



HAL
open science

A variant of the Kármán line theory facilitating the identification of a precise altitude boundary

Nicolas Bérend

► **To cite this version:**

Nicolas Bérend. A variant of the Kármán line theory facilitating the identification of a precise altitude boundary. International Astronautical Congress 2022 (IAC 2022), Sep 2022, Paris, France. hal-03798830v1

HAL Id: hal-03798830

<https://hal.science/hal-03798830v1>

Submitted on 5 Oct 2022 (v1), last revised 26 Oct 2022 (v2)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

A variant of the Kármán line theory facilitating the identification of a precise altitude boundary

N. Bérend

ONERA, The French Aerospace Lab, France

nicolas.berend@onera.fr

Abstract

The concept of Kármán Line was first proposed by Andrew G. Haley in a 1957 IAC space law paper, following Kármán's idea to rely on flight mechanics considerations to identify a boundary between atmospheric flight and space flight. The physical phenomenon highlighted by Kármán, and illustrated by Masson & Gazley's diagram, is the fact that, starting from a certain altitude domain, a near-orbital velocity is required to meet equilibrium glide conditions, in which case weight compensation is nearly completely ensured by centrifugal force and lift becomes negligible.

The Kármán line altitude was eventually set by FAI to 100 km in 1960, for the purpose separating aeronautics and astronautics records. Although we know that this altitude was set in accordance with Kármán's idea, it is at least partly an arbitrary choice, as Kármán's guidelines alone are insufficient to define a very precise altitude. Indeed, in a 1961 conference (i.e. one year after he contributed to FAI's choice), Kármán confirms that his proposal for the boundary of space is based on the Masson & Gazley diagram and the phenomenon of centrifugal force being "dominant" (consistently with Haley's and FAI's explanations) but he only points towards the approximate altitude range where this phenomenon is visible (300,000-400,000 ft, or 91-122 km) on the "lift barrier" part of the diagram, rather than a very precise altitude.

In this paper, we propose a new solution to identify a unique altitude threshold while being consistent with Kármán's guidelines. The proposed solution is based on the fact, apparently never mentioned before in literature about the Kármán line, that the equilibrium velocity actually goes through a maximum when altitude increases. The corresponding altitude de facto separates two regions: an atmospheric flight region, where lifting flight behaviour prevails (equilibrium velocity increases with altitude), and a spaceflight region, where orbital flight behaviour prevails (equilibrium velocity decreases with altitude).

We demonstrate, both numerically and theoretically, that this altitude boundary is relatively stable with regards to the vehicle's lift performance, and that it necessarily yields near orbital velocity conditions (which fits Kármán's idea). Numerically, this variant of the Kármán line definition is 110 km, which is relatively close to FAI's setting, but with the advantage of being precisely set through flight mechanics considerations. Legal aspects of the boundary of space definition(s) are outside the scope of this paper, which focuses only on flight mechanics/astrodynamics aspects.

Keywords: Kármán line, spaceflight, atmospheric flight, aeronautics, astronautics

1. Introduction

The Kármán line is one convention for defining the "boundary of space". It is based on Kármán's idea to define such a boundary through flight mechanics considerations, involving the respective roles of centrifugal force and lift in the sustained flight of a lifting vehicle when altitude varies [1]. Other definitions or proposals for the boundary of space exist, based on different ideas, like the USAF/NASA altitude criterion for "astronaut wings", apparently based on aerodynamic controllability considerations [2], and McDowell's recent proposal, based on considerations of drag and orbit stability for satellites [3].

The Kármán line concept emerged in space law discussions in the 1950's. The name was coined by Andrew G. Haley space law pioneer, in a 1957 IAC paper

[4], in which he proposed to define a "jurisdictional boundary" between atmospheric flight and spaceflight, based on Kármán's scientific guidelines.

The Kármán line was adopted in 1960 by FAI (Fédération Aéronautique Internationale) as the boundary separating aeronautic flight records and spaceflight records, and set to 100 km by a group of scientists led by Kármán [5]. To this date, this is the only application for which the Kármán line is recognized at international level.

In section 2, we detail the phenomenon highlighted by Kármán and we provide the equation of the "lift barrier" curve he uses to illustrate his point. We also remind Kármán's own view about the altitude of the boundary of space, according to the most direct source, which is a conference he made in 1961 (i.e. after the choice of the 100 km altitude boundary by FAI).

In this conference, Kármán proposes an altitude range, rather than a precise altitude [1].

In section 3, we propose an addition to the Kármán line theory, based on a previously unexploited feature of the "lift barrier" curve, which allows identifying more precisely an altitude boundary, based only on flight mechanics considerations.

2. The original Kármán line definition

2.1 General principle and Kármán's guidelines

Kármán's idea to identify a boundary of space is based on a graphics by Masson & Gazley [1][4].

