is proposed by estimating only 3 infinitesimal angles. This method is applied to the determination of the diurnal and subdiurnal variations in the Earth's orientation from the VLBI campaign from 11 to 24 January 1994. It has been implemented in the $SteelBreeze$ software for VLBI data processing at the Main Astronomical Observatory of the Ukraine. The high-frequency variations of the instantaneous rotation vector are then calculated and compared with the Herring and Dong model for these variations after being referred to the instantaneous rotation axis.

Key words: astrometry – Earth – ephemerides

1. Introduction

The classical way of determining the Earth orientation from Very Long Base Line Interferometry (VLBI) observations is to estimate 5 parameters, two for the motion of the Celestial Ephemeris Pole (CEP) within the Earth (x, y), one for the Earth's angle of rotation and two associated with the motion of the CEP in space, as for instance corrections to the nutation angles with respect to a model in the J2000.0 equatorial frame. However VLBI now provides data with very short temporal resolution, up to 15 minutes. In such estimations, the definition of the CEP as given by the the IAU (Lieske et al. 1977), becomes unclear because the distinction between high frequency polar motion and nutation is no longer possible (Brzezinski & Capitaine 1993). Therefore the physical interpretation of the Earth's orientation parameters (EOP) referred to the CEP becomes very difficult. However the ambiguity could be partly removed. This leads to a "classical" way for estimating high frequency Earth's orientation which is described in Sect. 2.

Send offprint requests to: Ch. Bizouard

⋆ *Present address:* National Ukrainian Academy of Science, Main Astronomical Observatory, Kiev, Ukraine
e-mail: bolotin@mao.gluk.apc.org

It is clear that in such VLBI data processing, one determines diurnal and subdiurnal perturbations of the global orientation of the terrestrial frame with respect to the celestial frame. Let Q be the matrix for the coordinate transformation from the TRS (Terrestrial Reference System) to the CRS (Celestial Reference System). Q includes a known part Q_{ref} and an unknown part. To estimate the classical EOP, one can directly estimate an infinitesimal rotation A corresponding to this unknown part of Q. A simple way also allows us to calculate the instantaneous rotation vector from matrix Q, and then to determine the corrections of this vector, $\boldsymbol{\omega}$, which in the terrestrial frame is given by:

$$\boldsymbol{\omega} = \Omega\,[m_1\hat{x} + m_2\hat{y} + (1 + m_3)\hat{z}]\,. \tag{1}$$

The parameters of this axis have a clear physical interpretation, whereas it is not the case of the estimated orientation parameters. The theoretical and conceptual aspects of this method are presented in Sect. 3. In Sect. 4 we describe the VLBI data used in the estimation. Sect. 5 is devoted to the implementation of this method in the VLBI data processing software $SteelBreeze$ developed at the Main Astronomical Observatory of Ukraine. Then an analysis of the VLBI data obtained during the VLBI campaign of January 1994 has been utilized. This has been performed with the parameters of the infinitesimal rotation A. The results (infinitesimal angles, deduced instantaneous angular velocity and classical estimations) are presented in Sect. 6 and a comparison with the empirical model of Herring and Dong (1994) is presented.

2. Use of the classical Earth's Orientation Parameters

The *realised CEP* is defined through the computed components of the mean geographic axis Gz in the CRS as given by the IAU precession-nutation model (Lieske et al. 1977). Actually it contains the long periodic forced motions of the Gz axis (longer than 4.5 days) induced by the variable tidal forces of the Moon, and Sun on the equatorial bulge of the Earth, calculated by Wahr (1981) from the rigid Earth's nutation series provided by (Kinoshita 1977). The computed parameters of the CEP are

the precession angles, the nutations in longitude and obliquity $(\Delta\psi, \Delta\varepsilon)$ or its coordinates (X, Y) in the CRS.

We have to distinguish the realised CEP from the CEP provided by astrometry, itself independant of dynamical theories. The difference between this realised CEP and the CEP is specified by the "celestial pole offset" $(\delta X, \delta Y)$, measurable by astrometric observations. The spatial orientation of the Gz axis can thus be obtained from the realised CEP by adding the celestial pole offsets, and the complementary terms which are conventionally expressed in the TRF. These are the terrestrial coordinates $p = x - iy$ of the CEP. From a physical point of view, the offset between the CEP and the Gz axis has two sources:

- a deficient model of the forced motions of the Gz axis induced by the gravitional influence of the external bodies (planetary effects neglected, gravitationally induced deformation mismodeled, deficient model for the Earth's interior);
- a motion of the Gz axis induced by pure geophysical processes.

In order to define clearly the CEP we have to decide which components of this offset belong to the celestial pole offsets, the remainder corresponding to the polar motion p.

The astrometric analyses are referred to the CEP through the matrix transformation Q between the CRS and the TRF. Two forms of Q are used, the classical one given by:

$$Q = P\ N(\Delta\psi + \delta\psi, \Delta\varepsilon + \delta\varepsilon) R(GST) W(x, y)\ ,$$

and the "non-classical" one referred to the non-rotating origine (Capitaine 1986) given by:

$$Q = \widetilde{PN}(X + \delta X, Y + \delta Y) \tilde{R}(\Omega UT1) \tilde{W}(x, y).$$

In both cases PN is associated with the precession-nutation model and the celestial pole offsets, R to the angle of rotation around the CEP and W to the terrestrial position of the CEP.

Current VLBI analysis allows us to estimate daily corrections $(\delta X, \delta Y)$, $UT1 - UTC$ and (x, y). These daily estimations provide both corrections to the precession-nutation model for long periodic terms (greater than 2 days), to the Earth rotation angle $(UT1 - UTC)$, and to the long periodic components of (x, y), called polhody, that is to say on the spatial diurnal oscillation of the Gz axis.

We can split the offset between the realised CEP and the Gz axis into prograde and retrograde circular motions at a given frequency (complex Fourier transform) and then describe it with respect to two frequency bands:

- $F_1^{CRS} = [-0.5\ \Omega, 0.5\ \Omega]$ containing all its long periodic component in the CRS (greater than 2 days);
- $F_2^{CRS} = [-\infty, -0.5\ \Omega] \bigcup [0.5\ \Omega, \infty]$ containing its diurnal and subdiurnal components

where Ω is the standard Earth's angular velocity.

