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VIBROACOUSTIC SIMILITUDE LAWS FOR FLUID-LOADED PLATES EXCITED BY POINT FORCES

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Vibroacoustic experiments on large complex structures radiating sound in a heavy fluid (such as naval structures) can be costly, time-consuming and difficult to implement. There is a need for methods that overcome these constraints. Similitude theory provides a methodology which links two systems in order to predict the response of one to the other by scaling all of their parameters (e.g. materials, size, loading conditions). Previous studies demonstrated the potential of similitude methods to predict the vibration and sound radiation of plates in a light fluid from measurements on a scaled-down system. The challenge here is to determine similitude laws for bending plates excited by a point force and loaded by a heavy fluid. This paper derives the vibroacoustic similitude laws for such a situation. By taking into account the wall pressure acting on the plate, new similitude conditions need to be respected in order to obtain perfect similitude for both the vibration and acoustic response. These conditions are used to adequately choose the scaled system. Then, two examples are considered and numerical results illustrate the similitude laws previously developed. The first example highlights the impact of fluid-structure coupling on the choice of the scaled system. The second example validates the laws taking into account this coupling and the corresponding similitude conditions. These examples show that structural response and radiated acoustic power of a reference plate can be recovered by using the related scaling laws.

Keywords: similitude laws, fluid-structure coupling, vibration, acoustic radiation, flat plates.

1. Introduction

Experimental tests can be costly in time and money, but also difficult to implement. This is particularly the case when the systems of interest are large complex structures, such as in the fields of aerospace, naval or railway transport. Similitude theory provides a methodology which links two systems in order to predict the response of one to the other by defining laws depending on their properties (e.g. materials, size, loading conditions).

The use of similitude methods is common in many engineering fields in order to obtain, by simulations or measurements, results from scaled systems. Recent reviews of these methods can be found in [1, 2] for various problems of mechanical engineering and in [3] for plate vibroacoustics. The existing literature on similitude laws concerns structural and acoustic responses for possibly complex structures but the problem of structures immersed in a heavy fluid, such as water, has not been addressed. However, many papers or books demonstrate the influence of fluid on plate vibrations [4, 5, 6], needs therefore to be accounted for in the naval domain.

The present study deals with similitude laws applied to simply-supported plates excited by a point force and loaded by a heavy fluid. The paper is organised as follows. The derivation of the similitude laws related to the vibroacoustic response of the fluid-loaded plate is presented in Section 2. Some conditions are derived to obtain perfect similitude laws on the plate velocity and the radiated acoustic power. Then, numerical simulations are reported in Section 3 to verify these laws and to discuss the conditions associated to these laws. Finally, the main points of this work are summarised in the conclusion.

2. Theoretical development

2.1 Governing equation and scaling parameters

Two rectangular thin plates inserted in a baffle (rigid surface), immersed in a fluid on one side and excited by a harmonic normal point force are considered. These systems are shown in Fig. 1. One of these plates is called the reference plate whereas the other one is called the scaled plate. For both of them, the boundary conditions on the four edges are simply supported and the plate materials are supposed elastic and isotropic. The material properties can however be different for the two plates. Let us consider the reference plate, of dimensions $l_x \times l_y \times h$. Its material properties are defined by the density ρ , the Young's modulus E , the Poisson's ratio ν and the damping coefficient η . The fluid properties are ρ_0 and c_0 for the fluid density and the sound velocity respectively. Under Love-Kirchhoff assumptions, the transverse velocity v of the plate when excited by a harmonic force of angular frequency ω , of amplitude F_0 and applied at point \mathbf{x}_0 is described by the governing equation

$$-\rho h \omega^2 v(\mathbf{x}; \omega) + D \nabla^4 v(\mathbf{x}; \omega) = j\omega F_0 \delta(\mathbf{x} - \mathbf{x}_0) - j\omega p(\mathbf{x}; \omega), \quad (1)$$

with $D = \frac{Eh^3}{12(1-\nu^2)}$ the flexural rigidity and $\nabla^4 = \left(\frac{\partial^4}{\partial x^4} + \frac{\partial^4}{\partial y^4} + 2 \frac{\partial^4}{\partial x^2 \partial y^2} \right)$ the bi-Laplacian operator. The $p(\mathbf{x}; \omega)$ term is the wall acoustic pressure due to fluid-structure coupling and is defined by the Rayleigh integral

$$p(\mathbf{x}; \omega) = j\rho_0 \omega \iint_S v(\mathbf{x}'; \omega) \frac{e^{-jk\|\mathbf{x}-\mathbf{x}'\|}}{2\pi \|\mathbf{x}-\mathbf{x}'\|} d\mathbf{x}', \quad (2)$$

where k is the wavenumber, \mathbf{x} a receiving point in the acoustic medium and S the plate surface (\mathbf{x}' is the integration variable).