This diagram (Figure 1 from [6]) shows the "corridor of continuous flight" which is the accessible flight domain, represented in altitude vs. velocity coordinates considering technological constraints in aerodynamics (upper curve) and materials (lower curve).

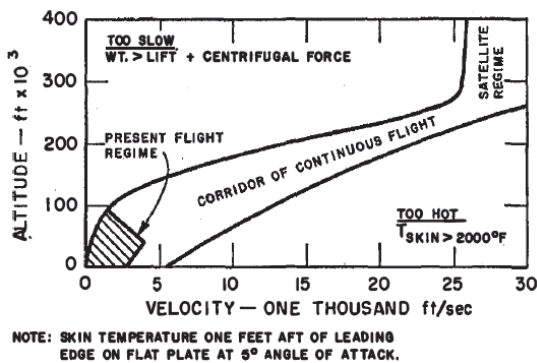


Figure 1 – “Masson & Gazley diagram” showing the corridor of continuous flight [6] (reproduced in Kármán's papers [1] and [9])

The lower curve corresponds to a temperature limitation of the materials. It is usually called the “temperature barrier” [7] or “heat barrier” [9]. It shows, for a given altitude, the maximum velocity a vehicle can fly without exceeding the materials temperature limit.

The upper curve, which is the one that is relevant concerning Kármán's proposition, is the “lift barrier” [7]. It shows, for a given altitude, the minimum velocity that is required to allow a stable horizontal flight for a lifting vehicle. For convenience, this velocity will be called the “equilibrium velocity” in the following, a short designation reminding that it corresponds to equilibrium flight, i.e. the balance between weight and the sum of lift and centrifugal forces. As will be detailed later, the “lift barrier” curve depends on a hypothesis on the maximum lifting performance that is technologically achievable.

This representation of the “corridor of flight” diagram actually predates Masson & Gazley's paper, which references a 1953 Fritz Haber paper as the source of the graphics [7].

While F. Haber's paper, in turn, provides no references to other sources, the “lift barrier” curve appears to be much older. The earliest document we could identify showing this curve is a 1933 book by Eugen Sänger [8]. Like Kármán, both Haber and Sänger highlight the fact that the relative contributions of lift and centrifugal forces to weight compensation change when we move along the curve [7][8]. Kármán brought the idea of defining an altitude boundary of spaceflight based on this phenomenon.

In [1], Kármán explains:

Gazley considered that continuous flight is limited by two extreme conditions. One condition is that the vehicle is too slow to fly because the weight of the vehicle is larger than the sum of the lift force and centrifugal force (...).

It is evident that when the centrifugal force becomes dominant, a vehicle is no longer an airplane but a satellite. It is this point which may be considered the beginning of space. Hence I propose that space begins at 300 or 400,000 ft [91.44 or 121.92 km].

As Figure 1 shows, the 300,000-400,000 ft (~91-122 km) altitude range corresponds to the domain where the lift barrier curve gets suddenly very steep while velocity gets very close to orbital velocity, and tends asymptotically towards it when altitude increases. As will be shown analytically later, this phenomenon also means that the contribution of centrifugal force to the compensation of weight gets very close to 100 %, while the contribution of lift gets close to 0 % (but never gets strictly null, as long as there remains a residual atmospheric density).

We assume that reference [1], cited above, is the most reliable source about Kármán's own view on the subject of the boundary of space. Indeed, there exist very few original documents explaining Kármán's view on this subject and nearly all of them are indirect sources, not authored by Kármán himself. Reference [1] is actually the only source we could identify that is undoubtedly from Kármán himself (and alone). It is the transcript of one of Kármán's latest conferences, which, interestingly, occurred in 1961, i.e. one year after the choice of the 100 km altitude limit by FAI.

The other “most direct” sources are:

- Haley's papers ([4] and later), which contains some quotes of Kármán, but are not co-authored by him. Haley mentions the altitude of 275,000 ft (83.82 km), but he precise that is condition is “illustrative” of the Kármán line and “may be changed in the future”.

- Kármán's autobiography [10], which was posthumously completed by Lee Edson "using the rough material left by the scientist" and published in 1967. The section about the boundary of space appears to be made of excerpts from Haley's papers, save for the fact that the altitude mentioned is 300,000 ft (91.44 km).
- FAI's 2004 web article [2] about the Kármán line, written more than 40 years after the setting of the 100 km boundary.

While all these sources consistently mention the role of centrifugal force and lift in the Kármán line theory, they do not explain clearly how this actually leads to the mentioned altitude boundary.