From a terrestrial point of view, all the band F_2^{CRS} has to be shifted by the diurnal angular velocity and gives the frequency band $F_2^{TRS} = [-\infty, -1.5\ \Omega] \bigcup [-0.5\ \Omega, \infty]$. The classical astrometric analysis are thus referred to a CEP where celestial pole offsets belong to the band F_1^{CRS}, and polar motion contains only the long periodic terms of F_2^{TRS}.

In the present analysis, the VLBI observations are very dense and the former parameters can be obtained continuously in time (around 15 minutes between two successive and independant estimations). But we have to be aware that simultaneous estimations of the celestial pole offsets and the polar motion would be quite impossible to interpret because the orientation's corrections in the band F_2^{CRS} would be split arbitrarily into celestial pole offsets and polar motion.

In order to analyse correctly the dense VLBI observations, we have to start from the most precise spatial position in the band F_1^{CRS}, that is to take into account both the IAU nutation model and the celestial pole offsets as given for instance by the IERS (International Earth Rotation Service) series. These celestial pole offsets can be obtained, for instance, from current VLBI observations (daily estimation). Then the residual orientation of the Earth will correspond only to terrestrial motion of the CEP in band F_2^{TRS} and variations in $UT1$. Only three parameters are therefore corrected: $UT1$, x and y. In fact, a small error remains in the spatial orientation of the Gz axis in the band F_1^{CRS} because of the error on the celestial pole offset $(\delta X + i\delta Y$ of the order of 0.1 mas). In our estimation, this kind of offset will appear, with the same amplitude, as a retrograde motion δp^r in the estimated polar motion . The link between $\delta X + i\delta Y$ and δp^r is given, to first order by:

$$\delta p^r \simeq (\delta X + i\delta Y)\ e^{-i\Omega UT},$$

where UT is the universal time.

The use of the classical parameters referred to the CEP, is therefore possible as it has been done by Herring and Dong (1994). However the involved separation between spatial and terrestrial motion of the Gz axis is artificial. Thus the physical interpretation of the EOP referred to it may lead to inevitable ambiguities. This is the reason for referring to the instantaneous axis of rotation, as it is developed in the next sections.

3. New parameters for estimating high frequency variations of Earth Rotation

3.1. Estimation of an infinitesimal rotation

Classical VLBI data reduction consists of estimating corrections to the parameters of the matrix Q such as $[CRS] = Q[TRS]$. The other way implies the direct fitting of an infinitesimal matrix A to be applied to Q. Let Q_{ref} be the reference coordinate transformation and A be defined by the matrix product:

$$A = R_x(\alpha_1) R_y(\alpha_2) R_z(\alpha_3) \tag{2}$$

where α_1, α_2 and α_3 are angle smaller than 10 mas.

The matrix $Q_{\text{ref}} A$ has to minimise the $O - C$ according to the least-squares criterion. This method for estimating the Earth's orientation is independent of the way of expressing Q_{ref}.

In the present case, only the corrections δx, δy, and $\delta UT1$ are estimated, as explained in the previous section. These can be identified to the first order with the estimated infinitesimal

angles. Indeed the total matrix transformation including corrections can be written to first order:

$$\begin{aligned} Q &= PN(X_{\rm ref}, Y_{\rm ref})R_3(-\Omega(UT1_{\rm ref} + \delta UT1)) \\ &\quad R_1(y_{\rm ref} + \delta y)R_2(x_{\rm ref} + \delta x) \\ &= PN(X_{\rm ref}, Y_{\rm ref})R_3(-\Omega\, UT1_{\rm ref})R_1(y_{\rm ref})R_2(x_{\rm ref}) \\ &\quad R_1(\delta x)R_2(\delta y)R_3(-\Omega\, \delta UT1)\ . \end{aligned} \tag{3}$$

It appears immediately that:

$$\begin{aligned} \alpha_1 &= \delta y\ , \\ \alpha_2 &= \delta x\ , \\ \alpha_3 &= -\Omega\, \delta UT1\ . \end{aligned} \tag{4}$$

3.2. Determination of the instantaneous vector of rotation

The components of the instantaneous rotation vector can be calculated from the product of matrices $\dot{Q}$ and Q^{-1} where Q is the coordinate transformation from the TRS to the CRS. Indeed one shows very easily that the components of $\boldsymbol{\omega}$ in the terrestrial frame ($Oxyz$) are given by the antisymmetric matrix (Kinoshita et al. 1979):

$$Q^{-1}\dot{Q} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}\ . \tag{5}$$

In the inertial frame ($OXYZ$), the components of the instantaneous rotation vector are given by the inverse product:

$$\dot{Q}Q^{-1} = \begin{bmatrix} 0 & -\omega_Z & \omega_Y \\ \omega_Z & 0 & -\omega_X \\ -\omega_Y & \omega_X & 0 \end{bmatrix}\ . \tag{6}$$

The coordinates of the instantaneous pole of rotation are then given by $m_1 = \omega_x/\Omega, m_2 = \omega_y/\Omega, 1 + m_3 = \omega_z/\Omega$ in the terrestrial frame and $X = \omega_X/\Omega, Y = \omega_Y/\Omega, \omega_Z/\Omega$ in the CRS.

We wish only to derive the components of spatial orientation of the TRS belonging to the frequency band $[-\infty, -1.5\ \Omega] \bigcup [0.5\ \Omega, \infty]$. $Q_{\rm ref}$ has to contain the precession-nutation model and the set of the celestial pole offsets, polhode and angle of rotation as estimated by current observations. Therefore we use the coordinate transformation associated with the IAU model of nutation as well as the IERS previous values for the corrections to this model and for polar motion $p = x - iy$. The matrix A is expected to contain all the remaining motion, and its angles can then be used to derive the corrections to the instantaneous rotation vector.