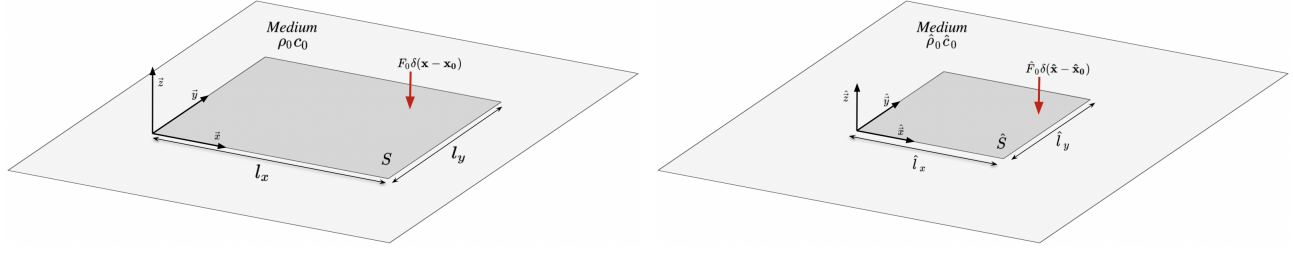


Figure 1: Illustration of the two systems : (left) the reference system, (right) the scaled system.

In order to obtain the vibroacoustic response of the system, Eq. (1) is expressed in the plate modal basis [6],

$$\left(\mathbf{diag} [\rho h (\omega_N^2 - \omega^2) N_N] + j\omega \mathbf{Z}(\omega) \right) \mathbf{V}(\omega) = j\omega \mathbf{F}(\omega), \quad (3)$$

with $\mathbf{diag}[\cdot]$ a diagonal matrix, $N_N = \iint_S W_N^2(\mathbf{x}) d\mathbf{x}$ the norm of the mode N , $\mathbf{V}(\omega)$ the modal velocities, $\mathbf{F}(\omega)$ the modal forces and \mathbf{Z} the modal radiation impedance matrix, each term $Z_{N,P}$ being defined by

$$Z_{N,P}(\omega) = j\rho_0\omega \iint_S \iint_S W_N(\mathbf{x}') \frac{e^{-jk\|\mathbf{x}-\mathbf{x}'\|}}{2\pi \|\mathbf{x}-\mathbf{x}'\|} W_P(\mathbf{x}) d\mathbf{x}d\mathbf{x}', \quad (4)$$

where W_N and W_P are the eigenshapes of modes N and P respectively, and $N = (m, n)$ and $P = (p, q)$ are couples of integers. In the case of a thin rectangular plate with simply-supported conditions, the modal frequencies and the mode shapes are given by Leissa [7] :

$$\begin{aligned} \omega_N &= \sqrt{\frac{D}{\rho h}} \left[\left(\frac{m\pi}{l_x} \right)^2 + \left(\frac{n\pi}{l_y} \right)^2 \right], \\ W_N(\mathbf{x}) &= \sin \left(\frac{m\pi}{l_x} x \right) \sin \left(\frac{n\pi}{l_y} y \right). \end{aligned} \quad (5)$$

Let us now consider the scaled plate, of dimensions $\hat{l}_x \times \hat{l}_y \times \hat{h}$. Its parameters are noted with the symbol $\hat{\cdot}$. Its governing equation, expressed in its modal basis, is given by Eq. (6)

$$\left(\mathbf{diag} [\hat{\rho} \hat{h} (\hat{\omega}_N^2 - \hat{\omega}^2) \hat{N}_N] + j\hat{\omega} \hat{\mathbf{Z}}(\hat{\omega}) \right) \hat{\mathbf{V}}(\hat{\omega}) = j\hat{\omega} \hat{\mathbf{F}}(\hat{\omega}). \quad (6)$$