Haley's 275,000 ft altitude (~84 km) is sometimes presented as the original altitude calculated by Kármán [11][12]. This appears to be a misconception, originating from some ambiguity in Haley's papers about what parts of the proposition are from Kármán himself, and also from the fact that indirect sources usually omit that this altitude is presented as "illustrative". In fact, Haley's altitude is not in the domain proposed by Kármán. It is located where the "lift barrier" curve starts only to be steep, while Kármán propose a higher altitude range, where the curve is nearly vertical.

On the other hand, the altitude mentioned in Kármán's posthumous autobiography (300,000 ft or ~91 km), is partly consistent with the altitude range from [1], as it is its lower bound. A possible explanation is that the biography's co-author may have chosen this figure as the best compromise between Haley's papers (from which some sentences are reproduced identically) and Kármán's conference.

Finally, FAI's altitude (100 km) is definitely within the altitude domain proposed by Kármán in [1], and we know that it was retained partly because it was a round figure, so this is consistent.

Therefore, according to reference [1], and unless previously unknown documents from Kármán surface, it appears that, from the purely scientific point of view, Kármán did not identify a precise altitude for the boundary of space but rather a rough altitude range. This range corresponds to domain where the "lift barrier" curve gets very steep, which is a visual sign that the weight is nearly entirely balanced by centrifugal force and lift is nearly null.

2.2 Analytical study of the "lift barrier curve"

As explained in the legends of Masson & Gazley's figure (Figure 1), in Kármán explanations and also in other papers reproducing the same figure [13][14], the equation that describes the upper limit of the "corridor of continuous flight" is the equation of the balance between weight and the sum of lift and centrifugal force:

$$\text{Weight} = \text{Lift} + \text{Centrifugal force} \quad \text{Eq. [1]}$$

Or, in terms of acceleration:

$$g(h) = \frac{1}{2} \frac{\rho(h) V^2}{\left(\frac{M}{S \cdot C_L} \right)} + \left(\frac{V^2}{r_E + h} \right) \quad \text{Eq. [2]}$$

with:

h = altitude

$g(h)$ = gravity acceleration at altitude h

$\rho(h)$ = atmospheric density at altitude h

V = velocity

r_E = Earth radius

M = mass (kg).

C_L = lift coefficient (-)

S = aerodynamic surface (m²).

$M/(S \cdot C_L)$ is the "lift parameter" [15] expressed in kg/m². This coefficient is the link between the ambient dynamic pressure to the lift acceleration, so it characterizes the lift performance of the vehicle. The lower this coefficient is, the better the lift performance is. Therefore, reducing the lift parameter makes the lift barrier curve higher on the velocity/altitude diagram, which enlarges the "corridor of continuous flight". In the context of the "lift barrier" the lift parameter should represent the technological limit.

The lift parameter can be expressed as a function of other key design parameters. Indeed, it is ratio between the wing loading (M/S) and the lift coefficient (C_L), and also between the ballistic coefficient $M/(S \cdot C_D)$ and the lift-to-drag ratio (C_L/C_D):

$$\frac{M}{S \cdot C_L} = \frac{\left(\frac{M}{S} \right)}{C_L} = \frac{\left(\frac{M}{S \cdot C_D} \right)}{\left(\frac{C_L}{C_D} \right)} \quad \text{Eq. [3]}$$

The most direct way to plot the lift barrier curve in the velocity/altitude reference frame is to extract velocity from Eq. [2]:

$$V = \sqrt{\frac{(r_E + h) \cdot g(h)}{1 + \frac{(r_E + h) \cdot \rho(h)}{2 \left(\frac{M}{S \cdot C_L} \right)}}} \quad \text{Eq. [4]}$$

Alternatively, another way to plot the curve is to set velocity and to solve Eq. [2] for the altitude, but it would need to be done numerically, as there is no closed-form solution.

Velocity V , as expressed by Eq. [4], is the equilibrium velocity at altitude h , i.e. the exact velocity that is required for a stable, locally horizontal flight.

Indeed, Eq. [2] is a particular case of the state dynamics equation of the flight path angle (γ) [15] for $\gamma=0^\circ$:

$$V \cdot \dot{\gamma} = \cos \gamma \left[\left(\frac{V^2}{r_E + h} \right) - g(h) \right] + \frac{1}{2} \frac{\rho(h) V^2}{\left(\frac{M}{S \cdot C_L} \right)} \quad \text{Eq. [5]}$$

As precised in Masson & Gazley's paper [6] (and also in the Fritz Haber paper that is referenced as the source of the graphics [7]), the "lift barrier" curve is plotted considering a 80 psf lift parameter, which is around 400 kg/m² (390.6 kg/m²). This value is "a very low figure by today's standards", wrote Fritz Haber in this 1953 article [7].

Figure 2 allows to verify that Masson & Gazley's lift barrier curve (red curve, which is digitized from the version appearing in Haley's paper [4]) indeed coincides with the curve described by Eq. [4] and a 80 psf lift parameter (dark blue curve).

