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Control-Oriented Modeling of a Planar Cable-Driven Parallel Robot with Non-Straight Cables



Abstract

The purpose of this work is the control-oriented modeling of cable-driven parallel manipulators when the cables are not considered as straight but show transverse vibrations due to fast movements of the platform. The model has been derived from Lagrange equations with multipliers in order to deal with the kinematic constraints (DAE model). The number of generalized coordinates have been reduced by performing a transformation of the DAE model to an ODE model. The ODE model has been linearized and an \mathcal{H}_∞ controller has been synthesized to control the cable tension and enable the platform to track a reference trajectory. The results in detailed have been published in [Saadaoui et al., 2022].

Model (2) can be recast to obtain the nonlinear differential-algebraic equations (DAE)

$$\underbrace{\begin{bmatrix} M(q) & -A(q)^T \\ A(q) & \mathbb{O} \end{bmatrix}}_V \underbrace{\begin{bmatrix} \ddot{q} \\ \lambda \end{bmatrix}}_F = \underbrace{\begin{bmatrix} C(q, \dot{q}) + Q(q) - \dot{M}(q, \dot{q}) \dot{q} \\ -\dot{A}(q, \dot{q}) \dot{q} \end{bmatrix}}_F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}. \quad (3)$$

Two approaches of linearization of the DAE have been followed to obtain a linear model suitable for control:

- A1. The left-hand side matrix in the model (3) can be inverted in order to reveal a second order ODE model $\ddot{q} = f(q, \dot{q}, u)$ depending on $3n + 3$ state variables and linearized at the point $x_e = q_e, \dot{q}_e, u_e$.

$$\begin{bmatrix} \delta \ddot{q} \\ \delta \lambda \end{bmatrix} = \underbrace{\frac{\partial(V^{-1}F)}{\partial q}}_{A_1} \bigg|_{x_e} \delta q + \underbrace{\frac{\partial(V^{-1}F)}{\partial \dot{q}}}_{A_2} \bigg|_{x_e} \delta \dot{q} + \underbrace{\frac{\partial(V^{-1}F)}{\partial u}}_{B_1} \bigg|_{x_e} \delta u \quad (4)$$

where u are the inputs of the robot (are the winder's torques).

The state space representation is given:

$$\begin{bmatrix} \dot{q} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbb{O} & \mathbb{I} \\ A_{1 \times k} & A_{2 \times k} \end{bmatrix} \begin{bmatrix} q \\ \lambda \end{bmatrix} + \begin{bmatrix} \mathbb{O}_{k \times n} \\ B_{1 \times n} \end{bmatrix} u. \quad (k = 3n + 3) \quad (5)$$

- A2. The model (3) can be directly linearized as a linear descriptor model ([Duan, 2010]) at the point $x_e = q_e, \dot{q}_e, \lambda_e, u_e$.

$$\underbrace{M}_{M_0} \bigg|_{x_e} \delta \ddot{q} = \underbrace{\frac{\partial F_1}{\partial q}}_{A_1} \bigg|_{x_e} \delta q + \underbrace{\frac{\partial F_1}{\partial \dot{q}}}_{A_2} \bigg|_{x_e} \delta \dot{q} + \underbrace{\frac{\partial F_1}{\partial \lambda}}_{A_3} \bigg|_{x_e} \delta \lambda + \underbrace{\frac{\partial F_1}{\partial u}}_{B_1} \bigg|_{x_e} \delta u. \quad (6)$$

$$\underbrace{A}_{A_0} \bigg|_{x_e} \delta \ddot{q} = \underbrace{\frac{\partial F_2}{\partial q}}_{A_4} \bigg|_{x_e} \delta q + \underbrace{\frac{\partial F_2}{\partial \dot{q}}}_{A_5} \bigg|_{x_e} \delta \dot{q}. \quad (7)$$

The state space representation is given:

$$\begin{bmatrix} \mathbb{I} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & M_0 & \mathbb{O} \\ \mathbb{O} & A_0 & \mathbb{O} \end{bmatrix} \begin{bmatrix} \dot{q} \\ \ddot{q} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbb{O} & \mathbb{I} & \mathbb{O} \\ A_1 & A_2 & A_3 \\ A_4 & A_5 & \mathbb{O} \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \\ \lambda \end{bmatrix} + \begin{bmatrix} \mathbb{O} \\ \mathbb{O} \\ B_1 \end{bmatrix} u. \quad (8)$$

The obtained linear models in (5) and (8) were found non-controllable and non-observable due to the variables dependency.