The derivation of similitude laws is based on the scaling parameters for geometry, material properties, fluid properties, excitation and frequency defined in [8], [9] and [10] :

$$\begin{aligned} r_x &= \frac{\hat{l}_x}{l_x}, \quad r_y = \frac{\hat{l}_y}{l_y}, \quad r_h = \frac{\hat{h}}{h}, \\ r_\rho &= \frac{\hat{\rho}}{\rho}, \quad r_E = \frac{\hat{E}}{E}, \quad r_\nu = \frac{\hat{\nu}}{\nu}, \quad r_\eta = \frac{\hat{\eta}}{\eta}, \\ r_{\rho_0} &= \frac{\hat{\rho}_0}{\rho_0}, \quad r_{c_0} = \frac{\hat{c}_0}{c_0}, \\ r_F &= \frac{\hat{F}}{F}, \quad r_\omega = \frac{r_E^{1/2} r_h}{r_x^2 r_\rho^{1/2}}. \end{aligned} \quad (7)$$

In order to derive laws for perfect similitude, the following hypotheses are assumed :

- $r_x = r_y$ (proportional sides), the length and width are identically scaled;
- $r_\nu = 1$ and $r_\eta = 1$, which means the Poisson's ratio and the damping coefficient between the two systems remain unchanged;
- $\hat{W}_N(\hat{\mathbf{x}}) = W_N(\mathbf{x})$, the mode shapes remain the same for the two plates as the boundary conditions are the same.

2.2 Similitude for the structural response

The aim of similitude methods is to predict the response of a reference system from the response of a scaled one. Here, the velocity responses $\hat{\mathbf{V}}(\hat{\omega})$ and $\mathbf{V}(\omega)$ have to be linked. In this end, Eq. (6) is rewritten using the scaling parameters defined previously (Eq. (7)), hence

$$\left(r_\rho r_h r_\omega^2 r_x^2 \mathbf{diag}[\rho h(\omega_N^2 - \omega^2)N_N] + r_\omega j \omega \hat{\mathbf{Z}}(\hat{\omega}) \right) \hat{\mathbf{V}}(\hat{\omega}) = r_\omega r_F j \omega \mathbf{F}(\omega). \quad (8)$$

In order to relate the velocity responses of the systems, the modal radiation impedance $\hat{\mathbf{Z}}$ need to be put in similitude. Each term of $\hat{\mathbf{Z}}$ is given by

$$\hat{Z}_{N,P}(\hat{\omega}) = j \hat{\rho}_0 \hat{\omega} \iint_{\hat{S}} \iint_{\hat{S}} \hat{W}_N(\hat{\mathbf{x}}') \frac{e^{-jk\|\hat{\mathbf{x}}-\hat{\mathbf{x}}'\|}}{2\pi \|\hat{\mathbf{x}}-\hat{\mathbf{x}}'\|} \hat{W}_P(\hat{\mathbf{x}}) d\hat{\mathbf{x}} d\hat{\mathbf{x}}'. \quad (9)$$

Introducing the various scaling parameters and given the hypothesis of identical eigenfunctions, $\hat{Z}_{N,P}$ becomes

$$\hat{Z}_{N,P}(\hat{\omega}) = r_{\rho_0} r_\omega j \rho_0 \omega \iint_S \iint_S W_N(\mathbf{x}') \frac{e^{-\frac{r_x r_\omega}{r_{c_0}} jk\|\mathbf{x}-\mathbf{x}'\|}}{2\pi r_x \|\mathbf{x}-\mathbf{x}'\|} W_P(\mathbf{x}) r_x^4 d\mathbf{x} d\mathbf{x}'. \quad (10)$$

It is clear that the only possibility to relate $\hat{Z}_{N,P}$ and $Z_{N,P}$ is to equal the arguments of the exponentials in Eqs. (4) and (10), which brings up the following similitude condition :

$$\frac{r_x r_\omega}{r_{c_0}} = 1. \quad (11)$$

If the previous condition is respected, the similitude law is obtained for the modal radiation impedance $\hat{Z}_{N,P} = r_{\rho_0} r_\omega r_x^3 Z_{N,P}$. Introducing this new law in Eq. (8),

$$\left(r_\rho r_h r_\omega^2 r_x^2 \mathbf{diag}[\rho h(\omega_N^2 - \omega^2)N_N] + r_{\rho_0} r_\omega^2 r_x^3 j \omega \mathbf{Z}(\omega) \right) \hat{\mathbf{V}}(\hat{\omega}) = r_\omega r_F j \omega \mathbf{F}(\omega). \quad (12)$$

