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► **To cite this version:**

Pierre Suquet, Jean-Claude Michel. Variational problems in plasticity and fracture of composite materials. 11th European Solid Mechanics Conference (ESMC 2022), Jul 2022, Galway, Ireland. hal-03797505

**HAL Id: hal-03797505**

**<https://hal.science/hal-03797505>**

Submitted on 4 Oct 2022

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# Variational problems in plasticity and fracture of composite materials

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*Keywords:* Homogenization, plasticity, fracture.

**Homogenization of limit loads.** When the strength of the constituents of a heterogeneous material is limited and characterized by a convex domain  $P^{(r)}$  (in phase  $r$ ) where the stress tensor is constrained to stay, the composite itself has a limited strength characterized by a homogenized strength domain  $P^{\text{hom}}$ . This domain is characterized by the dual function of its indicator function, the homogenized plasticity dissipation  $P^{\text{hom}} = \{\Sigma \text{ such that } \Sigma : \mathbf{E} \leq \Pi^{\text{hom}}(\mathbf{E}) \text{ for all } \mathbf{E}\}$ , where

$$\pi^{\text{hom}}(\dot{\mathbf{E}}) = \inf_{\mathbf{u} = \dot{\mathbf{E}} \cdot \mathbf{x} + \mathbf{u}^*} \langle \pi(\varepsilon(\mathbf{u})) \rangle, \quad \pi(\mathbf{x}, \mathbf{e}) = \sup_{\boldsymbol{\sigma} \in P(\mathbf{x})} \boldsymbol{\sigma} : \mathbf{e}, \quad (1)$$

where  $\mathbf{u}$  is a velocity field,  $\mathbf{u}^*$  is its fluctuation around the affine field  $\dot{\mathbf{E}} \cdot \mathbf{x}$  subjected to appropriate boundary conditions (for instance  $\mathbf{u}^*$  has to be periodic when the medium is periodic),  $\varepsilon$  is the symmetric gradient of  $\mathbf{u}$ . An important feature of  $\pi$  (also shared by  $\pi^{\text{hom}}$ ) is that it is homogeneous of degree 1 in  $\mathbf{e}$  and non differentiable at the origin. Several computational methods have been developed along the years to solve the variational problem (1), either by direct methods or by solving an associated elasto-plastic problem, and they are now well established (Michel *et al* [1]).

**Homogenization of effective toughness of elastic brittle materials.** In a recent work Schneider [2] proposed a variational definition of the effective toughness of heterogeneous materials made from individual constituents with toughness  $\gamma(\mathbf{x})$  based on available mathematical homogenization results, leading to the following homogenized *surface energy* beginequation

$$\gamma^{\text{hom}}(\boldsymbol{\nu}) = \inf_{\mathbf{u} = \mathbf{u}^* + \boldsymbol{\nu} \cdot \mathbf{x}} \langle \gamma(\mathbf{x}) |\nabla \mathbf{u}| \rangle. \quad (2)$$

Again the function to be minimized is positively homogeneous of degree 1. The similarity between the problems (1) and (2) can be exploited from a computational standpoint and the above mentioned standard techniques for limit loads may be applied to (2). It also open the way to the application of nonlinear homogenization techniques ([3]) to derive bounds on effective energy potentials. Finally the paper discusses the relations between this effective *surface energy* (2) and the effective *toughness* of the composite.

[1] J.C. Michel, H. Moulinec, and P. Suquet, *Effective properties of composite materials with periodic microstructure: a computational approach. Comp. Meth. Appl. Mech. Engng.*, **172**:109–143, 1999.

[2] M. Schneider, *An FFT-based method for computing weighted minimal surfaces in microstructures with applications to the computational homogenization of brittle fracture. International Journal for Numerical Methods in Engineering*, 2020. <https://onlinelibrary.wiley.com/doi/abs/10.1002/nme.6270>.

[3] P. Ponte Castañeda and P. Suquet, *Nonlinear composites. In E. Van der Giessen and T.Y. Wu, editors, Advances in Applied Mechanics, volume 34, pages 171–302. Academic Press, New York, 1997.*