



**HAL**  
open science

## Stochastic Modulational Instability in Optical Fibers with Colored Random Dispersion

Andrea Armaroli, Alexandre Kudlinski, Arnaud Mussot, Matteo Conforti,  
Stefano Trillo, Guillaume Dujardin, Stephan de Bièvre

► **To cite this version:**

Andrea Armaroli, Alexandre Kudlinski, Arnaud Mussot, Matteo Conforti, Stefano Trillo, et al..  
Stochastic Modulational Instability in Optical Fibers with Colored Random Dispersion. Advanced  
Photonic Congress, Jul 2022, Maastricht, Netherlands. hal-03796524

**HAL Id: hal-03796524**

**<https://hal.science/hal-03796524>**

Submitted on 4 Oct 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Stochastic Modulational Instability in Optical Fibers with Colored Random Dispersion

Andrea Armaroli, Alexandre Kudlinski, Arnaud Mussot, Matteo Conforti

Univ. Lille, CNRS, UMR 8523-PhLAM-Physique des Lasers Atomes et Molécules, F-59000 Lille, France

andrea.armaroli@univ-lille.fr

Stefano Trillo

Department of Engineering, University of Ferrara, I-44122 Ferrara, Italy

Guillaume Dujardin, Stephan De Bièvre

Univ. Lille, Inria, CNRS, UMR 8524 - Laboratoire Paul Painlevé, F-59000 Lille, France

**Abstract:** We study modulational instability in optical fibers with random group-velocity dispersion fluctuations. We consider Gaussian and dichotomous colored stochastic processes and characterize analytically and numerically the MI sidebands in average normal dispersion regime. © 2022 The Author(s)

## 1. Introduction

Modulational instability (MI) is a ubiquitous physical phenomenon in nonlinear dispersive systems. In optical fibers it is conventionally observed if the group velocity dispersion (GVD) is anomalous. It can be modelled at the lowest order by the nonlinear Schrödinger equation (NLSE).

MI can be observed also in the normal GVD regime if the waveguiding parameters are changed along the propagation direction, see e.g. [1], where a random variation of GVD modelled by white noise was studied. This random process allows one to derive simple closed mathematical formulas, but is not physically realizable, because it consists in infinitely abrupt variations of GVD on an infinitely small scale.

We show here the effect of GVD fluctuations generated by colored random processes. By means of analytical and numerical approaches we obtain several families of MI sidebands in normal GVD that differ from both the white noise limit and well-known MI obtained for periodic GVD variations in dispersion-oscillating fibers (DOFs) [2].

## 2. Model and results

Our model reads

$$i\partial_z U - \frac{1}{2}\beta_2(z)\partial_{tt}U + \gamma|U|^2U = 0, \quad (1)$$

with  $U$  the complex envelope of the optical field,  $(t,z)$  the physical time and propagation distance in a frame moving at the group velocity of the fiber mode,  $\gamma$  the (constant) nonlinear coefficient, and  $\beta_2(z) = \beta_2^0 + \delta\beta(z)$  is the GVD. We write it as a random process  $\delta\beta(z)$  of zero mean around  $\beta_2^0 > 0$  (normal GVD). We consider stationary (in  $z$ ) Gaussian and dichotomous processes. The associated power spectral density (PSD) belongs to one of the two families: low-pass ( $S_{\delta\beta}(\kappa) = \frac{N_0}{2} \frac{B^2}{B^2 + \kappa^2}$ , with  $B$  the wavenumber bandwidth and  $N_0/2$  the central value, or band-pass ( $S_{\delta\beta}(\kappa) = \frac{N_0}{2} \left[ \frac{B^2}{B^2 + (\kappa - \kappa_0)^2} + \frac{B^2}{B^2 + (\kappa + \kappa_0)^2} \right]$ , modulated around  $\kappa_0$ ). Both families are in principle physically implementable by continuous variation of fiber radius of fiber splicing.

Equation (1) is linearized around the continuous wave solution  $U_0(z) = \sqrt{P} \exp(i\gamma P z)$  and a linear system resembling a harmonic oscillator with a randomly fluctuating frequency is obtained (multiplicative noise). Two mathematical techniques are then alternatively employed: the cumulant expansion [3] and the Furutsu-Novikov-Shapiro-Loginov-Donsker (FNSLD) formulas [4]. As known for the harmonic oscillator, the first moments of the dependent variables (MI sidebands) do not exhibit any instability and the we have to resort to second moments to achieve non-null MI gain predictions. This means that a net energy transfer from carrier to sidebands occurs on average, even though their mean value vanishes. We also perform direct numerical solution in a Monte Carlo spirit, which is nevertheless essential to assess the reliability and the limits of application of the two analytic approaches. We present a detailed analysis of the various MI regimes occurring in normal GVD [5].