Ordinary differential equation model (ODE)

An ODE model has been derived by transforming the original parametrization q to a reduced set of parameters $q_r = [w_1 \dots w_n \ x_p \ y_p \ \alpha]^T$ by using the geometric constraints that express some variables as functions of others. This leads to a model with $n + 3$ independent variables as:

$$\ddot{q}_r = M_r^{-1} (C_r + Q_r - \dot{M}_r \dot{q}_r) = \Gamma_r. \quad (9)$$

The model has been linearized at the point

$$x_{r_e} = q_{r_e}, \dot{q}_{r_e}, u_{r_e}.$$

$$\delta \ddot{q}_r = \underbrace{\frac{\partial \Gamma_r}{\partial q_r}}_{A_{r1}} \bigg|_{x_{r_e}} \delta q_r + \underbrace{\frac{\partial \Gamma_r}{\partial \dot{q}_r}}_{A_{r2}} \bigg|_{x_{r_e}} \delta \dot{q}_r + \underbrace{\frac{\partial \Gamma_r}{\partial u}}_{B_{r1}} \bigg|_{x_{r_e}} \delta u. \quad (10)$$

The state space representation is given:

$$\begin{bmatrix} \dot{q}_r \\ \ddot{q}_r \end{bmatrix} = \begin{bmatrix} \mathbb{O} & \mathbb{I} \\ A_{r1} & A_{r2} \end{bmatrix} \begin{bmatrix} q_r \\ \dot{q}_r \end{bmatrix} + \begin{bmatrix} \mathbb{O} \\ B_{r1} \end{bmatrix} u. \quad (11)$$

The ODE linear model was found controllable and observable.

\mathcal{H}_∞ synthesis

The \mathcal{H}_∞ control approach (see Figure 2) aims to find a controller $K(s)$ which makes the closed-loop system stable and minimizes the \mathcal{H}_∞ norm of the transfer function $T_{zv}(K)$ as:

