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Spacetime Curvature and Hawking Radiation, two results from the complex mass/energy wave function Entropy

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Abstract

The main objective of this paper is to understand the entropy of the mass/energy objects (Signals Information SI) wave functions. As the wave functions are complex, we computed a complex entropy that is equivalent to the Shannon one to calculate the deformation due to the space and time summation or the interferences or what we have demonstrated to be the Gravity effect. The imaginary part of this complex entropy has been shown to describe the spacetime curvature. In this paper we will prove that the real part of this entropy is the thermodynamical entropy effect one defined by the Hawking Radiation.

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I. INTRODUCTION

In our last paper "*Spacetime curvature, a potential result of the quantum mass/energy Information Entropy*" submitted to Foundation of Physics, We have defined that everything may be described by a time-dimensional signal-Information that travels certainly in time dimension if it exists in term of mass/energy. The SI are defined by the wave functions. We assumed also that the quantum vacuum space is a sea where potential particle-antiparticle couples may be created in their respective fields (see Quantum Field Theory QFT for example [2],[1]...). In our last work ([3] and [4]) we may have demonstrated that the interference or the space and time summation of Signal Information (SI) wave functions is the Gravity effect. The deformation due to the summation in the space and time of the signals explains the spacetime curvature as result of the existence of the mass/energy object. This deformation has been computed thanks to a complex entropy of the SI because of the complexity of the wave functions, that have a real part and an imaginary part. This complex entropy has an imaginary part that has been proved to explain the spacetime deformation or the spacetime curvature. However the real part has been only assumed to be the thermodynamical energy one. The objective of this paper is to prove this assumption.

A. Vacuum spacetime curvature and Heisenberg energy-time principle

At each point N_2 and time t , we have supposed that in any quantum field type, the wave functions are summed over time Δt and space $\partial\Omega$. The space time summation is supposed to be the gravity effect [3]. We remind that the total signal-Information function received during Δt , the incertitude duration in Heisenberg principle respective to each quantum field, at N_2 space location with virtual surface boundary $\partial\Omega$ is :

$$\psi(N_2, t_f) = \int_{\partial\Omega} \int_{\Delta t} |\psi(N^r, t)| e^{i \frac{1}{\hbar} \int^t \frac{1}{2} mc^2 du - \delta_{inh,r,t} \pi + \mathcal{L}_{m,r}} D[q_r(t)] dt dr \quad (1)$$

where $\frac{1}{2}mc^2$ is the *time* kinetic energy of virtual particle or virtual antiparticle in the time paths that may exist during Δt thanks to the Heisenberg principle. mc^2 is the kinetic energy of virtual particle + virtual

antiparticle. $-\delta_{inh,r,t}\pi$ is the effect of the virtual antiparticles with the mean of $\delta_{inh,r,t}$ is $\frac{1}{2} \times \frac{1}{4\pi\Delta t}$ (see [3]). $\delta_{inh,r,t}$ describes the virtual antiparticles effect and it is equal to 1 if the time path *brings* virtual antiparticles and $\delta_{inh,r,t} = 0$ if the time path q_r is excitatory i.e. brings virtual particles. $D[q_r(t)]dtdr$ is the measure of the time path q_r arriving with incidence r in the N_2 point. One may suppose that it is a constant if all the paths are equivalent. $|\psi(N^r, t)|$ is the initial wave function module coming from the virtual point N^r respectively to the path q_r since time t where we recall that $|\psi(N^r, t)| = 1_T + |\psi(N_x, t)|$ with $1_T = 1_{r, Time\ Dimensions}$ in the Time dimensions the wave function of the Vacuum virtual particles/antiparticles (thanks to Heisenberg time-energy principle). In the case where there is no real incoming mass/energy from neighboring spatial location $|\psi(N^r, t)| = 1_T$. $|\psi(N_x, t)|$ is the module of the Signal Information (SI) existing in 3D space neighboring locations N_x at time t . \mathbf{L}_{m_r} is the standard model Lagrangian density divided by \hbar that describes the particles located at the neighboring positions N_x at time t and all their interactions with the other quantum fields. If at N_x there is an electron, \mathbf{L}_{m_r} may represent the electron and all its all possible interactions with the other fields (electromagnetic field, Higgs field...).

As we have already done in the last paper, we set the Δt and we let mc^2 varies to $m(\frac{dq_r}{du})^2$. We set the Δt for the effect of the virtual antiparticle in the $\delta_{inh,r,t}\pi$ part as the real particles are almost surely not antiparticles. If Δt is transformed to shorter $\Delta t'$ in presence of a non-null \mathbf{L}_{m_r} , $\frac{1}{2}m(\frac{dq_r}{du})^2$ will be higher than $\frac{1}{2}mc^2$ which includes not only the time kinetic energy but also the spatial kinetic energy of the real particle that may have been moved from N_x to N_2 .