At this stage, the only possibility to obtain a similitude law between $\hat{\mathbf{V}}(\hat{\omega})$ and $\mathbf{V}(\omega)$ is to fulfill another similitude condition :

$$r_\rho r_h r_\omega^2 r_x^2 = r_{\rho_0} r_\omega^2 r_x^3 \quad \Leftrightarrow \quad \frac{r_\rho r_h}{r_x r_{\rho_0}} = 1. \quad (13)$$

In this case, the following similitude law is obtained :

$$\hat{v}(\hat{\mathbf{x}}; \hat{\omega}) = r_v v(\mathbf{x}; \omega) \quad \text{with} \quad r_v = \frac{r_F}{r_\rho r_h r_\omega r_x^2}. \quad (14)$$

Focusing on the spatial average of the squared velocity on the plate, we get :

$$\langle \hat{v}^2(\hat{\omega}) \rangle = \frac{1}{2\hat{S}} \iint_{\hat{S}} \hat{v}(\hat{\mathbf{x}}; \hat{\omega}) \hat{v}^*(\hat{\mathbf{x}}; \hat{\omega}) d\hat{\mathbf{x}} = r_v^2 \langle v^2(\omega) \rangle, \quad (15)$$

where * is the complex conjugate. The similitude law for the velocity response is the same than the one in the literature when the fluid-structure coupling is not accounted for [10]. However, in order to obtain perfect similitude, the two similitude conditions defined in Eqs. (11) and (13) have to be satisfied.

2.3 Similitude for the radiated acoustic power

The radiated acoustic power depends on acoustic intensity \mathbf{I} on the plate surface which is a function of the normal vibratory velocity $v = \mathbf{v} \cdot \mathbf{n}$ and the acoustic pressure p . In the case of the scaled system it is defined by

$$\hat{\Pi}(\hat{\omega}) = \frac{1}{2} \iint_{\hat{S}} \hat{\mathbf{I}} \cdot \hat{\mathbf{n}} \, d\hat{\mathbf{x}} = \frac{1}{2} \iint_{\hat{S}} \Re\{\hat{p}(\hat{\mathbf{x}}; \hat{\omega}) \hat{v}^*(\hat{\mathbf{x}}; \hat{\omega})\} \, d\hat{\mathbf{x}}, \quad (16)$$

where $\hat{\mathbf{n}}$ is the normal vector to the scaled plate and $*$ the complex conjugate. The acoustic pressure $\hat{p}(\hat{\mathbf{x}}; \hat{\omega})$ is the one defined by Eq. (2) in the case of the scaled plate. A similitude law can easily be derived using the scaling parameters previously defined,

$$\hat{p}(\hat{\mathbf{x}}; \hat{\omega}) = r_{\rho_0} r_{\omega} j \rho_0 \omega \iint_S r_v v(\mathbf{x}'; \omega) \frac{e^{-\frac{r_x r_{\omega}}{r_{c_0}} jk \|\mathbf{x} - \mathbf{x}'\|}}{2\pi r_x \|\mathbf{x} - \mathbf{x}'\|} r_x^2 d\mathbf{x}' = r_{\rho_0} r_{\omega} r_v r_x p(\mathbf{x}; \omega). \quad (17)$$

Applying the similitude laws derived for the velocity response (Eq. (14)) and for the acoustic pressure (Eq. (17)), the similitude law for the radiated acoustic power takes the form

$$\hat{\Pi}(\hat{\omega}) = r_{\rho_0} r_{\omega} r_v^2 r_x^3 \Pi(\omega). \quad (18)$$

Given the dependency of this similitude law on the velocity similitude law r_v , the conditions defined in Eqs. (11) and (13) have to be satisfied in order to have perfect similitude for the radiated acoustic power.

3. Numerical validation of the method

3.1 Description of the plates

Three plates are considered in order to verify the methodology derived in Section 2. Their characteristics are reported in Table 1. Plates A and B are made of steel and plate C is made of zinc. Details of their properties can be found in Table 2. The scaling parameters are also reported in these tables, with r_{ω} (Eq. (7)) depending on geometry and material properties. They are related to plate A, which is considered as the reference plate in this study. For the three plates, the damping coefficient (considered constant over the frequency range) and the Poisson's ratio are equal to 0.01 and 0.3 respectively. Moreover, the three plates are immersed in water ($\rho_0 = 1030 \text{ kg/m}^3$, $c_0 = 1500 \text{ m/s}$) on one side : $r_{\rho_0} = 1$ and $r_{c_0} = 1$. The magnitude of the point force is the same for all the plates ($r_F = 1$) and is set to 1 N.