$$K(s) \min_{\text{stabilize}} \|T_{zv}(K)\|_\infty. \quad (12)$$

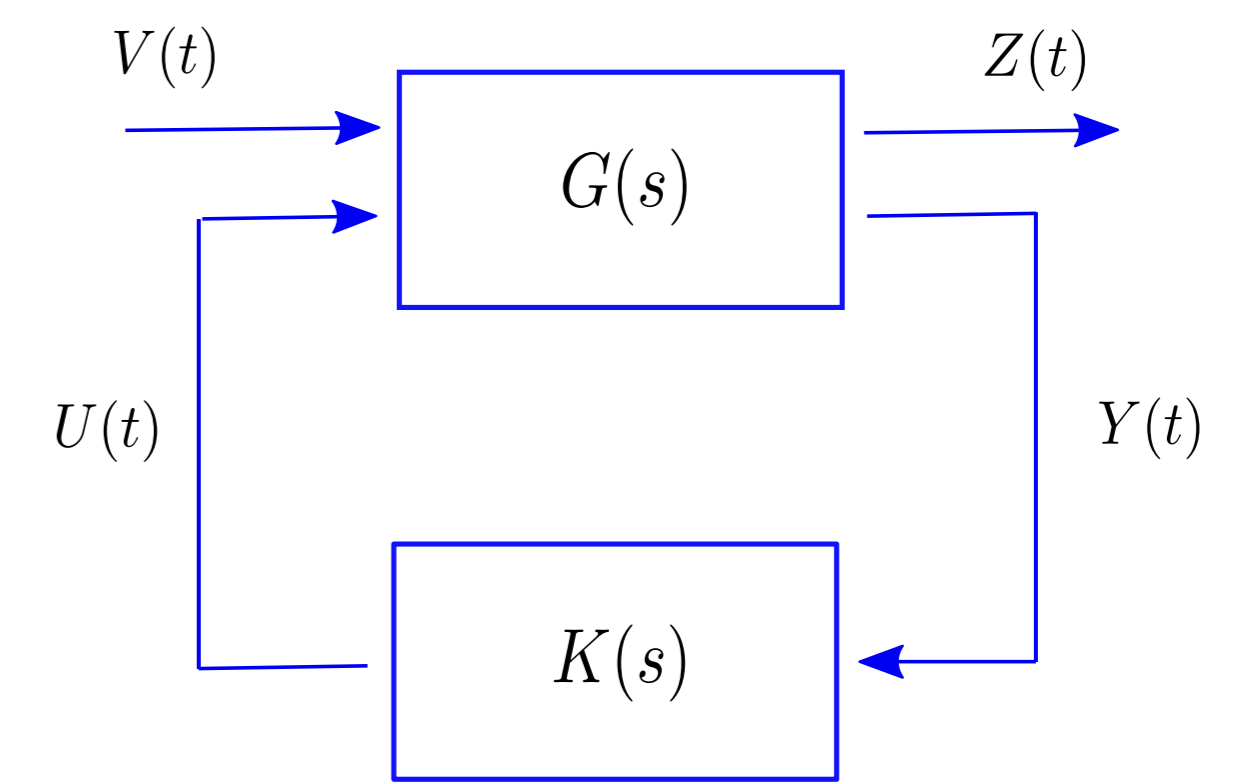


Figure 2: \mathcal{H}_∞ control schema.

Simulation and results

- The linear ODE model has been implemented to synthesize an \mathcal{H}_∞ controller to control the cables mean tension and the platform position and rotation (x_p, y_p, α) .
- The results in Figure 3 and Figure 4 show a good trajectories tracking and disturbance rejection.

Overview of cable-driven parallel robots (CDPRs)

CDPRs are robots that have a moving platform connected to the ground via n several cables. Winding and unwinding of the cables allows the displacement of the platform over large workspaces with possibly a high dynamics (see Figure 1).