Substracting the matrix of the reference angular velocity $Q_{\rm ref}^{-1}\dot{Q}_{\rm ref}$ from the matrix of total angular velocity in the TRS (see Eq. 5), one gets the corrections to $\boldsymbol{\omega}$ in the TRS:

$$\begin{aligned} \begin{bmatrix} 0 & -\delta\omega_z & \delta\omega_y \\ \delta\omega_z & 0 & -\delta\omega_x \\ -\delta\omega_y & \delta\omega_x & 0 \end{bmatrix} &= A^{-1}Q_{\rm ref}^{-1}\dot{Q}_{\rm ref}A \\ &\quad + A^{-1}\dot{A} - Q_{\rm ref}^{-1}\dot{Q}_{\rm ref}\ . \end{aligned} \tag{7}$$

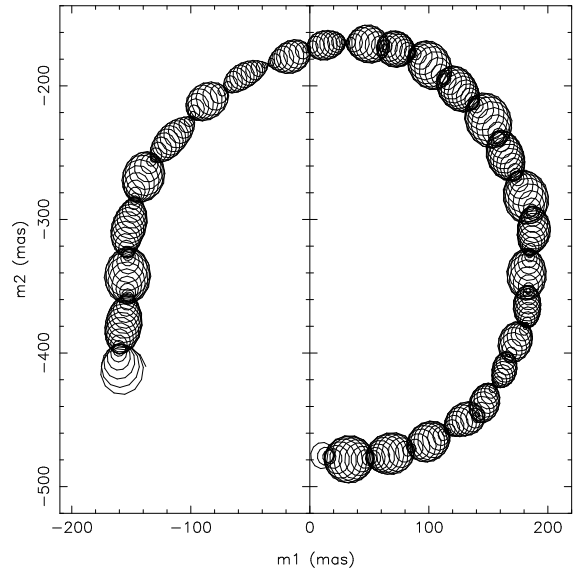

Fig. 1. Motion of the instantaneous axis of rotation in the TRS from 1/01/94 (extremity at the right) to 31/12/94 (extremity at the left) (as deduced from the IAU nutation model and the IERS combined series EOP (IERS) 90 C 04 (celestial pole offsets and polar motion). The polhody and the diurnal nutations provide a retrograde epicycloïdal path modulated with a period of 13.66 days. Our work has provided the diurnal and subdiurnal corrections to this motion.

From Eq. (6) one gets the corrections to $\boldsymbol{\omega}$ in the CRS:

$$\begin{bmatrix} 0 & -\delta\omega_Z & \delta\omega_Y \\ \delta\omega_Z & 0 & -\delta\omega_X \\ -\delta\omega_Y & \delta\omega_X & 0 \end{bmatrix} = Q_{\rm ref}\dot{A}A^{-1}Q_{\rm ref}^{-1}\ . \tag{8}$$

3.3. Terrestrial motion of the instantaneous rotation vector and link with the estimated polar motion

Let (Ψ_f, θ_f) be the Euler angles of the Gz axis in the J2000.0 equatorial frame. The kinematic Euler equations allow us to determine $m(t)$ from the spatial motion of Gz:

$$\dot{\theta_f} + i\dot{\Psi}_f \sin\theta = -m\Omega\, e^{i\Phi} \tag{9}$$

where $m = m_1 + im_2$ and $\Phi \simeq \Omega\, UT$ is the Earth rotation angle in space and UT is the universal time.

First, the long-periodic spatial motion of the Gz axis (frequency band F_1^{CRS}) induces the so-called *diurnal nutations* composed of retrograde quasi-diurnal motions of the instantaneous rotation axis R in the terrestrial frame (frequency band $R_1^{TRS} = [-1.5\ \Omega, -0.5\ \Omega]$). The sum of these motions, corresponding in the CRS to the *Oppolzer terms*, has an amplitude reaching 13 mas and a modulation of 13.66 days (see Fig. 1).

Secondly, the diurnal and subdiurnal components of the Gz axis (frequency band F_2^{CRS}) give motions of R belonging to the band $R_2^{TRS} = [-\infty, -1.5\ \Omega] \bigcup [-0.5\ \Omega, \infty]$ in the TRS. They include long periodic motion (polhode), prograde quasi-diurnal and subdiurnal components.

In summary, the position of $\boldsymbol{w}$ in the Earth appears as the combination of the polhody ($0.1 - 0.5$"), the retrograde diurnal

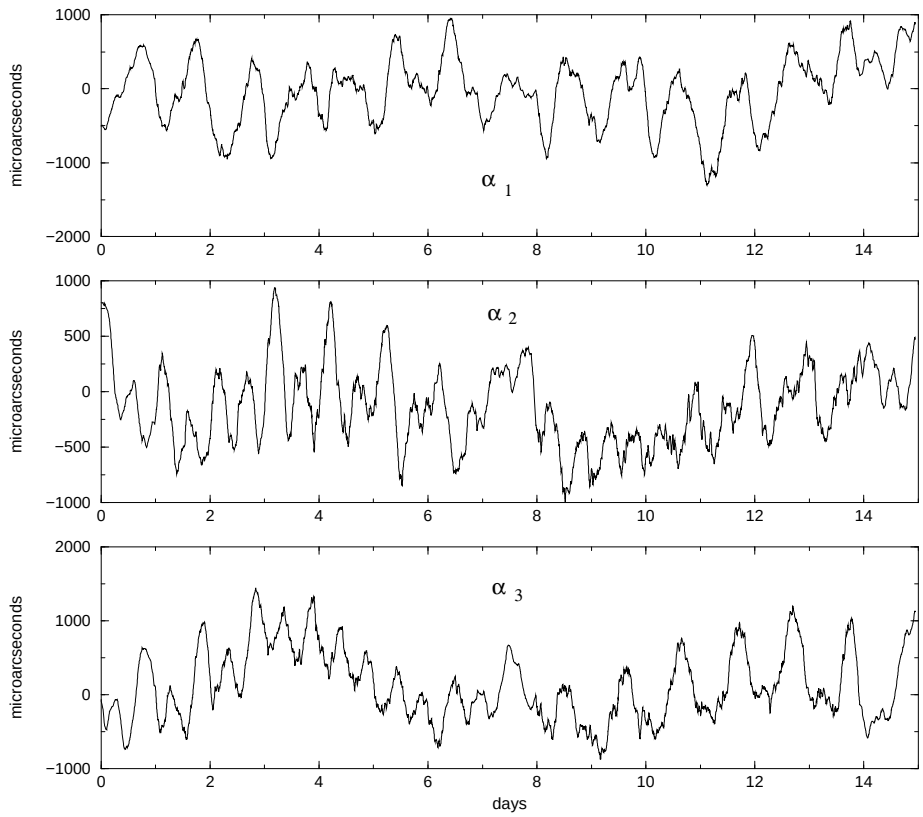

Fig. 2. Infinitesimal angles α_1, α_2 and α_3.

nutations (0-13 mas), and the prograde diurnal and subdiurnal components of m.