Table 1: Characteristics of the plates and corresponding scaling parameters

Plate	Length [mm]	Width [mm]	Thickness [mm]	Material	$r_x = r_y$	r_h	r_{ω}
A	740	660	12.8	Steel	-	-	-
B	518	462	8.96	Steel	0.7	0.7	1.43
C	370	330	8.96	Zinc	0.5	0.7	2.01

3.2 Fulfillment of the similitude conditions

Contrary to the case of a light fluid dealt with in the literature [8, 11, 12], taking the fluid-structure coupling into account reveals new similitude conditions to be satisfied (Eqs. (11) and (13)). The choice

Table 2: Material properties and corresponding scaling parameters

Plate	Material name	Density [kg/m ³]	Young's modulus [GPa]	r_ρ	r_E
A	Steel	7900	210	-	-
B	Steel	7900	210	1.0	1.0
C	Zinc	5700	78	0.72	0.37

of a scaled system to perform measurements or simulations depends on these conditions and is not as simple as in the case of a light fluid. These conditions depend on the material properties of the reference and the scaled plates and on their dimensions. In the examples described previously, the reference plate (plate A) is made of steel, plate B is made of the same material and plate C is made of a zinc alloy.

The scaling parameters between plate A and plate B for the material properties are $r_\rho = 1$ and $r_E = 1$, which implies $r_x = r_h$ (see Eq. (13)). In this case, the similitude conditions Eq. (11) and Eq. (13) are satisfied : $\frac{r_x r_\omega}{r_{c_0}} = 1$ and $\frac{r_\rho r_h}{r_x r_{\rho_0}} = 1$. The material properties for plate C are standard zinc alloy properties (see Table 2). The corresponding scaling parameters are $r_\rho = 0.72$ and $r_E = 0.37$. The similitude conditions Eq. (11) and Eq. (13) are also approximately satisfied : $\frac{r_x r_\omega}{r_{c_0}} = 1.004$ and $\frac{r_\rho r_h}{r_x r_{\rho_0}} = 1.008$.

The similitude laws can be used to predict the response of plate A from plate B and plate C. It is also possible to consider plate B or plate C as the reference plate. In the case of a light fluid, the conditions Eq. (11) and Eq. (13) do not have to be respected to use the similitude laws and predict the vibroacoustic responses. For example, any material properties can be used for the scaled plate provided frequency scaling r_ω is correctly calculated whereas with the consideration of the fluid-structure coupling Eqs. (11) and (13) need to be satisfied and impose conditions on the material properties of the scaled plate.

3.3 Vibroacoustic response under a point force

Numerical simulations are presented here to verify the similitude laws developed previously. The mean square velocity (MSV) and the acoustic power of the three plates described above are calculated and provided in the upper parts of Figs. 2 and 3 respectively. The similitude laws are then applied in order to predict the behaviour (vibratory and acoustic responses) of plate A from plates B and C. The remodulated responses are provided in the lower parts of Figs. 2 and 3.

It can be observed in the upper parts of Figs 2 and 3 that the plate responses are quite different, especially in terms of frequency distribution. However, when the similitude laws are applied on the responses of plates B and C, both in terms of frequency and response amplitude, the response of plate A can be recovered. The observation applies for both the vibratory and acoustic responses. Moreover, it has to be noted that the responses of plates B and C have to be calculated for frequencies up to 4290 Hz and 6030 Hz respectively to predict the response of plate A on the frequency range 0 – 3000 Hz (r_ω applied).

4. Conclusion

This work defines similitude laws for the velocity response and the acoustic power of simply supported rectangular, isotropic, thin plates excited by a point force and loaded by a heavy fluid. The consideration of the fluid-structure coupling in the development of the similitude laws showed that additional similitude conditions have to be satisfied to obtain perfect similitude, with respect to the uncoupled problem. These conditions restrain the choice of the scaled system. However, if they are satisfied the response of one plate can be perfectly recovered from the response of another plate, with different dimensions and a different material. Numerical results for three plates validate the method developed in this paper.

