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Large Modulational Instability Gain in Random Dispersion-Managed Fibers

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Abstract: We study modulational instability in fibers where group velocity dispersion alternates between two different signs over random lengths. We apply both semi-analytical and numerical calculations to estimate predict large instability gains. © 2022 The Author(s)

1. Introduction

Dispersive nonlinear systems may exhibit modulational instability (MI). This effect has been known for decades in nonlinear optical fibers if light propagates in anomalous group velocity dispersion (GVD). The nonlinear Schrödinger equation (NLSE) well captures its main features. If other physical effects are taken into account, MI can appear also in normal dispersion. A typical condition is the (periodic or random) oscillation of GVD along the propagation direction [1]. Dispersion management (DM), i.e., using an alternation of fiber segments of opposite GVD, was proposed as an effective way to compensate pulse distortion in long-haul transmission links. The study of MI in DM systems showed that a threshold exist for the period of alternation below which no MI is observed [3]. The propagation of Gaussian pulses in DM system featuring a random variation of the interval length has been considered by means of variational techniques [5]. Large random fluctuations of GVD were studied in Ref. [4], in the form of localised abrupt changes (kicks) around an average normal GVD. We apply the method developed in our previous work to compute the MI gain in a DM system where the length of each segment fluctuates randomly with a certain probability distribution.

2. Model and results

We write the NLSE as

$$i\partial_z U - \frac{1}{2}\beta_2(z)\partial_t^2 U + \gamma|U|^2 U = 0, \quad (1)$$

with U the complex envelope of the optical field, (t, z) the physical time and propagation distance in a frame moving at the group velocity of the fiber mode, γ the (constant) nonlinear coefficient, and $\beta_2(z) = \pm\beta_2^0$ is the GVD, which takes only two values. The sign changes occur at z_1, z_2, \dots , where $z_n = z_{n-1} + \Delta L_n$, $n = 1, 2, \dots$ and $z_0 = 0$. The lengths ΔL_n are independent identically distributed random variables with uniform probability distribution function in $[\bar{L}(1 - \varepsilon), \bar{L}(1 + \varepsilon)]$, where \bar{L} is the average span length and ε the amplitude of its fluctuation.

Following [2, 4], Eq. (1) is linearized around the continuous wave solution $U_0(z) = \sqrt{P}\exp(i\gamma Pz)$ and the resulting system for sideband phase-quadrature variables (x_1, x_2) is written as a product of random matrices, depending on the random variables ΔL_n . It can be shown that the Lyapunov exponent of this random linear map is related to the MI gain. As common in random dynamical systems, the gain, denoted as $G_1(\omega)$ (resp. $G_2(\omega)$) with ω the frequency detuning from the carrier, estimated from the first (resp. second) moments of the dependent variables, are in general different. Recall that for a deterministic dispersion profile $G_1 = G_2$. In the periodic case $\varepsilon = 0$, for $\gamma P = 1$, the threshold for MI is $\bar{L} \approx 1.07$ [3], and the MI sidebands appear around a frequency which depends on β_2^0 .

By letting M^\pm the matrices governing to the evolution of the power of the perturbations $(x_1^2, x_2^2, x_1 x_2)$ for GVD values $\pm\beta_2^0$ (respectively, a rotation and a dilation matrix), we obtain, by averaging over ΔL_n , the following expression for the MI gain:

$$G_2(\omega) = \frac{1}{4\bar{L}} \log |\lambda|, \text{ with } \lambda \text{ the maximum magnitude eigenvalue of } \langle M^+ M^- \rangle, \quad (2)$$

the angle brackets denoting the expectation operation.

The MI equations are also directly solved numerically, by taking a fixed number, $2N = 100$, of fiber segments, and averaging the output sideband power over $N_{\text{iter}} = 5 \times 10^5$ segment distributions.

In Fig. 1 we show two examples obtained for $\beta_2^0 = \gamma P = 1$. In Fig. 1(a), we consider $\bar{L} < 1.07$, so the MI is

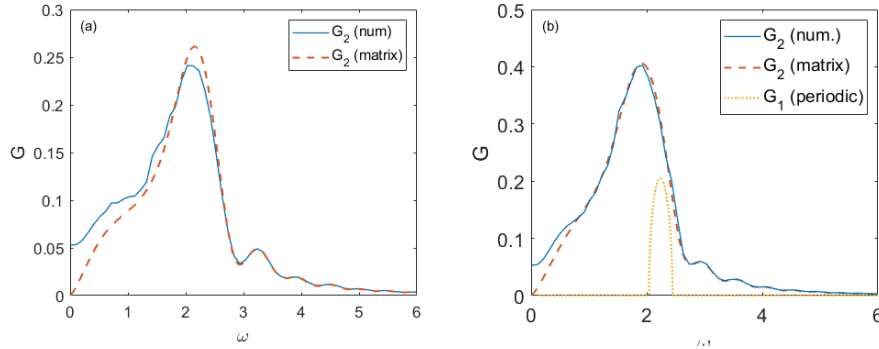


Fig. 1. Comparison of numerical growth rate with G_2 estimated from the eigenvalues of the averaged matrix (a) $\bar{L} = 1$, $\varepsilon = 0.3$; (b) $\bar{L} = 1.1$, $\varepsilon = 0.45$. In (b) the MI gain $G_1 = G_2$ for the periodic case is also included.

of purely stochastic origin. The MI sidebands are close to the carrier and exhibit a relatively large gain (1/4 of the conventional value observed in a uniform anomalous GVD fiber). Eq. (2) matches almost perfectly with the numerical results. In Fig. 1(b), we consider instead $\bar{L} = 1.1 > 1.07$. For a sufficiently small value of ε , only the conventional parametric MI appears and $G_1 \approx G_2$ (not shown). For $\varepsilon > 0.4$, a broad MI sidelobe is observed, resembling the shape of Fig. 1(a), but with a larger maximum value (40% of the conventional MI). In Fig. 1(b), the parametric MI is also included: importantly, contrary to the effect of white noise [1], which spreads the MI sideband and diminishes the maximum gain, here the random fluctuations of periodicity greatly enhance the MI gain. This effect persists also for larger $\bar{L} < 3$. Above this value the parametric MI dominates again. We found that Eq. (2) provides a satisfactory estimate of the numerically computed mean gain in all the simulation we have performed.

3. Conclusions

We studied the modulational instability in a dispersion-managed fiber with randomly fluctuating interval lengths. The instability gain is computed numerically and estimated by a simple yet accurate analytical method. An instability gain much broader and larger than its deterministic counterpart is predicted. This sheds light on the effect of large random fluctuations and the effective methods to analyze them in nonlinear optics.

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