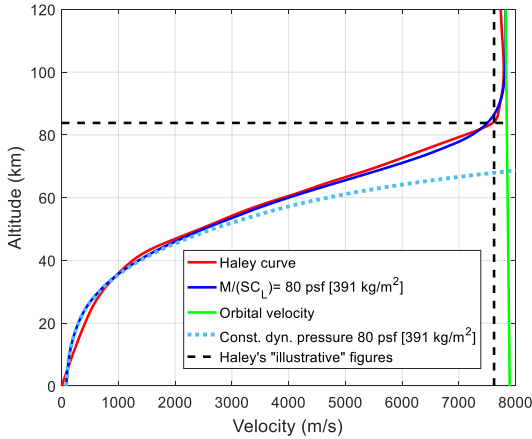


Figure 2 – Re-creation of the "lift barrier" curve using Eq. [7]

The 80 psf curve has been re-calculated considering the following models:

- The atmospheric density model ($\rho(h)$) from the ARDC 1956 model [16], which was the standard at that time.
- A simple Kepler gravity acceleration model ($g(h)$), defined as follows:

$$g(h) = g_0 \left(\frac{r_E}{r_E + h} \right)^2 \quad \text{Eq. [6]}$$

where g_0 and r_E are set consistently with the atmosphere model [16]:

- $g_0 = 9.80665 \text{ m/s}^2 = \text{sea-level gravity acceleration at latitude } 45^\circ 32' 40''$
- $r_E = 6356.755 \text{ m} = \text{effective Earth radius at latitude } 45^\circ 32' 40''$

Using Eq. [6], Eq. [4] becomes:

$$V = \sqrt{\frac{r_E \cdot g_0}{\left(1 + \frac{h}{r_E} \right) \cdot \left(1 + \frac{(r_E + h) \cdot \rho(h)}{2 \left(\frac{M}{S \cdot C_L} \right)} \right)}} \quad \text{Eq. [7]}$$

Figure 2 also illustrates two phenomena that occurs in the low velocity and high velocity domain, respectively:

- At low velocity: the lift barrier curve tends to be the same as the constant 80 psf dynamic pressure curve (light blue dotted curve), which illustrates the fact that in this domain, the weight is balanced nearly entirely by lift, while the centrifugal force is negligible.
- Conversely, at high velocity: the lift barrier curve tends to be the same as the orbital velocity curve (green), which illustrates the fact the weight is balanced nearly entirely by centrifugal force lift, while the contribution lift is negligible.

The link between the lift barrier curve and the phenomenon of "centrifugal force taking over the lift" appears more clearly by introducing the ratio between centrifugal force and weight, that will be denoted k :

$$k = \frac{\left(\frac{V^2}{r_E + h} \right)}{g(h)} \quad \text{Eq. [8]}$$

Since orbital velocity at altitude h is:

$$V_{orb} = \sqrt{(r_E + h) \cdot g(h)} \quad \text{Eq. [9]}$$

k can be expressed as a function of the ratio between velocity and orbital velocity:

$$k = \left(\frac{V}{V_{orb}} \right)^2 \quad \text{Eq. [10]}$$

Eq. [10] illustrates the fact that, when velocity tends towards orbital speed, the contribution of centrifugal force to the compensation of weight tends towards 100 % (while the contribution of lift tends towards 0 %).

Transforming Eq. [4] using Eq. [9], we get:

$$\frac{V}{V_{orb}} = \sqrt{\frac{1}{1 + \frac{(r_E + h) \cdot \rho(h)}{2 \left(\frac{M}{S \cdot C_L} \right)}}} \quad \text{Eq. [11]}$$

Then, with Eq. [10]:

$$k = \frac{1}{1 + \frac{(r_E + h) \cdot \rho(h)}{2 \left(\frac{M}{S \cdot C_L} \right)}} \quad \text{Eq. [12]}$$

Eq. [12] shows that the ratio between centrifugal force and weight increases with the altitude (i.e. when we move upward along the “lift barrier curve”) and tends towards 100 % (while the contribution of lift tends towards 0 %) : this is the precise phenomenon highlighted by Kármán (and also, as noted earlier, by other authors [7][8]).

A detailed study of the variation of the altitude boundary as a function of k and the lift parameter, is given in a previous paper [17].

It is important to note that k (then V/V_{orb}) never gets strictly equal to 100 % along the lift barrier curve, as long as we consider that atmospheric density never gets strictly null. Since we cannot make this assumption (which would have answered the question of the boundary of space, in the first place !), the hypothesis behind the lift barrier curve excludes the possibility of reaching exactly the orbital velocity or compensating 100 % of the weight using centrifugal force.