Let us determine now the contribution of the estimated polar motion on the terrestrial motion of the instantaneous rotation axis. Considering the Eq. (7), and replacing A by its expression as a function of α_1, α_2, and α_3 and the product $Q_{\rm ref}^{-1}\dot{Q}_{\rm ref}$ by its expression as a function of $\omega_x^{\rm ref}$, $\omega_y^{\rm ref}$, $\omega_z^{\rm ref}$, one obtains:

$$\begin{aligned}\delta\omega_x &= \alpha_2\omega_z^{\rm ref} - \alpha_3\omega_y^{\rm ref} - \dot{\alpha_1}\ ,\\ \delta\omega_y &= -\alpha_1\omega_z^{\rm ref} + \alpha_3\omega_x^{\rm ref} - \dot{\alpha_2}\ ,\\ \delta\omega_z &= \alpha_1\omega_y^{\rm ref} - \alpha_2\omega_x^{\rm ref} - \dot{\alpha_3}\ .\end{aligned} \tag{10}$$

As the products $\alpha_3\omega_y^{\rm ref}$, $\alpha_3\omega_x^{\rm ref}$, $\alpha_2\omega_x^{\rm ref}$, $\alpha_1\omega_y^{\rm ref}$ are second order terms, we have:

$$\begin{aligned}\delta\omega_x &= \alpha_2\omega_z^{\rm ref} - \dot{\alpha_1}\ ,\\ \delta\omega_y &= -\alpha_1\omega_z^{\rm ref} - \dot{\alpha_2}\ ,\\ \delta\omega_z &= -\dot{\alpha_3}\ .\end{aligned} \tag{11}$$

From Eq. (4) we have to first order:

$$\begin{aligned}\delta\omega_x &= \delta x\ \omega_z^{\rm ref} - \delta\dot{y}\ ,\\ \delta\omega_y &= -\delta y\ \omega_z^{\rm ref} - \delta\dot{x}\ ,\\ \delta\omega_z &= \Omega\ \delta U\dot{T}1\ .\end{aligned} \tag{12}$$

The former equation holds for any *a priori* value of $UT1$, x, and y. Thus the corrections to these parameters can designate the complete additional part on the rotation angle $\Omega\,UT$, and the complete polar motion $p = x - iy$. Looking then at the two first components, these appear as the complex equation:

$$\delta m = \delta p - \frac{i}{\Omega}\delta\dot{p}\ , \tag{13}$$

which provides the contribution δm of the estimated polar motion δp to the terrestrial motion m of the instantaneous rotation axis. This relation has been already given by Brzezinski and Capitaine (1993).

Taking the Fourier transform of this equation, we obtain the relation between the frequency components of the estimated polar motion ($P(\sigma)$) and those of the associated motion for the instantaneous rotation axis ($M(\sigma)$):

$$M(\sigma) = (1 + \frac{\sigma}{\Omega})P(\sigma)\ . \tag{14}$$

Looking at each significant frequency band in the TRS, resulting from Eq. (14):

- For the retrograde quasi-diurnal band ($\sigma = -\Omega + \lambda', \lambda' \ll \Omega$), we have $M = \dfrac{\lambda'}{\Omega}P$. This shows that an estimated retrograde component δp^r in the TRS has to be interpreted as long periodic precession-nutation of the Gz axis.
- For the long periods ($|\sigma| \ll \Omega$) the motions of the CEP and R are both of the order of 1 mas. Indeed if we consider a Chandlerian motion having an amplitude of 0".5 (the maximum), the difference reaches 1 mas. By contrast to precession-nutation, the polhode (prograde quasi-diurnal sway of Gz axis in space) is widely unpredictable.
- For the prograde quasi-diurnal band($\sigma = \Omega + \lambda', \lambda' \ll \Omega$), we have $M \simeq 2\ P$.
- For the prograde quasi-semi-diurnal band ($\sigma = 2\Omega + \lambda', \lambda' \ll \Omega$), we have $M \simeq 3\ P$.
- For the retrograde quasi-semi-diurnal band ($\sigma = -2\Omega + \lambda', \lambda' \ll \Omega$), we have $M \simeq -\ P$.

In order to derive the corrections δm to the position of the instantaneous rotation axis from the estimated polar motion δp, we have to use the Eq. (13).

4. Description of the VLBI data used in the estimation

The extensive VLBI/GPS campaign in January 1994 was conducted by NASA GSFC VLBI group in order to compare Earth orientation results (hourly and daily values) derived by VLBI networks and the IGS network. It was assumed that the VLBI stations would produce nearly continuous series of observations.

There were up to 24 VLBI stations that made observations simultaneously in three separate networks (NASA-R&D, VLBA and NAVEX-G). We have processed observations of one of these networks (NASA-R&D). There were 7 stations: Fairbanks (Alaska), Kokee Park (Hawaii), Westford (Massachusets), Los Alamos VLBA (New Mexico), Wettzell (Germany), Onsala (Sweden) and Fort Davis VLBA (Texas) observing 41 radio sources. The NASA-R&D observations were conducted from 12 January to 25 January and there is a one-day gap in observations on 18/19 January. Due to this gap, the session 18 January from the NEOS-A network was added to the NASA-R&D observations. The sessions 11 January and 25 January from the NEOS-A network were added to get a longer history of observations. These sessions utilized 5 stations: Fairbanks (Alaska), NRAO85-3, Green Bank (West Virginia), Fortaleza (Brazil), Wettzell (Germany), Kokee Park (Hawaii) and 28 radio sources. Overall we have 15 days of continuous observations with 9 stations and 49 radio sources.