We therefore rewrite the SI function as following:

$$\psi(N_2, t_f) = \int_{\partial\Omega} \int_{\Delta t} |\psi(N^r, t)| e^{i\frac{1}{\hbar} \int^t \frac{1}{2} m(\frac{dq_r(u)}{du})^2 du - \delta_{inh,r,t}\pi + \mathbf{L}_{m_r}} D[q_r(t)] dtdr \quad (2)$$

As any kind of information has its entropy, we have considered in our last paper [3] a complex entropy where its imaginary part describes the paths entropy or in other words the spacetime entropy. This time entropy has been demonstrated to be equivalent to the spacetime curvature described in General Relativity.

In fact, the equation (1 Vacuum spacetime curvature and Heisenberg energy-time principle equation.1.1) can be seen as a conditional expected value of the function $D[q_R(T)]$ knowing that the trajectory with incidence R is either excitatory or inhibitory (knowing the value of $\delta_{inh,r,t}$), with complex density where R and T are the random variables defined as follows:

- R represents the random variable of the signal existence or the Time path existence coming from the virtual point N^R , a virtual projection of the real point N_x including the virtual signals of the virtual particles and antiparticles of the Vacuum.
- T is the random variable of the summation possibility of signals coming from the neighboring points i.e it represents the time summation capacity or the interference capacity in time. This depends on the quantum field type and on the presence of not of the neighboring signals information.

Hence we can see mathematically $\psi(N_2, t_f)$ as:

$$\psi(N_2, t_f) = \mathbb{E}[D(q_R(T)) | \delta_{inh,R,T}] \quad (3)$$

We have already introduced an entropy in [3] similar to Shannon entropy in information theory [6]. We recall that Shannon entropy measures the information quantity sent by a information source. The more different information the source emits, the greater the entropy (or uncertainty about what the source emits). In quantum mechanics, we work with wave functions, a particular probability functions. Any complex wave function $\psi(r, t)$ can be rewritten as following $\psi(r, t) = R(r, t)e^{i\theta(r, t)}$ with the probability density $p(r, t) = R^2(r, t)$ and its phase $\theta(r, t)$. Recall that $\psi(N^r, t)$ in our study is the wave function of the signal-Information coming to N_2 at time t from the particular Time path with incidence r , and r is a point on the virtual surface border $\partial\Omega$ of N_2 . Nalewajski in [7] gives a definition of the complex entropy by first defining the *overall entropy*:

$$\S[D(q_R(T))] = \int -p(r, t)[\ln(p(r, t)) + 2i\theta(r, t)]dtdr \equiv s(p) + i s(\theta), \quad (4)$$

where the integral is overall the surface $\partial\Omega$ of the N_2 virtual border and over a time duration Δt . Here we note $p(r, t) = p(N^r, t)$.

The imaginary part of the complex entropy (4 Vacuum spacetime curvature and Heisenberg energy-time principle equation.1.4) was demonstrated to be equivalent to the spacetime deformation due to the matter/energy presence. The real part has been supposed to be the thermodynamical entropy part and this is will be shown in this section. This complex entropy may be reformulated as following:

$$\Im[D(q_R(T))] \simeq \int_{\Delta t} \int_{\partial\Omega} -p(r,t) \ln(p(r,t)) dr dt + i \frac{M_p^2 l_p^3}{S_p t_p \hbar} \left(\frac{\mathcal{R}}{2} - \Lambda \right) \sqrt{|g|} \quad (5)$$

$\frac{M_p^2 l_p^3}{S_p t_p \hbar} = \frac{c^2}{8\pi G} = \frac{1}{\kappa}$ or in energy information entropy, the previous expression is rewritten as following:

$$\Im[D(q_R(T))] \simeq c^2 \int -p(r,t) \ln(p(r,t)) dr dt + i \frac{c^4}{8\pi G} \left(\frac{\mathcal{R}}{2} - \Lambda \right) \sqrt{|g|} \quad (6)$$

The imaginary part defines the Einstein-Hilbert action [5] including the cosmological constant part that is explained by the virtual antiparticles effect $-\pi \delta_{inh,r,t}$. But what does the real part of this entropy define?

II. THE REAL PART OF THE COMPLEX INFORMATION ENTROPY DEFINITION: THE HAWKING RADIATION EFFECT FOR THE PLANCK DIMENSION BLACK HOLES

In this section we want to explain the definition of the real part of the complex information entropy (6 Vacuum spacetime curvature and Heisenberg energy-time principle equation.1.6) of the signal-Information received *virtually* by the point N_2 in other words the entropy of $D(q_R(T))$.

p is the arrived massive information per time and per surface to the virtual border $\partial Omega$. We transform p to P to renormalize the real part by multiplying and dividing by $\frac{M_p}{S_p t_p}$, where we recall that M_p is the Planck mass, S_p is the Planck surface and t_p is the Planck time:

$$\Im[D(q_R(T))] \simeq \frac{M_p}{S_p t_p} c^2 \int -P(r,t) \ln(p(r,t)) dr dt + i \frac{c^4}{8\pi G} \left(\frac{\mathcal{R}}{2} - \Lambda \right) \sqrt{|g|} \quad (7)$$