With this respect, Haley’s paper [4] and FAI’s article [5] appear to be inexact, which seems to be the origin of a common misconception about the Kármán line. Indeed, they associate the crossing of the Kármán line to the vanishing of lift (“Lift is gone” [4]) and a velocity reaching or even exceeding the orbital velocity (“[The speed required for lift] could be close or even bigger than the circular orbital speed at that altitude” [5]). The contradiction is actually obvious in Haley’s paper, as the “illustrative” conditions mentioned are a velocity of 25,000 ft/s (7620 m/s) and an altitude of 275,000 ft (~84 km): these are suborbital conditions (suborbital velocity, hence requiring a contribution of lift to ensure a sustained horizontal flight).

The many contradictions in the early literature about the Kármán line altitude have been highlighted by Gangale in [12], who suggests that “von Kármán himself never proposed a boundary at 100 kilometers” and that “reference to the 100-kilometer line as the von Kármán line is a misnomer” and an “urban legend of the space age”. Our findings (namely reference [1]) are partly in line with the first part of these conclusions, as they tend to confirm that the 100 km is not – by itself – the result of a precise *calculation* by Kármán. But reference [1] also proves that Kármán himself undoubtedly proposed an altitude range based on the same flight mechanics phenomenon (and graphics) as described in Haley’s paper [4], and this range is definitely consistent with the 100 km altitude, which, as we know, was eventually set to a round number (“apparently [by] Von Karman himself”, according to [5]). Conversely, the 84 km/275,000 ft altitude should not be attributed to Kármán, as this was no more than an “illustrative” figure [4] provided by Haley (a lawyer), a precision that is,

unfortunately, often omitted. Incidentally, when we identify these facts, the mention by Haley in [18] that “[FAI’s 100 km altitude] coincides with the Kármán theory” appears no longer as a contradiction compared to his earlier paper [4]. It is rather a further confirmation that Kármán’s work is indeed at the origin of the 100 km limit, even though the eventual choice of a round number is partly arbitrary. The purpose of the variant of the Kármán line definition that will be presented in the next section, is, precisely, to avoid resorting to an arbitrary choice to get a unique altitude.

3. Proposed variant of the Kármán line definition

3.1 The “Maximum Equilibrium Velocity (MEV) altitude”

The “lift barrier” curve on Masson & Gazley’s diagram (Figure 1) appears to be simplified compared to the actual curve obtained using Eq. [4] (Figure 2).

Indeed, in the high velocity domain, it shows a vertical asymptote, with the altitude continuously increasing when velocity increases.

This is actually a simplified representation of the asymptote, which, as noted in the previous section, corresponds to orbital velocity (green curve on Figure 2).

The orbital velocity curve is not vertical but slightly tilted, since orbital velocity decreases as altitude increases. Conversely, when we move along the lift barrier towards high altitudes, velocity gets closer to orbital velocity but it actually decreases in absolute value, while it is increasing with altitude in the lower altitude domain of the graphics.

This can be observed more clearly on Figure 3, which is a zoomed version of Figure 2 around the domain where this phenomenon occurs.

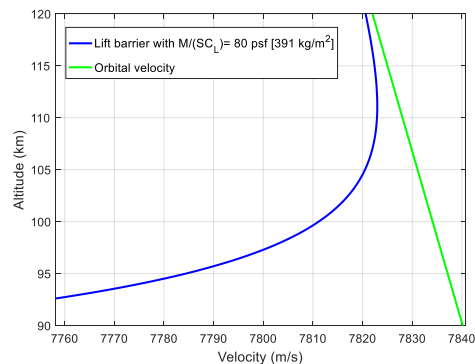


Figure 3 – Zoom on the asymptote of the lift barrier

As a consequence (and it will be later demonstrated analytically using a simplified atmospheric density model), the equilibrium velocity goes through a maximum value. This maximum occurs around 110 km (with the same ~400 kg/m² lift parameter hypothesis).

Note that this is quite close to the middle of the altitude range identified by Kármán (while the FAI 100 km altitude is comparatively farther).

A corollary of the existence of a Maximum Equilibrium Velocity altitude (now abbreviated as “MEV altitude” for convenience) is that it separates two altitude domains:

- At lower altitudes: a domain where the equilibrium velocity increases with altitude. This property can be seen as an “aeronautical behaviour” (for a lifting vehicle, a higher altitude means a higher velocity).
- At higher altitudes: a domain where the equilibrium velocity decreases with altitude. This property may be described as “spaceflight” or “orbital behaviour” (for a space vehicle in a circular orbit, a higher altitudes means a lower velocity).

With the MEV altitude, we now have identified a physical phenomenon that is:

- fully consistent with Kármán’s ideas (as will be demonstrated later, it necessarily occurs at conditions where centrifugal force is dominant and lift negligible).
- occurs at a definite altitude (while Kármán’s proposal was a limited to a rough altitude range).