The low-pass processes yield MI sidebands at low frequency  $\omega$  around the carrier, similarly to the white noise limit. In Fig. 1 we compare the numerical results to the analytical approximations. Special care is taken to insure

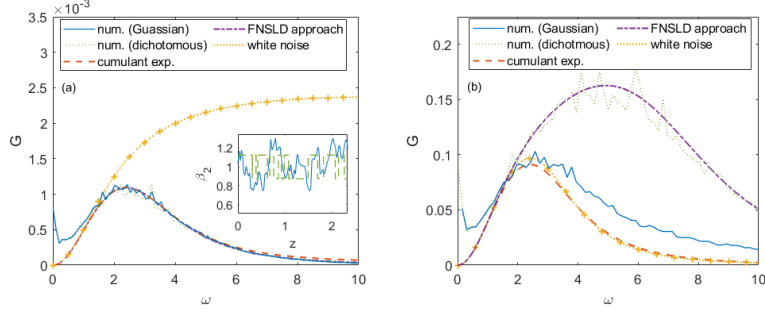


Fig. 1. MI gain for the low-pass processes. Comparison of numerical results and analytical estimates. We let  $\gamma P = \beta_2^0 = 1$  and (a)  $N_0 = 0.005$ ,  $B = 4\pi$ , (b)  $N_0 = 0.4$ ,  $B = 4\pi$ . The inset in (a) shows a sample realization of  $\beta_2(z)$ .

the statistical convergence of the averaged quantities. The cumulant expansion requires a small  $\varepsilon \equiv \frac{\omega^2}{4} \sqrt{\frac{N_0}{B}}$ , while the FNSLD approach is accurate for the dichotomous process only, independently of  $\varepsilon$ . Thus, in Fig. 1(a), the two approaches both behave well and predict a small MI gain. The maximum gain in Fig. 1(b) is obtained from a dichotomous process and is about 1/6 of conventional MI gain in anomalous dispersion.

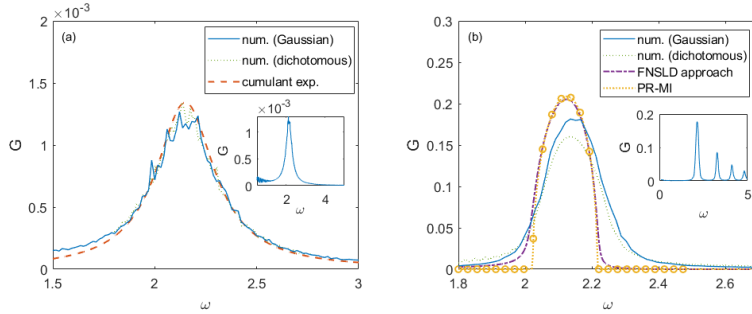


Fig. 2. MI gain for the band-pass processes. Comparison of numerical results and analytical estimates. We let  $\gamma P = \beta_2^0 = 1$  and (a)  $N_0 = 0.005$ ,  $B = \pi/4$ , (b)  $N_0 = 3.2$ ,  $B = \pi/32$ . The insets show a larger  $\omega$  range.

Band-pass processes yield MI sidebands localized around one or more frequencies given by a parametric resonance (PR) condition like in DOFs,  $\beta_2^0 \omega^2 (\beta_2^0 \omega^2 + 4\gamma P) = m^2 \kappa_0^2$ , with integer  $m$ , see Fig. 2. For small  $\varepsilon$  [Fig. 2(a)], the cumulant expansion correctly models the single MI sideband pair, the maximum of which depends on  $N_0$ . For larger  $N_0$ , the maximum gain scales with  $\sqrt{N_0 B}$  and FNSLD has to be employed combined to a nearly-resonant expansion, see Fig. 2(b). Several MI sidebands appear, that resemble those observed in DOFs, but exhibit smoother tails.

### 3. Conclusions

We studied modulational instability due to *colored* random fluctuations of dispersion around an average positive value. We observe the transition from instability sidebands located at small frequency detuning to a set of peaks around a discrete set of frequencies. Two analytical methods were employed to correctly model the possible different behaviors.

### References

1. F. K. Abdullaev, S. A. Darmanyan, A. Kobayakov, and F. Lederer, “Modulational instability in optical fibers with variable dispersion” *Phys. Lett. A*, 220, (1996), 213–218.
2. A. Mussot, M. Conforti, S. Trillo, F. Copie, and A. Kudlinski, . Modulation instability in dispersion oscillating fibers. *Advances in Optics and Photonics*, 10(1) (2018), 1–42
3. N. G. Van Kampen, *Stochastic Processes in Physics and Chemistry*, Elsevier (2007)
4. M. Gitterman, *The Noisy Oscillator: Random Mass, Frequency, Damping*, World Scientific Publishing Company (2005)
5. A. Armaroli, G. Dujardin, A. Kudlinski, A. Mussot, S. Trillo, S. De Bièvre, et M. Conforti, “Stochastic modulational instability in the nonlinear Schrödinger equation with colored random dispersion”, *Phys. Rev. A* **105**, (2022), 013511.