We then multiply and divide by \hbar and M_p the real part of the complex information entropy:

$$\Im[D(q_R(T))] \simeq \frac{M_p^2}{S_p t_p \hbar} c^2 \frac{\hbar}{M_p} \int -P(r,t) \ln(p(r,t)) dr dt + i \frac{c^4}{8\pi G} \left(\frac{\mathcal{R}}{2} - \Lambda \right) \sqrt{|g|} \quad (8)$$

To normalize $dr dt$ by dV , we need to divide $dr dt$ by l_p^3 and multiply by c to transform time dt to a length and so multiply by $\frac{l_p^3}{c}$ the integral i.e. $dV = \frac{dr \times c dt}{l_p^3}$:

$$\Im[D(q_R(T))] \simeq \frac{M_p^2 \times l_p^3}{S_p t_p \hbar} c^2 \frac{\hbar}{c \times M_p} \int -P(r,t) \ln(p(r,t)) dV + i \frac{c^4}{8\pi G} \left(\frac{\mathcal{R}}{2} - \Lambda \right) \sqrt{|g|} \quad (9)$$

We add the Boltzman constant k_B by multiplying and dividing the real part integral to display the thermodynamical entropy:

$$\Im[D(q_R(T))] \simeq \frac{M_p^2 \times l_p^3}{S_p t_p \hbar} c^2 \frac{\hbar}{c \times M_p k_B} \times k_B \int -P(r,t) \ln(p(r,t)) dV + i \frac{c^4}{8\pi G} \left(\frac{\mathcal{R}}{2} - \Lambda \right) \sqrt{|g|} \quad (10)$$

Using the fact that $\frac{c^2 M_p^2 l_p^3}{S_p t_p \hbar} = \frac{c^4}{8\pi G}$, we have therefore:

$$\Im[D(q_R(T))] \simeq \frac{c^3 \hbar}{8\pi G k_B M_p} \int -k_B P(r,t) \ln(p(r,t)) dV + i \frac{c^4}{8\pi G} \left(\frac{\mathcal{R}}{2} - \Lambda \right) \sqrt{|g|} \quad (11)$$

One can note that $\frac{c^3 \hbar}{8\pi G k_B M_p}$ is the formula of the Hawking temperature of a black hole with mass $\frac{M_p}{J_{\Delta t'}}$ which means the relative mass of the particular particle of the studied quantum field. The lower the $\Delta t'$ the higher the mass which is equivalent to the time-energy Heisenberg uncertainty principle. $\int -k_B P(r,t) \ln(p(r,t)) dV$ defines the thermodynamical entropy depending on the information created in this position.

The real part of this entropy is therefore the heat/energy of the real particle by this particular *black hole* or the energy of the real particle recreated by the black hole located at N_2 thanks to the Hawking radiation [8]. The $\partial\Omega$ of N_2 position is therefore the surface of the black hole created by the transformation of the virtual particles-antiparticles from the quantum vacuum to real particles thanks to space and time summation described in our previous paper as the gravity effect. These black holes are created in space dimension location N_2 as soon as any ϵ of energy/mass Information is present in the neighboring position depending on

the quantity of the energy arriving at N_2 . If any ϵ of energy arrives to a punctual position (if we supposed that there is no such Planck scales) then a black hole is created with a surface equivalent to this ϵ energy. In the presence of any ϵ energy/mass, a black hole is created in the quantum field and its surface transform the virtual particle-antiparticles present in Time dimension or in the Vacuum to real ones during Δt . And thanks to the space and time summation of the gravity effect, these real particles are added during Δt over $\partial\Omega$ to delete this black hole and becomes virtual particles to fix the black hole or to fix their respective quantum field. The virtual particles fix the space dimension: the black holes disappears almost instantly during Δt or less.

III. CONCLUSION

The complex entropy of any signal Information (SI), defined by its wave function, describes the spacetime curvature that is defined by the General Relativity and the Hawking Radiation energy. As any complex number, the SI complex entropy has the a real part and an imaginary part. The imaginay part has been already shown that it describes the spacetime curvature part by the Einstein-Hilbert action in our last paper [3]. In this paper we have demonstrated that the real part of this entropy describes the energy of the Hawking Radiation of the black holes. In fact in any quantum field, in presence of any real or virtual energy/mass Signal Information a black hole is created in Planck scale, which produces real particles from the virtual particles and antiparticles of the quantum vacuum. These particles will be added around the black hole surface thanks to the space and time summation, a property of the gravity effect [3], and hence delete the black hole and fix the space and time dimensions by returning back to their respective quantum field. Spacetime dimensions are hence all the quantum fields, the fields of different virtual particles and antiparticles.

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