Due do these properties, the MEV altitude can be seen as a variant – or, more exactly, an addition - to Kármán’s guidelines for the definition of an altitude boundary between atmospheric flight and spaceflight.

Following these observations, the following question naturally arises: could this “variant” actually be what Kármán had in mind ? All the elements available tend to show that the answer is “no” and that this particular proposition for the boundary of space definition is indeed new. Indeed, no document in Kármán line literature refers to the particular phenomenon we highlight (equilibrium glide velocity going through a maximum when altitude varies). In particular, this phenomenon is not mentioned at all in the most direct sources we have about the Kármán line theory (by Haley, Kármán or FAI).

All these sources provide only qualitative explanations about the relative roles of centrifugal force and lift, but no precise mathematical or physical definition allowing to identify a precise altitude.

Furthermore, in its 1961 conference, still illustrates his theory using the original “Masson & Gazley” diagram (Figure 1), which shows a simplified version of the “lift barrier” curve (with a vertical asymptote, thus hiding the existence of a MEV altitude). If Kármán had wanted to highlight the concept of MEV altitude, he would had shown a “regular” version of the lift barrier curve on

which the MEV is visible and he would have mentioned this phenomenon.

For the sake of completeness, it shall be noted that Haley’s version of the diagram (which he says to have “unskillfully redone” [4]) actually shows a qualitatively correct bending of the lift barrier asymptote. However, as analysed in a previous paper [17], a likely explanation is that Haley (who was not a scientist but a lawyer) has mixed the original Masson & Gazley curves with elements from a similar flight domain diagram by Sänger (also reproduced in the same Haley paper). Beside the graphic itself, Haley’s explanations never refers to the existence of an altitude where the equilibrium velocity is minimum, and the “illustrative” altitude/velocity conditions he provides are placed much lower on the graphics.

3.2 Sensitivity to the lift parameter

The lift barrier curve, hence the MEV altitude, depends on an estimation of the minimum lift parameter ($M/(S.C_L)$) that is technically achievable.

As explained previously, the original Masson & Gazley figure considered a ~ 400 kg/m² (80 psf) lift parameter. Today, this value is still low compared to today’s technology, so it seems to be still relevant as an “optimistic value” representing the technology limitations.

Nevertheless, such an estimation is, by definition, uncertain, so it is interesting to observe how the MEV altitude varies with it.

Figure 4 shows the lift barrier curves for different lift parameters between 100 and 2000 kg/m² (which is very large range). For each curve, the MEV point is indicated with a red diamond.

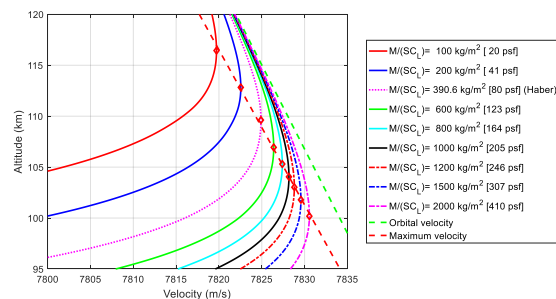


Figure 4 – Zoom on the lift barrier curve for different lift parameter hypotheses

If we consider a factor 2 uncertainty on the ~ 400 kg/m² hypothesis (which seems a reasonable assumption), we get:

- For 200 kg/m² (= half the original hypothesis): MEV altitude ~ 113 km
- For 800 kg/m² (= twice the original hypothesis): MEV altitude ~ 105 km

It shall be noted that changing the lift parameter hypothesis by a factor 2 (in either ways) induces a relatively small change in the MEV altitude (less than 5 km), compared to its original 110 km altitude. That rate of change of the MEV altitude is around 1.3 km for each 200 kg/m² variation.

This relative stability of the MEV altitude with regards to the technological hypothesis make it an interesting candidate for the definition of an altitude boundary based on scientific considerations.

3.3 Closed-formed estimation

The MEV altitude (h_{MEV}) is the altitude that maximizes the equilibrium velocity, as expressed by Eq. [7], so we have:

$$h_{MEV} = \underset{h}{\operatorname{argmax}} \sqrt{\frac{r_E \cdot g_0}{\left(1 + \frac{h}{r_E}\right) \cdot \left(1 + \frac{(r_E + h) \cdot \rho(h)}{2 \left(\frac{M}{S \cdot C_L}\right)}\right)}} \quad \text{Eq. [13]}$$

An equivalent expression is:

$$h_{MEV} = \underset{h}{\operatorname{argmin}} \left[(r_E + h) \cdot \left(1 + \frac{(r_E + h) \cdot \rho(h)}{2 \left(\frac{M}{S \cdot C_L}\right)}\right) \right] \quad \text{Eq. [14]}$$