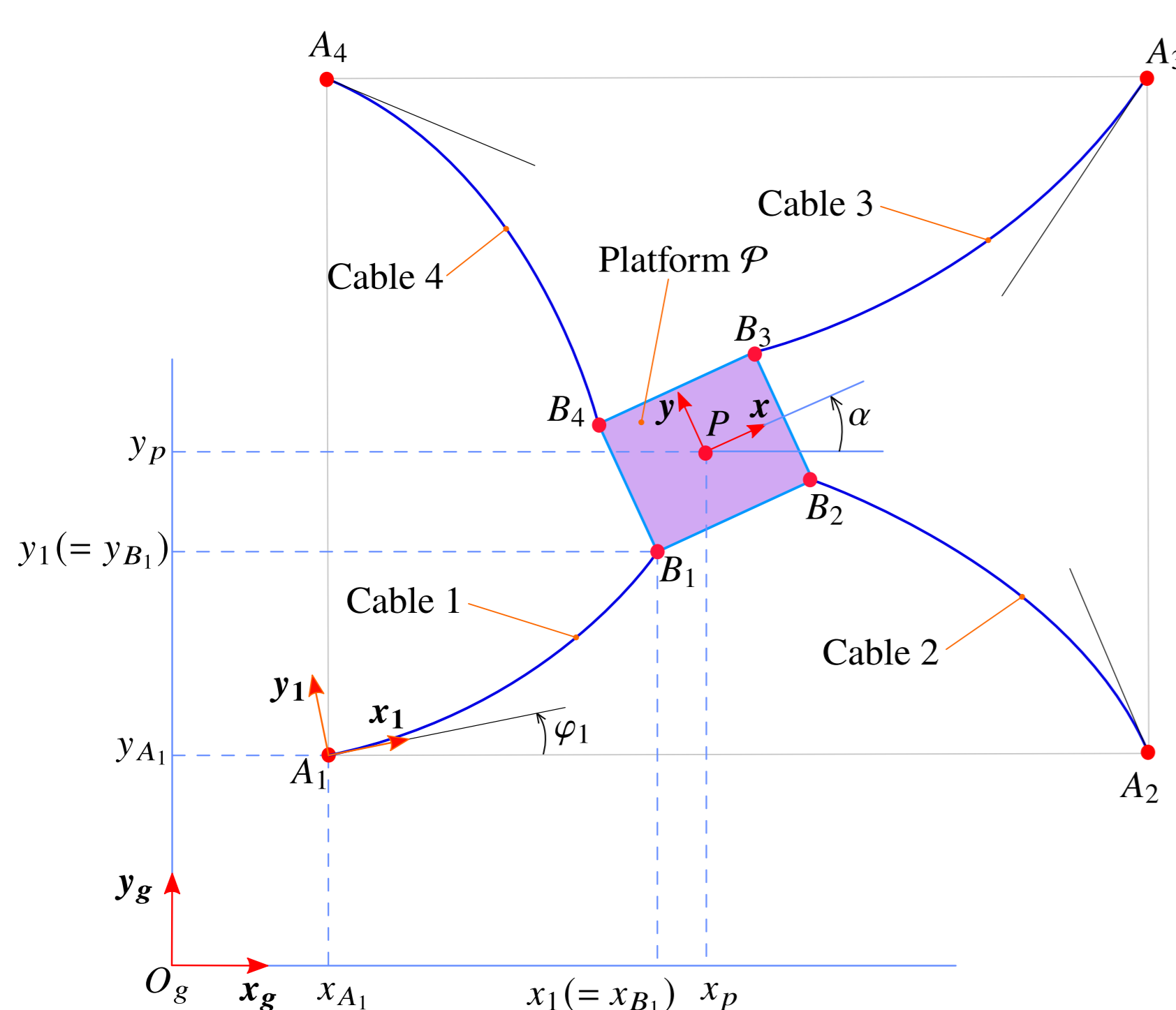


Figure 1. Schematic representation of a planar four cable robot.

Modeling approach

Parameterization and cables deformation

The platform configuration (x_p, y_p, α) and the variables describing each cables (length l_i , flexible mode w_i , orientation φ_i) are gathered in $q = [l_1 \dots l_n \ w_1 \dots w_n \ \varphi_1 \dots \varphi_n \ x_p \ y_p \ \alpha]^T$ that forms the generalized coordinate vector of the system.

Cables are flexible elements which deform during the movement. flexibilities are modeled by a combination of polynomial functions using the method of assumed modes [Meirovitch, 1967] as :

$$Y(x, t) = \sum_{j=1}^N \phi_j(x) w_{ji}(t). \quad (i = 1, \dots, n) \quad (1)$$

with N is the number of flexible modes w_i and $x \in [0, l_i]$.

Differential-algebraic model (DAE)

The dynamic model was obtained using Lagrange formalism [Goldstein et al., 2002] as:

$$\underbrace{M \ddot{q} + \dot{M} \dot{q}}_{\frac{d}{dt} \frac{\partial T}{\partial \dot{q}}} = \underbrace{C}_{\frac{\partial T}{\partial q}} + Q(q) + A^T \lambda. \quad (2)$$

with q verifying the constraints $\dot{A} \dot{q} = 0$ and T is the kinetic energy.