5. Description of the numerical computations

5.1. Models and a priori values

To obtain the instantaneous axis of the Earth's rotation we have used the software SteelBreeze 1.01 developed at the main astronomical Observatory of Ukraine. This software is based on the IERS Standards (1992) and the SRIF-technique described in Biermann (1977).

The ITRF92 (solution SSC(IERS)93C01) with its velocity field and ICRF92 (solution RSC(IERS)93C01) were used as a realization of the Terrestrial Reference frame and Celestial Reference Frame respectively. The IAU 1980 Nutation model and the initial values of EOP(IERS)90C04 were used for matrix Q_{ref}. Q_{ref} has been calculated using the matrix representation linked to the non-rotating origin (see *e.g.* Capitaine, 1986, 1990).

Tropospheric refraction in the local zenith direction caused by hydrostatic and water vapour components of the neutral atmosphere was modelled according to Saastamoinen (1972). The zenith delays were mapped to line of sight elevations with the MTT mapping function of Herring (1992) for both hydrostatic and wet components. The ionospheric effect was eliminated by the use of simultaneous dual frequency observations. Calibration data collected at each observing site were applied to correct for variations in the electrical length of the cables between the radio receivers and the data acquision systems.

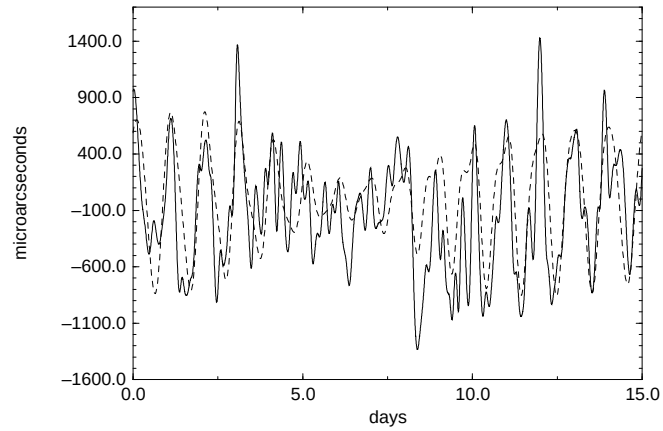

Fig. 3. High frequency motion of the instantaneous rotation axis: m_1; filtered observations (full line) and Herring and Dong model transformed to the instantaneous axis of rotation (dashed line).

5.2. Numerical method for the estimation of the parameters

The solution is divided into two steps.

In the first step, the stations' coordinates have been treated as global parameters. The clock model and the wet component of tropospheric delay for each observing site have been treated as stochastic parameters. We have modelled the stochastic process as a random walk with the following magnitudes of power spectral density (PSD) for white noise:

- for clock model: 18^2 ps^2/hour;

- for wet zenith delay: 1.0 cm^2/hour; together with the zenith delay and clock offset for each session. The estimated station coordinates for 12-25 January 1994 are given in Table 1.

The second step has been performed for the three angles of additive rotation (α_1, α_2 and α_3). They have been calculated with the same models, the same *a priori* values and the new solution for station coordinates.

The clock model and the wet component of tropospheric delay for each observing site have been treated as stochastic process with the following magnitudes of PSD for white noise:

- for clock model: 18^2 ps^2/hour;

- for wet zenith delay: 1.0 cm^2/hour.

The additive rotation angles have been treated as a stochastic process with the following magnitude of PSD for white noise:

- for α_1, α_2 and α_3: 0.18^2 mas^2/hour.

The clock parameter and wet zenith delay at each observing site have been reset as soon as a new observing session was started. Therefore continuity has not been assumed in the behaviour of these parameters at the boundaries of sessions. The α_1, α_2 and α_3 parameters have not been reset and these parameters have been processed as a continuous series for the whole interval of observations.

Altogether 38,355 group delays with quality factor 0 only have been processed for each case. The final weighted rms post-fit residuals are 11.45 ps. The results for the estimated parameters α_1, α_2, and α_3 in mas are shown in Fig. 2.

Table 1. Solution for station coordinates.

Station name	x-coordinate (m)	y-coordinate (m)	z-coordinate (m)	Uncert. (cm) x	y	z	Number of observations
FD-VLBA	-1324009.05279	-5332181.99337	3231962.50658	0.24	0.26	0.42	11692
FORTLEZA*	4985370.04994	-3955020.27960	-428472.34803	0.53	0.45	0.33	730
GILCREEK*	-2281547.22439	-1453645.08744	5756993.19394	0.20	0.26	0.41	12689
KOKEE	-5543837.57133	-2054568.14914	2387851.80394	0.26	0.46	0.69	9272
LA-VLBA	-1449752.28587	-4975298.62897	3709123.96912	0.23	0.25	0.41	11467
NRAO85 3	882325.63925	-4925137.99850	3943397.68722	0.26	0.50	0.46	755
ONSALA60*	3370606.05775	711917.44648	5349830.69628	0.18	0.17	0.23	8586
WESTFORD*	1492206.67025	-4458130.51290	4296015.54609	0.22	0.16	0.27	11072
WETTZELL*	4075539.91945	931735.21647	4801629.32473	0.19	0.17	0.26	10362

* station coordinates have been used to link new TRF to the ITRF92 with an a priori standard deviation $0.5cm$

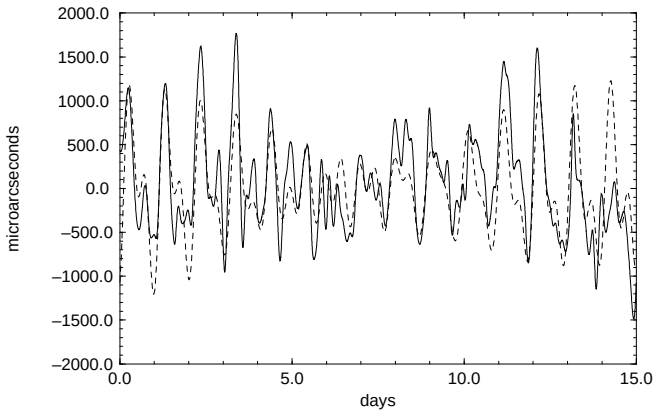

Fig. 4. High frequency motion of the instantaneous rotation axis: m_2; filtered observations (full line) and Herring and Dong model transformed to the instantaneous axis of rotation (dashed line).