So h_{MEV} satisfies:

$$\frac{d}{dh} \left[(r_E + h) \cdot \left(1 + \frac{(r_E + h) \cdot \rho(h)}{2 \left(\frac{M}{S \cdot C_L}\right)}\right) \right] = 0 \quad \text{Eq. [15]}$$

or:

$$\left(1 + \frac{\rho(h) \cdot (r_E + h)}{2 \left(\frac{M}{S \cdot C_L}\right)}\right) + \frac{(r_E + h)}{2 \left(\frac{M}{S \cdot C_L}\right)} \cdot \left(\frac{d\rho(h)}{dh} \cdot (r_E + h) + \rho(h)\right) = 0 \quad \text{Eq. [16]}$$

Let's now consider an exponential atmospheric density model:

$$\rho(h) = \rho_0 \cdot e^{-\frac{h}{L}} \quad \text{Eq. [17]}$$

with:

ρ_0 = atmospheric density at sea-level
 L = scale height

With this model, Eq. [16] becomes:

$$\frac{M}{S \cdot C_L} = \rho_0 \cdot e^{-\frac{h}{L}} \cdot (r_E + h) \left(\frac{(r_E + h)}{2L} - 1\right) \quad \text{Eq. [18]}$$

Considering that $r_E \gg h$ and $r_E \gg 2L$, we get:

$$\frac{M}{S \cdot C_L} \cong \frac{\rho_0 \cdot e^{-\frac{h}{L}} \cdot r_E^2}{2L} \quad \text{Eq. [19]}$$

Then, solving Eq. [19], we get a closed form estimation of the MEV altitude:

$$h_{MEV} \cong L \cdot \ln \left(\frac{\rho_0 \cdot r_E^2}{2L \left(\frac{M}{S \cdot C_L}\right)} \right) \quad \text{Eq. [20]}$$

This formula allows to get easily an estimation of altitude h_{MEV} using only a few parameters:

- A technical parameter: an estimation of the minimum achievable lift parameter ($M/(S \cdot C_L)$).
- Three parameters related to the planetoid model (r_E) and the atmosphere model (ρ_0 and L), respectively.

With the same simplifying assumption, we can now get an estimation of the ratio between centrifugal force to weight ratio at h_{MEV} boundary (k_{MEV}).

From Eq. [19], we have:

$$\frac{\rho(h_{MEV})}{\left(\frac{M}{S \cdot C_L}\right)} \cong \frac{2L}{r_E^2} \quad \text{Eq. [21]}$$

Which, after replacement in Eq. [12], yields:

$$k_{MEV} \cong \frac{1}{1 + \frac{L}{r_E}} \quad \text{Eq. [22]}$$

Considering $L = 6.7$ km for the exponential atmospheric density for Earth [19], we have $L/r_E \cong 1.05 \cdot 10^{-3}$. Since $L \ll r_E$, Eq. [21] shows that k_{MEV} is necessarily close to 1 (≈ 99.88 %), and, due to Eq. [10], that V/V_{orb} is even closer to 1 (≈ 99.95 %).

In other words, these closed form expressions demonstrate that the maximum equilibrium velocity is necessarily close to orbital velocity, with a very small (but non zero) contribution of lift. This feature is consistent with Kármán's general guidelines about the region on of the lift barrier curve where the "boundary of space" should be.

Using the same lift parameter hypothesis (390.6 kg/m² or 80 psf), we verify that Eq. [20] provides a valid estimation of h_{MEV} : 110.01 km, which is nearly identical to the value we get with the regular atmospheric density model and complete numerical solving (cf. previous section).

Out of curiosity and for the purpose of comparison, we have also computed the estimations of h_{MEV} for Mars and Venus (using the same lift parameter hypothesis).

The results for the three planets (as well as the model parameters) are given in Table 1. The sources for the parameter models are [19] for Earth and [20] for Mars and Venus.

Planet	ρ_0 (kg/m ²)	r_E (km)	L (km)	h_{MEV} (km)
Earth	1.752	6356.766	6.7	110.01
Mars	0.02	3389.5	11.1	113.05
Venus	65	6051.8	15.9	303.23

Table 1 – Comparison of the MEV altitude for Earth, Mars and Venus (with $M/(S.C_L) = 390.6 \text{ kg/m}^2$ or 80 psf).

Surprisingly, the MEV altitude for Mars (113 km) is very close to the one of Earth, while it is much higher for Venus (303 km).

4. Conclusion

We have proposed a variant – or, more exactly an addition - to the original Kármán line theory. It is defined as the altitude where equilibrium velocity (which is the velocity allowing a locally horizontal flight for a lifting vehicle) is maximum.

It separates two altitude domains that are relevant to the distinction between atmospheric flight and spaceflight: a low altitude domain where “aeronautics behaviour” prevails (the equilibrium velocity increases with altitude) and a high altitude domain where the “spaceflight behaviour” prevails (opposite effect).