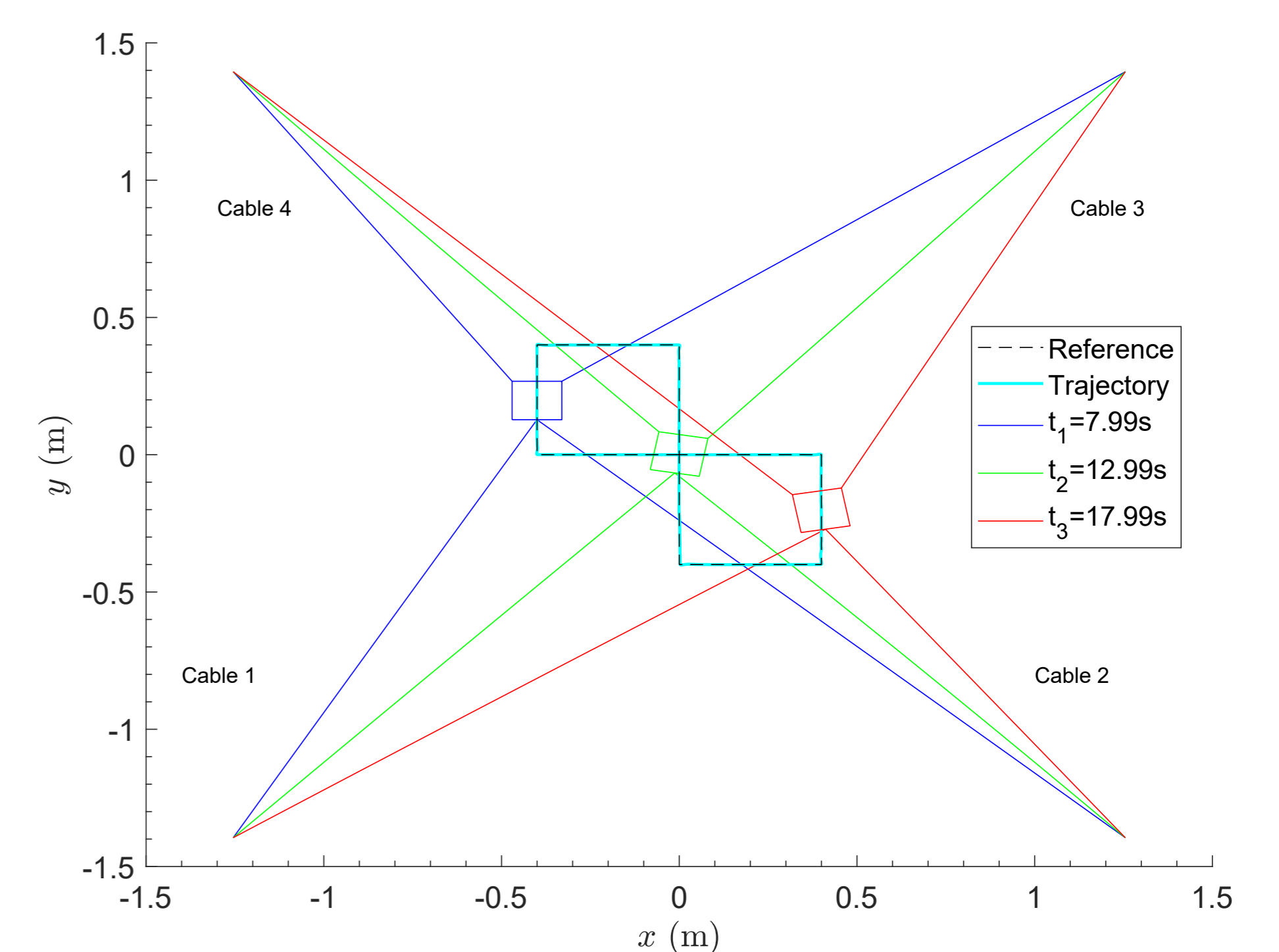


Figure 3: Position and rotation trajectories tracking of the platform.