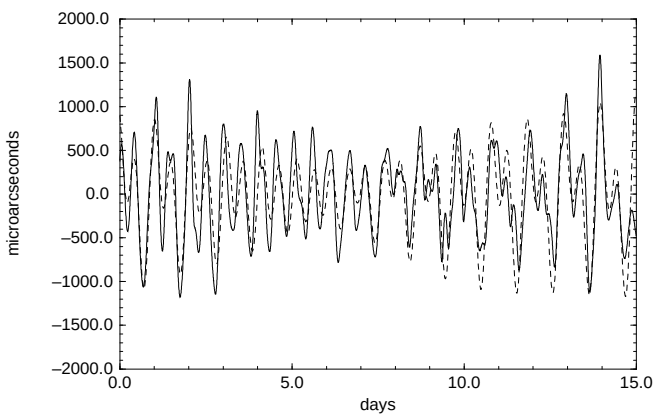

Fig. 5. High frequency motion of the instantaneous angular velocity: m_3; filtered observations (full line) and Herring and Dong model transformed to the instantaneous angular velocity (dashed line).

5.3. Computations of the parameters of the instantaneous vector of rotation

In order to compute the corrections to the coordinates of the instantaneous axis of rotation, we begin to estimate the derivatives $\dot{\alpha}_k$ of the infinitesimal angles taking:

$$\dot{\alpha}_k(t_i) = \frac{\alpha_k(t_{i+1}) - \alpha_k(t_i)}{t_{i+1} - t_i}. \tag{15}$$

These values, together with the α_k values are then used in Eq. (7) in order to obtain the corrections $\delta\boldsymbol{\omega}$ in the TRS. The $\dot{\alpha}_k$, and thus the $\delta\omega_k$ exhibit some large high-frequency noise as expected after the derivation step. This is removed by applying an appropriate filter. We use a Vondrak filter (Vondrak 1977) with the half response period of the filter set at 0.25 days (for 0.57 days 99% of signal remains). With this filter semi-diurnal periods are very weakly affected and the diurnal ones not at all. The high-frequency noise is greatly removed. The filtered results are presented in Figs. 3, 4, and 5. The phenomenom appears modulated. This means there are several frequencies close to 1 cpsd (cycles per sideral day) as indicated by the theory (effect of the ocean tides in the diurnal and semi-diurnal bands as predicted in (Brosche et al. 1989).

Finally we have calculated the total motion of the instantaneous rotation axis within the Earth with a temporal resolution of 6 hours. The resulting trajectory is shown in Fig. 7, together with its reference.

6. Comparison with the empirical model of Herring and Dong

Herring and Dong (1994) have estimated from VLBI observations spreading over two years the amplitudes of 11 diurnal and 11 semi-diurnal waves both in polar motion and in UT1. Their work provides an empirical model for the diurnal and subdiurnal variations of these parameters. The most likely explanation for the largest part of the prograde diurnal and semi-diurnal variations of the CEP is the ocean tides (Gross 1993).

The parametrization of these effects as given by Brosche et al. (1989) is the following. A general tide argument j is given by:

$$\phi_j = a_j l + b_j l' + c_j F + d_j D + e_j \Omega + f_j(GST + \pi) \ , \tag{16}$$

where a_j, b_j, c_j, d_j, e_j are integer constants which multiply the conventional arguments of the Moon and the Sun, and f_j is an integer specifying the frequency band of the tide ($f_j = 1, -2, 2$ for prograde diurnal, retrograde semi-diurnal, and prograde semi-diurnal waves respectively). The prograde diurnal and semi-diurnal variations in the rotation of the Earth can then be ex-

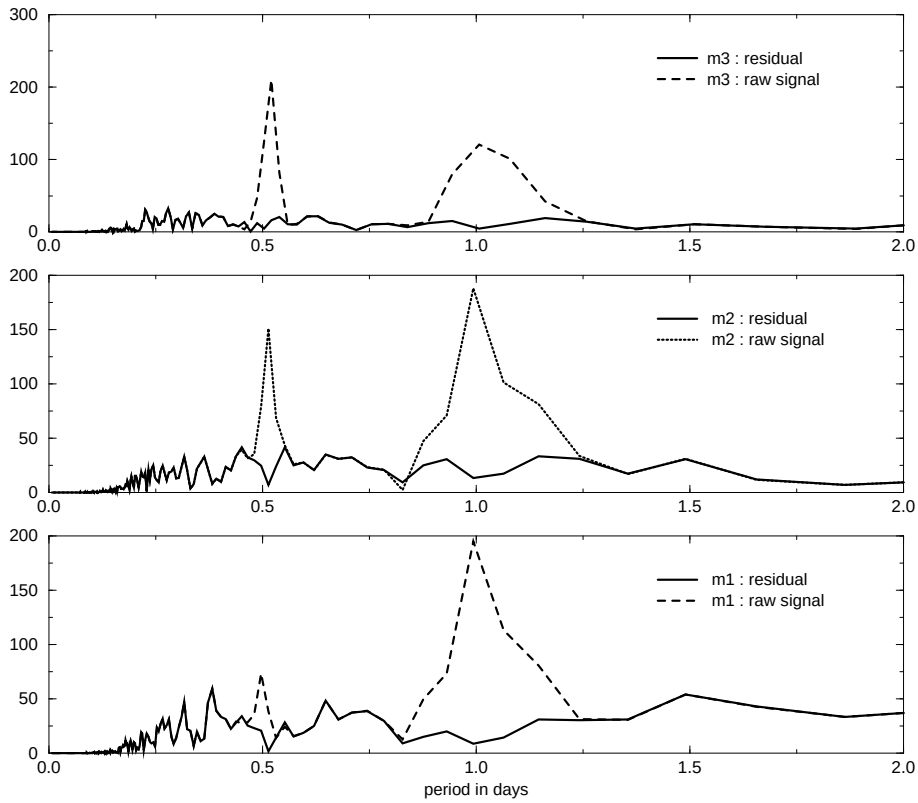

Fig. 6. Spectrum of the residuals after estimation (full line) and of the raw signals (dashed line).

pressed as:

$$\delta UT1 = \sum_j u_j^c \cos\phi_j + u_j^s \sin\phi_j \ , \tag{17}$$

$$\delta p = \delta x - i\delta y = \sum_j -(p_j^c + ip_j^s)e^{i\phi_j} \ , \tag{18}$$

where $\delta UT1$, δx, δy are the tidally coherent variations in UT1 and polar motion and the index j loops over N terms in both the diurnal and semi-diurnal bands.