The interest of this proposal is twofold:

- It is fully compatible with Kármán's original idea (relying on the "lift barrier curve" and the phenomenon of "centrifugal force taking over") and with the altitude range he proposed (92-122 km).
- It provides a method to identify a *precise* altitude boundary using flight mechanics considerations and a technical hypothesis (minimum achievable lift parameter), while Kármán proposed only a rough altitude interval [1] and FAI selected arbitrarily a round number (100 km) in the domain pointed by Kármán [5].

According to this definition, and using the same lift parameter hypothesis as the original "lift barrier curve" used by Kármán ($\sim 400 \text{ kg/m}^2$), the altitude for the boundary of space is located around **110 km**. This altitude is relatively stable when the lift parameter hypothesis varies.

This proposal should be seen primarily as an answer to the scientific question: “is there a physical phenomenon, pertaining to flight mechanics, allowing to identify an altitude boundary between atmospheric flight and space flight ?”

While this proposal may contribute to the debates, the question of setting (or adapting) a precise “boundary of space” altitude for particular applications (space law, space records, astronaut wings, etc.) is outside the scope of this paper, as, in any case, it is a matter of convention.

References

- [1] Th. von Kármán, From Low Speed Aerodynamics to Astronautics, Pergamon Press, 1961
- [2] D. Jenkins, A Word about the definition of space, NASA Dryden Flight Research Center website, October 21, 2005
https://www.nasa.gov/centers/dryden/news/X-Press/stories/2005/102105_Schneider.html
- [3] J. McDowell, The edge of space: Revisiting the Kármán Line, Acta Astronautica 151 (2018) 668–677
- [4] A. G. Haley, Space Law-The Development of Jurisdictional Concept, VIIIth International Astronautical Congress (Barcelona 1957), 1958, Springer
- [5] S. Sanz Fernández de Córdoba, 100km Altitude Boundary for Astronautics (Last update: 21-06-2004), FAI website:
<https://www.fai.org/page/icare-boundary/>
- [6] D. J. Masson, C. G. Gazley, “Surface Protection And Cooling Systems For High-Speed Flight” (preprint), IAS National Summer Meeting, Aerodynamic Heating-Aerodynamic Aspects session, IAS, Los Angeles, June 18-21, 1956
- [7] Fritz. Haber, The Heat Barrier, Scientific American, December 1953
- [8] E. Sänger, Rocket Flight Engineering (Translation of Translation of “Raketenflugtechnik”, Verlag von R. Oldenbourg, Munich and Berlin, 1933), NASA TT F-223, NASA, September 1965.
- [9] T. Von Kármán, Aerodynamic Heating-The Temperature Barrier In Aeronautics, Proceedings of the Symposium on “High Temperature-A Tool for the Future” (Berkeley, California, June 25-27, 1956), Stanford Research Institute, University of California at Berkeley, 1956
- [10] Th. von Kármán, L. Edson, The Wind and Beyond: Theodore von Kármán-Pioneer in Aviation and Pathfinder in Space, Little Brown & Co., 1967
- [11] R. Goedhart, The never ending dispute : delimitation of air space and outer space, Forum for air and space law ; vol. 4 ; Editions Frontières, Paris, 1996
- [12] Th. Gangale, The Non Kármán Line: An Urban-Legend of the Space Age, Journal of Space Law, Vol. 41, No. 2, 2017, University of Mississippi School of Law, 2017
- [13] R. Hermann, Problems of Hypersonic Flight at the Re-Entry of Satellite Vehicles, IXth International

- Astronautical Congress (Amsterdam 1958), 1959, Springer
- [14] R. Hermann, Re-Entry Trajectories and Problems of Hypersonic Flow, Space Trajectory, 1960, Academic Press
- [15] E. H. Hirschel, C. Weiland, Selected Aerothermodynamic Design Problems of Hypersonics Flight Vehicles, Springer-Verlag, 2009
- [16] R. A. Minzner, W. S. Ripley, The ARDC atmosphere model 1956, Air Force Surveys in Geophysics n°88, USAF, December 1956
- [17] N. Bérend, The Missing Calculation behind the Original Kármán Line Definition -A Credible Hypothesis, IAC-21-E4.2.4, 72nd International Astronautical Congress, 25-29 October 2021, Dubai, United Arab Emirates
- [18] A. G. Haley, Survey of Legal Opinion on Extraterrestrial Jurisdiction, XIth Int. Astronautical Congress (Stockholm 1960), 1961, Springer
- [19] U. Walter, Astronautics - The Physics of Space Flight (3rd Edition), Springer, 2018
<https://doi.org/10.1007/978-3-319-74373-8>
- [20] NASA – Lunar and Planetary Science on NASA Space Science Data Coordinated Archive
<https://nssdc.gsfc.nasa.gov/planetary/>