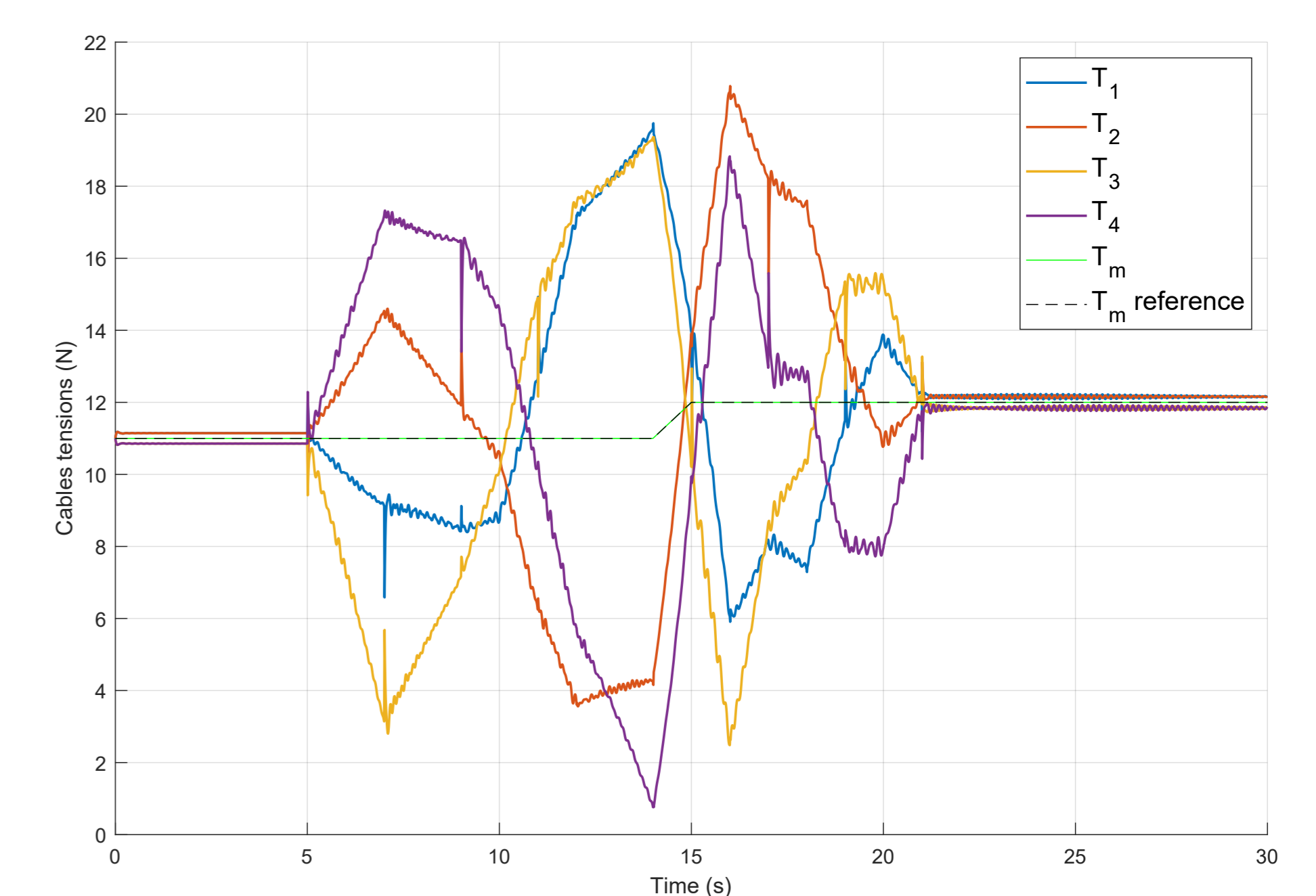


Figure 4: Cables tensions and mean tension reference.

Conclusion

In this work, the dynamic modeling of a planar CDPR with non-straight cables is considered. The assumed modes approach is used to model the flexibility of cables. The obtained DAE model has been transformed into an ODE model and an \mathcal{H}_∞ controller has been synthesized to control the translation and rotation of the platform, in addition to the mean tension of cables.

References

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