Using the relation (13) together with Eq. (17) allows us to calculate the corresponding corrections to the instantaneous angular velocity vector:

$$\delta m_3 = \sum_j m_{3j}^c \cos\phi_j + m_{3j}^s \sin\phi_j \ , \tag{19}$$

$$\delta m = \delta m_1 + i\delta m_2 = \sum_j (m_j^c + im_j^s)e^{i\phi_j} \ , \tag{20}$$

with

$$m_{3j}^c = -\dot{\phi}_j u_j^s \ , \tag{21}$$

$$m_{3j}^s = \dot{\phi}_j u_j^c \ , \tag{22}$$

$$m_j^c + im_j^s = -(1 + \frac{\dot{\phi}_j}{\Omega})(p_j^c + ip_j^s) \ . \tag{23}$$

Using the relations (19) and (21), we have estimated the parameters m_j^c, m_j^s, m_{3j}^c and m_{3j}^s for our results minus the model of Herring and Dong transformed to the instantaneous axis of rotation. In Figs. 3, 4, and 5, we have also plotted the corrections δm_1, δm_2 and δm_3 provided by Herring and Dong model. Due to the shortness of our dataset (around fifteen days), we could not estimate all the diurnal and semi-diurnal waves. We have chosen to estimate one wave for each of the following major group of tides:

- $O1$, $K1(S1, P1)$, $\eta 1(OO1)$ for the diurnal band,
- $M2(N2)$, $K2(S2)$ for the semi-diurnal band.

The results for the prograde diurnal and semi-diurnal bands, together with Herring and Dong model transformed as described above are listed in Table 2. We have also estimated retrograde diurnal waves, which give an estimation of the corrections on the *a priori* diurnal nutations (or precession-nutation) of the instantaneous rotation axis. Our estimations show discrepancies with respect to Herring and Dong waves up to 100 μas for a few waves whereas the standard deviation of these estimates are better than 5 μas. A spectrum of the residuals, displayed in Fig. 6, shows that the major part of the raw signal in the diurnal and semidiurnal band has been removed by Herring and Dong model and our additional corrections. The root mean square of the residuals is about 55% of the rms of the raw signal where it is 65% for the raw signals minus Herring and Dong model (see Table 3). So the diurnal and semidiurnal waves have been removed, but the global rms remains high. Three reasons can be put forward: the shortness of our dataset does not enable estimation of the corrections to the tidal displacements of the VLBI sites, non-geophysical noise, and irregular geophysical effects at short time scales, which estimation of the tidal coherent waves cannot display.

Table 2. Herring and Dong model for high frequency polar motion and rotation angle transformed to instantaneous rotation vector and additional corrections estimated from the VLBI campaign of January 1994.

name	Arguments						Herring and Dong model *				Additional corrections *			
	l	l'	F	D	Ω	$GST+\pi$	m_j^c	m_j^s	m_{3j}^c	m_{3j}^s	m_j^c	m_j^s	m_{3j}^c	m_{3j}^s
$Q1$	-1	0	-2	0	-2	1	62	-21	-	-	-	-	-	-
$O1$	0	0	-2	0	-2	1	343	-172	-	-	117±3	-18±3	-	-
$M1$	-1	0	0	0	0	1	-8	-6	-	-	-	-	-	-
$P1$	0	0	-2	2	-2	1	98	-70	-	-	-	-	-	-
$S1$	0	-1	0	0	0	1	-44	-16	-	-	-	-	-	-
$K1$	0	0	0	0	0	1	-266	148	-	-	74±4	87±4	-	-
$\Psi1$	0	1	0	0	0	1	16	-14	-	-	-	-	-	-
$\Phi1$	0	0	2	-2	2	1	-32	-10	-	-	-	-	-	-
$J1$	1	0	0	0	0	1	41	-18	-	-	-	-	-	-
$OO1$	0	0	2	0	2	1	-29	35	-	-	-	-	-	-
$\eta1$	1	0	2	0	2	1	8	38	-	-	71±4	-50±4	-	-
$Q1$	1	0	2	0	2	-1	-	-	-57.6	41.5	-	-	-	-
$O1$	0	0	2	0	2	-1	-	-	-224.5	241.2	-6±3	1±3	68±4	-28±4
$M1$	1	0	0	0	0	-1	-	-	33.3	-26.1	-	-	-	-
$P1$	0	0	2	-2	2	-1	-	-	-88.3	58.3	-	-	-	-
$S1$	0	1	0	0	0	-1	-	-	19.5	-27.0	-	-	-	-
$K1$	0	0	0	0	0	-1	-	-	267.7	-97.8	17±4	-23±4	-3±4	89±4
$\Psi1$	0	-1	0	0	0	-1	-	-	4.5	-22.6	-	-	-	-
$\Phi1$	0	0	-2	2	-2	-1	-	-	-10.6	-37.8	-	-	-	-
$J1$	-1	0	0	0	0	-1	-	-	34.3	-1.6	-	-	-	-
$OO1$	0	0	-2	0	-2	-1	-	-	-3.2	-19.4	-	-	-	-
$\eta1$	-1	0	-2	0	-2	-1	-	-	-8.3	25.0	8±4	4±4	-8±4	-61±4
	-1	0	-2	0	-2	-2	-2	-12	12.7	-12.7	-1±4	-43±4	-28±4	54±4
	0	0	-2	0	-2	-2	-8	4	-9.4	-6.2	-	-	-	-
	-1	0	0	0	0	-2	1	-2	9.2	3.1	-	-	-	-
	0	0	-2	2	-2	-2	2	-10	9.0	-6.0	-	-	-	-
	0	-1	0	0	0	-2	6	4	3.0	9.0	-	-	-	-
$K2$	0	0	0	0	0	-2	-26	16	111.3	-24.1	63±4	-47±4	-85±4	3±4
$R2$	0	1	0	0	0	-2	-5	9	18.0	24.0	-	-	-	-
$S2$	0	0	2	-2	2	-2	-67	98	258.0	3.0	-	-	-	-
	1	0	0	0	0	-2	-5	-4	0.0	11.8	-	-	-	-
$M2$	0	0	2	0	2	-2	-9	246	414.5	313.0	-7±3	45±3	17±4	-7±4
$N2$	1	0	2	0	2	-2	-9	42	79.6	45.5	-	-	-	-
$N2$	-1	0	-2	0	-2	2	-35	35	-	-	-	-	-	-
$M2$	0	0	-2	0	-2	2	-3	170	-	-	13±3	-39±3	-	-
	-1	0	0	0	0	2	36	0	-	-	-	-	-	-
$S2$	0	0	-2	2	-2	2	-6	36	-	-	-	-	-	-
$R2$	0	-1	0	0	0	2	36	-21	-	-	-	-	-	-
$K2$	0	0	0	0	0	2	-117	15	-	-	-28±4	3±4	-	-
	0	1	0	0	0	2	39	-6	-	-	-	-	-	-
	0	0	2	-2	2	2	6	-18	-	-	-	-	-	-
	1	0	0	0	0	2	-15	-18	-	-	-	-	-	-
	0	0	2	0	2	2	-3	0	-	-	-	-	-	-
	1	0	2	0	2	2	-12	-40	-	-	4±4	13±4	-	-