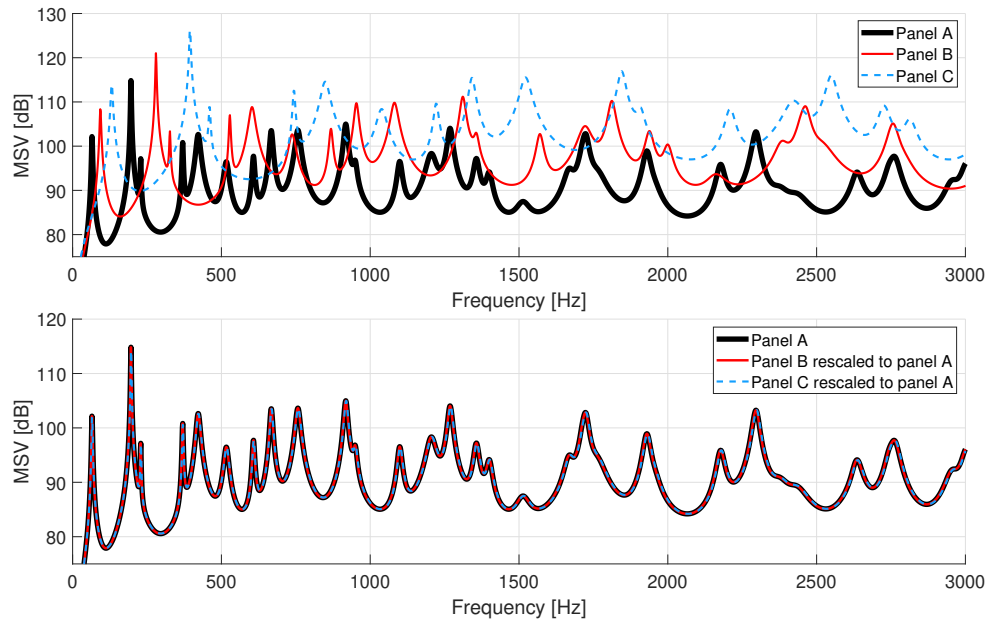


Figure 2: Mean Square Velocity (MSV) [dB ref 10^{-9} m/s] : (upper part) unscaled results, (lower part) results for plate A and those from plates B and C rescaled to plate A.

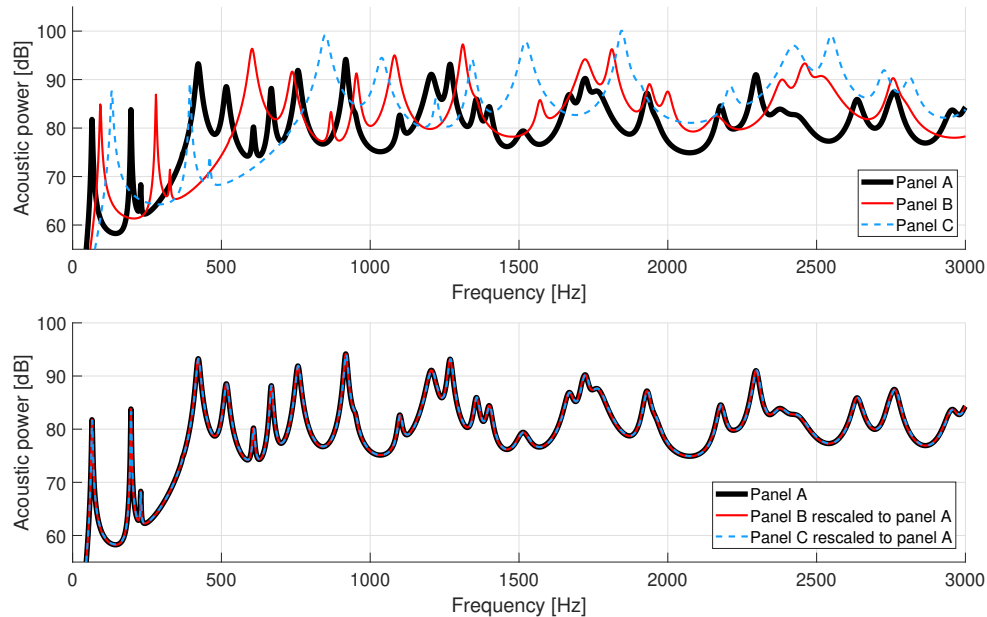


Figure 3: Acoustic power [dB ref 10^{-12} W] : (upper part) unscaled results, (lower part) results for plate A and those from plates B and C rescaled to plate A.

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