* All the given values are in μas

7. Conclusion

In this paper a clarification has been given concerning the classical Earth orientation parameters for high frequency variations of the Earth's rotation. Three infinitesimal rotation angles have been chosen to estimate such variations during the VLBI campaign from 11 to 24 January 1994 treated by the *SteelBreeze* software developed at the Main Observatory of the Ukraine. Corresponding variations of the instantaneous axis of rotation have been derived up to 6 hours. These variations contain mainly prograde diurnal waves and are globally consistent with the empirical model of Herring and Dong (1994) transformed to the instantaneous angular velocity vector. Corrections to the tidally coherent waves of this model have been also estimated.

This study has shown the possibility for calculating the diurnal and subdiurnal components of the instantaneous rotation vector

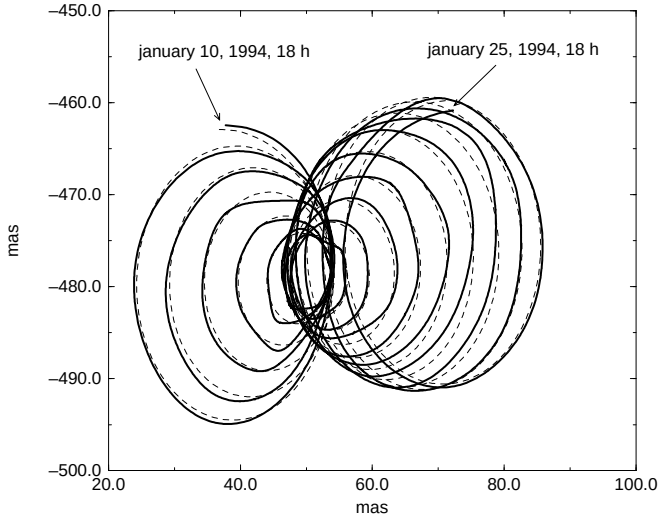

Fig. 7. Trajectory of the instantaneous rotation axis with respect to the terrestrial crust from the 10th to the 25th of January 1994 (full line) with a temporal resolution of 6 hours the reference motion (dashed line) has been deduced from the precession-nutation model IAU 1980 and the series EOP (IERS)90 04.

Table 3. Root mean squares in μas: (a) raw signals; (b) raw signals minus Herring and Dong model; (c) residuals after estimation

	(a)	(b)	(c)
m_1	468	319	266
m_2	505	300	262
m_3	485	230	195

of the Earth from VLBI observations. Unless the determination of this vector is not direct and is limited by the high frequency noise, it allows us a straightforward geophysical interpretation of the observations.

Acknowledgements. For a part of this work, one of us (S. Bolotin) has been supported by a long-term visitor support at the Observatory of Paris. For another part, he was partly supported by the International Science Foundation under grant $U4S000$.

References

Biermann, G.J., 1977, Factorization Methods for Discrete Sequential Estimation, V128, Mathematics in Science and Engineering Series, Academic Press

Brosche, P., Seiler, U., Sünderman, J., Wünsch, 1989, A&A, 220, 318-320

Brzezinski, A., Capitaine, N. 1993, J. Geophys. Res., 98, B4, 6667-6675.

Capitaine, N. 1986, A&A, 162, 323-329.

Capitaine, N. 1990, Celes. Mech. and Dyn. Ast., 48, 127-143.

Gross, R.S. 1993, Geophys. Res. Lett., 20, 293-296.

Herring, T.A. 1992, Proceedings of the symposium on Refraction of Transatmospheric Signals in Geodesy, ed. by J.C. De Munch and T.A.Th. Spoelstra, Netherland Geodetic Commision, Delft, Netherlands, Publicatioms on Geodesy, 36, 157-164.

Herring, T.A., Dong, D. 1994, J. Geophys. Res., 99, 18051-18071.

International Earth Rotation Service 1992, IERS Standards, IERS Technical Note 13, Ed. Dr. McCarthy, Paris, France.

Kinoshita, H. 1977, Celest. Mech., 15, 277-326.

Kinoshita, H., Nakajima, K., Kubo, Y., et al. 1979, Publ. Int. Latitude Obs. Mizusawa, 12, 71-108.

Lieske, J. H., Lederle, T., Fricke, W., & Morando, B. 1977, A&A, 58, 1

Saastamoinen, J., The Use of Artificial Satellite for Geodesy, Geophys. Monogr. Ser. 15 (S.W. Henriksen et al., eds.), AGU, Washington, D.C., 247-251.

Vondrak, J. 1977, Bull. Astron. Inst. Czech., 28, 84-89.

Wahr, J.M. 1981, Geophys. J.R. Astron. Soc., 64, 705-727.

This article was processed by the author using Springer-Verlag LaTeX A&A style file *L-AA* version 3.